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Performance prediction of interactive telemedicine

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ABSTRACT

Telemedicine is a remote medical practice, which utilizes information and communication technology to provide healthcare and exchange of health information for distance places. Diagnosis and prescription for rural and remote area patients are the main advantages of this system. The major categories of telemedicine are store-and-forward, remote patient monitoring and interactive telemedicine. Interactive telemedicine is a real-time interaction between patients and doctors. This method can be conducted in a nursing home or at the patient's house via a video-conferencing system. Moreover, this is more convenient for patients, than to obtain health care with a caretaker in person. However, Internet-based networks are congested with many users. We considered interactive telemedicine as a queueing system and analyzed it by using the method of supplementary variables. We obtained the performance measures through a probability generating function. The numerical results of the queueing system may be useful to provide service satisfactorily and update the utility function of interactive telemedicine.

1. Introduction

In medical systems, patients have to go to a hospital or clinic for consultation and treatment; therefore, it is inconvenient for patients, and for patients residing in rural and remote areas [1]. Telemedicine is a remote medical practice, which utilizes advanced telecommunications and information technologies for the delivery of healthcare and the exchange of health information across distances [2,3]. Health care organizations showing interest in the implementation of telemedicine technology to develop the health care facilities [4–6]. It bridges the gap between patients from rural areas and medical professionals located in city centers. The main advantages of telemedicine are specialist care, reducing long travels and increasing health care quality in rural and remote areas [7–9]. It is a natural growth of healthcare in the digital world and a convenient way to access because of internet-based communication applications [10–12].

The major categories of telemedicine are remote patient monitoring, store-and-forward and interactive method [13]. In remote patient monitoring, patients are monitored through the use of advanced technologies like a smart watch and medical devices [14,15]. Store and forward method helps in sharing of medical information from one location to another location [16,17]. Interactive telemedicine is a real-time interaction between patients and doctors separated by distance [18–20].

Performance measures of queueing systems using supplementary variable technique can be found in Ref. [21]. Queueing systems with repeated trial have been successfully used to model many practical problems in telecommunication networks, packet switching networks, collision avoidance and computer systems etc. [22–24]. New customers arrive from outside the system to get service. If they find the server is busy at the time of their arrival is obliged to leave the service area and repeat the demand after some random time. Between trials, unsatisfied customers may join the pool of unsatisfied customers called a retrial group or may not join in retrial group [25,26].

In some queueing situation, all arriving customers need first essential service but some of them only need second optional service. As soon as first essential service is completed, the customer may opt for second optional service or may leave from service area [27–30]. Once the queue gets cleared the server may leave for a secondary job. On completion of a secondary job, if the queue is empty, then the server may wait for the new customer to arrive or leaves for another secondary job [31–33]. One of the important thing in the practical situation is perfectly reliable servers do not exist and it may breakdown while serving the customers [34–36]. The customer just being served before breakdown waits for the remaining service to complete [37]. The failed server will immediately send to a repair facility. In repair station, there is a chance to start repair after some period of time due to some reasons [38–40].

According to our knowledge, less number of works carried out for performance measures in telemedicine. Priya et al. [41] have derived the performance measures for medical E-commerce shopping cart abandonment in the cloud by $M^X/G/1$ queueing model with reneging through

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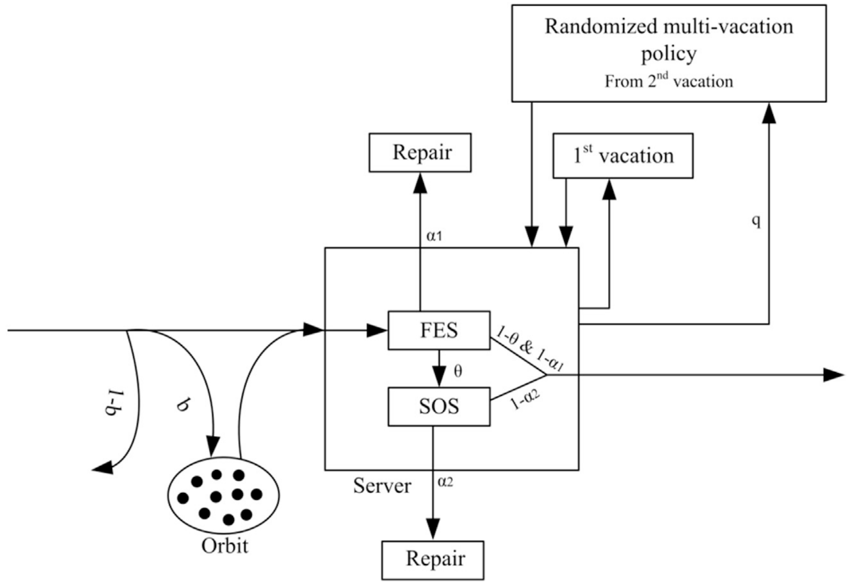

Fig. 1. Schematic representation of the proposed model.

probability generating functions. Similarly, we considered interactive telemedicine as batch arrival retrial queueing system with two phases of service, balking, delaying repair and randomized multi-vacation policy. The objective of this study is to obtain the performance measures of interactive telemedicine through probability generating function by the method of supplementary variables.

This paper structured as follows: First, we have given a detailed description of telemedicine. Second, we developed the differential-difference equations for interactive telemedicine. Third, we have derived the system performance measures. Finally, some numerical examples are presented for the proposed model.

2. Model description

Patients are arriving in batches to get a consultation by the interactive method. If the doctor is available while trying for consultation, patients will get consultation immediately. If the doctor is not available or busy, the patients may retry for service after some random time. Once the interactive session got over, patients have the option for treatment. The system may fail for short interval of time and immediately will send to a repair facility. If the orbit length is empty at a service completion instant, the server will move to a secondary job. If the orbit length is empty at the end of a vacation, the server either remains idle with probability p or leaves for another vacation with probability $q\ (=1-p)$. When one or more patients arrive during server idle, the server immediately starts providing service for the patients. The various stochastic processes are involved in this system and can be modeled as $M^X/G/1$ retrial queueing system with balking, delaying repair and randomized multi-secondary job. Schematic representation of the proposed model is given in Fig. 1.

Telemedicine has been well-defined with the following assumptions.

2.1 Patients are arriving in groups of size k according to compound Poisson process with the rate λ. Let be X_k the size of the k^{th} arrival batch and $X(z)$ be its probability generating function.

2.2 If the server is free at the time of patients arrival, the arriving patients begins to be served one by one according to the order of their arrival. Otherwise, if the server is busy, breakdown or on vacation, the patients may become a source of repeated patients with probability b or may leave with probability $1-b$. The source of repeated patients viewed as a sort of queue. Consecutive retrial times form a sequence of independent random variables having continuous distribution function $\{O(x)\}$ with the first-order differential equation $\delta(x)dx = \frac{dO(x)}{1-O(x)}, \forall x \ge 0$.

2.3 Telemedicine centers provide heterogeneous service in two phases. First one is video conferencing (First Essential Service) and the other is a treatment (Second Optional Service). First essential service is common for all patients and the other is patients choice. Successive service times form a sequence of independent random variables having continuous distribution function $\{S_i(x)\}$ with the first-order differential equation $\mu_i(x)dx = \frac{dS_i(x)}{1-S_i(x)}, \forall x \ge 0$.

2.4 If the orbit size is empty at service completion instant, the server will move for a secondary job. After a vacation, if the orbit is empty, the server may opt for one more vacation with probability q. Consecutive secondary jobs having continuous distribution function $\{V(x)\}$ with the first-order differential equation $w(x)dx = \frac{dV(x)}{1-V(x)}, \forall x \ge 0$.

2.5 The server may breakdown for short interval of time in both phases with respective probabilities α_1 and α_2.

2.6 The failed system immediately will send to repair facility and sometimes repair may start after some period of time due to some reasons. Successive delay times form a sequence of independent random variables having continuous distribution function $\{D_i(y)\}$ with the first-order differential equation $\eta_i(y)dy = \frac{dD_i(y)}{1-D_i(y)}, \forall y \ge 0$.

2.7 The successive repair times of breakdown server form a sequence of independent random variables having continuous distribution function $\{R_i(y)\}$ with the first-order differential equation $\gamma_i(y)dy = \frac{dR_i(y)}{1-R_i(y)}, \forall y \ge 0$.

2.8 Patients leave the service area once service completed successfully.

3. Mathematical formulation

The following steady state differential-difference equations are obtained by using the supplementary variable technique

$$\lambda I_0 = p\int_0^\infty \sum_{n=0}^{\infty} \pi_{j,0}(x)w(x)dx \tag{1}$$

$$\frac{dI_n(x)}{dx} = -(\lambda+\delta(x))I_n(x),\ x>0,\ n\ge 1 \tag{2}$$

Table 1
The effect of p on L_q and W_q for the values of $\lambda = 5; \theta, \alpha_1$ & $\alpha_2 = 0.5; w = 1; \delta = 9; \mu_1, \eta_1$ & $\gamma_1 = 5; \mu_2, \gamma_2$ & $\eta_2 = 6$.

p	b = 0.1		b = 0.5		b = 1	
	L_q	W_q	L_q	W_q	L_q	W_q
0.1	0.0355	0.0071	5.8372	1.1674	61.2167	12.2433
0.2	0.0307	0.0061	5.6866	1.1373	60.8624	12.1725
0.3	0.0263	0.0053	5.5396	1.1079	60.5110	12.1022
0.4	0.0222	0.0044	5.3962	1.0792	60.1624	12.0325
0.5	0.0184	0.0037	5.2560	1.0512	59.8167	11.9633

Table 2
The effect of δ on L_q and W_q for the values of $\lambda = 5; \theta, b, \alpha_1$ & $\alpha_2 = 0.5; w = 1; \mu_1, \eta_1$ & $\gamma_1 = 5; \mu_2, \eta_2$ & $\gamma_2 = 6$.

δ	p = 0.1		p = 0.5		p = 1	
	L_q	W_q	L_q	W_q	L_q	W_q
1	8.4260	1.6852	7.7865	1.5573	7.0903	1.4181
2	7.8933	1.5787	7.3062	1.4612	6.6603	1.3321
3	7.5024	1.5005	6.9416	1.3883	6.3205	1.2641
4	7.1824	1.4365	6.6340	1.3268	6.0237	1.2047
5	6.8993	1.3799	6.3545	1.2709	5.7461	1.1492

$$\frac{d\Omega_{i,0}(x)}{dx} = -(\lambda + \alpha_i + \mu_i(x))\Omega_{i,0}(x) + \lambda(1-b)\Omega_{i,0}(x) + \int_0^\infty \Psi_{i,0}(x,y)\gamma_i(y)dy,\ x > 0,\ i = 1,2 \tag{3}$$

$$\frac{d\Omega_{i,n}(x)}{dx} = -(\lambda + \alpha_i + \mu_i(x))\Omega_{i,n}(x) + \lambda(1-b)\Omega_{i,n}(x) + \lambda b\sum_{k=1}^{n}\chi_k\Omega_{i,n-k}(x) + \int_0^\infty \Psi_{i,n}(x,y)\gamma_i(y)dy, \tag{4}$$

$x > 0,\ n \geq 1,\ i = 1,2$

$$\frac{dQ_{i,0}(x,y)}{dx} = -(\lambda + \eta_i(y))Q_{i,0}(x,y) + \lambda(1-b)Q_{i,0}(x,y), \tag{5}$$

$x > 0,\ y > 0,\ i = 1,2$

$$\frac{dQ_{i,n}(x,y)}{dx} = -(\lambda + \eta_i(y))Q_{i,n}(x,y) + \lambda(1-b)Q_{i,n}(x,y) + \lambda b\sum_{k=1}^{n}\chi_k Q_{i,n-k}(x,y), \tag{6}$$

$x > 0,\ y > 0,\ n \geq 1,\ i = 1,2$

$$\frac{d\Psi_{i,0}(x,y)}{dx} = -(\lambda + \gamma_i(y))\Psi_{i,0}(x,y) + \lambda(1-b)\Psi_{i,0}(x,y), \tag{7}$$

$x > 0,\ y > 0,\ i = 1,2$

Table 3
The effect of p on L_q and W_q for the values of $\lambda = 5; b = 0.1; \theta, \alpha_1$ & $\alpha_2 = 0.5; c = 0.7; w = 1; \delta = 9; \mu_1, \eta_1$ & $\gamma_1 = 5; \mu_2, \eta_2$ & $\gamma_2 = 6$.

p	Exp		Erlang - 2 stage		Hyp - Exp	
	L_q	W_q	L_q	W_q	L_q	W_q
0.1	19.8435	3.9687	6.0404	1.2081	1.8778	0.3756
0.2	19.7593	3.9519	5.8998	1.1800	1.8498	0.3700
0.3	19.6799	3.9360	5.7631	1.1526	1.8226	0.3645
0.4	19.6050	3.9210	5.6302	1.1260	1.7962	0.3592
0.5	19.5343	3.9069	5.5009	1.1002	1.7704	0.3541

Table 4
The effect of b on L_q and W_q for the values of $\lambda = 1.5; \theta$ & $p = 0.1; c = 0.9; \alpha_1$ & $\alpha_2 = 0.5; w = 8; \delta = 9; \mu_1, \gamma_1$ & $\eta_1 = 8; \mu_2, \gamma_2$ & $\eta_2 = 6$.

b	Exp		Erlang- 2 stage		Hyp-Exp	
	L_q	W_q	L_q	W_q	L_q	W_q
0.1	0.0231	0.0154	0.0125	0.0083	0.0065	0.0043
0.2	0.0907	0.0605	0.0464	0.0309	0.0461	0.0307
0.3	0.2004	0.1336	0.0987	0.0658	0.1407	0.0938
0.4	0.3508	0.2339	0.1705	0.1137	0.3056	0.2037
0.5	0.5412	0.3608	0.2688	0.1792	0.5531	0.3687

Table 5
The effect of γ_2 on L_q and W_q for the values of $\lambda = 2; \theta, p, b, \alpha_1$ & $\alpha_2 = 0.5; c = 0.7; w = 1; \delta = 4; \mu_1, \gamma_1$ & $\eta_1 = 5; \mu_2$ & $\eta_2 = 6$.

γ_2	Exp		Erlang- 2 stage		Hyp-Exp	
	L_q	W_q	L_q	W_q	L_q	W_q
1	0.5514	0.2757	1.1169	0.5584	0.2647	0.1324
2	0.4116	0.2058	0.9457	0.4729	0.2033	0.1017
3	0.3767	0.1884	0.8725	0.4363	0.1890	0.0945
4	0.3609	0.1805	0.8330	0.4165	0.1827	0.0914
5	0.3519	0.1759	0.8084	0.4042	0.1792	0.0896

Table 6
The effect of α_2 on L_q and W_q for the values of $\lambda = 2; \theta, p, b$ & $\alpha_1 = 0.5; c = 0.9; w = 1; \delta = 5; \mu_1, \gamma_1$ & $\eta_1 = 5; \mu_2, \gamma_2$ & $\eta_2 = 6$.

α_2	Exp		Erlang- 2 stage		Hyp-Exp	
	L_q	W_q	L_q	W_q	L_q	W_q
0.1	0.3076	0.1538	0.6109	0.3054	0.1640	0.0820
0.2	0.3181	0.1591	0.6456	0.3228	0.1691	0.0845
0.3	0.3287	0.1644	0.6800	0.3400	0.1742	0.0871
0.4	0.3394	0.1697	0.7142	0.3571	0.1793	0.0897
0.5	0.3502	0.1751	0.7480	0.3740	0.1845	0.0923

$$\frac{d\Psi_{i,n}(x,y)}{dx} = -(\lambda + \gamma_i(y))\Psi_{i,n}(x,y) + \lambda(1-b)\Psi_{i,n}(x,y) + \lambda b\sum_{k=1}^{n}\chi_k\Psi_{i,n-k}(x,y), \tag{8}$$

$x > 0,\ y > 0,\ n \geq 1,\ i = 1,2$

$$\frac{d\Pi_{j,0}(x)}{dx} = -(\lambda + w(x))\Pi_{j,0}(x) + \lambda(1-b)\Pi_{j,0}(x), \tag{9}$$

$x > 0,\ j = 1,2,\cdots,\infty$

Table 7
The effect of η_1 on L_q and W_q for the values of $\lambda = 2$; p, b, θ, α_1 & $\alpha_2 = 0.5$; $w = 1$; $c = 0.7$; $\delta = 4$; μ_1 & $\gamma_1 = 5$; μ_2, γ_2 & $\eta_2 = 6$.

η_1	Exp		Erlang- 2 stage		Hyp-Exp	
	L_q	W_q	L_q	W_q	L_q	W_q
1	0.6839	0.3419	0.9645	0.4823	0.2914	0.1457
2	0.4262	0.2131	0.8827	0.4414	0.2044	0.1022
3	0.3770	0.1885	0.8362	0.4181	0.1877	0.0938
4	0.3569	0.1784	0.8091	0.4045	0.1807	0.0904
5	0.3461	0.1730	0.7916	0.3958	0.1769	0.0885

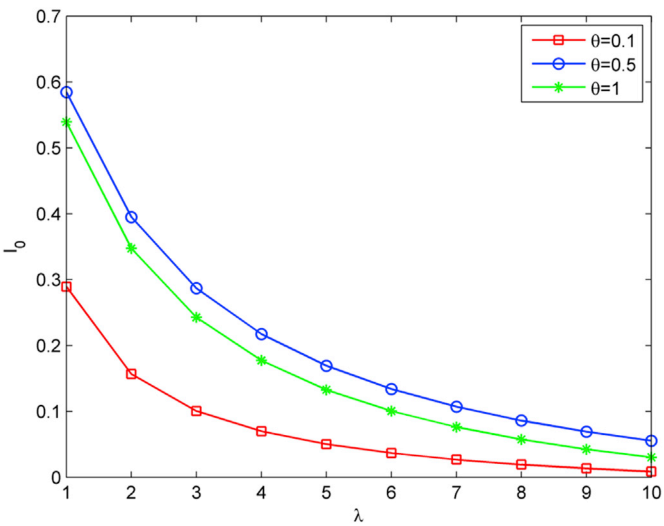

Fig. 2. λ versus I_0 for $\theta = 0.1,\ 0.5,\ 1$.

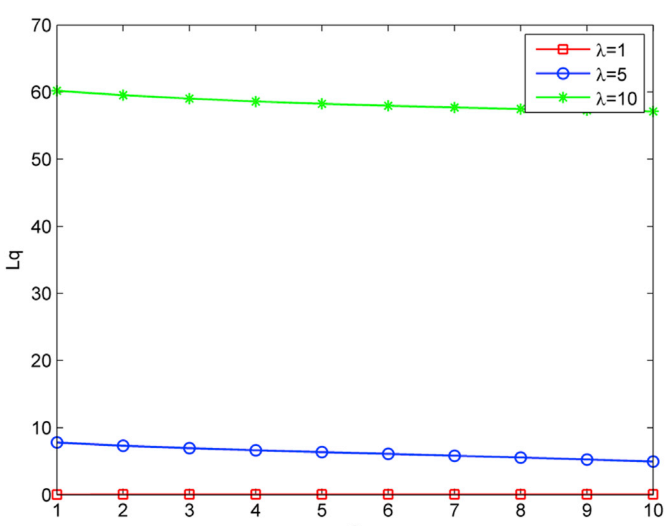

Fig. 3. δ versus L_q for $\lambda = 1,\ 5,\ 10$.

$$\frac{d\Pi_{j,n}(x)}{dx} = -(\lambda + w(x))\Pi_{j,n}(x) + \lambda(1-b)\Pi_{j,n}(x) + \lambda b\sum_{k=1}^{n}\chi_k\Pi_{j,n-k}(x), \tag{10}$$

$x > 0,\ n \geq 1,\ j = 1, 2, \cdots\infty$

The boundary conditions are,

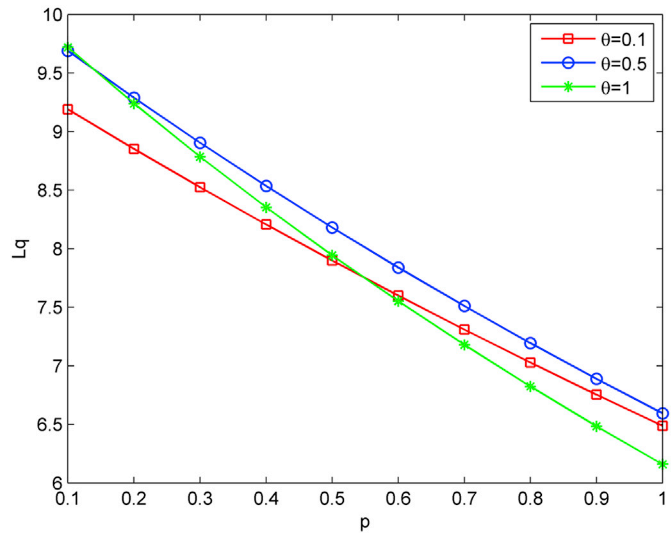

Fig. 4. p versus L_q for $\theta = 0.1,\ 0.5,\ 1$.

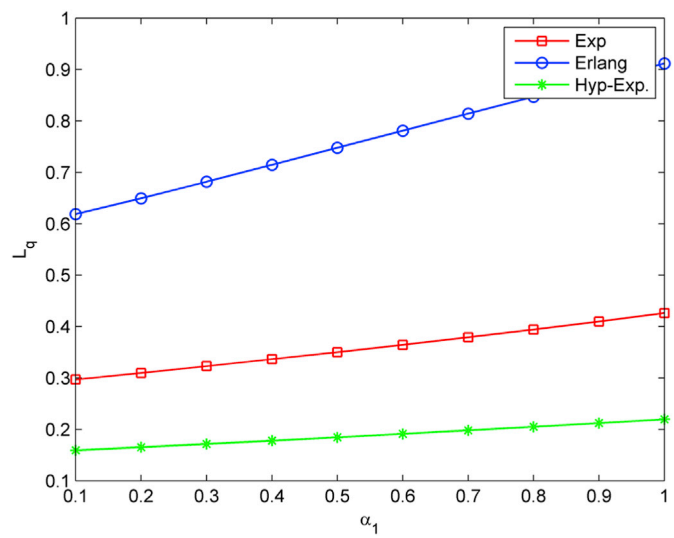

Fig. 5. α_1 versus L_q.

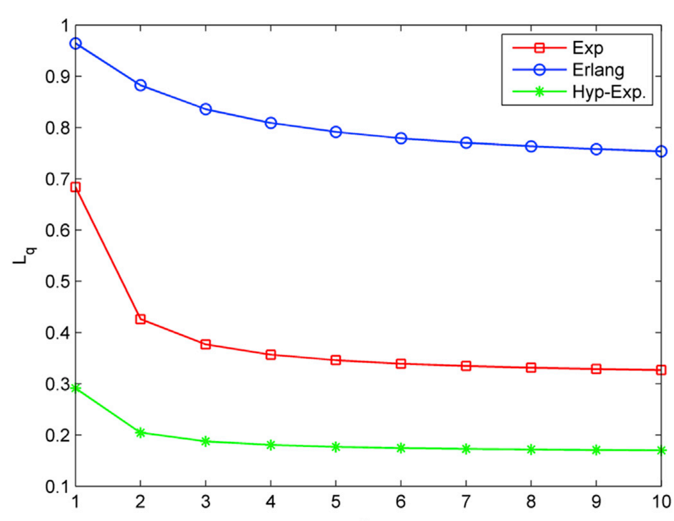

Fig. 6. γ_1 versus L_q.

$$I_n(0) = \int_0^\infty \Pi_{j,n}(x)w(x)dx + \int_0^\infty \Omega_{2,n}(x)\mu_2(x)dx + (1-\theta)\int_0^\infty \Omega_{1,n}(x)\mu_1(x)dx,\ n \ge 1 \tag{11}$$

$$\Omega_{1,0}(0) = \int_0^\infty I_1(x)\delta(x)dx + \lambda I_0 \tag{12}$$

$$\Omega_{1,n}(0) = \lambda\sum_{k=1}^{n}\chi_k\int_0^\infty I_{n-k+1}(x)dx + \int_0^\infty I_{n+1}(x)\delta(x)dx + \lambda\chi_{n+1}I_0,\ n \ge 1 \tag{13}$$

$$\Omega_{2,n}(0) = \theta\int_0^\infty \Omega_{1,n}(x)\mu_1(x)dx,\ n \ge 0 \tag{14}$$

$$\Pi_{1,n}(0) = \begin{cases} (1-\theta)\int_0^\infty \Omega_{1,n}(x)\mu_1(x)dx + \int_0^\infty \Omega_{2,n}(x)\mu_2(x)dx & n=0 \\ 0 & n \ge 1 \end{cases} \tag{15}$$

$$\Pi_{j,n}(0) = \begin{cases} \int_0^\infty q\,\Pi_{j-1,n}(x)w(x)dx & n=0 \\ 0 & n \ge 1,\ j=1,..,\infty \end{cases} \tag{16}$$

at $y = 0$

$$Q_{i,n}(x,0) = \alpha_i\Omega_{i,n}(x),\ x>0,\ n \ge 0,\ i=1,2 \text{ and} \tag{17}$$

$$\Psi_{i,n}(x,0) = \int_0^\infty Q_{i,n}(x,y)\eta_i(y)dy,\quad x>0,\ n \ge 0,\ i=1,2 \tag{18}$$

$$P(z) = I_0 + \sum_{n=1}^{\infty}\int_0^\infty I_n(x)dx + \sum_{n=0}^{\infty}\int_0^\infty \Omega_{i,n}(x)dx + \sum_{n=0}^{\infty}\int_0^\infty Q_{i,n}(x)dx + \sum_{n=0}^{\infty}\int_0^\infty \Psi_{i,n}(x)dx + \sum_{n=0}^{\infty}\int_0^\infty \Pi_{j,n}(x)dx,\quad j = 1,2,3,\cdots,\infty,\ i=1,2 \tag{19}$$

From normalizing condition, we obtained the following probability generating function of orbit length

$$P(z) = \frac{Nr}{Dr} \tag{20}$$

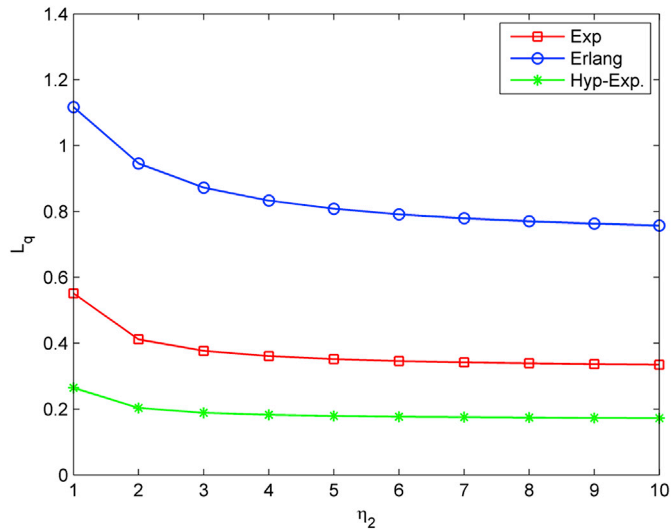

Fig. 7. η_2 versus L_q.

$$Dr = \lambda b(1-X(z))A_1(z)A_2(z)\left[\begin{array}{c} z - \left(O^*(\lambda) + X(z)\left(1-O^*(\lambda)\right)\right) \\ \left((1-\theta)S_1^*(A_1(z)) + \theta S_1^*(A_1(z))S_2^*(A_2(z))\right)\end{array}\right]$$

$$A_i(z) = \lambda b(1-X(z)) + \alpha_i\left(1 - R_i^*(\lambda b(1-X(z)))D_i^*(\lambda b(1-X(z)))\right),\quad i=1,2$$

4. System performance measures

4.1. Expected number of patients in retrial group

The expected number of patients in the orbit is obtained by differentiating $P(z)$ at $z=1$ and by using L'Hospital rule

$$L_q = \frac{Nr^{(V)}Dr^{(IV)} - Nr^{(IV)}Dr^{(V)}}{5{Dr^{(IV)}}^2} \tag{21}$$

$$Nr = \lambda I_0\Bigg\{\left[\left(\frac{V^*(\lambda b(1-X(z)))-1}{pV^*(\lambda b)} - 1\right)(O^*(\lambda) + X(z)(1-O^*(\lambda))) + X(z)\right]$$
$$\left[\alpha_1A_2(z)\left(1-S_1^*(A_1(z))\right)\left(1-R_1^*(\lambda b(1-X(z)))D_1^*(\lambda b(1-X(z)))\right) + \theta\alpha_2A_1(z)S_1^*(A_1(z))\left(1-S_2^*(A_2(z))\right)\right.$$
$$\left.\left(1-R_2^*(\lambda b(1-X(z)))D_2^*(\lambda b(1-X(z)))\right) + \lambda b(1-X(z))\left(A_2(z)\left(1-S_1^*(A_1(z))\right) + \theta A_1(z)S_1^*(A_1(z))\left(1-S_2^*(A_2(z))\right)\right)\right] +$$
$$b(1-X(z))A_1(z)A_2(z)\left[zO^*(\lambda) - O^*(\lambda)\left((1-\theta)S_1^*(A_1(z)) + \theta S_1^*(A_1(z))S_2^*(A_2(z))\right) + z(1-O^*(\lambda))\left(\frac{V^*(\lambda b(1-X(z)))-1}{pV^*(\lambda b)}\right)\right] +$$
$$A_1(z)A_2(z)\left(\frac{1-V^*(\lambda b(1-X(z)))}{pV^*(\lambda b)}\right)\left[z - (O^*(\lambda) + X(z)(1-O^*(\lambda)))\left((1-\theta)S_1^*(A_1(z)) + \theta S_1^*(A_1(z))S_2^*(A_2(z))\right)\right]\Bigg\}$$

$$Nr^{(IV)} = \frac{T_1E(V)}{pV^*(\lambda b)}\left(1 - E(X)\left(1 - O^*(\lambda)\right)\right) + \quad T_1K_1E(X)$$
$$+\ bE(X) \quad \left(O^*(\lambda)(1 - T_1K_1) + \frac{T_1E(V)}{pV^*(\lambda b)}\left(1 - O^*(\lambda)\right)\right)$$

$$K_1 = \rho_1E(S_1) + \theta\rho_2E(S_2)K_2 = \rho_1M_2 + \rho_2M_1K3$$
$$= \rho_1^2E\left(S_1^2\right) + \theta\rho_2^2E\left(S_2^2\right) + 2\theta\rho_1\rho_2E(S_1)E(S_2)$$

$$M_1 = T_2\rho_1 + \alpha_1T_1^2\left(E\left(R_1^2\right) + E\left(D_1^2\right) + 2E(R_1)E(D_1)\right)$$

$$Nr^{(V)} = T_1^2\rho_1\rho_2K_1\left(\frac{T_2E(V) + T_1^2E\left(V^2\right)}{pV^*(\lambda b)} + 2\frac{T_1E(V)E(X)}{pV^*(\lambda b)}(1 - O^*(\lambda)) + E\left(X^2\right)\right) +$$

$$\left(T_1\rho_1\rho_2bE\left(X^2\right) + bE(X)K_2\right)\left(O^*(\lambda)(1 - T_1K_1) + \frac{T_1E(V)}{pV^*(\lambda b)}(1 - O^*(\lambda))\right) + T_1\rho_1\rho_2bE(X)$$

$$\left(\frac{T_2E(V) + T_1^2E\left(V^2\right)}{pV^*(\lambda b)}(1 - O^*(\lambda)) + 2\frac{T_1E(V)}{pV^*(\lambda b)}(1 - O^*(\lambda)) - O^*(\lambda)\left(M_1E(S_1) + \theta M_2E(S_2) + \ T_1^2K_3\right)\right) +$$

$$(1 - E(X)(1 - O^*(\lambda)) - T_1K_1)\left(\frac{T_1E(V)K_2}{pV^*(\lambda b)} + T_1\rho_1\rho_2\frac{T_2E(V) + T_1^2E\left(V^2\right)}{pV^*(\lambda b)}\right) - T_1\rho_1\rho_2\ \frac{T_1E(V)}{pV^*(\lambda b)}$$

$$\left(E\left(X^2\right)(1 - O^*(\lambda)) + 2T_1K_1E(X)(1 - O^*) + M_1E(S_1) + \theta M_2E(S_2) + T_1^2K_3\right) + T_1\left(\frac{T_1E(V)}{pV^*(\lambda b)} +\ E(X)\right)$$

$$\left[\rho_1\rho_2(\theta\alpha_2E(S_2)(T_2(E(R_2) + E(D_2)) + T_1^2\left(E\left(R_2^2\right) + E\left(D_2^2\right) + 2E(R_2)E(D_2)\right)\right) + \alpha_1E(S_1)\right]$$

$$\left(T_2(E(R_1) + E(D_1)) + T_1^2\left(E\left(R_1^2\right) + E\left(D_1^2\right) + 2E(R_1)E(D_1)\right)\right) + T_2(E(S_1) + \theta E(S_2)) + T_1^2K_3) + K_1K_2]$$

$$Dr^{(IV)} = 1 - E(X)\left(1 - O^*(\lambda)\right) - T_1K_1$$

$$Dr^{(V)} = (K_2 + T_2\rho_1\rho_2)(1 - E(X)(1 - O^*(\lambda)) - T_1K_1) -$$
$$T_1\rho_1\rho_2(E\left(X^2\right)(1 - O^*(\lambda)) + 2T_1K_1E(X)$$
$$(1 - O^*(\lambda)) + M_1E(S_1) + \theta M_2E(S_2) + T_1^2K_3\big)$$

$$I_0 = \frac{Nr_{I_0}}{Dr_{I_0}}Nr_{I_0} = \frac{T_1}{\lambda}\left(1 - E(X)\left(1 - O^*(\lambda)\right) - T_1K_1\right)Dr_{I_0}$$
$$= \frac{T_1E(V)}{pV^*(\lambda b)} \quad \left(1 - E(X)\left(1 - O^*(\lambda)\right) + bE(X)\left(1 - O^*(\lambda)\right)\right)$$
$$+ bE(X)O^*(\lambda) + T_1K_1E(X)\left(1 - bO^*(\lambda)\right)$$

$$\rho_1 = 1 + \alpha_1(E(R_1) + E(D_1))$$

$$\rho_2 = 1 + \alpha_2(E(R_2) + E(D_2))$$

$$T_1 = \lambda bE(X)$$

$$T_2 = \lambda bE\left(X^2\right)$$

$$M_2 = T_2\rho_2 + \alpha_2T_1^2\left(E\left(R_2^2\right) + E\left(D_2^2\right) + 2E(R_2)E(D_2)\right)$$

4.2. *The probability that the server is free during the retrial time*

Let I_1 be the probability that the server is idle during retrial time

$$I_1 = \frac{I_0(1 - O^*(\lambda))\left[\begin{array}{c} E(X) - 1 + T_1 \\ \left(\rho_1E(S_1) + \theta\rho_2E(S_2) + \dfrac{E(V)}{pV^*(\lambda b)}\right)\end{array}\right]}{1 - E(X)(1 - O^*(\lambda)) - T_1(E(S_1)\rho_1 + \theta E(S_2)\rho_2)} \tag{22}$$

4.3. *The probability that the server is busy with service in first phase*

Let Ω_1 be the probability that the server is busy with service in first phase

$$\Omega_1 = \frac{\lambda I_0E(S_1)\left[\frac{T_1E(V)}{pV^*(\lambda b)} + E(X)O^*(\lambda)\right]}{1 - E(X)(1 - O^*(\lambda)) - T_1(E(S_1)\rho_1 + \theta E(S_2)\rho_2)} \tag{23}$$

4.4. *The probability that the server is busy with service in second phase*

Let Ω_2 be the probability that the server is busy with service in second phase

$$\Omega_2 = \frac{\lambda I_0 \theta E(S_2)\left[\frac{T_1 E(V)}{pV^*(\lambda b)} + E(X)O^*(\lambda)\right]}{1 - E(X)(1 - O^*(\lambda)) - T_1(E(S_1)\rho_1 + \theta E(S_2)\rho_2)} \tag{24}$$

4.5. The probability of delay time in first phase

Let Q_1 be the delay probability in first phase

$$Q_1 = \frac{\lambda I_0 \alpha_1 E(S_1)E(D_1)\left[\frac{T_1 E(V)}{pV^*(\lambda b)} + E(X)O^*(\lambda)\right]}{1 - E(X)(1 - O^*(\lambda)) - T_1(E(S_1)\rho_1 + \theta E(S_2)\rho_2)} \tag{25}$$

4.6. The probability of delay time in second phase

Let Q_2 be the delay probability in second phase

$$Q_2 = \frac{\lambda I_0 \alpha_2 \theta E(D_2)E(S_2)\left[\frac{T_1 E(V)}{pV^*(\lambda b)} + E(X)O^*(\lambda)\right]}{1 - E(X)(1 - O^*(\lambda)) - T_1(E(S_1)\rho_1 + \theta E(S_2)\rho_2)} \tag{26}$$

4.7. The probability of repair time in first phase

Let Ψ_1 be the repair probability in first phase

$$\Psi_1 = \frac{\lambda I_0 \alpha_1 E(R_1)E(S_1)\left[\frac{T_1 E(V)}{pV^*(\lambda b)} + E(X)O^*(\lambda)\right]}{1 - E(X)(1 - O^*(\lambda)) - T_1(E(S_1)\rho_1 + \theta E(S_2)\rho_2)} \tag{27}$$

4.8. The probability of repair time in second phase

Let Ψ_2 be the repair probability in second phase

$$\Psi_2 = \frac{\lambda I_0 \alpha_2 \theta E(R_2)E(S_2)\left[\frac{T_1 E(V)}{pV^*(\lambda b)} + E(X)O^*(\lambda)\right]}{1 - E(X)(1 - O^*(\lambda)) - T_1(E(S_1)\rho_1 + \theta E(S_2)\rho_2)} \tag{28}$$

4.9. The probability that the server is on secondary job

Let Π be the probability that the server is on secondary job

$$\Pi = \left(qV^*(\lambda b)\right)^{j-1} \frac{\lambda I_0 E(V)(1 - qV^*(\lambda b))}{pV^*(\lambda b)} \tag{29}$$

4.10. Expected waiting time in the orbit

The expected waiting time in the orbit (W_q) is obtained by using Little's formula [42].

$$W_q = \frac{L_q}{\lambda E(X)} \tag{30}$$

L_q is given in Eqn. (21)

5. Results

In this section, we perform the numerical results in order to illustrate the effect of various parameters on performance measures of interactive telemedicine using MATLAB. We considered three different distributions for analysis namely: (i) exponential distribution; (ii) 2-stage Erlang distribution; and (iii) 2-stage hyper-exponential distribution. The numerical results are computed by choosing the arbitrary values to the parameters of the performance measures.

6. Discussion

In this paper, we derived the orbit size distribution for $M^X/G/1$ retrial queueing system with balking, delaying repair and randomized multi-vacation policy. Such a model was considered by Priya et al. [41] without retrial, repair and secondary jobs. Tables 1–3 shows that the expected number of patients in the orbit and waiting time in the orbit decreases with an increasing the values of p. Tables 1 and 4 shows that the expected number of patients in the orbit and waiting time in the orbit increases with an increasing the values of b. Table 2 shows that the expected number of patients in the orbit and waiting time in the orbit decreases with an increasing the retrial rate δ. Table 5 shows that the expected number of patients in the orbit and waiting time in the orbit decreases with an increasing the repair rate γ_2. Table 6 shows that the expected number of patients in the orbit and waiting time in the orbit increases with an increasing the breakdown probability α_2. Table 7 shows that the expected number of patients in the orbit and waiting time in the orbit decreases with an increasing the delay rate η_1.

Fig. 2 shows that the idle probability I_0 decreases with an increasing the arrival rate λ and varying optional service probability θ. Fig. 3 shows that the expected number of patients in the orbit L_q is increases with an increasing the retrial rate δ and varying arrival rate λ. The expected number of patients in the orbit decreases with an increasing the probability values of p and varying optional service probability θ is shown in Fig. 4. The expected number of patients in the orbit increases with an increasing the breakdown probability α_1 is shown in Fig. 5. The expected number of patients in the orbit decreases with an increasing the repair rate γ_1 is shown in Fig. 6. The expected number of patients in the orbit decreases with an increasing the delay rate η_2 is shown in Fig. 7.

7. Conclusion

To tackle the difficulties of interactive telemedicine, the queueing system was framed by including the novel features namely provision of unreliable repairable retrial queueing system which is capable of providing the first essential service and second optional service, recovery of the failed server via. delayed repair, balking, randomized multi-secondary job and bulk arrivals. The steady state system equations are derived by using the method of supplementary variables. Also, we have derived the performance measures for the proposed queueing model. Further, some numerical results are performed to study the effect of various parameters of interactive telemedicine. The focus of our contribution is providing information by analyzing the queuing system and update their utility functions. The present retrial queueing system can be extended to other types of secondary job, negative customers, and repairs such as multi-optional repairs.

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Conflicts of interest

The authors do not have any conflict of interests.

References

[1] Rezaeibagha F, Mu Y. Practical and secure telemedicine systems for user mobility. J Biomed Inform 2017;78:24–32. https://doi.org/10.1016/j.jbi.2017.12.011.
[2] Sood S, Mbarika V, Jugoo S, Dookhy R, Doarn CR, Prakash N, et al. What is telemedicine? A collection of 104 peer-reviewed perspectives and theoretical underpinnings. Telemed E-Health 2007;13:573–90.
[3] Kamsu-Foguem B, Tiako PF, Fotso LP, Foguem C. Modeling for effective collaboration in telemedicine. Telemat Informatics 2015;32:776–86. https://doi.org/10.1016/j.tele.2015.03.009.
[4] Klaassen B, van Beijnum BJF, Hermens HJ. Usability in telemedicine systems—a literature survey. Int J Med Inform 2016;93:57–69.
[5] AlDossary S, Martin-Khan MG, Bradford NK, Smith AC. A systematic review of the methodologies used to evaluate telemedicine service initiatives in hospital facilities. Int J Med Inform 2017;97:171–94.
[6] Raskas MD, Gali K, Schinasi DA, Telemedicine Vyas S, Urgent Pediatric. Care: a vision into the future. Clin Pediatr Emerg Med 2017;18:24–31.
[7] Broens THF, in't Veld RMHA, Vollenbroek-Hutten MMR, Hermens HJ, van Halteren AT, Nieuwenhuis LJM. Determinants of successful telemedicine implementations: a literature study. J Telemed Telecare 2007;13:303–9.

[8] Jennett PA, Hall LA, Hailey D, Ohinmaa A, Anderson C, Thomas R, et al. The socio-economic impact of telehealth: a systematic review. J Telemed Telecare 2003;9:311–20.
[9] Dullet NW, Geraghty EM, Kaufman T, Kissee JL, King J, Dharmar M, et al. Impact of a university-based outpatient telemedicine program on time savings, travel costs, and environmental pollutants. Value Heal 2017;20:542–6.
[10] The american telemedicine association, what isTelemedicine?. 2017 [n.d].
[11] Whan P, Brown NA, Wootton R. A bibliographic snapshot of the telemedicine citation literature. J Telemed Telecare 2006;12:95–102.
[12] Komkhao M, Halang WA. Recommender systems in telemedicine. IFAC Proc Vol 2013;46:28–33.
[13] Hasan J. Effective telemedicine project in Bangladesh: special focus on diabetes health care delivery in a tertiary care in Bangladesh. Telemat Informatics 2012;29:211–8.
[14] Chakraborty C, Gupta B, Ghosh SK. A review on telemedicine-based WBAN framework for patient monitoring. Telemed E-Health 2013;19:619–26.
[15] Chaudhry SI, Phillips CO, Stewart SS, Riegel B, Mattera JA, Jerant AF, et al. Telemonitoring for patients with chronic heart failure: a systematic review. J Card Fail 2007;13:56–62.
[16] Nelson CA, Takeshita J, Wanat KA, Bream KDW, Holmes JH, Koenig HC, et al. Impact of store-and-forward (SAF) teledermatology on outpatient dermatologic care: a prospective study in an underserved urban primary care setting. J Am Acad Dermatol 2016;74:484–90.
[17] Kokesh J, Ferguson AS, Patricoski C. Preoperative planning for ear surgery using store-and-forward telemedicine. Otolaryngol Neck Surg 2010;143:253–7.
[18] Salles N, Lafargue A, Cressot V, Glenisson L, Barateau M, Thiel E, et al. Global geriatric evaluation is feasible during interactive telemedicine in nursing homes. Eur Res Telemedicine/La Rech Eur En Télémédecine 2017;6:59–65.
[19] Segato F, Masella C. Telemedicine services: how to make them last over time. Heal Policy Technol 2017;6:268–78.
[20] Drago A, Winding TN, Antypa N. Videoconferencing in psychiatry, a meta-analysis of assessment and treatment. Eur Psychiatry 2016;36:29–37.
[21] Cox DR. The analysis of non-Markovian stochastic processes by the inclusion of supplementary variables. Math. Proc. Cambridge Philos. Soc. 1955;51:433–41.
[22] Gross D. Fundamentals of queueing theory. John Wiley & Sons; 2008.
[23] Artalejo JR, Gómez-Corral A. Retrial queueing systems. Math Comput Model 1999; 30 [xiii–xv].
[24] Yang T, Templeton JGC. A survey on retrial queues. Queueing Syst 1987;2:201–33.
[25] Arivudainambi D, Godhandaraman P. Analysis of an MX/G/1 retrial queue with two phases of service, balking, feedback and K optional vacations. Int J Inf Manag Sci 2012:199–215.
[26] Arivudainambi D, Godhandaraman P. Retrial queueing system with balking, optional service and vacation. Ann Oper Res 2015. https://doi.org/10.1007/s10479-014-1765-5.
[27] Madan KC. An M/G/1 queue with second optional service. Queueing Syst 2000;34:37–46.
[28] Choudhury G, Deka K. An M/G/1 retrial queueing system with two phases of service subject to the server breakdown and repair. Perform Eval 2008;65:714–24.
[29] Choudhury G, Tadj L, Deka K. A batch arrival retrial queueing system with two phases of service and service interruption. Comput Math Appl 2010;59:437–50.
[30] Choudhury G, Deka K. An M X/G/1 unreliable retrial queue with two phases of service and Bernoulli admission mechanism. Appl Math Comput 2009;215:936–49.
[31] Ke J, Huang K, Lea W. The performance measures and randomized optimization for an unreliable server M [x]/G/1 vacation system. Appl Math Comput 2011;217:8277–90. https://doi.org/10.1016/j.amc.2011.03.008.
[32] Ke J, Huang K, Lea W. A batch arrival queue under randomised multi-vacation policy with unreliable server and repair, vol. 43; 2012. p. 552–65.
[33] Ke J-C, Yang D-Y. Analysis of a batch arrival queue with vacation policy and exceptional service. Commun Stat Methods 2016;45:3637–57.
[34] Choudhury G, Deka M. A single server queueing system with two phases of service subject to server breakdown and Bernoulli vacation. Appl Math Model 2012;36:6050–60. https://doi.org/10.1016/j.apm.2012.01.047.
[35] Ke, JC; Huang K, Bin, Analysis of batch arrival queue with randomized vacation policy and an un-reliable server. J Syst Sci Complex n.d.:1–19.
[36] Yang D, Chang F, Ke J. On an unreliable retrial queue with general repeated attempts and J optional vacations, vol. 40; 2016. p. 3275–88. https://doi.org/10.1016/j.apm.2015.10.023.
[37] Wang J. An M/G/1 queue with second optional service and server breakdowns. Comput Math Appl 2004;47:1713–23.
[38] Ke J-C, Huang K-B. Analysis of an unreliable server M [x]/G/1 system with a randomized vacation policy and delayed repair. Stoch Model 2010;26:212–41.
[39] Choudhury G, Deka M. A batch arrival unreliable server delaying repair queue with two phases of service and Bernoulli vacation under multiple vacation policy. Qual Technol Quant Manag 2016:1–30.
[40] Singh CJ, Jain M, Kaur S. Performance analysis of bulk arrival queue with balking, optional service, delayed repair and multi-phase repair. Ain Shams Eng J 2017:1–11. https://doi.org/10.1016/j.asej.2016.08.025.
[41] Priya V, Subha S, Balamurugan B. Analysis of performance measures to handle medical E-commerce shopping cart abandonment in cloud. Informatics Med Unlocked 2017;8:32–41. https://doi.org/10.1016/j.imu.2017.03.003.
[42] Little JDC, Graves SC. Little's law. Build. Intuit., Springer; 2008. p. 81–100.