

Optimum Detector for Spatial Modulation using Sparsity Recovery in Compressive Sensing

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Abstract

Objective: Spatial Modulation (SM) is proposed for next generation green communications due its high spectral and energy efficiency. SM is incorporated with massive MIMO structures to leverage its high potential in the application for future generation wireless networks. **Methods/statistical Analysis:** The optimal ML detector for SM-MIMO systems requires enormous computational complexity, which makes the implementation infeasible in practice. However, the greedy low complexity detectors suffer from inferior performance and have huge performance gap from the optimal detectors. In this paper we propose new transmission schemes and detector structures for SM-MIMO systems. **Findings:** In particular the transmitter imposes certain structures, known as joint sparse, and the receiver exploits the information in detecting the symbols. We have shown that our proposed detector performs better than other greedy algorithms in the literature and performs close to the ML solution. We establish theoretical recovery guarantees for our proposed approach and compare the performance in theoretical and simulation results. **Improvements:** The theoretical characterization shows significant improvement in the detection performance compared to the conventional schemes. It is shown in simulation that the proposed algorithm achieves a gain of 4 dB compared to the conventional detectors.

Keywords: SM-MIMO System, Wireless Networks

1. Introduction

Spatial Modulation (SM)^{1,2} is the technique of conveying the information through the index of the antenna used to transmit the information. In SM, the information is transmitted via both the symbol being transmitted and the antenna used to transmit the symbol. In particular SM incorporated with Multi Input Multi Output (MIMO) systems to get larger spectral gains from the additional degrees of freedom available in antenna index. SM-MIMO can be classified as, small-scale SM-MIMO, which provides limited gain in the spec-

tral efficiency and massive SM-MIMO, which is the one that of major interest in recent research directions for future green communications. Since the RF chains used in the transmitter consumes most of the power, in massive SM-MIMO only a single or few of the many transmit antennas used to transmit the information and thereby reducing the number of RF chains participating in the transmission. The transmitter employs many low cost antennas and the receiver structures are proposed to get spatial diversity with significantly low correlated channel. Massive SM-MIMO offers dramatic energy efficiency by a large number of transmits antennas and the conventional

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linear detectors converge to be the optimal detectors³. Though, the large transmit antennas in the conventional MIMO systems offer higher throughput they suffer from the increased energy consumption due to increased number of RF chains at the transmitter, which surpasses the benefit of massive MIMO structures. To overcome this issue, it is proposed in massive SM-MIMO that to activate one or a subset of transmit antennas to transmit the information. Further, additional information is conveyed by the indices of the transmit antennas being used to transmit the symbol. The receivers of SM-MIMO⁴, not only detect the information symbols transmitted but also the subset of the antennas used to transmit the symbol by prior knowledge of the channels between the each pair of individual transmit antennas and receiver antennas. Compressive sensing deals with the recovery of unknown vectors from underdetermined linear measurements. The number of measurements is far less than the dimension of the unknown vector. The key idea behind the unique recovery of the high dimensional unknown vector for the low dimensional measurement vector lies on the ground of two key aspects.

- The vector to be recovered is sparse, that is the number of non zero elements is far less than the dimension of the vector.
- The matrix used to measure the unknown vector known as sensing matrix, should possess certain properties.

There were several works proposed in the literature of compressive sensing which address the recovery of sparse vectors from compressed measurements. A set of signals is said to be jointly sparse⁵, if the support (location of the non zero elements) is same in all the vectors. Some of the works related to recovery of joint sparse signals in compressive sensing can be seen in the literature⁶. In this paper, we propose a transmission scheme which collects the transmission in groups. In each group the same set of antennas are activated to convey the spatial constellation symbol. As the number of transmit antennas is quite large compared to the number of antennas that are activated, the transmission vector contains *sparse* structures and contain non zero values at the indices corresponding to transmit antennas. Further, as the same set of antennas

are activated in each group, the non zero elements appear at similar locations in all the transmission vectors. Hence, the set of transmitted vectors in each group become jointly sparse signals. In⁷ the receiver, we propose low complexity greedy algorithms to detect the information symbols along with the indices of active antennas in the framework of compressive sensing. In particular, we apply the principles joint sparse recovery in compressive sensing in the detection process at the receiver and establish their recovery guarantees. We also compare the performance of our proposed approach with the conventional techniques in the literature. Further, we establish the closeness of our theoretical development with the simulation results.

Notation:

Matrices/vectors are denoted by bold uppercase/lowercase letters, $\|\cdot\|$ for ℓ_2 norm, $\|\cdot\|_p$ norm by $\|\cdot\|_p$, Frobenius norm by $\|\cdot\|_F$, transpose by $(\cdot)^t$, hermitian by $(\cdot)^*$, column space of matrix \mathbf{A} by $R(\mathbf{A})$, sets by \mathbf{A} , cardinality of the set by $|\mathbf{A}|$, and set minus operation by $\mathbf{A} \setminus \mathbf{B}$.

2. System Model and Problem Statement

Let us consider an $N_t \times N_r$ MIMO system, i.e., the transmitter has N_t antennas and the receiver has N_r antennas. Let the transmit vector be $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ and each of the transmit symbol of $\{\mathbf{x}_i, i = 1, \dots, N_t\}$ is drawn from M -ary constellation. The observation or received vector is denoted as $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$. In essence,

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \tag{1}$$

Where \mathbf{H} is the channel matrix of size $N_r \times N_t$ and each of the element of \mathbf{H} , h_{ij} denotes the channel from j^{th} transmit antenna to i^{th} receive antenna. The additive noise $\mathbf{w} \in \mathbb{C}^{N_r \times 1}$ is $CN(0, \sigma^2)$. We consider the MIMO system to be spatially modulated as follows. In the transmitter, only N_a (out of N_t) number of antennas activated at any given time. The information is conveyed by means of both the transmitted symbol and the set of active antennas. As there are $\binom{N_t}{N_a}$ combinations are there to choose a set of

N_a antennas out of N_t , the number of bits that can be conveyed through the antenna indices is the $\lceil \log_2 (C_{N_a}^{N_t}) \rceil$. The information bits that are conveyed via the set of active antennas form the spatial constellation symbols and the M -ary symbols that are actually transmitted from the transmit constellation symbol. The following Figure 1 shows the systematic flow of how SM-MIMO works. Further, we consider the massive MIMO system where the number of transmit antennas is very large compared to the number of receive antennas and number of active antennas, i.e., $N_t \gg N_r > N_a$. As the number of active antennas is far less than total number of transmit antennas, the transmit vector \mathbf{x} contains the sparse structure, since the number of non zero elements in \mathbf{x} is quite low compared to the dimension of \mathbf{x} . Hence, the problem becomes

recovery of sparse signals from underdetermined linear measurements, i.e., recovery of sparse \mathbf{x} given \mathbf{y} from the underdetermined and can be solved in compressive sensing framework.

3. Detector Structure

In this section we apply the Orthogonal Matching Pursuit (OMP)⁸ algorithm in the compressive sensing literature to the system model. Let us interpret the channel matrix as

$$H = [h_1, h_2, \dots, h_{N_t}] \tag{2}$$

$$h_i = [h_{i1}, h_{i2}, \dots, h_{iN_r}] \tag{3}$$

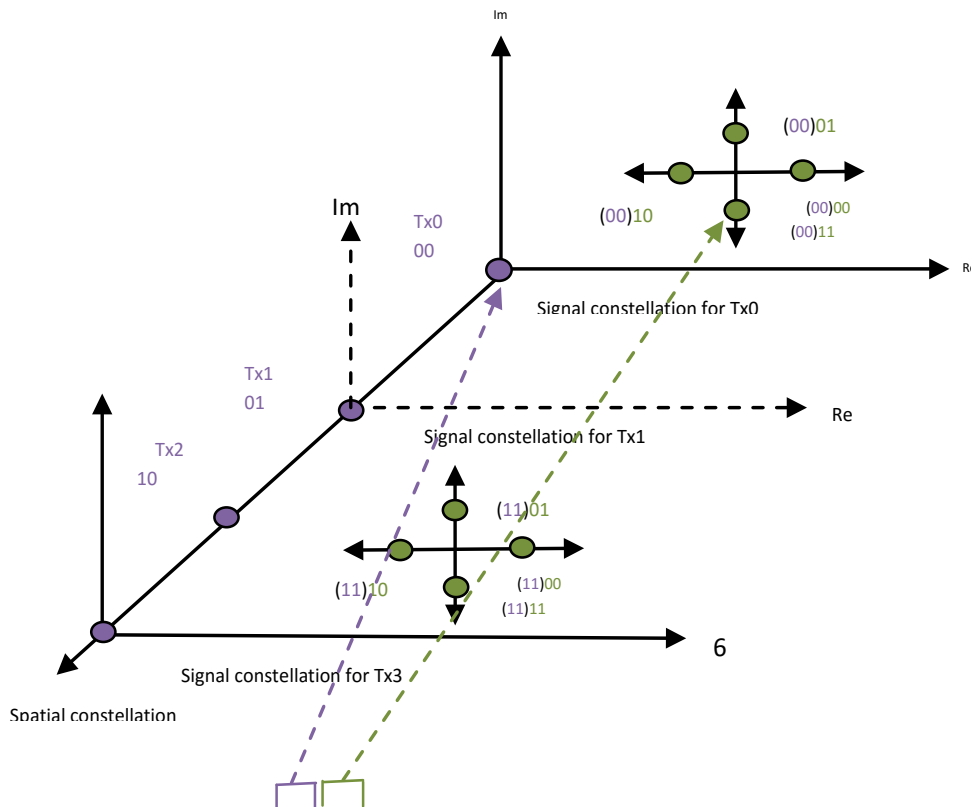


Figure 1. Spatial modulation schematic diagram.

In essence, the i^{th} column of channel matrix \mathbf{H} contains the channel gains from i^{th} transmit antenna to all the receive antennas. Further, the observation vector \mathbf{y} at the receiver can be interpreted as,

$$\mathbf{y} = x_1 \mathbf{h}_1 + x_2 \mathbf{h}_2 + \dots + x_{N_t} \mathbf{h}_{N_t} \quad (4)$$

Where, the transmit vector $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_{N_t}]^T$. As only N_a antennas are active out of N_t antennas at the transmitter, the observation vector \mathbf{y} becomes the linear combination of N_a vectors out of N_t column vectors in \mathbf{H} . The OMP algorithm[] is modified to the spatial modulated OMP (SM-OMP) as follows.

Step 1. Initialize $t = 1, \hat{\mathbf{A}} = \emptyset, \Phi = [] , \mathbf{r}_0 = \mathbf{y}$.

Step 2. Find the antenna index a_t at the t^{th} iteration such that $a_t = \arg \max_{k=1-N_t} |\mathbf{h}_k^* \mathbf{r}|$

Step 3. Update the detected active antenna index $\hat{\mathbf{A}} \leftarrow \hat{\mathbf{A}} \cup a_t$

Step 4. Compute the least squares estimate of the \mathbf{x} at the t^{th} iteration as follows.

$$\Phi_t = [\Phi_{t-1} \ \mathbf{h}_{a_t}]$$

$$\tilde{\mathbf{x}} = (\Phi_t^* \Phi_t)^{-1} \Phi_t^* \mathbf{y}$$

Step 5. From the estimated $\tilde{\mathbf{x}}$ detect the transmitted symbol from the M -ary constellation using the minimum distance decoder as follows.

Step 6. Update the residual vector $\mathbf{r}_t \leftarrow \mathbf{y} - \Phi_t \tilde{\mathbf{x}}$ and $t \leftarrow t + 1$.

Step 7. Repeat until all the N_a active antennas and their transmit symbols are detected.

If $t < N_a$ go to Step 2 else STOP.

To summarize, the SM-OMP chooses the transmit antenna index that is most likely by correlating all the columns in the channel matrix in Step 2. At each iteration the detected components are estimated using least squares in Step 4. It should be noted that the main difference between the conventional OMP and SM-OMP stems from the fact that in conventional OMP the least squares estimated component is removed from the observation as given in Step 6. But in the case of SM-MIMO, the values or entries of the transmit vector \mathbf{x} is from a finite alphabet constellation symbols. Hence, the detected values of the vector \mathbf{x} are replaced by their closest symbol in the transmit constellation using a minimum distance decoder^{9,10} as given in Step 5. This drastically improves the detection performance of SM-OMP than directly applying the conventional OMP to the system model in. Before proceeding to characterizing the perfect recovery of spatial constellation, let us define the following important parameter which is related to the correlation^{11,12} of the channel and hence to the spatial diversity.

Definition 1. The channel correlation ρ of a given MIMO system is defined as the minimum angle between any two columns of its channel matrix.

$$\rho = \max_{i,k,i \neq k} \frac{\mathbf{h}_i^* \mathbf{h}_k}{\|\mathbf{h}_i\| \|\mathbf{h}_k\|} \quad (5)$$

As the value of ρ approaches zero, the system achieves full spatial diversity¹³⁻¹⁵. The following theorem characterizes the recovery guarantee of the spatial constellation symbol and symbol error rate for the spatially detection performance.

Theorem 1.

The spatial modulated OMP (SM-OMP) algorithm perfectly recovers the spatial constellation symbols with the additive noise being zero mean Gaussian with variance σ^2 if

$$\rho < 1 / \sqrt{2N} \sqrt{1 - 4\sigma / (N_a \|\mathbf{h}_1\|_{\min})} \sqrt{2} \sqrt{(P_{\min})} \quad (6)$$

and the probability of spatial constellation symbol error P_{SER} is given by

$$P_{SER} \leq Prob(\|h_{\min}\|_2 < (1 - 8\sigma)/(2\rho N_t a \sqrt{P_{\min}})) \tag{7}$$

Where $\|h_{\min}\|_2 = \min_i \|h_i\|_2, i=1 \dots N_t$ is the minimum ℓ_2 norm of the columns of the channel matrix and P_{\min} is the minimum power of the transmit constellation symbols.

Proof: See Appendix

The following inferences can be made from the above theorem.

- It can be seen from (26) that the error free recovery of spatial constellation symbol depends on the channel correlation and in turn on the spatial diversity.

- Further, the spatial symbol error rate depends on the probability that the channel gain crosses below a particular threshold, which is generally referred as the deep fade event.
- The transmit power is same for all symbols in the case of M-ary phase shift keying signals and different for M-ary QAM signals (except M = 4 as 4-ary QAM is nothing but QPSK) and the maximum power of the transmit symbols plays an important role in the probability of symbol error rate.
- The symbol error rate is lesser for the M-ary PSK signals than M-ary QAM signals used for SM-MIMO systems while all other parameters are fixed.

4. Simulation Results

In this section we evaluate the performance of our proposed algorithm using Monte-Carlo simulations. We compare the performance of SM-OMP algorithm with traditional Linear Minimum Mean Square Error

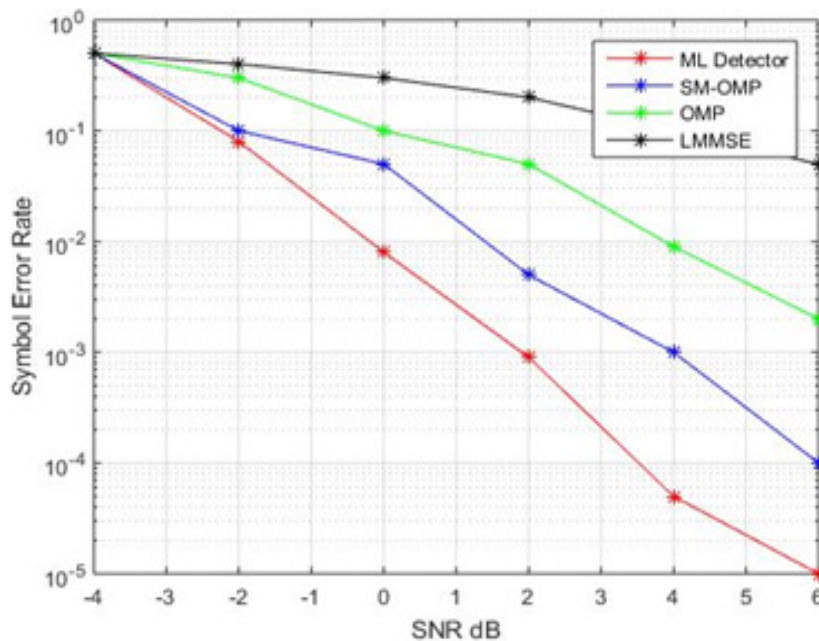


Figure 2. Performance of various detectors in Rayleigh fading channels.

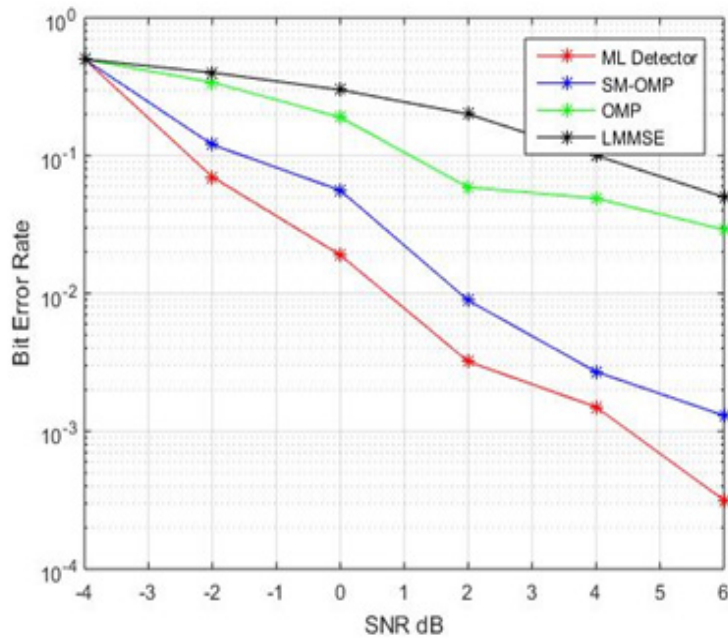


Figure 3. BER comparison of various detectors.

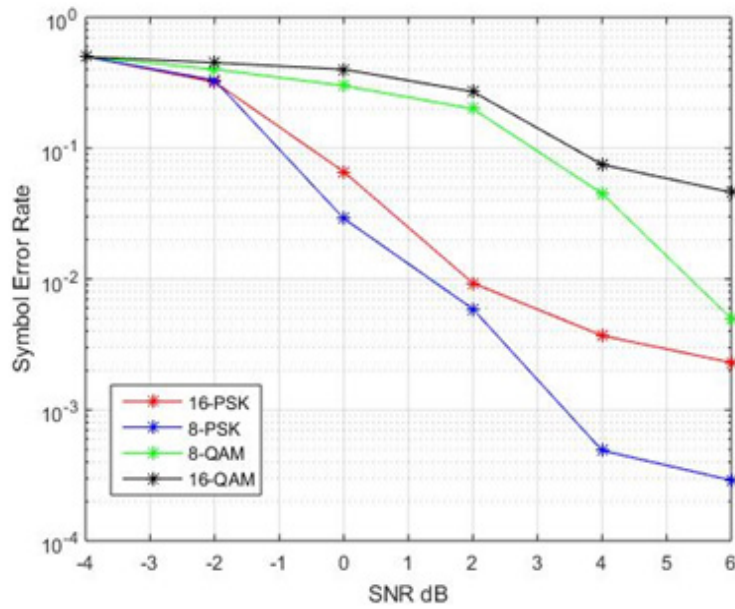


Figure 4. Performance of various detectors in different spatial constellation.

(LMMSE)^{16,17} detector and conventional OMP¹⁸ algorithm directly applied to our system model. Also we compare the performances of the optimal ML detector to gauge the performance of our proposed approach with the best possible detector. We consider Rayleigh fading MIMO channel and number of transmit antenna to be $N_t = 64$, number of receive antenna $N_r = 8$. The number of active antennas $N_a = 4$. We simulate the detection performance for two different transmit constellations, 8-PSK and 16-QAM. The additive noise is considered to be Gaussian with zero mean and variance σ^2 and independent of the channel and transmit symbols.

In Figure 2, we compare the performance of all spatial constellation symbols for the following algorithms, 1. LMMSE, 2. Conventional OMP 3. Our proposed SM-OMP and 4. Optimal ML detector. It can be seen that the perfect detection rate of our proposed approach performs significantly better than the conventional OMP and LMMSE detectors. Further, SM-OMP algorithm performs closely to the optimal detector. In Figure 3, we plot the symbol error rate performance of our proposed SM-OMP and the conventional OMP directly applied to the system model. It can be seen that as the SNR increases, the symbol error rate performance of our proposed approach significantly outperforms the other algorithms and perform similar to the optimal ML detector. In Figure 4, we simulate the performance for 8-PSK and 8-QAM signal constellations. As discussed in the Theorem 1, the symbol error rate performance is better in the case of PSK constellations than the QAM constellations. As the transmit power is equal for all the transmit symbols in the case of PSK constellations, the symbol error rate is low compared to the QAM constellation as the transmit power vary for different symbols.

5. Conclusion

We proposed a new SM-OMP algorithm for spatial modulation and characterized its theoretical performance in terms of the successful recovery of all the spatial constellation symbols and the probability of symbol error rate. Also, we compared the performance of our proposed approach with the conventional OMP algorithm directly applied to the SM-MIMO systems and showed the out-

performance of our proposed approach. Further, we have shown that our proposed approach performs close to the optimal ML detector.

6. References

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Appendix

Without loss of generality let us assume the following.

First N_a antennas are active in the transmit vector, i.e. the set of active antenna indices $A = \{1, 2, \dots, N_a\}$

The gains of the channel matrix H can be ordered as

Thus the observation vector y can be written as,

$$y = x_1 h_1 + x_2 h_2 + \dots + x_{N_t} h_{N_t} + w \dots\dots\dots(8)$$

In the first iteration the SM-OMP will correctly identify the first antenna index if

$$|h_1^* y| > |h_k^* y| \forall k \in A \dots\dots\dots(9)$$

The inner product on the left hand side of the above condition can be bounded as,

$$|h_1^* y| = \|h_1^* (x_1 h_1 + x_2 h_2 + \dots + x_{N_t} h_{N_t} + w)\|_2 \dots\dots(10)$$

$$= \left| \sum_{k=1}^{N_a} x_k h_1^* h_k + h_1^* w \right| \dots\dots(11)$$

By triangle inequality the above term can be lower bounded as,

$$|h_1^* y| = \left| \sum_{k=1}^{N_a} x_k h_1^* h_k + h_1^* w \right| \dots\dots(12)$$

$$\geq x_1 \|h_1\|_2^2 - \left\| \sum_{k=2}^{N_a} x_k h_1^* h_k + h_1^* w \right\|_2 \dots\dots(13)$$

$$\geq x_1 \|h_1\|_2^2 - \left(\sum_{k=2}^{N_a} \|x_k h_1^* h_k\|_2 + \|h_1^* w\|_2 \right) \dots(14)$$

From the definition of the channel correlation ρ given in (5)

$$|x_k h_1^* h_k| = \rho \|h_1\|_2 \|h_k\|_2 \leq \rho \|h_1\|_2^2 \dots\dots\dots(15)$$

since it is assumed without loss of generality that $\|h_1\|_2 \geq \|h_2\|_2 \geq \dots \geq \|h_{N_a}\|_2$. Further, the product $\|h_1^* w\|_2$ can be bounded as

$$\|h_1^* w\|_2 \leq \|h_1\|_2 \|w\|_2 \leq \|w\|_2 \dots\dots\dots(16)$$

by applying the Schwarz inequality and noting the fact that gain of the channel cannot exceed 1. Moreover, the noise vector w is Gaussian with zero mean and variance σ^2 . Therefore using the sigma rule for Gaussian, the ℓ_2 norm of the vector w is upper bounded by $\|w\|_2 \leq 4\sigma$ with very high probability. Therefore,

$$\|h_1^* w\|_2 \leq 4\sigma \dots\dots\dots(17)$$

Applying the results on the bounds obtained in (15) and (17) in (14) we get,

$$|h_1^* y| \geq |x_1| \|h_1\|_2^2 - (N_a - 1)\rho |x_1| \|h_1\|_2^2 - 4\sigma \dots(18)$$

Similarly, the right hand side of the condition in (9) can be bounded as follows.

$$|h_1^* y| = \left\| \sum_{i=0}^{N_a} h_k^* x_i h_i + h_k^* w \right\|_2 \dots\dots\dots(19)$$

$$\leq \sum_{i=0}^{N_a} x_i \|h_k^* h_i\|_2 + \|h_k^* w\|_2 \dots\dots(20)$$

Using the bounds in (15) and (17) in the above inequality, we obtain,

$$|h_1^* y| \leq (N_a)\rho |x_1| \|h_1\|_2^2 + 4\sigma \dots\dots(21)$$

By using the results in (18) and (21) in the recovery condition given in (9), the correct antenna index or the spatial constellation symbol is detected in the first iteration by SM-OMP if,

$$\|x_1\| \|h_1\|_2 - (N_a - 1)\rho \|x_1\| \|h_1\|_2 - 4\sigma > (N_a)\rho \|x_1\| \|h_1\|_2^2 + 4\sigma \quad \dots(22)$$

$$2(N_a - 1)\rho \|x_1\| \|h_1\|_2^2 < \|x_1\| \|h_1\|_2^2 - 8\sigma \quad 2(N_a - 1)\rho \|x_1\| \|h_1\|_2^2 < \|x_1\| \|h_1\|_2^2 - 8\sigma \quad \dots(23)$$

$$(2N_a)\rho \|x_1\| \|h_1\|_2^2 < \|x_1\| \|h_1\|_2^2 - 8\sigma \quad (2N_a)\rho \|x_1\| \|h_1\|_2^2 < \|x_1\| \|h_1\|_2^2 - 8\sigma \quad \dots(24)$$

$$\rho < \frac{1}{2N_a} - \frac{4\sigma}{N_a \|x_1\| \|h_1\|_2^2} \quad \dots\dots\dots(25)$$

The same argument can be carried out to show the condition in (25) is sufficient for the detection of correct spatial symbol in each iteration. Since, the detected columns of the channel matrix H are estimated and removed from the residual as given in Step 6 of SM-OMP, any column or any spatial information symbol to get detected twice. Thus considering the worst condition of all the iterations, SM-OMP perfectly detects the spatial constellation symbol if,

$$\rho < 1/(2N_a) - 4\sigma/(N_a \sqrt{(P_{\min})} \|h_{\min}\|_2^2) \quad \dots\dots\dots(26)$$

Which is the condition for perfect recovery of spatial constellation symbols given in (6) By rearranging equation (26),

$$(N_a + \rho)/N_a < 4\sigma/(N_a \sqrt{(P_{\min})} \|h_{\min}\|_2^2) \quad \dots(27)$$

$$1/(4N_a + \rho) < (\sqrt{(P_{\min})} \|h_{\min}\|_2^2)/4\sigma \quad \dots\dots\dots(28)$$

$$\|h_{\min}\|_2^2 < 4\sigma / (\sqrt{(N_a + \rho)} \sqrt{(P_{\min})}) \quad \|h_{\min}\|_2^2 < 4\sigma / (\sqrt{(N_a + \rho)} \sqrt{(P_{\min})}) \quad \dots\dots\dots(29)$$

As the condition (29) is sufficient but not necessary, the entire spatial constellation symbol error probability is upper bounded as given in (7).