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Packing chromatic number of certain fan and wheel related graphs

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Received 17 June 2015; accepted 20 May 2016

Available online xxxx

Abstract

The packing chromatic number $\chi_\rho(G)$ of a graph G is the smallest integer k for which there exists a mapping $\pi : V(G) \longrightarrow \{1, 2, \ldots, k\}$ such that any two vertices of color i are at distance at least $i + 1$. In this paper, we compute the packing chromatic number for certain fan and wheel related graphs.
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Keywords: Packing chromatic number; Uniform n-fan split graph; Uniform n-wheel split graph

1. Introduction

Let G be a connected graph and k be an integer, $k \geq 1$. A packing k-coloring of a graph G is a mapping $\pi : V(G) \longrightarrow \{1, 2, \ldots, k\}$ such that any two vertices of color i are at distance at least $i + 1$. The packing chromatic number $\chi_\rho(G)$ of G is the smallest integer k for which G has packing k-coloring. The concept of packing coloring comes from the area of frequency assignment in wireless networks and was introduced by Goddard et al. [1] under the name broadcast coloring. It has several applications, such as, in resource placement and biological diversity. The term packing chromatic number was introduced by Brešar et al. [2].

Goddard et al. [1] proved that the packing coloring problem is NP-complete for general graphs and Fiala and Golovach [3] proved that it is NP-complete even for trees. It is proved that the packing coloring problem is solvable in polynomial time for graphs whose treewidth and diameter are both bounded [3] and for cographs and split graphs [1]. Sloper [4] studied a special type of packing coloring, called eccentric coloring and proved that the infinite 3-regular tree has packing chromatic number 7. For the infinite planar square lattice $\mathbb{Z}^2$, $10 \leq \chi_\rho(\mathbb{Z}^2) \leq 17$ [5,6]. The packing coloring of distance graphs was studied in [7,8]. For the infinite hexagonal lattice $\mathbb{H}$, $\chi_\rho(\mathbb{H}) = 7$ [2].

Argiroffo et al. [9,10] proved that the packing coloring problem is solvable in polynomial time for the class of $(q, q - 4)$ graphs, partner limited graphs and for an infinite subclass of lobsters, including caterpillars. It is proved in [11,12] that the infinite, planar triangular lattice and the three dimensional square lattice have unbounded packing chromatic number. In this paper, we study the packing chromatic number of certain fan and wheel related graphs.

Peer review under responsibility of Kalasalingam University.
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http://dx.doi.org/10.1016/j.akcej.2016.11.001
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2. Main results

Let G_1 and G_2 be vertex disjoint graphs with $|V(G_1)| = n_1$, $|E(G_1)| = m_1$, $|V(G_2)| = n_2$ and $|E(G_2)| = m_2$.

Definition 2.1. The union of G_1 and G_2 is the graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$. It is denoted by $G_1 \cup G_2$. So, $G_1 \cup G_2$ has $n_1 + n_2$ vertices and $m_1 + m_2$ edges.

Definition 2.2. The sum or join of G_1 and G_2 is the graph obtained from $G_1 \cup G_2$ by joining every vertex of G_1 with G_2. It is denoted by $G_1 + G_2$. So, $G_1 + G_2$ has $n_1 + n_2$ vertices and $m_1 + m_2 + n_1 n_2$ edges.

Definition 2.3. A Fan graph F_n is defined as the graph $K_1 + P_n$, where K_1 is the singleton graph and P_n is the path on n vertices.

Definition 2.4. The wheel W_{n+1} is defined as the graph $K_1 + C_n$, where K_1 is the singleton graph and C_n is the cycle graph on n vertices.

Definition 2.5 (*[13]*). A uniform n-fan split graph SF_n^r contains a star S_{n+1} with hub at x such that the deletion of the n edges of S_{n+1} partitions the graph into n independent fans $F_r^i = P_r^i + K_1$, $1 \le i \le n$ and an isolated vertex. See Fig. 1.

Theorem 2.6. *For the uniform n-fan split graph SF_n^r, $n \ge 4$, $r \ge 5$, we have $\chi_\rho(SF_n^r) \ge 3 + n[r - \lceil \frac{r}{2} \rceil - 1]$.*

Proof. Let F_r^i, $1 \le i \le n$ be the fans of SF_n^r. Let $V(SF_n^r) = \{w_j^i, w_i, x : 1 \le i \le n, 1 \le j \le r\}$, where w_i is the hub of F_r^i and x is the hub of S_{n+1}. Since the diameter of SF_n^r is 4, colors greater than 3 can be assigned to only one vertex of SF_n^r.

Fact 1: If color 3 is assigned to vertex x, no other vertex of SF_n^r can receive color 3 because $d(x, w_j^i) = 2$ and $d(x, w_i) = 1$, $1 \le i \le n$, $1 \le j \le r$. Similarly, if color 3 is assigned to any vertex w_i, no other vertex of SF_n^r can receive color 3. And also, if color 3 is assigned to any vertex w_j^i of any F_r^i and since $\text{diam}(F_r^i) = 2$, no other vertex of chosen F_r^i can receive 3. There are n fans in SF_n^r. Since $d(w_j^i, w_m^l) = 4$, $i \ne l$, $1 \le i, l \le n$, $1 \le j, m \le r$, at most n vertices receive color 3. Thus, the maximum number of vertices that can receive color 3 is n.

Fact 2: Since $d(w_j^i, w_m^l) = 4$, $i \ne l$, $1 \le i, l \le n$, $1 \le j, m \le r$ and $d(w_i, w_m^l) = 3$, $i \ne l$, $1 \le i, l \le n$, $1 \le m \le r$, assigning color 2 to a vertex w_j^i or w_i, at most n vertices can receive 2. Thus, the maximum number of vertices that can receive color 2 is n.

Fact 3: If color 1 is assigned to any vertex w_i, at most $(n-1)\lceil \frac{r}{2} \rceil + 1$ vertices can receive 1. But, if color 1 is assigned to vertex x and alternative vertices of F_r^i with 1, at most $n\lceil \frac{r}{2} \rceil + 1$ vertices can receive 1. Thus, the maximum number of vertices that can receive color 1 is $n\lceil \frac{r}{2} \rceil + 1$.

There are $nr + n + 1$ vertices in SF_n^r and at most $n\lceil \frac{r}{2} \rceil + 1 + n + n$ vertices receive color 1, 2 and 3. Thus, at least $nr + n + 1 - [n + n + n\lceil \frac{r}{2} \rceil + 1] = n[r - \lceil \frac{r}{2} \rceil - 1]$ vertices should receive distinct colors starting from 4 to $3 + n[r - \lceil \frac{r}{2} \rceil - 1]$. Thus, $\chi_\rho(SF_n^r) \ge 3 + n[r - \lceil \frac{r}{2} \rceil - 1]$.

We give an algorithm to color the uniform n-fan split graph SF_n^r and prove that the bound is sharp.

Procedure PACKING COLORING SF_n^r, $n \ge 4$, $r \ge 5$

Input: A uniform n-fan split graph SF_n^r

Algorithm:

Step 1: Color the vertices w_{2j-1}^i, $1 \le i \le n$, $1 \le j \le \lceil \frac{r}{2} \rceil$ of F_r^i by 1.

Step 2: Color the vertices w_{2j}^i, $1 \le i \le n$, $1 \le j \le 2$ of F_r^i by $(1 + j)$.

Step 3: Color the hub vertex x by 1.

Step 4: Color remaining vertices of SF_n^r with distinct colors starting from 4 to $3 + n[r - \lceil \frac{r}{2} \rceil - 1]$.

Output: A packing $3 + n[r - \lceil \frac{r}{2} \rceil - 1]$-coloring of SF_n^r.

Proof of Correctness: The diameter of F_r^i is 2. Coloring the vertices w_{2j-1}^i, $1 \le i \le n$, $1 \le j \le \lceil \frac{r}{2} \rceil$ of any F_r^i by 1, at most $\lceil \frac{r}{2} \rceil$ vertices receive color 1. There are n fans in SF_n^r and since $d(w_j^i, w_m^l) = 4$, $i \ne l$, $1 \le i, l \le n$, $1 \le j, m \le r$, at most $n\lceil \frac{r}{2} \rceil$ vertices receive color 1. Since $\text{diam}(F_r^i) = 2$, colors greater than 1 cannot be used more than

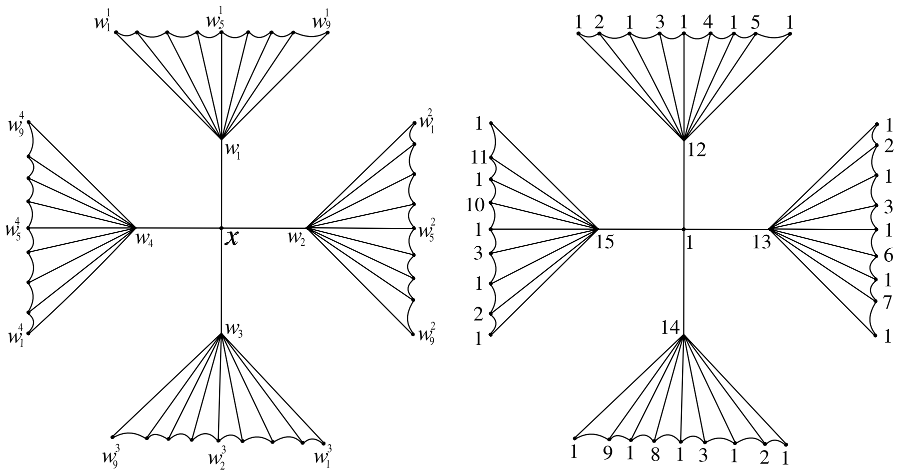

Fig. 1. A packing 15-coloring of SF_4^9.

once in F_r^i, so that remaining vertices of F_r^i receive distinct colors greater than 1. Therefore, at most $n[r-\lceil\frac{r}{2}\rceil]$ vertices receive distinct colors. Since $\operatorname{diam}(SF_n^r)=4$ and $d(w_j^i,w_m^l)=4, i\neq l, 1\leq i,l\leq n, 1\leq j,m\leq r$, coloring vertices $w_{2j}^i, 1\leq i\leq n, 1\leq j\leq 2$ of any F_r^i at most two vertices receive colors 2 and 3. There are n fans in SF_n^r. Therefore $2n$ vertices receive colors 2 and 3. Thus $n[r-\lceil\frac{r}{2}\rceil-2]$ vertices receive distinct colors greater than 3. By the coloring of F_r^i by 1, the vertex x receives color 1. There are n hub vertices w_i in SF_n^r. Thus, $n[r-\lceil\frac{r}{2}\rceil-2]+n=n[r-\lceil\frac{r}{2}\rceil-1]$ vertices receive distinct colors from 4 to $3+n[r-\lceil\frac{r}{2}\rceil-1]$. Hence $\chi_\rho(SF_n^r)=3+n[r-\lceil\frac{r}{2}\rceil-1]$. □

Definition 2.7 ([13]). Let $u_i, 1\leq i\leq n$ be the vertices of the complete graph K_n. Let $W_{r+1}^i=C_r^i+K_1$ be the wheels with hubs $w^i, 1\leq i\leq n$ respectively. Let $u_iw^i, 1\leq i\leq n$ be an edge. The graph constructed is called uniform n-wheel split graph and denoted by $KW(n,r)$.

Remark 2.8. A uniform n-wheel split graph $KW(n,r)$ is a graph in which the deletion of n edges $u_iw^i, 1\leq i\leq n$ partitions the graph into a complete graph and n independent wheels W_{r+1}. This graph can be thought of as a generalization of the standard split graph in the sense that the elements of the independent set are replaced by wheels here. The number of vertices in $KW(n,r)$ is $n(r+2)$ and the number of edges is $n(2r+\frac{n-1}{2}+1)$. The diameter of $KW(n,r)$ is 5. See Fig. 2.

Theorem 2.9. *For the uniform n-wheel split graph, we have* $\chi_\rho(KW(n,r))\geq 4+n[(r+1)-\lfloor\frac{r}{2}\rfloor-2]-1, n\geq 5, r\geq 6$.

Proof. Let $W_{r+1}^i, 1\leq i\leq n$ be the wheels of $KW(n,r)$. Let $V(W_{r+1}^i)=\{w_j^i,w^i : 1\leq i\leq n, 1\leq j\leq r\}$, where w^i is the hubs of W_{r+1}^i. Since the diameter of $KW(n,r)$ is 5, colors greater than 4 can be assigned to only one vertex of $KW(n,r)$.

Fact 1: If color 4 is assigned to any vertex u_i of K_n or hub w^i, no other vertex of $KW(n,r)$ can receive color 4 because $d(u_i,w_j^i)=2$ and $d(u_i,w^i)=1, 1\leq i\leq n, 1\leq j\leq r$. Therefore, we color one vertex w_j^i of any W_{r+1}^i by color 4. Since $\operatorname{diam}(W_{r+1}^i)=2, 1\leq i\leq n, 1\leq j\leq r$, no other vertex of chosen W_{r+1}^i can receive color 4. There are n wheels in $KW(n,r)$ and since $d(w_j^i,w_m^l)=5, i\neq l, 1\leq i,l\leq n, 1\leq j,m\leq r$, at most n vertices can receive color 4. Thus the maximum number of vertices that can receive color 4 is n.

Fact 2: If color 3 is assigned to any vertex u_i of K_n, no other vertex of $KW(n,r)$ can receive 3 because $d(u_i,w_j^i)=2$ and $d(u_i,w^i)=1, 1\leq i\leq n, 1\leq j\leq r$. And also, if color 3 is assigned to any vertex w^i or w_j^i, at most n vertices of $KW(n,r)$ can receive color 3. Thus, the maximum number of vertices that can receive color 3 is n.

Fact 3: If color 2 is assigned to any vertex u_i or w^i or w_j^i, at most n vertices can receive 2 because $d(w_j^i,w_m^l)=5$, $i\neq l, 1\leq i,l\leq n, 1\leq j,m\leq r$ and $d(w^i,w_m^l)=4, i\neq l, 1\leq i,l\leq n, 1\leq m\leq r$ and $d(u_i,w_m^l)=3$, $i\neq l, 1\leq i,l\leq n, 1\leq m\leq r$. Thus, the maximum number of vertices that can receive color 2 is n.

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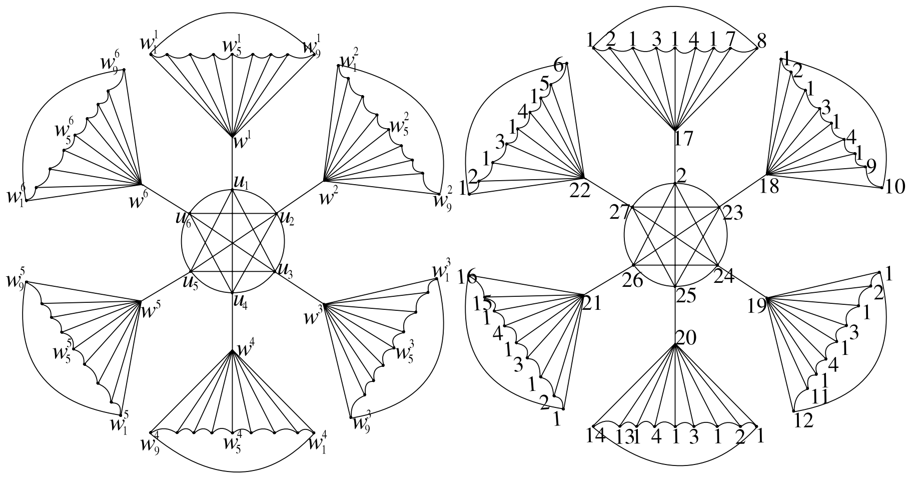

Fig. 2. A packing 27-coloring of $KW(6, 9)$.

Fact 4: If color 1 is assigned to any vertex w^i with 1, at most $(n-1)\lfloor\frac{r}{2}\rfloor + 2$ vertices receive 1. But, if color 1 is assigned to any vertex of u_i of K_n or w^i_j, at most $n\lfloor\frac{r}{2}\rfloor + 1$ vertices of $KW(n,r)$ can receive color 1 because $d(u_i, w^i_j) = 2$ and $\text{diam}(W^i_{r+1}) = 2$. Thus, the maximum number of vertices that can receive color 1 is $n\lfloor\frac{r}{2}\rfloor + 1$. There are $n[(r+2)]$ vertices in $KW(n,r)$ and at most $n\lfloor\frac{r}{2}\rfloor + 1 + n + n + n$ vertices receive color 1, 2, 3 and 4. Thus, at least $n[(r+2)] - [n+n+n+n\lfloor\frac{r}{2}\rfloor + 1] = n[(r+1) - \lfloor\frac{r}{2}\rfloor - 2] - 1$ vertices should receive distinct colors.

Thus, $\chi_\rho(KW(n,r)) \geq 4 + n[(r+1) - \lfloor\frac{r}{2}\rfloor - 2] - 1$.

We give an algorithm to color the uniform n-wheel split graph $KW(n,r)$ and prove that the bound is sharp.

Procedure PACKING COLORING $KW(n,r), n \geq 5, r \geq 6$

Input: A uniform n-wheel split graph $KW(n,r)$

Algorithm:

Step 1: Color the vertices $w^i_{2j-1}, 1 \leq i \leq n, 1 \leq j \leq \lfloor\frac{r}{2}\rfloor$ of W^i_{r+1} by 1.

Step 2: Color the vertices $w^i_{2j}, 1 \leq i \leq n, 1 \leq j \leq 3$ of W^i_{r+1} by $(1+j)$.

Step 3: Color any one vertex of K_n in $KW(n,r)$ by 1.

Step 4: Color remaining vertices of $KW(n,r)$ with distinct colors starting from 5 to $4 + n[(r+1) - \lfloor\frac{r}{2}\rfloor - 2] - 1$.

Output: A packing $4 + n[(r+1) - \lfloor\frac{r}{2}\rfloor - 2] - 1$-coloring of $KW(n,r)$.

Proof of Correctness: The diameter of W^i_{r+1} is 2. Coloring the vertices $w^i_{2j-1}, 1 \leq i \leq n, 1 \leq j \leq \lfloor\frac{r}{2}\rfloor$ of any W^i_{r+1} by 1, at most $\lfloor\frac{r}{2}\rfloor$ vertices receive color 1. There are n wheels in $KW(n,r)$ and since $d(w^i_j, w^l_m) = 5$, $i \neq l, 1 \leq i, l \leq n, 1 \leq j, m \leq r$, at most $n\lfloor\frac{r}{2}\rfloor$ vertices receive color 1. Since $\text{diam}(W^i_{r+1}) = 2$, color greater than 1 cannot be used more than once in W^i_{r+1}, so that remaining $[(r+1) - \lfloor\frac{r}{2}\rfloor]$ vertices of W^i_{r+1} receive distinct colors greater than 1. Therefore, at most $n[(r+1) - \lfloor\frac{r}{2}\rfloor]$ vertices receive distinct colors. Since $\text{diam}(KW(n,r)) = 5$ and $d(w^i_j, w^l_m) = 5, i \neq l, 1 \leq i, l \leq n, 1 \leq j, m \leq r$, coloring vertices $w^i_{2j}, 1 \leq i \leq n, 1 \leq j \leq 3$ of any W^i_{r+1} at most three vertices receive colors 2, 3 and 4. There are n wheels in $KW(n,r)$. Therefore, $3n$ vertices receive colors 2, 3 and 4. Thus, $n[(r+1) - \lfloor\frac{r}{2}\rfloor - 3]$ vertices receive distinct colors greater than 4. By the coloring of W^i_{r+1} by 1, at most one vertex of K_n receives color 1. The remaining $(n-1)$ vertices of K_n receive distinct colors in addition to $n[(r+1) - \lfloor\frac{r}{2}\rfloor - 3]$ vertices. Thus, $n[(r+1) - \lfloor\frac{r}{2}\rfloor - 3] + (n-1) = n[(r+1) - \lfloor\frac{r}{2}\rfloor - 2] - 1$ vertices receive distinct colors from 5 to $4 + n[(r+1) - \lfloor\frac{r}{2}\rfloor - 2] - 1$. Hence $\chi_\rho(KW(n,r)) = 4 + n[(r+1) - \lfloor\frac{r}{2}\rfloor - 2] - 1$. □

The proofs of Theorems 2.11, 2.13 and 2.15 are similar to that of Theorems 2.6 and 2.9.

Definition 2.10 ([13]). Let $u_i, 1 \leq i \leq n$ be the vertices of a star S_{n+1} with hub at x. Let $u_i w_i, 1 \leq i \leq n$ be an edge. Let $W^i_{r+1} = C^i_r + K_1$ be wheels with hubs $w_i, 1 \leq i \leq n$. The graph obtained is denoted by $SW(n,r)$.

The number of vertices in $SW(n,r)$ is $n(r+2)+1$ and the number of edge is $2n(r+1)$. The diameter of $SW(n,r)$ is 6. See Fig. 3.

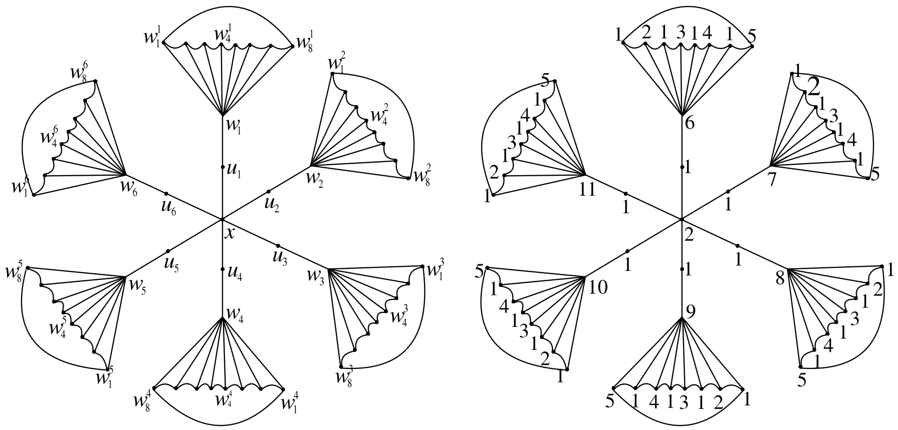

Fig. 3. A packing 11-coloring of $SW(6, 8)$.

Theorem 2.11. *The packing chromatic number of* $SW(n, r)$ *is given by* $\chi_\rho(SW(n, r)) = 5+n[(r+1)-\lfloor\frac{r}{2}\rfloor-4], n \geq 3, r \geq 8.$

The algorithm to color $SW(n, r)$ is given below:

Procedure PACKING COLORING $SW(n, r), n \geq 3, r \geq 8$

Input: A graph $SW(n, r)$

Algorithm

Step 1: Color the vertices $w^i_{2j-1}, 1 \leq i \leq n, 1 \leq j \leq \lfloor\frac{r}{2}\rfloor$ of W^i_{r+1} by 1.
Step 2: Color the vertices $w^i_{2j}, 1 \leq i \leq n, 1 \leq j \leq 4$ of W^i_{r+1} by $(1 + j)$.
Step 3: Color the hub vertex x by 2.
Step 4: Color the vertices $u_i, 1 \leq i \leq n$ by 1.
Step 5: Color remaining vertices of $SW(n, r)$ with distinct colors starting from 6 to $5 + n[(r + 1) - \lfloor\frac{r}{2}\rfloor - 4]$.
Output: A packing $5 + n[(r + 1) - \lfloor\frac{r}{2}\rfloor - 4]$-coloring of $SW(n, r)$.

Definition 2.12 (*[13]*)**.** Let $x_i, 1 \leq i \leq n$ be the vertices of the complete graph K_n. Let $W^i_{r+1} = C^i_r + K_1$ be wheels with hubs $w_i, 1 \leq i \leq n$. Let $x_i w_i, 1 \leq i \leq n$ be an edge. Subdivide each edge $x_i w_i$ by $u_i, 1 \leq i \leq n$. The graph obtained is denoted by $KDW(n, r)$.

The number of vertices in $KDW(n, r)$ is $n(r + 3)$ and the number of edge is $n(2r + 1) + n(\frac{n+1}{2})$. The diameter of $KDW(n, r)$ is 7. See Fig. 4.

Theorem 2.13. *The packing chromatic number of* $KDW(n, r)$ *is given by* $\chi_\rho(KDW(n, r)) = 6+n[(r+1)-\lfloor\frac{r}{2}\rfloor-4]-1, n \geq 4, r \geq 10.$

The algorithm to color $KDW(n, r)$ is given below:

Procedure PACKING COLORING $KDW(n, r), n \geq 4, r \geq 10$

Input: A graph $KDW(n, r)$

Algorithm

Step 1: Color the vertices $w^i_{2j-1}, 1 \leq i \leq n, 1 \leq j \leq \lfloor\frac{r}{2}\rfloor$ of W^i_{r+1} by 1.
Step 2: Color the vertices $w^i_{2j}, 1 \leq i \leq n, 1 \leq j \leq 5$ of W^i_{r+1} by $(1 + j)$.
Step 3: Color any one vertex of K_n in $KDW(n, r)$ by 2.
Step 4: Color the vertices $u_i, 1 \leq i \leq n$ by 1.
Step 5: Color remaining vertices of $KDW(n, r)$ with distinct colors starting from 7 to $6 + n[(r + 1) - \lfloor\frac{r}{2}\rfloor - 4] - 1$.
Output: A packing $6 + n[(r + 1) - \lfloor\frac{r}{2}\rfloor - 4] - 1$-coloring of $KDW(n, r)$.

Definition 2.14 (*[13]*)**.** The graph SW^r_n contains a star S_{n+1} with hub at x such that the deletion of the n edges of S_{n+1} partitions the graph into n independent wheels $W^i_{r+1} = C^i_r + K_1, 1 \leq i \leq n$ and an isolated vertex. See Fig. 5.

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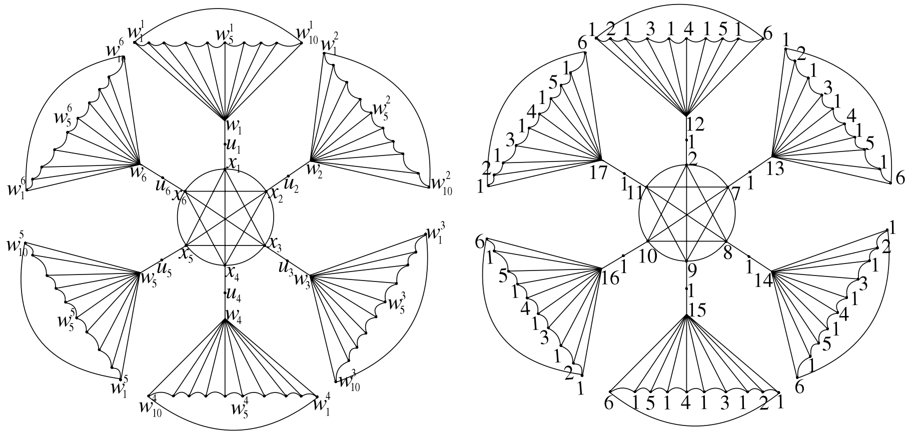

Fig. 4. A packing 17-coloring of $KDW(6, 10)$.

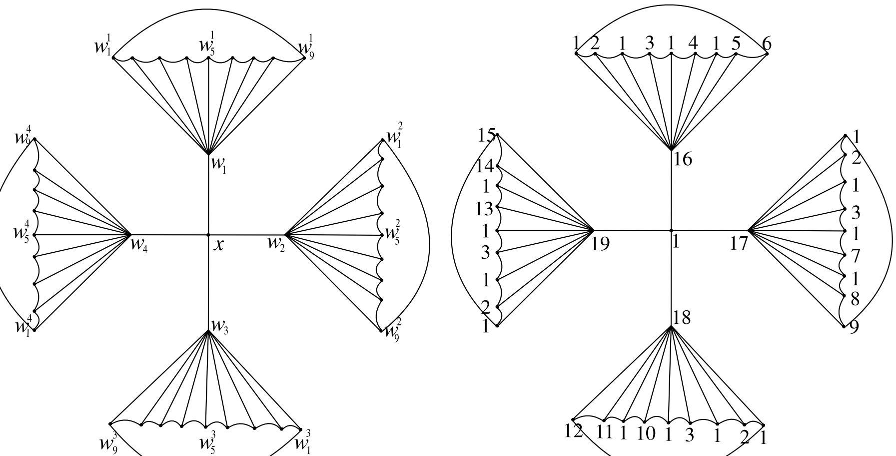

Fig. 5. A packing 19-coloring of SW_4^9.

Theorem 2.15. *The packing chromatic number of* $SW_n^r, n \geq 4, r \geq 5$ *is given by* $\chi_\rho(SW_n^r) = 3 + n[r - \lfloor \frac{r}{2} \rfloor - 1]$.

The algorithm to color SW_n^r is given below:

Procedure PACKING COLORING $SW_n^r, n \geq 4, r \geq 5$

Input: A graph SW_n^r

Algorithm

Step 1: Color the vertices $w_{2j-1}^i, 1 \leq i \leq n, 1 \leq j \leq \lfloor \frac{r}{2} \rfloor$ of W_{r+1}^i by color 1.
Step 2: Color the vertices $w_{2j}^i, 1 \leq i \leq n, 1 \leq j \leq 2$ of W_{r+1}^i by color $(1 + j)$.
Step 3: Color the hub vertex x by 1.
Step 4: Color remaining vertices of SW_n^r with distinct colors starting from 4 to $3 + n[r - \lfloor \frac{r}{2} \rfloor - 1]$.
Output: A packing $3 + n[r - \lfloor \frac{r}{2} \rfloor - 1]$-coloring of SW_n^r.

Acknowledgment

This work is supported by Maulana Azad National Fellowship F1-17.1/2011/MANF-CHR-TAM-2135 of the University Grants Commission, New Delhi, India.

References

[1] W. Goddard, S.M. Hedetniemi, S.T. Hedetniemi, J.M. Harris, D.F. Rall, Broadcast chromatic numbers of graphs, Ars Combin. 86 (2008) 33–49.
[2] B. Brešar, S. Klavžar, D.F. Rall, On the packing chromatic number of cartesian products, hexagonal lattice and trees, Discrete Appl. Math. 155 (2007) 2303–2311.
[3] J. Fiala, P.A. Golovach, Complexity of the packing coloring problem for trees, Discrete Appl. Math. 158 (2010) 771–778.
[4] C. Sloper, An eccentric coloring of trees, Australas. J. Combin. 29 (2004) 309–321.
[5] J. Fiala, S. Klavžar, B. Lidický, The packing chromatic number of infinite product graphs, European J. Combin. 30 (5) (2009) 1101–1113.
[6] P. Holub, R. Soukal, A note on packing chromatic number of the square lattice, Electron. J. Combin. 17 (2010) 1–7. Note 17.
[7] J. Ekstein, P. Holub, B. Lidický, Packing chromatic number of distance graphs, Discrete Appl. Math. 160 (2012) 518–524.
[8] O. Togni, On packing colorings of distance graphs, Discrete Appl. Math. 167 (2014) 280–289.
[9] G. Argiroffo, G. Nasini, P. Torres, The packing coloring problem for lobsters and partner limited graphs, Discrete Appl. Math. 164 (2014) 373–382.
[10] G. Argiroffo, G. Nasini, P. Torres, The packing coloring problem for $(q, q-4)$ graphs, Comb. Optim., Lect. Notes Comput. Sci. 7422 (2012) 309–319.
[11] A. Finbow, D.F. Rall, On the packing chromatic number of some lattices, Discrete Appl. Math. 158 (2010) 1224–1228.
[12] D. Rall, B. Brešar, A. Finbow, S. Klavžar, On the packing chromatic number of trees, cartesian products and some infinite graphs, Electron. Notes Discrete Math. 30 (2008) 57–61.
[13] Y. Kins, Radio labeling of certain graphs (Ph.D. thesis), University of Madras, India, November 2011.

Please cite this article in press as: S. Roy, Packing chromatic number of certain fan and wheel related graphs, AKCE International Journal of Graphs and Combinatorics (2016), http://dx.doi.org/10.1016/j.akcej.2016.11.001