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Research Article

Partial Sums of Generalized Class of Analytic Functions Involving Hurwitz-Lerch Zeta Function

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Let $f_n(z) = z + \sum_{k=2}^{n} a_k z^k$ be the sequence of partial sums of the analytic function $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$. In this paper, we determine sharp lower bounds for $\Re\{f(z)/f_n(z)\}, \Re\{f_n(z)/f(z)\}$, $\Re\{f'(z)/f'_n(z)\}$, and $\Re\{f'_n(z)/f'(z)\}$. The usefulness of the main result not only provides the unification of the results discussed in the literature but also generates certain new results.

1. Introduction and Preliminaries

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$

which are analytic and univalent in the open disc $U = \{z : |z| < 1\}$. We also consider T a subclass of $\mathcal{A}$ introduced and studied by Silverman [1], consisting of functions of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k, \quad z \in U. \tag{1.2}$$

For functions $f \in \mathcal{A}$ given by (1.1) and $g \in \mathcal{A}$ given by $g(z) = z + \sum_{k=2}^{\infty} b_k z^k$, we define the Hadamard product (or convolution) of f and g by

$$(f * g)(z) = z + \sum_{k=2}^{\infty} a_k b_k z^k, \quad z \in U. \tag{1.3}$$

We recall here a general Hurwitz-Lerch zeta function $\Phi(z, s, a)$ defined in [2] by

$$\Phi(z, s, a) := \sum_{k=0}^{\infty} \frac{z^k}{(k + a)^s} \tag{1.4}$$

($a \in \mathbb{C} \setminus \mathbb{Z}_0^-$; $s \in \mathbb{C}$, when $|z| < 1$; $\Re(s) > 1$, when $|z| = 1$), where, as usual, $\mathbb{Z}_0^- := \mathbb{Z} \setminus \mathbb{N}$, ($\mathbb{Z} := \{0, \pm 1, \pm 2, \pm 3, \ldots\}$); $\mathbb{N} := \{1, 2, 3, \ldots\}$. Several interesting properties and characteristics of the Hurwitz-Lerch Zeta function $\Phi(z, s, a)$ can be found in the recent investigations by Choi and Srivastava [3], Ferreira and López [4], Garg et al. [5], Lin and Srivastava [6], Lin et al. [7], and others. Srivastava and Attiya [8] (see also Răducanu and Srivastava [9] and Prajapat and Goyal [10]) introduced and investigated the linear operator $\mathcal{J}_{\mu,b} : \mathcal{A} \to \mathcal{A}$ defined in terms of the Hadamard product by

$$\mathcal{J}_{\mu,b} f(z) = \mathcal{G}_{b,\mu} * f(z) \quad \left(z \in U;\ b \in \mathbb{C} \setminus \mathbb{Z}_0^-;\ \mu \in \mathbb{C};\ f \in \mathcal{A}\right), \tag{1.5}$$

where, for convenience,

$$G_{\mu,b}(z) := (1 + b)^{\mu} \left[\Phi(z, \mu, b) - b^{-\mu}\right] \quad (z \in U). \tag{1.6}$$

We recall here the following relationships (given earlier by [9, 10]) which follow easily by using (1.1), (1.5), and (1.6):

$$\mathcal{J}_b^{\mu} f(z) = z + \sum_{k=2}^{\infty} \left(\frac{1 + b}{k + b}\right)^{\mu} a_k z^k. \tag{1.7}$$

Motivated essentially by the Srivastava-Attiya operator [8], we introduce the generalized integral operator

$$\mathcal{J}_{\mu,b}^{m,\eta} f(z) = z + \sum_{k=2}^{\infty} C_k^m(b, \mu) a_k z^k = F(z), \tag{1.8}$$

where

$$C_k^m(b, \mu) = \left| \left(\frac{1 + b}{k + b}\right)^{\mu} \frac{m!(k + \eta - 2)!}{(\eta - 2)!(k + m - 1)!} \right|, \tag{1.9}$$

and (throughout this paper unless otherwise mentioned) the parameters μ, b are constrained as $b \in \mathbb{C} \setminus \mathbb{Z}_0^-$; $\mu \in \mathbb{C}$, $\eta \geq 2$ and $m > -1$. It is of interest to note that $J_{\mu,b}^{1,2}$ is the

Srivastava-Attiya operator [8] and $J_{0,b}^{m,\eta}$ is the well-known Choi-Saigo-Srivastava operator (see [11, 12]). Suitably specializing the parameters m, η, μ, and b in $\mathcal{J}_{\mu,b}^{m,\eta}f(z)$, we can get various integral operators introduced by Alexander [13] and Bernardi [14]. Further more, we get the Jung-Kim-Srivastava integral operator [15] closely related to some multiplier transformation studied by Flett [16].

Motivated by Murugusundaramoorthy [17–19] and making use of the generalized Srivastava-Attiya operator $\mathcal{J}_{\mu,b}^{m,\eta}$, we define the following new subclass of analytic functions with negative coefficients.

For $\lambda \geq 0$, $-1 \leq \gamma < 1$, and $\beta \geq 0$, let $P_{\mu}^{\lambda}(\gamma,\beta)$ be the subclass of $\mathcal{A}$ consisting of functions of the form (1.1) and satisfying the analytic criterion

$$\Re\left\{\frac{z\left(\mathcal{J}_{\mu,b}^{m,\eta}f(z)\right)' + \lambda z^2\left(\mathcal{J}_{\mu,b}^{m,\eta}f(z)\right)''}{(1-\lambda)\mathcal{J}_{\mu,b}^{m,\eta}f(z) + \lambda z\left(\mathcal{J}_{\mu,b}^{m,\eta}f(z)\right)'} - \gamma\right\} > \beta\left|\frac{z\left(\mathcal{J}_{\mu,b}^{m,\eta}f(z)\right)' + \lambda z^2\left(\mathcal{J}_{\mu,b}^{m,\eta}f(z)\right)''}{(1-\lambda)\mathcal{J}_{\mu,b}^{m,\eta}f(z) + \lambda z\left(\mathcal{J}_{\mu,b}^{m,\eta}f(z)\right)'} - 1\right|, \tag{1.10}$$

where $z \in U$. Shortly we can state this condition by

$$-\Re\left\{\frac{zG'(z)}{G(z)} - \gamma\right\} > \beta\left|\frac{zG'(z)}{G(z)} - 1\right|, \tag{1.11}$$

where

$$G(z) = (1-\lambda)F(z) + \lambda zF'(z) = z + \sum_{k=2}^{\infty}[1+\lambda(k-1)]\left|C_k^m(b,\mu)\right|a_k z^k, \tag{1.12}$$

and $F(z) = \mathcal{J}_{\mu,b}^{m,\eta}f(z)$.

Recently, Silverman [20] determined sharp lower bounds on the real part of the quotients between the normalized starlike or convex functions and their sequences of partial sums. In the present paper and by following the earlier work by Silverman [20] (see [21–25]) on partial sums of analytic functions, we study the ratio of a function of the form (1.1) to its sequence of partial sums of the form

$$f_n(z) = z + \sum_{k=2}^{n} a_k z^k, \tag{1.13}$$

when the coefficients of $f(z)$ satisfy the condition (1.14). Also, we will determine sharp lower bounds for $\Re\{f(z)/f_n(z)\}$, $\Re\{f_n(z)/f(z)\}$, $\Re\{f'(z)/f_n'(z)\}$, and $\Re\{f_n'(z)/f'(z)\}$. It is seen that this study not only gives as a particular case, the results of Silverman [20], but also gives rise to several new results.

Before stating and proving our main results, we derive a sufficient condition giving the coefficient estimates for functions $f(z)$ to belong to this generalized function class.

Lemma 1.1. *A function $f(z)$ of the form* (1.1) *is in $P_{\mu}^{\lambda}(\gamma,\beta)$ if*

$$\sum_{k=2}^{\infty}(1+\lambda(k-1))\left[k(1+\beta) - (\gamma+\beta)\right]|a_k|C_k^m(b,\mu) \leq 1-\gamma, \tag{1.14}$$

where, for convenience,

$$\rho_k = \rho_k(\lambda, \gamma, \eta) = (1 + \lambda(k-1))[n(1+\beta\ ) - (\gamma+\beta)]C_k^m(b,\mu), \tag{1.15}$$

$0 \le \lambda \le 1$, $-1 \le \gamma < 1$, $\beta \ge 0$, *and* $C_k^m(b,\mu)$, *is given by* (1.9).

Proof. The proof of Lemma 1.1 is much akin to the proof of Theorem 1 obtained by Murugusundaramoorthy [17], hence we omit the details. □

2. Main Results

Theorem 2.1. *If f of the form* (1.1) *satisfies the condition* (1.14), *then*

$$\Re\left\{\frac{f(z)}{f_n(z)}\right\} \ge \frac{\rho_{n+1}(\lambda,\gamma,\eta) - 1 + \gamma}{\rho_{n+1}(\lambda,\gamma,\eta)} \quad (z \in U), \tag{2.1}$$

where

$$\rho_k = \rho_k(\lambda,\gamma,\eta) \ge \begin{cases} 1-\gamma & \text{if } k = 2,3,\ldots,n, \\ \rho_{n+1} & \text{if } k = n+1, n+2, \ldots. \end{cases} \tag{2.2}$$

The result (2.1) *is sharp with the function given by*

$$f(z) = z + \frac{1-\gamma}{\rho_{n+1}} z^{n+1}. \tag{2.3}$$

Proof. Define the function $w(z)$ by

$$\begin{aligned} \frac{1+w(z)}{1-w(z)} &= \frac{\rho_{n+1}}{1-\gamma}\left[\frac{f(z)}{f_n(z)} - \frac{\rho_{n+1} - 1 + \gamma}{\rho_{n+1}}\right] \\ &= \frac{1 + \sum_{k=2}^{n} a_k z^{k-1} + (\rho_{n+1}/(1-\gamma)) \sum_{k=n+1}^{\infty} a_k z^{k-1}}{1 + \sum_{k=2}^{n} a_k z^{k-1}}. \end{aligned} \tag{2.4}$$

It suffices to show that $|w(z)| \le 1$. Now, from (2.4) we can write

$$w(z) = \frac{(\rho_{n+1}/(1-\gamma)) \sum_{k=n+1}^{\infty} a_k z^{k-1}}{2 + 2\sum_{k=2}^{n} a_k z^{k-1} + (\rho_{n+1}/(1-\gamma)) \sum_{k=n+1}^{\infty} a_k z^{k-1}}. \tag{2.5}$$

Hence we obtain

$$|w(z)| \le \frac{(\rho_{n+1}/(1-\gamma)) \sum_{k=n+1}^{\infty} |a_k|}{2 - 2\sum_{k=2}^{n} |a_k| - (\rho_{n+1}/(1-\gamma)) \sum_{k=n+1}^{\infty} |a_k|}. \tag{2.6}$$

Now $|w(z)| \le 1$ if and only if

$$2\left(\frac{\rho_{n+1}}{1-\gamma}\right)\sum_{k=n+1}^{\infty}|a_k| \le 2-2\sum_{k=2}^{n}|a_k| \tag{2.7}$$

or, equivalently,

$$\sum_{k=2}^{n}|a_k| + \sum_{k=n+1}^{\infty}\frac{\rho_{n+1}}{1-\gamma}|a_k| \le 1. \tag{2.8}$$

From the condition (1.14), it is sufficient to show that

$$\sum_{k=2}^{n}|a_k| + \sum_{k=n+1}^{\infty}\frac{\rho_{n+1}}{1-\gamma}|a_k| \le \sum_{k=2}^{\infty}\frac{\rho_k}{1-\gamma}|a_k| \tag{2.9}$$

which is equivalent to

$$\sum_{k=2}^{n}\left(\frac{\rho_k-1+\gamma}{1-\gamma}\right)|a_k| + \sum_{k=n+1}^{\infty}\left(\frac{\rho_k-\rho_{n+1}}{1-\gamma}\right)|a_k| \ge 0. \tag{2.10}$$

To see that the function given by (2.3) gives the sharp result, we observe that for $z = re^{i\pi/n}$,

$$\begin{aligned}\frac{f(z)}{f_n(z)} &= 1+\frac{1-\gamma}{\rho_{n+1}}z^n \longrightarrow 1-\frac{1-\gamma}{\rho_{n+1}}\\ &= \frac{\rho_{n+1}-1+\gamma}{\rho_{n+1}} \quad \text{when} \quad r \longrightarrow 1^-.\end{aligned} \tag{2.11}$$

□

We next determine bounds for $f_n(z)/f(z)$.

Theorem 2.2. *If f of the form* (1.1) *satisfies the condition* (1.14), *then*

$$\Re\left\{\frac{f_n(z)}{f(z)}\right\} \ge \frac{\rho_{n+1}}{\rho_{n+1}+1-\gamma} \quad (z\in U), \tag{2.12}$$

where $\rho_{n+1} \ge 1-\gamma$ and

$$\rho_k \ge \begin{cases} 1-\gamma & \text{if } k=2,3,\ldots,n,\\ \rho_{n+1} & \text{if } k=n+1,n+2,\ldots. \end{cases} \tag{2.13}$$

The result (2.12) *is sharp with the function given by* (2.3).

Proof. We write

$$\begin{aligned}\frac{1+w(z)}{1-w(z)} &= \frac{\rho_{n+1}+1-\gamma}{1-\gamma}\left[\frac{f_n(z)}{f(z)}-\frac{\rho_{n+1}}{\rho_{n+1}+1-\gamma}\right] \\ &= \frac{1+\sum_{k=2}^{n} a_k z^{k-1}-\left(\rho_{n+1}/(1-\gamma)\right)\sum_{k=n+1}^{\infty} a_k z^{k-1}}{1+\sum_{k=2}^{\infty} a_k z^{k-1}},\end{aligned} \tag{2.14}$$

where

$$|w(z)| \leq \frac{\left((\rho_{n+1}+1-\gamma)/(1-\gamma)\right)\sum_{k=n+1}^{\infty}|a_k|}{2-2\sum_{k=2}^{n}|a_k|-\left((\rho_{n+1}-1+\gamma)/(1-\gamma)\right)\sum_{k=n+1}^{\infty}|a_k|} \leq 1. \tag{2.15}$$

This last inequality is equivalent to

$$\sum_{k=2}^{n}|a_k|+\sum_{k=n+1}^{\infty}\frac{\rho_{n+1}}{1-\gamma}|a_k| \leq 1. \tag{2.16}$$

We are making use of (1.14) to get (2.10). Finally, equality holds in (2.12) for the extremal function $f(z)$ given by (2.3). □

We next turns to ratios involving derivatives.

Theorem 2.3. *If f of the form* (1.1) *satisfies the condition* (1.14)*, then*

$$\Re\left\{\frac{f'(z)}{f_n'(z)}\right\} \geq \frac{\rho_{n+1}-(n+1)(1-\gamma)}{\rho_{n+1}} \quad (z \in U), \tag{2.17}$$

$$\Re\left\{\frac{f_n'(z)}{f'(z)}\right\} \geq \frac{\rho_{n+1}}{\rho_{n+1}+(n+1)(1-\gamma)} \quad (z \in U), \tag{2.18}$$

where $\rho_{n+1} \geq (n+1)(1-\gamma)$ and

$$\rho_k \geq \begin{cases} k(1-\gamma) & \text{if } k=2,3,\ldots,n, \\ k\left(\dfrac{\rho_{n+1}}{n+1}\right) & \text{if } k=n+1,n+2,\ldots. \end{cases} \tag{2.19}$$

The results are sharp with the function given by (2.3).

Proof. We write

$$\frac{1+w(z)}{1-w(z)} = \frac{\rho_{n+1}}{(n+1)(1-\gamma)}\left[\frac{f'(z)}{f_n'(z)}-\left(\frac{\rho_{n+1}-(n+1)(1-\gamma)}{\rho_{n+1}}\right)\right], \tag{2.20}$$

where

$$w(z) = \frac{\left(\rho_{n+1}/((n+1)(1-\gamma))\right)\sum_{k=n+1}^{\infty} a_k z^{k-1}}{2 + 2\sum_{k=2}^{n} k a_k z^{k-1} + \left(\rho_{n+1}/((n+1)(1-\gamma))\right)\sum_{k=n+1}^{\infty} k a_k z^{k-1}}. \tag{2.21}$$

Now $|w(z)| \le 1$ if and only if

$$\sum_{k=2}^{n} k|a_k| + \frac{\rho_{n+1}}{(n+1)(1-\gamma)}\sum_{k=n+1}^{\infty} k|a_k| \le 1. \tag{2.22}$$

From the condition (1.14), it is sufficient to show that

$$\sum_{k=2}^{n} k|a_k| + \frac{\rho_{n+1}}{(n+1)(1-\gamma)}\sum_{k=n+1}^{\infty} k|a_k| \le \sum_{k=2}^{\infty}\frac{\rho_k}{1-\gamma}|a_k| \tag{2.23}$$

which is equivalent to

$$\sum_{k=2}^{n}\left(\frac{\rho_k - (1-\gamma)k}{1-\gamma}\right)|a_k| + \sum_{k=n+1}^{\infty}\frac{(n+1)\rho_k - k\rho_{n+1}}{(n+1)(1-\gamma)}|a_k| \ge 0. \tag{2.24}$$

To prove the result (2.18), define the function $w(z)$ by

$$\frac{1+w(z)}{1-w(z)} = \frac{(n+1)(1-\gamma)+\rho_{n+1}}{(1-\gamma)(n+1)}\left[\frac{f_n'(z)}{f'(z)} - \frac{\rho_{n+1}}{(n+1)(1-\gamma)+\rho_{n+1}}\right], \tag{2.25}$$

where

$$w(z) = \frac{-\left(1+\rho_{n+1}/((n+1)(1-\gamma))\right)\sum_{k=n+1}^{\infty} k a_k z^{k-1}}{2 + 2\sum_{k=2}^{n} k a_k z^{k-1} + \left(1-\rho_{n+1}/((n+1)(1-\gamma))\right)\sum_{k=n+1}^{\infty} k a_k z^{k-1}}. \tag{2.26}$$

Now $|w(z)| \le 1$ if and only if

$$\sum_{k=2}^{n} k|a_k| + \left(\frac{\rho_{n+1}}{(n+1)(1-\gamma)}\right)\sum_{k=n+1}^{\infty} k|a_k| \le 1. \tag{2.27}$$

It suffices to show that the left hand side of (2.27) is bounded previously by the condition

$$\sum_{k=2}^{\infty}\frac{\rho_k}{1-\gamma}|a_k|, \tag{2.28}$$

which is equivalent to

$$\sum_{k=2}^{n}\left(\frac{\rho_k}{1-\gamma}-k\right)|a_k|+\sum_{k=n+1}^{\infty}\left(\frac{\rho_k}{1-\gamma}-\frac{\rho_{n+1}}{(n+1)(1-\gamma)}\right)k|a_k|\geq 0. \tag{2.29}$$

□

Remark 2.4. As a special case of the previous theorems, we can determine new sharp lower bounds for $\Re\{f(z)/f_n(z)\}$, $\Re\{f_n(z)/f(z)\}$, $\Re\{f'(z)/f_n'(z)\}$, and $\Re\{f_n'(z)/f'(z)\}$ for various function classes involving the Alexander integral operator [13] and Bernardi integral operators [14], Jung-Kim-Srivastava integral operator [15] and Choi-Saigo-Srivastava operator (see [11, 12]) on specializing the values of η, m, μ, and b.

Some other work related to partial sums and also related to zeta function can be seen in ([26–29]) for further views and ideas.

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