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Parallel decoding for lattice reduction-aided MIMO Receiver

Madurakavi KARTHIKEYAN*, **Djagadeesan SARASWADY**
Department of Electronics and Communication, Pondicherry Engineering College, Puducherry, India

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Abstract: In this paper, we propose a modified lattice reduction (LR) for parallel decoding (PD) in multiple input and multiple output (MIMO) systems. Compared with the conventional methods, without any performance loss, the symbols can be estimated in parallel using this LR-PD algorithm. This parallel decoding is achieved through the proposed postrotation matrix. Among the various LR algorithms, LLL has been considered in the analysis of this postrotation matrix. However, this can be applied to any other LR algorithms without any modifications. In simulations, we show that with a minimal number of extra arithmetic operations and without any significant loss in orthogonal defect and bit error rate performance, the parallel decoding becomes possible in a real as well as complex-valued MIMO system model.

Key words: MIMO decoding, LR-PD, LLL, lattice reduction, parallel decoding

1. Introduction

High spectral efficiency in wireless communication can be achieved with the help of MIMO systems [1]. The complexity reduction of MIMO receive algorithms are being continuously focused on among researchers in implementing MIMO in real-time systems. Particularly, a system employing tens to hundreds of antennas, termed as large MIMO, requires low complexity receive algorithms without sacrificing the near-maximum likelihood (ML) performance [2]. The existing ML sphere decoder (SD) provides the optimum performance. However, its complexity is exponential in the dimension of the number of antennas [3]. Many algorithms were developed as a variant of the original SD, such as inter searching radius control (ISRC) [4], probability tree pruning (PTP-SD) [5], layer pruning (LP-SD) [6], etc. However, due to the exponential complexity of the SD, the maximum dimension is restricted to 32 [3].

For MIMO systems with large antenna arrays, linear receivers such as zero forcing (ZF) and minimum mean square error (MMSE) could be exploited due to the advantage of their low computational complexity. However, the poor BER is its major drawback and near-ML performance cannot be achieved. The poor performance of linear receivers is due to the fact that the channel matrices are poorly conditioned. Lattice reduction (LR) is a powerful preprocessing technique for achieving a near-ML performance in MIMO systems. This preprocessing can be used with linear receivers and successive interference cancellation (SIC) [7] methods. The LR-aided MIMO receiver first finds the set of small, nearly orthogonal matrices for the given channel matrix and decodes the symbols using this matrix rather than the original channel matrix. Different LR algorithms such as LLL [8] or that of Seysen [9] can be used to produce a near orthogonal matrix for the given lattice.

Recently developed algorithms such as element-based LR (ELR) [10] and improved LR (ILR) [11] improve

*Correspondence: karthikeyan2709@pec.edu

the efficiency of LR-aided MIMO decoders through complexity reduction compared to conventional techniques. In this paper, we increase the decoding efficiency using modified lattice reduction, which is suitable for parallel decoding of the transmitted symbols. The parallel decoding structure is obtained using combined LR and a postrotation matrix. This structure provides a way to conduct parallel decoding without any performance loss and with a minimal number of extra arithmetic operations. This paper is organized as follows. Section 2 describes the MIMO system model. The conventional and proposed LR with rotation matrix is explained in Section 3. Section 4 demonstrates the simulation results. Section 5 concludes this paper.

2. System model

Consider the $N_t \times N_r$ MIMO system with N_t transmit and N_r receive antennas. For this system model, the complex-valued received signal vector (y_c) can be written as

$$y_c = H_c x_c + n_c, \tag{1}$$

where H_c is the complex channel matrix with $N_r \times N_t$ entries and n_c is the additive white Gaussian noise vector (AWGN). x_c is the transmitted complex symbol vector whose real and imaginary values are drawn from the QAM constellation (Ω) . The real-valued system model for MIMO is obtained as

$$y = \begin{bmatrix} \Re(y_c) \\ \Im(y_c) \end{bmatrix}, x = \begin{bmatrix} \Re(x_c) \\ \Im(x_c) \end{bmatrix},$$

$$n = \begin{bmatrix} \Re(n_c) \\ \Im(n_c) \end{bmatrix},$$

$$H = \begin{bmatrix} \Re(H_c) & -\Im(H_c) \\ \Im(H_c) & \Re(H_c) \end{bmatrix}. \tag{2}$$

We can observe from Eq. (2) that the dimension of the system doubles that of the complex-valued model with $n_t = 2N_t$, $n_r = 2N_r$. The ML decoding for this system model, by assuming that the receiver has perfect knowledge of the channel matrix, can be formulated as

$$x = \arg\min_{x \in \Omega^{n_t}} \|y - Hx\|^2. \tag{3}$$

In Eq. (3), an exhaustive search is performed over an entire constellation set to find the estimate of the transmitting vector. This leads to nondeterministic polynomial (NP) hard time complexity. Hence, we are interested in approximate methods of MIMO decoding with an acceptable level of computational complexity. We use MATLAB notations to represent the equations throughout this paper. For example, $R_{k,k:m} x_{k:m} = \{R(k,k) \times x\,(k)\} + \{R(k,k+1) \times x\,(k+1)\} +, ..., \{R(k,m) \times x(m)\}$

3. Lattice reduction for parallel reduction

The prime reason for the poor performance loss in linear receivers is the high orthogonality defect in the obtained channel matrix. Hence, performance improvement can be achieved if MIMO decoding is performed on the nearly orthogonal matrix [7]. In this paper, to explain the proposed parallel decoding structure, the LLL algorithm [8] has been considered. Hence, LLL-PD and LR-PD are interchangeably used in this paper. However, it is straightforward to apply the proposed rotation matrix with any other lattice reduction techniques.

3.1. Conventional LR-aided SIC

In lattice reduction-based MIMO decoding, the columns of the channel matrix $H = [h_1, h_2, ..., h_{n_t}]$ can be considered as a basis vector of an $n_r \times n_t$ lattice. Then H is transformed into a better conditioned matrix using $G = HT$, where T is unimodular, such that all the entries are integers. Algorithm 1 is the LLL algorithm proposed in [8], which has been widely adopted to obtain the unimodular matrix. Therefore, the system model after LR is given as

$$y = HTT^{-1}x + n = Gs + n, \tag{4}$$

where $s = T^{-1}x$ becomes the effective symbol vector. In the SIC method, QR decomposition may be performed on effective channel matrix G and the received symbol vector can be given as

$$y = QRs + n, \tag{5}$$

$$\tilde{y} = Q^H y = Rs + Q^H n. \tag{6}$$

Eq. (6) can be written in matrix form as

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_{n_r} \end{bmatrix} = \begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,n_t} \\ 0 & R_{2,2} & \cdots & R_{2,n_t} \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & R_{n_r,n_t} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_{n_t} \end{bmatrix}. \tag{7}$$

This matrix form finds application in most of the MIMO decoding techniques, such as SIC, SD [3], the tabu search method [2], etc. Let us consider how LR-aided SIC decoding proceeds using Eq. (7). In SIC, the upper triangular structure of R can be used to find the symbol corresponding to s_{n_t} first and then symbol decoding takes place in the order of $s_{n_t-1}, s_{n_t-2}, ..., s_1$. This is expressed as

$$s_{n_t} = (\tilde{y}_{n_t}) / R_{n_t,n_t}, \tag{8}$$

$$s_{n_t-1} = (\tilde{y}_{n_t-1} - R_{n_t-1,n_t}, s_{n_t}) / R_{n_t-1,n_t-1}, \tag{9}$$

$$\vdots$$

$$s_1 = (\tilde{y}_1 - R_{1,2:n_t} s_{2:n_t}) / R_{1,1}. \tag{10}$$

After obtaining all the effective symbols $s_1, s_2, ..., s_{n_t}$, the original transmitted vector x can be found by applying a proper scaling and shifting on s as given in [10] and multiplying with T (i.e. $\hat{x} = Ts$).

3.2. Proposed postrotation matrix

The major constraint imposed for successive decoding by conventional LR is that the estimate of symbol vectors has to be done serially from one level to another. In Eq. (9), s_{n_t-1} is found with the knowledge of s_{n_t} obtained from Eq. (8). The main contribution of this paper is that a modified lattice reduction is proposed to provide a structure suitable for parallel decoding of successive symbols. In [12], the reordered representation of the ordinary channel matrix was proposed for SD, which paves the way for parallel decoding. In this paper, we propose the reordered representation for LR-aided MIMO decoding using a postrotation matrix. The proposed 2×2 rotation matrix (Θ) is given as

Algorithm 1. LLL lattice reduction.

Input: H

Output: Q, R, T

1. $[Q, R] = qr(H)$.
2. $\Delta = \{0.25, 1\}$.
3. m=size(H,2), T=I$_m$, k=2.
4. while k<=m
5.   for n=k-1:-1:1
6.   u=round($R_{n,k}/R_{n,n}$).
7.   if u~=0
8. $R_{1:n,k} = R_{1:n,k} - uR_{1:n,n}$.
9. $T_{:,k} = T_{:,k} - uT_{:,n}$.
10.   end
11.   end
12. if $\Delta \left|R_{k-1,k-1}\right|^2 > \left|R_{k,k}\right|^2 + \left|R_{k-1,k}\right|^2$
13.   swap the *k-1* and k columns of R and T .
14.   Given's rotation matrix $\Phi = \begin{vmatrix} \alpha 1 & \beta \\ -\beta & \alpha \end{vmatrix}$.
15.   Where $\alpha = \frac{R_{k-1,k-1}}{\|R_{k-1:k,k-1}\|^2}$, $\beta = \frac{R_{k,k-1}}{\|R_{k-1:k,k-1}\|^2}$, $\alpha 1 = \alpha^*$.
16. $R_{k-1:k,k-1:m} = \Phi R_{k-1:k,k-1:m}$.
17. $Q_{:,k-1:k} = Q_{:,k-1:k}\Phi^T$.
18. else
19. $k = k + 1$.
20. end if
21. end while
22. Output Q, R, T.

$$\Theta = \begin{pmatrix} 1 & \frac{-R_{k-1,k}}{R_{k,k}} \\ 0 & 1 \end{pmatrix}, k = 2, 4, ..., m. \tag{11}$$

This 2×2 submatrix after multiplying with the R and Q matrix produces the required structure for parallel decoding. This is illustrated using the example given below. Consider the 2×2 Rayleigh flat fading channel for simplicity:

$$H_c = \begin{bmatrix} 0.1958 + 0.4913j & 0.0687 + 0.6719j \\ 0.0326 + 0.2242j & 0.5823 + 0.0244j \end{bmatrix}. \tag{12}$$

Converting this to a real-valued matrix and performing LR-PD yields

$$H = \begin{bmatrix} 0.1958 & 0.0687 & -0.4913 & -0.6719 \\ 0.0326 & 0.5823 & -0.2242 & -0.0244 \\ 0.4913 & 0.6719 & 0.1958 & 0.0687 \\ 0.2242 & 0.0244 & 0.0326 & 0.5823 \end{bmatrix},$$

$$Q = \begin{bmatrix} -0.3651 & 0.2396 & 0.8547 & -0.2335 \\ 0.0340 & -0.8780 & 0.3495 & 0.3580 \\ -0.8330 & -0.2022 & -0.3926 & -0.3171 \\ -0.4270 & 0.3617 & 0.0629 & 0.8466 \end{bmatrix},$$

$$R = \begin{bmatrix} -0.5754 & 0 & -0.0053 & -0.0561 \\ 0 & -0.6218 & 0.0514 & 0.0057 \\ 0 & 0 & -0.5731 & 0 \\ 0 & 0 & 0 & 0.6194 \end{bmatrix},$$

$$T = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Observe in the R matrix obtained with the proposed method that the values of R(1,2) and R(3,4) are zero. The difference between the conventional and proposed decoding sequences of symbols for this example is shown in Figure 1. Hence, at each time instant, two symbols can be decoded independently. The detailed pseudocode of this proposed method is given Algorithms 2 and 3.

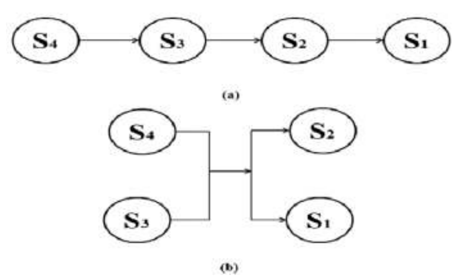

Figure 1. a) Conventional decoding, b) proposed decoding sequence of 2 × 2 MIMO system.

Some important remarks about the proposed rotation matrix are:

- Θ is designed arbitrarily in such a way that after the multiplication of Θ with $2 \times n_t$ the submatrix produces $R_{i,j} = 0$ for $j = i+1, i = 1,3,5,...,n_t - 1$.
- Multiplying Θ with the submatrix mentioned above will not affect the other rows; only the elements corresponding to the submatrix are changed.
- This rotation matrix can be directly applied to a complex system model without any modification.

Algorithm 2. Lattice reduction-aided parallel ZF-SIC decoding.

Input: H, y

Output: $\hat{x}$

1. [Q,R,T]=Algorithm 3 (H).
2. $\tilde{y} = Q^H y$.
3. $s_{n_t} = \frac{\tilde{y}_{n_t}}{R_{n_t,n_t}}$ and $s_{n_t-1} = \frac{\tilde{y}_{n_t-1}}{R_{n_t-1,n_t-1}}$.
4. $s_{n_t-2} = \frac{\tilde{y}_{n_t-2} - R_{n_t-2,n_t-1} s_{n_t-1} - R_{n_t-2,n_t} s_{n_t}}{R_{n_t-2,n_t-2}}$ and $s_{n_t-3} = \frac{\tilde{y}_{n_t-3} - R_{n_t-3,n_t-1} s_{n_t-1} - R_{n_t-2,n_t} s_{n_t}}{R_{n_t-3,n_t-3}}$.
5. s_{n_t-4} and $s_{n_t-5}, \ldots,$ s_2 and s_1 are decoded in parallel.
6. Output $\hat{x} = Ts$.

Algorithm 3. Lattice reduction for parallel decoding (LR-PD).

Input: H

Output: Q, R, T

1. [Q,R,T]=Algorithm 1 (H).
2. for k=2:2:n_t

   $\Theta = \begin{pmatrix} 1 & -R_{k-1,k}/R_{k,k} \\ 0 & 1 \end{pmatrix}$

   $R_{k-1:k,k-1:m} = \Theta R_{k-1:k,k-1:m}, Q_{:,k-1:k} = Q_{:,k-1:k} \Theta^T$

   end
3. Output Q, R, T.

4. Simulation results

In this section, we examine the BER performance, complexity, and variation in the orthogonality defect of the reduced matrix. The simulations were performed with QAM transmission over the Rayleigh flat fading channel in different MIMO configurations. For each case, 20,000 channels were realized to average the error rate performance.

4.1. BER performance

The BER performance of the proposed LR-PD in comparison with other traditional methods for a 4×4 MIMO system with 4-, 16-, and 64-QAM is given in Figure 2. LR-PD along with the MMSE receiver is also simulated. It can be noticed that for all the modulation levels, the proposed method achieves the same BER performance without any performance loss. In Figure 3, we have simulated an 8×8 MIMO system using complex LLL [13], i.e. without converting to real. It is clear from Figure 3 that in the complex domain the BER performance resembles that of the conventional methods.

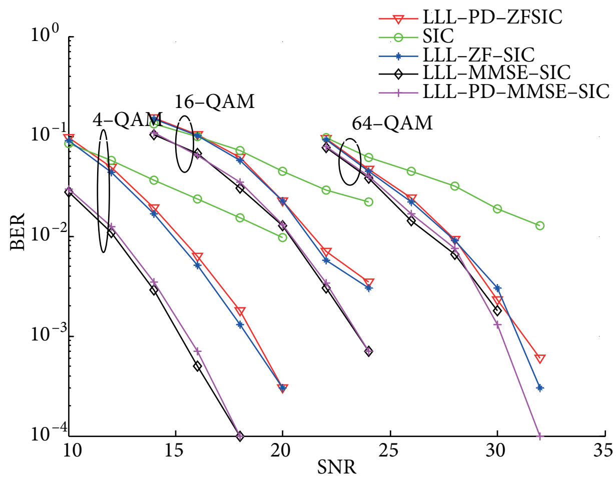

Figure 2. BER comparison of LR-PD and conventional methods for 4×4 with different QAM modulation levels. MIMO detection is performed on the real-valued system model.

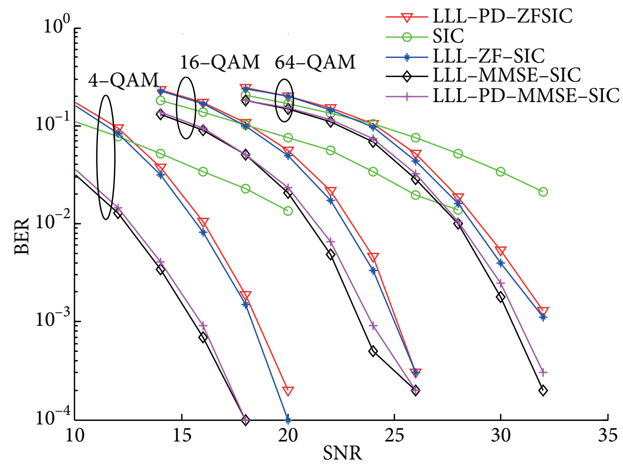

Figure 3. BER comparison of LR-PD and conventional methods for 8×8 with different QAM modulation levels. MIMO detection is performed on a complex-valued system model.

4.2. Orthogonality defect

The orthogonality defect is one of the prime factors in determining the quality of lattice reduction techniques [7]. The perfect orthogonal matrix will lead the linear receiver towards a near-ML performance. Hence, the lesser the orthogonality defect, the higher the BER performance. However, due to the postrotation matrix, the orthogonality defect of the resultant effective channel matrix gets increased as shown below in Tables 1 and 2. Note that although the LLL-PD algorithm produces an effective channel matrix with a slightly higher orthogonal defect, the BER performance is the same as with original LLL-aided detectors. This is due to the fact that, for lattice decoding, it is sufficient to have a weaker version of lattice reduction for MIMO detection [14]. The orthogonal deficiency of the reduced matrix G is defined in [13] as

Table 1. Orthogonality defect for real-valued channel matrix.

Dimension	Original	LLL	LR-PD
4	0.9931	0.2695	0.2809
6	0.9999	0.6014	0.6317
8	1.0000	0.8475	0.8729

Table 2. Orthogonality defect for complex-valued channel matrix.

Dimension	Original	LLL	LR-PD
2	0.4961	0.1153	0.1153
4	0.9053	0.5848	0.6305
8	0.9976	0.9866	0.9932

$$OD(G) = 1 - \frac{\det(G^H G)}{\Pi_{t=1}^{n_t} \|g_t\|^2}, \tag{13}$$

where g_t is the t_{th} column of G. As mentioned above, the lower value of OD results in a nearly orthogonal matrix, i.e. if OD$(G) = 0$, then G is perfectly orthogonal, or if OD$(G) = 1$, then it is singular. We experimented on 1000 real and complex-valued random matrices. The average orthogonal defects for various dimensions are tabulated in Tables 1 and 2. Additionally, from the tables, it can be observed that OD (G) is approaching towards OD (H) as the dimension increases. Due to this, a large performance gap between the ML performance and LR-aided linear receivers exists in large MIMO systems. However, the other algorithms developed recently, such as ELR [10] and ILR [11], provide near-ML performance at some extra complexity for large MIMO systems. One can directly apply our proposed postrotation matrix to any such methods and make use of the resulting parallel structure.

4.3. Complexity analysis

The LR-PD works by multiplying the rotation matrix with R and Q after the conventional lattice reduction is done. Hence, these processes involve some minimal amount of extra multiplications and additions. The extra complexity incurred by the LR-PD is calculated as follows. At one rotation, the proposed 2×2 rotation matrix is multiplied with the $2 \times n_t$ submatrix of R and Q. In general, for this matrix multiplication $8n_t$ real multiplications and $4n_t$ real additions are required. This step will be repeated for every $2 \times n_t$ matrices. Therefore, this rotation has to be performed in total $n_t/2$ times on each R and Q. Then the number of manipulations (C) needed per matrix, i.e. R or Q, is

$$C = \frac{n_t}{2}\left[\underbrace{8n_t}_{Multiplications} + \underbrace{4n_t}_{Additions}\right]. \tag{14}$$

In total, to achieve parallelism, K number of arithmetic operations (K includes rotation needed by both Q and R) is needed, more than that of conventional LR techniques.

$$K = 2\frac{n_t}{2}\left[\; 8n_t \quad + \quad 4n_t \;\right] = n_t\left[\; 8n_t \quad + \quad 4n_t \;\right] \tag{15}$$

Additionally, the parallel processing after LR given in Algorithm 2 eliminates one multiplication and one subtraction per two n_t symbols. For n_t symbols, K_1 arithmetic operations are reduced in LLL-PD. Finally, the total expense in terms of arithmetic operations (A) for achieving parallelism is given by

$$\begin{aligned} A &= K - K_1 \\ &= n_t\left[8n_t + 4n_t\right] - n_t\left[1+1\right] \\ &= n_t\left[\underbrace{(8n_t - 1)}_{Multiplications} + \underbrace{4n_t}_{Additions} - \underbrace{1}_{Subtractions}\right]. \end{aligned} \tag{16}$$

In Figure 4, we plot the time taken for decoding 2000 channel realizations by various methods, including the proposed and conventional methods. This LLL-PD time is measured by omitting the time taken by parallel symbols in order to observe how much time can be saved if decoding is done in parallel.

We can observe from the graph that approximately 82% of the time taken by conventional methods is reduced in the proposed parallel decoding. In Figure 5, we have plotted the time taken to estimate one transmitted vector for an 8×8 MIMO system with 16-QAM and observed approximately 82% reduction in time consumption. This time graph is simulated in MATLAB 7.14 on an Intel Core i5 3.10 GHz with 2 GB RAM PC running Windows 7.

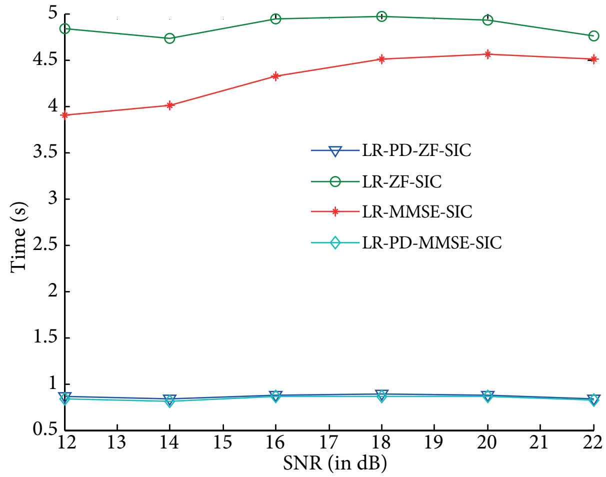

Figure 4. Time taken for 2000 transmitted vectors by LLL-PD and conventional methods for 4 × 4 using 16-QAM modulation levels. LR-PD is plotted by considering only one symbol decoding time in parallel decoding.

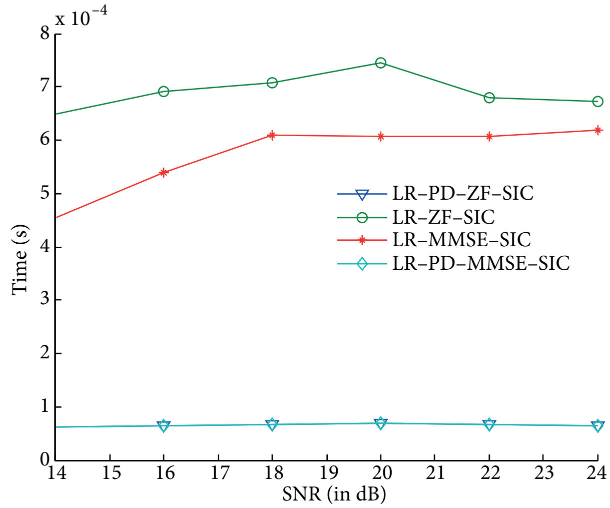

Figure 5. Time taken for one transmitted vector by LLL-PD and conventional methods for 8 × 8 using 16-QAM modulation levels. LR-PD is plotted by considering only one symbol decoding time in parallel decoding.

5. Conclusion

We proposed a parallel decoding algorithm for MIMO systems. This LLL-PD can achieve an 82% reduction in time when compared to original methods if a processor with the parallel processing capability is used with the proposed LR-PD. With the advancement in parallel processing architecture in real-time processors, this parallel decoding structure of MIMO finds its usefulness in the implementation point of view. Moreover, there is a negligible performance loss, as shown in the simulation results. This method is analyzed with the basic receivers in this paper. Hence, the directions for future work are the application of LR-PD to SD and large MIMO receivers. The decoding of large MIMO receivers becomes simple if parallel decoding is possible.

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