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ONE-THREE JOIN: A GRAPH OPERATION AND ITS CONSEQUENCES

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Abstract

In this paper, we introduce a graph operation, namely *one-three join*. We show that the graph G admits a one-three join if and only if either G is one of the basic graphs (bipartite, complement of bipartite, split graph) or G admits a constrained homogeneous set or a bipartite-join or a join. Next, we define $\mathcal{M}_H$ as the class of all graphs generated from the induced subgraphs of an odd hole-free graph H that contains an odd anti-hole as an induced subgraph by using one-three join and co-join recursively and show that the maximum independent set problem, the maximum clique problem, the minimum coloring problem, and the minimum clique cover problem can be solved efficiently for $\mathcal{M}_H$.

Keywords: one-three join, bipartite-join, homogeneous set, odd hole-free graphs.

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