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Performance of Concatenated Kernel Code in Cognitive Radio Networks

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Abstract

Error correction coding is extensively used to achieve reliability in digital communication especially in Physical Layer. In this paper, construction of concatenated kernel code defined over algebraic structure group is discussed. Minimal trellis representation is given for constructed concatenated kernel codeword. Constructed code is tested in cognitive radio network environment subjected to continuous interference due to the behavior of primary users. Proposed code is tested through simulations and its performance is analyzed in terms of Bit Error Rate (BER) in mitigating the effect of continuous interference. Maximum likelihood graph Viterbi decoding technique is used to decode the concatenated kernel code.

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1. Introduction

Reliability in information transmission is achieved through adding redundancy during transmission. Error correction coding deals with such well defined techniques of adding redundancy to information so that errors in transmission, if any, can be corrected with the help of added redundancy. Study and research of such techniques is broadly called as Channel coding. The process of adding redundancy to information by a specified technique is called encoding and thus formed encoded message is called codeword. Similarly, certain techniques are used to reconstruct the actual message from the codeword and the process is called Decoding. Redundancy bits added to information are also termed as parity bits.

Mathematically, a message m of length k from a message space defined over alphabet Σ is encoded into a codeword of length n such that $k < n$. Encoding E can be equivalently given as an injective map from message space Σ^k to the

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codeword space Σ^n[19]. Message space M includes all messages of length k to be encoded. Similarly, codeword space $\mathbb{C}$ includes all the codewords of length n to be transmitted over channel. Encoding E is given as a map as follows:

$$E : \Sigma^k \rightarrow \Sigma^n \tag{1}$$

where, Σ^k denotes message m of length k and Σ^n denotes codeword $\mathbb{C}$ of length n.

Similarly, decoding function D is given as a map as follows:

$$D : \Sigma^n \rightarrow \Sigma^k \tag{2}$$

Encoded message or codeword from codeword space Σ^n is transmitted over channel that can introduce errors η and $\eta \in \Sigma^n$.

In presence of channel noise $\eta \in \Sigma^n$, decoding function D is given as a mapping as follows:

$$D : \Sigma^{(n+\eta)} \rightarrow \Sigma^k \tag{3}$$

Majority of error correcting codes used in present day digital communication are defined over algebraic structure vector space with underlying field [14]. Error correcting codes defined over algebraic structure group has received less attention by researchers [2].

1.1. Concatenated Codes

Concatenated codes were extensively used in space missions lead by NASA in the late 60s and early 70s to achieve better reliability over traditional error correcting codes [4]. Forney introduced concatenated codes with inner code and outer code [7]. Concatenated codes doesnt belong to any specific family of codes [19] and careful selection of inner and outer code results in significant improvement in performance. Concatenated codes achieved goals of both communicating with rate (R) less than the channel capacity (C), i.e., $R < C$ (Shannon Model) [17] and correcting errors that occur during transmission over noisy channel (Hamming model) [8].

1.2. Trellis of a code

Graphs were used to represent codewords and trellis graphs are extensively used to represent codewords of a code $\mathbb{C}$. A trellis T representing a code $\mathbb{C}$ is given as $\mathbb{C}(T)$ [20].

Kschischang and Sorokine defined [11], Trellis for a block code C of length n is an edge labeled directed graph with a distinguished "root" vertex having in-degree zero and a distinguished "goal" vertex having out-degree zero, and with the following properties:

1. all vertices can be reached from the root;
2. the goal can be reached from all vertices;
3. the number of edges traversed in passing from the root to the goal along any path is n; and
4. the set of n-tuples obtained by 'reading off' the edge labels encountered in traversing all paths from the root to the goal is $\mathbb{C}$.

Many popular graph decoding algorithms such as Viterbi graph decoder [21] and Bahl, Cocke, Jelinek and Raviv (also called as BCJR algorithm) [1] are employed to decode the codewords represented as trellis.

1.3. Cognitive Radio Network

Cognitive Radio Network has emerged as a promising solution for the spectrum scarcity and inefficient utilization of licenced spectrum problems [22, 13]. Cognitive Radio Networks provide opportunity for the secondary users (i.e., unlicenced users) to use the spectrum when primary users (i.e., licenced users) are not using it. Since, secondary users are not the licenced users, priority should be given to the primary user transmission as and when they appear. Such appearance of primary users' transmissions create continuous interference for secondary user transmission. In such scenario, error correcting codes helps secondary users to reliably transmit information mitigating continuous interference from primary users [10, 5].

1.4. Our Contribution

To address the challenges of reliable communication in cognitive radio network, we propose concatenated kernel code defined over algebraic structure group with inner and outer code similar to construction of Forney. Its performance is analysed in cognitive radio network framework through simulations in terms of BER.

Our contributions are summarised as follows:

1. Concatenated kernel code is constructed over algebraic structure group.
2. Minimum trellis is constructed for the concatenated kernel code. Maximum likelihood graph Viterbi decoder is employed to decode the codeword from the constructed minimum trellis.
3. Performance of algebraically constructed concatenated kernel code is tested through simulation in cognitive radio network where secondary users are subjected to continuous interference from primary users.

Present paper is organized as follows: In section 2, algebraic construction of concatenated kernel code is given and also algorithm to compute concatenated kernel code is given. In section 3, an example construction of concatenated kernel code is discussed along with the minimum trellis for the constructed code. In section 4 some observations made from the constructed code is discussed. In section 5, system model to evaluate the concatenated kernel code are discussed. In section 6, performance evaluation of constructed code is made in terms of Bit Error Rate (BER). BER is considered as a Quality of Service (QoS) parameter to evaluate the performance [6, 5]. Section 7 deals with the conclusion.

2. Concatenated Kernel code construction

Concatenated code is constructed by combining two families (separate or same) of error correcting code [7]. Combination of inner Viterbi convolutional code with Reed Solomon outer code was successfully used in Voyager space mission [4]. Concatenated kernel code is constructed on similar lines with inner code and outer code.

2.1. Kernel code

Let $G_1, G_2, \ldots, G_n$ be finite groups and S be an abelain group with identity element e. The kernel of homomorphism μ

$$\mu : G_1 \times G_2 \times \cdots \times G_n \to S$$

is defined as $\mu(g_1, g_2, \ldots, g_n) = \mu_1(g_1)\mu_2(g_2)\ldots\mu_n(g_n)$ is called kernel code, where μ_i is a homomorphism from $G_i \to S$ where, $G_i = G_1 \times G_2 \times \ldots \times G_n$. μ is a homomorphism [16] and homomorphism mapped to identity element of abelian group is called kernel of homomorphism [14].

2.2. Concatenated Kernel code

Concatenated kernel code is constructed with inner code and outer code.

Inner Code:
Let $\mu_i : G \to S$ for $i = 1, 2, \ldots, k$ be homomorphisms and S be an abelian group. The kernel of homomorphism μ ($\mu : G^k \to S$) is defined as $\mu(g_1, g_2, \ldots, g_k) = \mu_1(g_1)\mu_2(g_2)\ldots\mu_k(g_k)$. Kernel of homomorphism μ is defined as inner code.

Outer Code:
Let G be an abelian group and $\mu' : G^k \to G^n$ be a homomorphism defined as $\mu'(g_1, g_2, \ldots, g_k) = ((g_1, g_2, \ldots, g_k), h_1(g_1, g_2, \ldots, g_k), \ldots, h_{n-k}(g_1, g_2, \ldots, g_k))$. h_i 's are chosen as applicable or required to channel. Images of μ' is defined as outer code.

Concatenated kernel code is the images of elements of kernel $\mu \circ \mu'$ under μ'. Algorithm 1 computes the concatenated kernel code from the given finite groups and homomorphism.

Algorithm 1 Concatenated Kernel Code

Input: Finite Groups and Abelian group
Output: Concatenated Kernel Code
for $i = 1$ *to* n **do**
  μ_i = *select values applicable for channel*
Compute set K of Cartesian products of Groups G_1, G_2 ,G_3 ,..., G_n
for $i = 1$ *to* n **do** ▷ On set K of Cartesian products
  if $\mu_1(g_1)\mu_2(g_2)\mu_3(g_3)...\mu_n(g_n) = 0$ **then**
    add to set P
    return P ▷ Set of Kernel codes
for $i = 1$ *to* n **do**
  $C = ((g_1, g_2, \ldots, g_k), h_1(g_1, g_2, \ldots, g_k), \ldots, h_{n-k}(g_1, g_2, \ldots, g_k))$
  add to set C
  return C ▷ Set of Concatenated Kernel codeword

2.3. Rate of Kernel code

On similar lines with linear code [2], the rate of kernel code is given by $R = \frac{1}{n} log|G|$ where, n is the word length and $|G|$ is the number of codewords, i.e., the number of elements in the kernel of homomorphism.

3. Example of Concatenated Kernel code

Let $< Z_2, + >$, $< Z_3, + >$ and $< Z_5, + >$ be finite groups with elements $Z_2 = \{0, 1\}$, $Z_3 = \{0, 1, 2\}$ and $Z_5 = \{0, 1, 2, 3, 4\}$. Concatenated kernel code is defined over $Z_2 \times Z_3 \times Z_5 \to Z_2$ with homomorphisms μ_i is defined as $\mu_1(0) = 0$, $\mu_1(1) = 1$, $\mu_2(0) = 0$, $\mu_2(1) = 0$, $\mu_3(0) = 0$, $\mu_3(1) = 1$, $\mu_3(2) = 1$, $\mu_4(0) = 0$.

From the defined homomorphism and using the algorithm 1, we obtain {000, 004, 010, 014, 020, 024, 101, 102, 103, 111, 112, 113, 121, 122, 123, 124} as kernel code.

Homomorphism for outer code is defined as $\mu_4(0) = 0$, $\mu_4(1) = 1$, $\mu_4(2) = 0$, $\mu_4(3) = 1$, $\mu_5(0) = 0$, $\mu_5(1) = 0$, $\mu_5(2) = 1$, $\mu_5(3) = 1$, $\mu_5(4) = 0$.

From the defined values of μ_i's we obtain $\mathbb{C}$ = {00000, 00400, 01010, 01410, 02001, 02400, 10110, 10211, 10311, 11101, 11201, 11300, 12111, 12210, 12310, 12410} as concatenated kernel code.

Minimum trellis representation for constructed concatenated kernel code is given in Fig 1. Trellis representation of code preserve the loss of valuable information that occurs during channel output symbol quantization that usually occurs in channel output symbol quantization over field in algebraic decoding [15]. Any standard trellis construction procedure can be used to construct trellis for a codeword $\mathbb{C}$ and the trellis T representing a code $\mathbb{C}$ of length n is given as as $\mathbb{C}(T)$ [20].

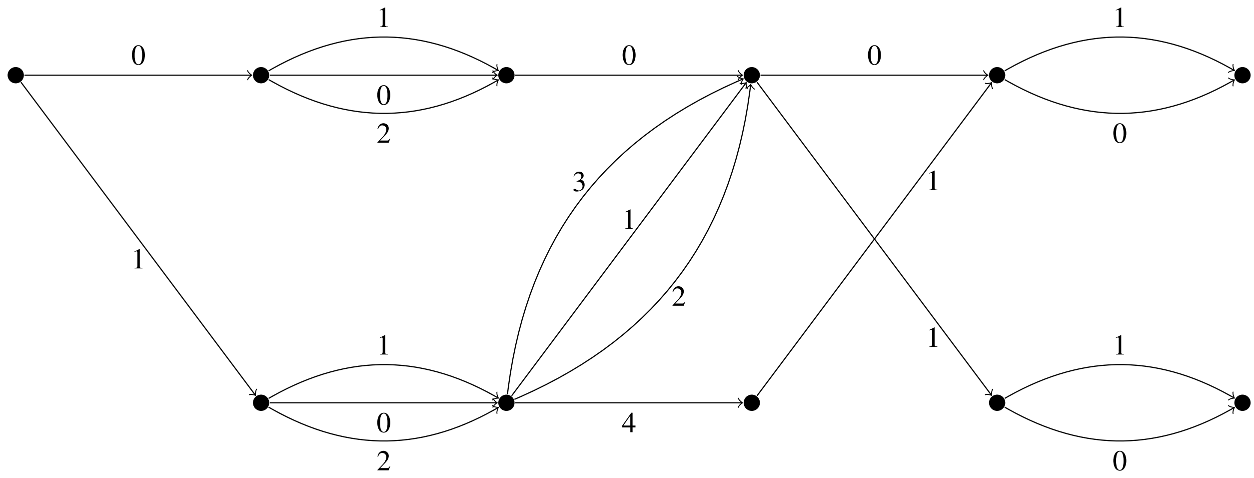

Fig. 1. Trellis for concatenated kernel code in example of section 3

4. Some observations from the Constructed code

Observation 4.1. *Construction of kernel code is clearly the mapping of homomorphisms from finite groups to the identity element of the abelian group.*

Observation 4.2. *Kernel is the normal sub group of the group mapped onto (i.e., surjective mapping).*

Observation 4.3. *In a large set of codewords constructed over a finite set of groups, the presence of homomorphism can be tested by the Blum-Luby-Rubinfeld Linearity Test (also called as BLR test) [3].*

Observation 4.4. *Low Density Parity Check (LDPC)*[1] *like sparse codes can be constructed algebraically from Kernel codes [12] and concatenated kernel codes by selecting more zero homomorphisms, i.e., the values of μ's.*

Observation 4.5. *Concatenated kernel codes are mapping of kernel codes to other groups, such mapping is bijective in nature. Bijective mapping guarantees the unique decoding of codewords with the decoding failure bound.*

Observation 4.6. *Outer code in the Concatenated Kernel provides the error correction capability to codeword which is not present in case of just kernel code.*

5. System model to check performace of constructed code in Cognitive Radio Network environment

A simple cognitive radio network with a pair (transmitter and receiver) of primary and secondary users is considered. Secondary user transmitter and receiver pair exist within the transmission receiver range of primary users [9, 18]. An overview of communication framework is given in Fig 2.

Information bits are encoded using concatenated kernel codes. Further, the concatenated kernel code is BFSK modulated before transmitting over channel. Appearance of channel noise is assumed to be Additive White Gaussian

[1] LDPC codes are not algebraically constructed codes

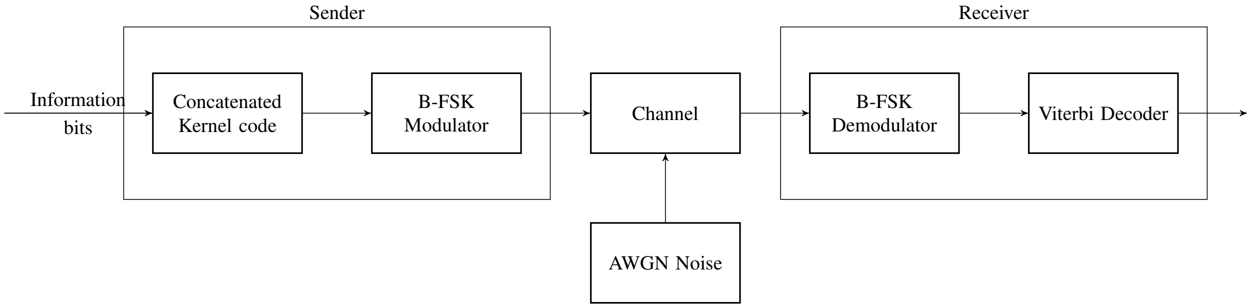

Fig. 2. Concatenated Kernel code with BFSK modulated communication system

Noise (AWGN) that is due to the behaviour of primary users' transmission. Similarly, on the receiver side, BFSK modulated signal is demodulated and further decoded using graph Viterbi hard decision decoding.

As in any BFSK modulation, symbols are modulated as sinusoidal waves given by

$$s_i(t) = \sqrt{\frac{2.E_s}{T_s}} cos(2\pi f_i t), 0 \leq t \leq T_s \tag{4}$$

$$f_i = f_0 + \frac{i-1}{T_s} \tag{5}$$

where, $i = 1, 2$ indicate two frequency states and E_s is the energy per modulated symbol.

At the receiver side, signal is demodulated and decoded using maximum likelihood graph Viterbi decoder [21].

6. Performance Evaluation

Cognitive Radio Network environment is implemented in MATLAB on Z420 work station with 8GB RAM, 1TB hard disk and Windows 7 operating system. Twelve bits are transmitted and BER is simulated thirty times per SNR to plot BER vs E_b/N_0 performance. Occurance of noise is simulated as Additive White Gaussian Noise (AWGN) that is due to the behaviour of primary users' transmissions. From the Figure 3, we can infer that concatenated kernel codes provides better performance over kernel codes with a coding gain of 0.25dB both at BER 10^{-3} and 10^{-2}.

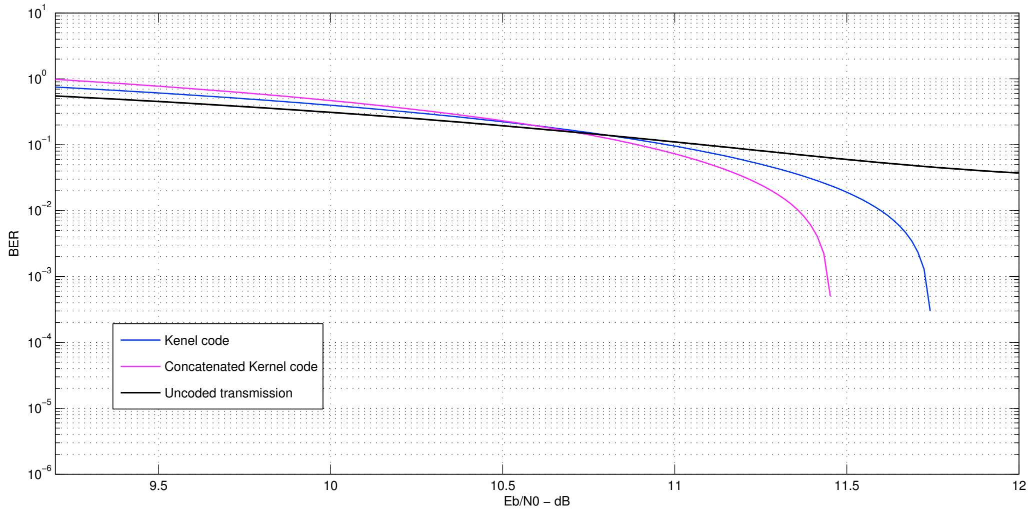

Fig. 3. BER performance of example code constructed in example 3

7. Conclusion

Construction and performance of concatenated kernel code over algebraic structure groups is discussed in the paper. Constructed concatenated kernel code meets the requirement of linear concatenated code construction. Trellis representation of constructed code is given to employ the Viterbi graph decoding algorithm. Algebraically constructed concatenated kernel code is tested for its performance in cognitive radio environment. Performance is measured in terms of BER indicating clear coding gain. Further, system theoretic properties of constructed concatenated kernel code and secure communication employing such codes and respective trellis can be studied as a future scope.

References

[1] Bahl, L., Cocke, J., Jelinek, F., Raviv, J., 1974. Optimal decoding of linear codes for minimizing symbol error rate (corresp.). IEEE Transactions on information theory 20, 284–287.
[2] Bailey, R.F., 2009. Error-correcting codes from permutation groups. Discrete Mathematics 309, 4253–4265.
[3] Blum, M., Luby, M., Rubinfeld, R., 1993. Self-testing/correcting with applications to numerical problems. Journal of Computer and System Sciences 47, 549–595.
[4] Costello, D.J., Forney, G.D., 2007. Channel coding: The road to channel capacity. Proceedings of the IEEE 95, 1150–1177.
[5] El-Bardan, R., Masazade, E., Ozdemir, O., Han, Y.S., Varshney, P.K., 2016. Permutation trellis coded multi-level FSK signaling to mitigate primary user interference in cognitive radio networks. IEEE Transactions on Communications 64, 104–116.
[6] El Bardan, R., Masazade, E., Ozdemir, O., Varshney, P.K., 2012. Performance of permutation trellis codes in cognitive radio networks, in: Sarnoff Symposium (SARNOFF), 2012 35th IEEE, IEEE. pp. 1–6.
[7] Forney, G.D., 1966. Concatenated codes. volume 11. Cambridge: MIT press.
[8] Hamming, R.W., 1950. Error detecting and error correcting codes. Bell System technical journal 29, 147–160.
[9] Hossain, E., Le, L., Devroye, N., Vu, M., 2009. Cognitive radio: from theory to practical network engineering, in: New directions in wireless communications research. Springer, pp. 251–289.
[10] Ko, J., Kim, C., 2011. Communication method between spectrum heterogeneous secondary users in ofdm-based cognitive radio. Electronics letters 47, 827–829.
[11] Kschischang, F.R., Sorokine, V., 1995. On the trellis structure of block codes. IEEE Transactions on Information Theory 41, 1924–1937.
[12] Kumar, C.P., Selvakumar, R., Bhattar, R.K., 2016. Construction of binary and nonbinary ldpc-like codes from kernel codes, in: Proceedings of the Second International Conference on Computer and Communication Technologies, Springer. pp. 459–466.
[13] Liang, Y.C., Chen, K.C., Li, G.Y., Mahonen, P., 2011. Cognitive radio networking and communications: An overview. IEEE Transactions on Vehicular Technology 60, 3386–3407.
[14] Moon, T.K., 2005. Error Correction Coding: Mathematical Methods and Algorithms. Wiley-Interscience.
[15] Priti, S., 2003. Codes and decoding on graphs. Current Science 85, 980–988.

[16] Selvakumar, R., Balasubramani, P., 2004. Kernel code and its controllability. Journal of Discrete Mathematical Sciences and Cryptography 7, 97–101.

[17] Shannon, C.E., 2001. A mathematical theory of communication. SIGMOBILE Mob. Comput. Commun. Rev. 5, 3–55. URL: `http://doi.acm.org/10.1145/584091.584093`, doi:10.1145/584091.584093.

[18] Stevenson, C.R., Chouinard, G., Lei, Z., Hu, W., Shellhammer, S.J., Caldwell, W., 2009. IEEE 802.22: The first cognitive radio wireless regional area network standard. IEEE communications magazine 47, 130–138.

[19] Sudan, M., 2001. Coding theory: tutorial & survey, in: Foundations of Computer Science, 2001. Proceedings. 42nd IEEE Symposium on, IEEE. pp. 36–53.

[20] Vardy, A., 1998. Trellis structure of codes. Handbook of coding theory 2, 1989–2117.

[21] Viterbi, A.J., 1967. Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. Information Theory, IEEE Transactions on 13, 260–269.

[22] Wang, B., Liu, K.R., 2011. Advances in cognitive radio networks: A survey. IEEE Journal of selected topics in signal processing 5, 5–23.