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Research paper

Oscillation of Generalized Second-Order Quasi Linear Difference Equations

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Abstract

Authorsipresent sufficientconditions for theioscillation of the generalizediperturbed quasilineariddifferenceiequation $\Delta_\ell\left(a((x-1)\ell+i)\left|\Delta_\ell v((x-1)\ell+i)\right|^{\gamma-1}\Delta_\ell v((x-1)\ell+i)\right)+F(x\ell+i,v(x\ell+i))=G(x\ell+i,v(x\ell+i),\Delta_\ell v(x\ell+i))$ where $0<\gamma<1$, $x\in[0,\infty)$ and $i=x-\left[\frac{x}{\ell}\right]\ell$. Examplesiillustrates the importanceiof our results are alsoiincluded.

Keywords: *Generalizedidifference equation; Oscillation; iQuasilinear,*

1. Introduction

Difference equations represent captivating mathematical field, has rich field of the applications in such diverse disciplines as population dynamics, operations research, ecology, economics, biology etc. For thelbackgroundlof differencelequations and its applicationslin diverselfields withlexamples, see [1]. The study of difference equations is based on the operator Δ defined as $\Delta u(x)=u(x+1)-u(x), x\in[0,\infty)$.

Thoughlmany authors [1],[16] have discussed the definition of Δ as

$$\Delta u(x)=u(x+\ell)-u(x), \quad \ell\in(0,\infty), \tag{1}$$

no notable progress have been taken on this line. Butlin [13] the authors took up the definition of Δ as given in (1), and given many important results and applications. They labeled the operator Δ defined by (1) as Δ_ℓ and its inversely Δ_ℓ^{-1}, many interesting results in number theory were obtained. Qualitativelproperties like rotatory, expanding, lshrinking, spiral and web like were established by extending theory ofl Δ_ℓ to complex function, for the solutions of difference equations involving Δ_ℓ in [2-15, 17-21].

In the sequel, in this paper we will be considered the generalized perturbed quasi linear difference equation for $x\in[0,\infty)$

$$\Delta_\ell\left(a((x-1)\ell+i)\left|\Delta_\ell v((x-1)\ell+i)\right|^{\gamma-1}\Delta_\ell v((x-1)\ell+i)\right)$$

$$+F(x\ell+i,v(x\ell+i))=G(x\ell+i,v(x\ell+i),\Delta_\ell v(x\ell+i)) \tag{2}$$

where $0<\gamma<1$, $a(x\ell+i)$ is an eventually positive real valued function, land Δ_ℓ is the generalized forward difference operator defined as $\Delta_\ell v(x\ell+i)=v((x+1)\ell+i)-v(x\ell+i)$.

By alsolution of (2), we mean a nontrivial real valued function $v(x\ell+i)$ satisfying (2) for $x\in[0,\infty)$. A solution $v(x\ell+i)$ is said to be oscillatory if it is neither eventually positive nor negative, and non oscillatory otherwise.

2. Main Results

In this paper we assume that there exist real valued functions $q(x\ell+i)$, $p(x\ell+i)$ andla function $f:R\to R$ such that

(i). $vf(v)>0$ for all $v\neq 0$;

(ii). $f(v)-f(w)=g(v,w)(v-w)$ for $v,w\neq 0$, where g is a nonnegativelfunction; land

(iii). $\frac{F(x\ell+i,v\ell+i)}{f(v\ell+i)}\geq q(x\ell+i)$,

$\frac{G(x\ell+i,v\ell+j,w\ell+i)}{f(v\ell+i)}\leq p(x\ell+i)$ for $v,w\neq 0$.

Thelfollowing conditionslare usedlthroughout thislpaper:

$$\sum\frac{1}{a^{1/\gamma}((x-1)\ell+i)}=\infty, \tag{3}$$

$$\int_\theta^\infty\frac{dx}{f(x)^{1/\gamma}}<\infty, \int_{-\theta}^{-\infty}\frac{dx}{f(x)^{1/\gamma}}<\infty \text{ forall } \theta>0 \tag{4}$$

$$\liminf_{x\to\infty}\sum_{r=x_0}^{x}(q(r\ell+i)-p(r\ell+i))\geq 0 \text{ for all large } x_0, \tag{5}$$

$$\int_0^\theta\frac{dx}{f(x)^{1/\gamma}}<\infty, \int_0^{-\theta}\frac{dx}{f(x)^{1/\gamma}}<\infty \text{ for all } \theta>0, \tag{6}$$

$$\sum\left[\frac{N}{a(r\ell+i)}-\frac{1}{a(r\ell+i)}\sum_{s=x_0}^{r}(q(s\ell+i)-p(s\ell+i))\right]=-\infty \qquad (7)$$

forlevery constantl$N$,

$$\limsup_{x\to\infty}\sum_{r=x_0}^{x}(q(r\ell+i)-p(r\ell+i))=\infty \text{ for allllarge } x_0, \qquad (8)$$

$$\limsup_{x\to\infty}\sum_{r=x_0}^{x}r(q(r\ell+i)-p(r\ell+i))=\infty \text{ forlall large } x_0, \qquad (9)$$

$$\sum(q(x\ell+i)-p(x\ell+i))R(x\ell+i,x_0\ell+i)=\infty \text{ where}$$

$$R(x\ell+i,x_0\ell+i)=\sum_{r=x_0}^{x}\frac{1}{a((r-1)\ell+i)}, \qquad (10)$$

$$\sum\frac{1}{(a(n-1)\ell+i)}<\infty, \qquad (11)$$

$$\sum(q(x\ell+i)-p(x\ell+i))T(x\ell+i,x_0\ell+i)=\infty \text{ where} \qquad (12)$$

$$T(x\ell+i,x_0\ell+i)=R((x-1)\ell+i,x_0\ell+i)=\sum_{r=x_0}^{x-1}\frac{1}{a((r-1)\ell+i)},$$

$$\sum\frac{1}{a((x-1)\ell+i)}=\infty, \qquad (13)$$

$$\frac{a(x\ell+i)}{a((x-1)\ell+i)}\le 1 \text{ for } x\ge 1. \qquad (14)$$

Theorem 1 *Supposal* $\gamma\ge 1$ *and (5)-(7) hold. Then, all solutions of (2) are oscillatory.*

Proof. Suppose that $v(x\ell+i)$ is alnonoscillatory solution of (2), say, $v(x\ell+i)>0$ for $k\ge k_0\ge 1$. Since (5) holds, $\Delta_\ell v(x\ell+i)$ does not oscillate. Welbegin withlthe followinglidentity

$$\Delta_\ell\left[\frac{a((x-1)\ell+i)\,|\Delta_\ell v((x-1)\ell+i)|^{\gamma-1}\Delta_\ell v((x-1)\ell+i)}{f(v((x-1)\ell+i))}\right]$$
$$=\frac{G(x\ell+i,v(x\ell+i),\Delta f(v(x\ell+i)))}{f(v(x\ell+i))}-\frac{F(x\ell+i,v(x\ell+i))}{f(v(x\ell+i))}$$
$$-\frac{a((x-1)\ell+i)g(v(x\ell+i),v((x-1)\ell+i))(\Delta_\ell v((x-1)\ell+i))^2}{f(v((x-1)\ell+i))f(v(x\ell+i))}$$
$$\times|\Delta_\ell v((x-1)\ell+i)|^{\gamma-1} \qquad (15)$$

whichlimplies

$$\Delta_\ell\left[\frac{a((x-1)\ell+i)\,|\Delta_\ell v((x-1)\ell+i)|^{\gamma-1}\Delta_\ell v((x-1)\ell+i)}{f(v((x-1)\ell+i))}\right]$$
$$\le p(x\ell+i)-q(x\ell+i). \qquad (16)$$

Case 1. Suppose that $\Delta_\ell v(x\ell+i)\ge 0$ for $x\ge x_1\ge x_0$. Summing (16) from (x_1+1) to x gives

$$\frac{a(x\ell+i)\left|\Delta_\ell v(x\ell+i)\right|^{\gamma-1}\Delta_\ell v(x\ell+i)}{f(v(x\ell+i))}$$
$$\le\frac{a(x_1\ell+i)\left|\Delta_\ell v(x_1\ell+i)\right|^{\gamma-1}\Delta_\ell v(x_1\ell+i)}{f(v(x_1\ell+i))}$$
$$-\sum_{r=x_1+1}^{x}(q(r\ell+i)-p(r\ell+i))$$

$$\frac{\left|\Delta_\ell v(x\ell+i)\right|^{\gamma-1}\Delta_\ell v(x\ell+i)}{f(v(x\ell+i))}\le\frac{N}{a(x\ell+i)}$$

$$-\frac{1}{a(x\ell+i)}\sum_{r=x_1+1}^{x}(q(r\ell+i)-p(r\ell+i)) \qquad (17)$$

where $X=a(x_1\ell+i)\,|\Delta_\ell v(x_1\ell+i)|^{\gamma-1}\Delta_\ell v(x_1\ell+i)/f(v(x_1\ell+i))$.
Again we sum (17) from (k_1+1) to k, to obtain

$$\sum_{r=x_1+1}^{x}\frac{\left|\Delta_\ell v(r\ell+i)\right|^{\gamma-1}\Delta_\ell v(r\ell+i)}{f(v(r\ell+i))} \qquad (18)$$

$$\le\sum_{s=x_1+1}^{x}\left[\frac{N}{a(r\ell+i)}-\frac{1}{a(r\ell+i)}\sum_{s=x_1+1}^{r}(q(s\ell+i)-p(s\ell+i))\right].$$

By (7), the right side of (18) tends to $-\infty$ as $x\to\infty$ where as the left side is nonnegative.

Case 2. Suppose that $\Delta_\ell v(x\ell+i)<0$ for $x\ge x_1\ge x_0$. Then, lfrom (18) welfind

$$-\sum_{r=x_1+1}^{x}\left[\frac{N}{a(r\ell+i)}-\frac{1}{a(r\ell+i)}\sum_{s=x_1+1}^{r}(q(s\ell+i)-p(s\ell+i))\right]$$
$$\le\sum_{r=x_1+1}^{x}\frac{|\Delta_\ell v(r\ell+i)|^{\gamma}}{f(v(r\ell+i))} \qquad (19)$$

$$\le\left[\sum_{r=x_1+1}^{x}\frac{|\Delta_\ell v(r\ell+i)|}{f(v(r\ell+i))^{1/\gamma}}\right]^{\gamma}$$

$$\le\left[\int_{v(x+1)}^{v(x_1+1)}\frac{du}{f(u)^{1/\gamma}}\right]^{\gamma}$$

$$\le\left[\int_{0}^{v(x_1+1)}\frac{du}{f(u)^{1/\gamma}}\right]^{\gamma}. \qquad (20)$$

By (7), the left side of (20) tends to ∞ asl $x\to\infty$ where as the right side is finite by (6).

Theorem 2 *Suppose* $a((x-1)\ell+i)\equiv 1$, $\gamma\ge 1$ l*and (4), (5), (9) hold. Then all solutions of (2) are oscillatory.*

Proof. Assume that $v(x\ell+i)$ is a nonsocial atory solution of (2), say, $v(x\ell+i)>0$ for $x\ge x_0\ge 1$. Since (5) holds, we see that $\Delta_\ell v(x\ell+i)$ does not oscillate.

Welbegin withlthe followinglidentity

$$\Delta_\ell\left[\frac{(x\ell+i)\,|\Delta_\ell v((x-1)\ell+i)|^{\gamma-1}\Delta_\ell v((x-1)\ell+i)}{f(v(x\ell+i))}\right]$$
$$=\frac{(x\ell+i)G(x\ell+i,v(x\ell+i),\Delta f(v(x\ell+i)))}{f(v(x\ell+i))}$$
$$-\frac{(x\ell+i)F(x\ell+i,v(x\ell+i))}{f(v(x\ell+i))}+\frac{|\Delta_\ell v(x\ell+i)|^{\gamma-1}\Delta_\ell v(x\ell+i)}{f(v((x+1)\ell+i))}$$
$$-\frac{(x\ell+i)g(v((x+1)\ell+i),v(x\ell+i))(\Delta_\ell v(x\ell+i))^2}{f(v(x\ell+i))}$$
$$\times\frac{|\Delta_\ell v(x\ell+i)|^{\gamma-1}}{f(v((x+1)\ell+i))}$$

Which give rise to

$$\Delta_\ell\left[\frac{(x\ell+i)\,|\Delta_\ell v((x-1)\ell+i)|^{\gamma-1}\Delta_\ell v((x-1)\ell+i)}{f(v(x\ell+i))}\right]$$

$$\leq (x\ell+i)(p(x\ell+i)-q(x\ell+i))+\frac{|\Delta_\ell v(x\ell+i)|^{\gamma-1}\Delta_\ell v(x\ell+i)}{f(v((x+1)\ell+i))}. \quad (21)$$

Case 1. Suppose that $\Delta_\ell v(x\ell+i)\geq 0$ for $x\geq x_1\geq x_0$. Summing (21) from (x_1+1) to x gives

$$\sum_{r=x_1+1}^{x}(r\ell+i)(q(r\ell+i)-p(r\ell+i))\leq \frac{((x_1+1)\ell+i)(\Delta_\ell v(x_1\ell+i))^\gamma}{f(v((x_1+1)\ell+i))}$$
$$-\frac{((x+1)\ell+i)(\Delta_\ell v(x\ell+i))^\gamma}{f(v((x+1)\ell+i))}+\sum_{r=x_1+1}^{x}\frac{(\Delta_\ell v(r\ell+i))^\gamma}{f(v((r+1)\ell+i))}$$
$$\leq \frac{((x_1+1)\ell+i)(\Delta_\ell v(x_1\ell+i))^\gamma}{f(v((x_1+1)\ell+i))}+\sum_{r=x_1+1}^{x}\frac{(\Delta_\ell v(r_1\ell+i))^\gamma}{f(v((r+1)\ell+i))}$$
$$\leq \frac{((x_1+1)\ell+i)(\Delta_\ell v(x_1\ell+i))^\gamma}{f(v((x_1+1)\ell+i))}+\left[\sum_{r=x_1+1}^{x}\frac{\Delta_\ell v(r\ell+i)}{f(v((r+1)\ell+i))^{1/\gamma}}\right]^\gamma$$
$$\leq \frac{((x_1+1)\ell+i)(\Delta_\ell v(x_1\ell+i))^\gamma}{f(v((x_1+1)\ell+i))}+\left[\int_{v(x_1+1)}^{v(x+1)}\frac{du}{f(u)^{1/\gamma}}\right]^\gamma. \quad (22)$$

By (9), the left side of (22) tends to ∞ as $x\to\infty$ whereas the right side is finite by (4).
Case 2. Suppose that $\Delta_\ell v(x\ell+i)<0$ for $k\geq k_1\geq k_0$. Condition (21) implies the existence of an integer $x_2\geq x_1$ such that

$$\sum_{r=x_1+1}^{x}(r\ell+i)(q(r\ell+i)-p(r\ell+i))\geq 0,\ x\geq x_2+1. \quad (23)$$

Multiplying l (2) by $x\ell+i$ land using l (iii), welobtain
$$(x\ell+i)(|\Delta_\ell v((x-1)\ell+i)|^{\gamma-1}\Delta_\ell v((x-1)\ell+i))$$
$$\leq (x\ell+i)f(v(x\ell+i))(p(x\ell+i)-q(x\ell+i))$$
Which on summing by parts from (x_2+1) to x provides
$$((x+1)\ell+i)|\Delta_\ell v(x\ell+i)|^{\gamma-1}\Delta_\ell v(x\ell+i)$$
$$\leq ((x_2+1)\ell+i)|\Delta_\ell v(x_2\ell+i)|^{\gamma-1}\Delta_\ell v(x_2\ell+i)$$
$$+\sum_{r=x_1+1}^{x}|\Delta_\ell v(r\ell+i)|^{\gamma-1}\Delta_\ell v(r\ell+i)$$
$$-\sum_{r=x_1+1}^{x}(r\ell+i)(q(r\ell+i)-p(r\ell+i))$$
$$=((x_2+1)\ell+i)|\Delta_\ell v(x_2\ell+i)|^{\gamma-1}\Delta_\ell v(x_2\ell+i)$$
$$+\sum_{r=x_1+1}^{x}|\Delta_\ell v(r\ell+i)|^{\gamma-1}\Delta_\ell v(r\ell+i)-f(v((x+1)\ell+i))$$
$$\times\sum_{r=x_1+1}^{x}(r\ell+i)(q(r\ell+i)-p(r\ell+i))$$
$$+\sum_{r=x_1+1}^{x}\Delta_\ell f(v(r\ell+i))\left[\sum_{s=x_1+1}^{r}(s\ell+i)(q(s\ell+i)-p(s\ell+i))\right]$$
$$=((x_2+1)\ell+i)|\Delta_\ell v(x_2\ell+i)|^{\gamma-1}\Delta_\ell v(x_2\ell+i)$$
$$+\sum_{r=x_1+1}^{x}|\Delta_\ell v(r\ell+i)|^{\gamma-1}\Delta_\ell v(r\ell+i)$$
$$-f(v((x+1)\ell+i))\sum_{r=x_1+1}^{x}(r\ell+i)(q(r\ell+i)-p(r\ell+i))$$
$$+\sum_{r=x_1+1}^{x}g(v((r+1)\ell+i),v(r\ell+i))\Delta_\ell v(r\ell+i)$$
$$\times\left[\sum_{s=x_1+1}^{r}(s\ell+i)(q(s\ell+i)-p(s\ell+i))\right]$$
$$\leq ((x_2+1)\ell+i)|\Delta_\ell v(x_2\ell+i)|^{\gamma-1}\Delta_\ell v(x_2\ell+i)$$
Where we have also used (23) in the last inequality. It followslthat

$$\Delta_\ell v(x\ell+i)\leq \frac{-((x_2+1)\ell+i)^{1/\gamma}|\Delta_\ell v(x_2\ell+i)|}{((x+1)\ell+i)^{1/\gamma}}, \quad (24)$$

For $x\geq x_2+1$. Once again we sum (24) from (x_2+1) to x to get

$$v((x+1)\ell+i)\leq v((x_2+1)\ell+i)$$
$$-((x_2+1)\ell+i)^{1/\gamma}|\Delta_\ell v(x_2\ell+i)|\sum_{r=x_2+1}^{x}\frac{1}{((r+1)\ell+i)^{1/\gamma}}. \quad (25)$$

The right side of (25) tends to $-\infty$ as $x\to\infty$, this contradicts the assumption that $v(x\ell+i)$ is eventually positive.

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