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PERIPHERAL LAYER VISCOSITY ON THE STENOTIC BLOOD VESSELS FOR HERSCHEL-BULKLEY FLUID MODEL

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Abstract

This paper deals with a theoretical investigation of blood flow in an arterial fragment with the existence of stenosis. The stream-wise blood is treated as steady and it is composed of two layers (the central core and plasma). The blood is taken to be non-Newtonian liquid described with help of Herschel-Bulkley fluid model. The artery is simulated as a cylindrical tube. Flow of blood is considered as steady. An extensive quantitative exploration has been performed through numerical computations of the flow physical parameters (the velocity, mass flux and shear stress). It is found that the mass-flux reduced as the consistency of peripheral layer fluid decreases, this happens due to the enhancement of pseudo plastic nature of the blood.

Keywords: Stenosis, Herschel-Bulkley fluid model, Peripheral layer, Pseudo plastic

1. Introduction

To our knowledge in the literature, it has been observed that, in some experimental studies done by Whitemore [1], Forrestor and Young [2], Shukla et al. [3], Jain et al. [4] and Neeraja et al. [5, 6] on blood flows in tubes of smaller diameters indicates that under certain flow conditions mainly at low shear rate blood behaves like a non-Newtonian fluid. The non-Newtonian behavior of blood is due to the suspension of RBCs in plasma. An analysis by Kapur [7] indicates that Casson fluid model and Herschel-Bulkley fluid (H-B fluid) model have non-zero yield stress and exhibits a non-Newtonian fluid behavior. Hence, these models are more suitable to study blood flows in narrow arteries. It is reported that, in recent studies, the existence of peripheral layer plays a significant role in functioning of the unhealthy or diseased arterial system (see Shukla et al. [8], Cocklet and Goldsmith [9], Pralhad and Schultz [10], Sharan and Propel [11], Srivastava [12], Srivastava and Saxena [13] and Chakravarty and Dutta

[14]). Shankar and Usik Lee [15] studied two-fluid Herschel-Bulkley model for blood flow in catheterized arteries and studied the effect of catheter ratio with peripheral layer thickness. Charkravarthy et al. [16] studied the effect of peripheral layer viscosity on the unsteady two-layered pulsatile blood flow in a stenosed flexible artery. Shukla et al [17] examined the influence of peripheral layer viscosity on flow with slight stenosis by considering blood as a power-law fluid model. It is found that the wall shear stress decreases as peripheral layer viscosity decreases.

Researchers in biomedical engineering and scientific field have explored extraordinary results in the flow and heat transfer of non-Newtonian fluid from different geometries because of its ever increasing biomedical industrial applications in endoscopy, heart beat controlling processes, polymer technology, metallurgy, petroleum industry, cooling of nuclear reactors, power generation, etc. Due to this significance the authors [20-24] discussed the non-Newtonian flow characteristics over various geometries and physical effects. With this they concluded that non-Newtonian fluids have tendency to control the flow behavior.

Motivated by the above evidences an attempt is made to study the effect of peripheral layer viscosity in a mild stenosed artery by considering blood as a Herschel – Bulkley fluid model. The various physical parameters of the flow velocity, volume flow rate and shear stress are investigated for the variations in yield stress, viscosity and stenosis height.

2. Mathematical Formulation

The following assumptions were made in the formulation of the physical problem:

- The artery is assumed to be a mild stenotic cylindrical tube.

- The stenotic protuberance is assumed to be an axisymmetric surface produced by a cosine curve following Young [18].
- Blood is considered to be Herschel – Bulkley fluid model
- The blood flow is assumed to be steady and laminar and fully developed.

The radius of the stenosis can be described mathematically in the form:

$$\frac{R(\bar{z})}{R_0} = 1-\frac{\delta_s}{2R_0}\left[1+\cos\frac{2\pi}{L_0}\left(\bar{z}-d-\frac{L_0}{2}\right)\right], \quad d \le \bar{z} \le L_0 + d$$

$$= 1, \qquad \text{elsewhere} \qquad (1)$$

where $R(\bar{z})$ is the radius of the stenosedartery, R_0 is the constant radius , L_0 is the length of the stenosis and δ_B is its maximum height ($\delta_s \langle\langle R_0$, Fig.1).

To see the effects of peripheral layer viscosity on the flow behavior, we consider the viscosity function $\mu(r)$ as follows Lih [19]:

$$\mu(r) = \mu_1 , \qquad 0 \le r \le R_1(z),$$

$$= \mu_2 ; \qquad R_1(z) \le r \le R(z), \qquad (2)$$

where $\mu_{1,}$, μ_2 are the viscosities of the central and the peripheral layers respectively. The function $R_1(z)$ represents the geometry of the central layer which is taken to be

$$\frac{R_1(z)}{R_0} = \alpha - \frac{\delta_B}{2R_0}\left[1+\cos\frac{2\pi}{L_0}(z-d-\frac{L_0}{2}\right], \qquad d \le z \le L_0 + d$$

$$= \alpha \qquad , \quad \text{elsewhere} \qquad (3)$$

where α is the ratio of central core radius to the tube radius in the non- stenotic region. The constant δ_B is the maximum bulging of the interface at z = d +(L_0/2) due to the presence of stenosis.

The basic equations governing the flow is (see Shukla et al. [8])

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau) = -\frac{dp}{dz}, \tag{4}$$

The constitutive equation for Herschel – Bulkley fluid model (see Kapur [7]) is given by

$$\tau = \mu(r)^{-\frac{1}{n}}\left(-\frac{\partial u}{\partial r}\right)^{\frac{1}{n}} + \theta, \qquad if\ \tau \geq \theta,$$

$$\frac{\partial u}{\partial r} = 0 \qquad , \qquad if\ \tau \leq \theta \tag{5}$$

where θ is the yield stress of the fluid, n is the power law index.

The appropriate boundary conditions are

$u_f = 0$ at $r = R(z)$, $u_p = 0$ at $r = R_1(z)$,

$$\tau_p = \tau_f \text{ at } r = R_1(z), \quad \frac{\partial u_p}{\partial r} = 0 \text{ at } r = 0. \tag{6}$$

Where u_p is the core velocity and u_f is the peripheral velocity, τ_p and τ_f are shear stresses in the core and peripheral regions respectively.

3. Method of Solution

Using equation (2) and equations (3) – (6) we obtain the core velocity u_p and the peripheral velocity u_f as follows

$$u_p = -\frac{2\mu_1}{P(n+1)}\left[\left(\frac{rP}{2}-\theta\right)^{(n+1)} - \left(\frac{RP}{2}-\theta\right)^{(n+1)}\right], \tag{7}$$

$$u_f = -\frac{2\mu_2}{P(n+1)}\left[\left(\frac{R_1P}{2}-\theta\right)^{(n+1)} - \left(\frac{RP}{2}-\theta\right)^{(n+1)}\right], \quad (8)$$

where $P = -\frac{dp}{dz}$.

The shear stress at the wall is given by

$$\tau_w\big|_{r=R(z)} = \mu_1 \frac{\partial u}{\partial r}\bigg|_{r=R(z)}, \quad (9)$$

The volumetric flow rate Q is given by

$$Q = Q_p + Q_f, \quad (10)$$

$$Q_p = \int_0^{R_1} 2\pi r \mu_1 \, dr = -\frac{4\pi\mu_1}{P(n+1)}[A_1 + A_2],$$

$$Q_f = \int_{R_1}^{R} 2\pi r \mu_2 \, dr = -\frac{4\pi\mu_2}{P(n+1)}[A_3 + A_4],$$

where

$$\varepsilon_1 = \delta_s / 2R_0, \; u_1 = \frac{a_{16}}{2n+1}\left(R^{1+2n} - r^{1+2n}\right) + \frac{a_{15}}{2+2n}\left(R^{2+2n} - r^{2+2n}\right) + \frac{a_9}{n}\left(R^n - r^n\right) + \frac{a_{13}}{n+1}\left(R^{1+n} - r^{1+n}\right)$$

$$, A_2 = \frac{4\theta}{P^2(n+2)(n+3)}, \; A_3 = \frac{2R}{P(n+2)}\left(\frac{RP}{2}-\theta\right)^{n+2} - \frac{2R_1}{P(n+2)}\left(\frac{R_1P}{2}-\theta\right)^{n+2},$$

$$A_4 = \frac{4}{P^2(n+2)(n+3)}\left(\frac{R_1P}{2}-\theta\right)^{n+3} - \frac{4}{P^2(n+2)(n+3)}\left(\frac{RP}{2}-\theta\right)^{n+3}.$$

4. Results and Discussion

In this model, we investigated the physical flow characteristics of Herschel – Bulkley fluid model of blood flow in an artery with mild stenosis in the presence of peripheral layer viscosity. We presented the numerical solutions for Aorta. The power-law index n = 0.95, the center of the stenosis is taken as $\delta_s = 0.2,\ \alpha = 1.05$ the viscosity of the Herschel – Bulkely fluid is considered as $\mu_1 = 3.5c.p$ and radius of the Aorta R_0 = 1.7 cm. Fig. 2 shows the effect of θ on core velocity profiles shown graphically. It is seen that velocity decreases with the increase in θ and the velocity profiles takes parabolic curve. At the axis of the artery the velocity is more and keeps reducing when reaches the artery wall. Due to this cause we saw decrement in the velocity profiles.

Figs. 3 and 4 illustrate the influence of yield stress θ and ε_1 on the peripheral velocity u_f. In Fig.3, it is clear that the peripheral velocity profiles u_f enhances with increasing values of yield stress θ. The velocity achieves its maximum at z = 0.2. The velocity profiles are shown mixed performance, initially the velocity fields increases up to z = 0.2 and retains reducing up to z = 0.4 and again jumps repeating the same action for the advanced values of z. This is because of the arterial wall is considered as a cosine curve i.e. the velocity is acts periodic nature in the peripheral region. In Fig. 4 it is observed that the opposite performance with increasing values of ε_1. Figs. 5 and 6 represent the volumetric flow rate Q profiles for various values of θ versus ε_1. It is observed that Q declines drastically as ε_1 increases. This is happens because of the stenosis, the gap or the area in which the flow diminishes in the channel. But the volumetric flow rate is growing with increasing values of θ. An opposite action is seen in the case of viscosity ratio as displayed in Fig.6. These explanations are accurate for any physiological situation.

Figs. 7–8 describe the shear stress profiles τ versus z. It is seen that the shear stress dispersion is periodic nature at the stenotic wall r = R(z). The higher values (0 to 0.2) of axial distance *z* shear stress increases and then it is decreases as z increases from 0.2 to 0.4, this actions indicates in the flow is periodic in nature. The supreme shear stress happens at the center of the stenosis. In Fig.7 shear stress dispersion with deviations in yield stress is plotted. The rise in yield stress value increases the shear stress as perceived Fig.8. The similar behavior is detected as an increase in ε_1. The volume flow rate (Q) versus pressure gradient (P) is tabulated for various numerical values of μ_2 and θ in Table 1. It is observed from that the volume flow rate rises with the higher values of P. Moreover, similar to the preceding deviations of Q with θ and ε_1 the volumetric flow goes on decreasing with increase values of the viscosity ratio μ_2. But, Q rises with the increasing in θ. From the numerical solutions presented in Table 2, it is noted that volume flow rate is low in two-layered model compared to the single-layered model. The similar observation is given by Shukla et al [8].

5. Conclusions

1. The existence of peripheral layer will be useful for understanding the functioning of the diseased arterial system

2. The present study reveals that the physiological model can be applied to stenosis affected small vessels where the non-Newtonian behavior is more prominent such as carotid artery, femoral arteries, coronaries and arterioles.

3. The mass-flux reduced as the consistency of peripheral layer fluid decreases, this happens due to the enhancement of pseudo plastic nature of the blood.

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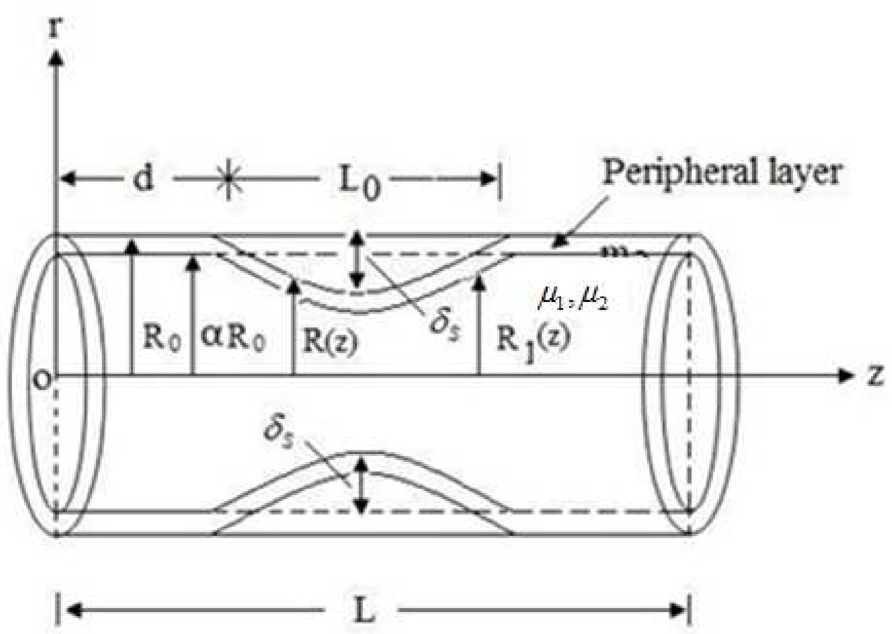

Fig. 1: Geometry of a stenosed artery with peripheral layer

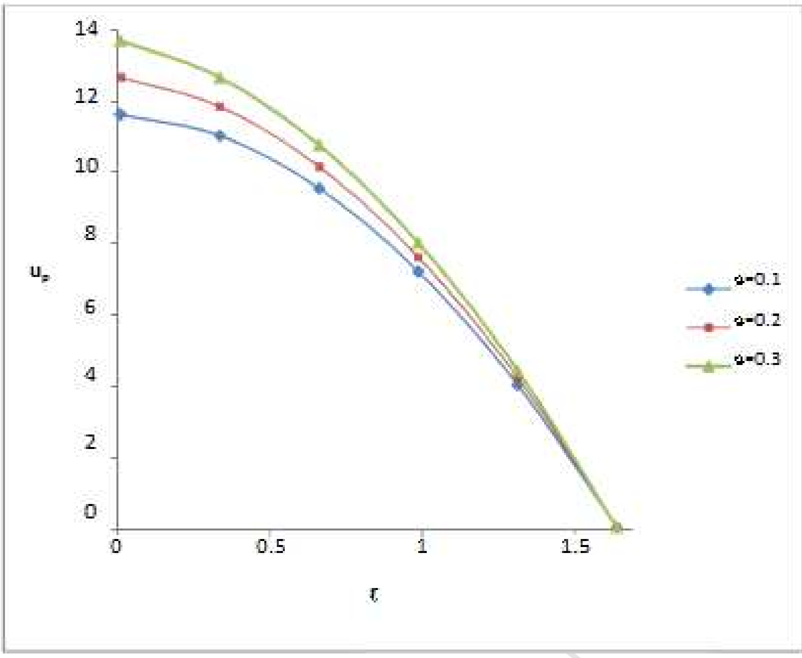

Fig. 2: Plug core velocity u_p versus r profiles for values θ when $\mu_2 = \mu_1 / 10$ ε_1 =0.2

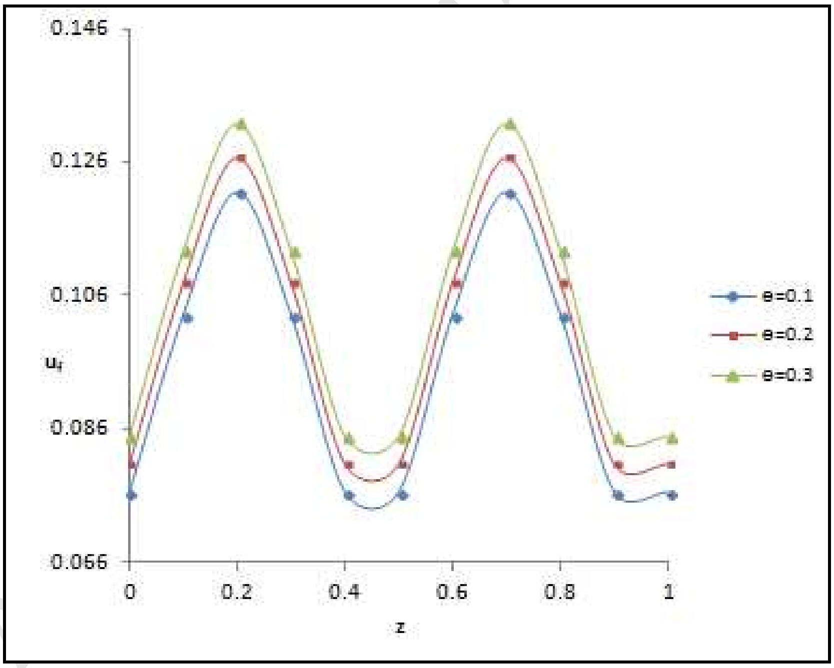

Fig. 3: Peripheral velocity u_f versus z profiles for values 6 when $\mu_2 = \mu_1 / 10$ ε_1 =0.2

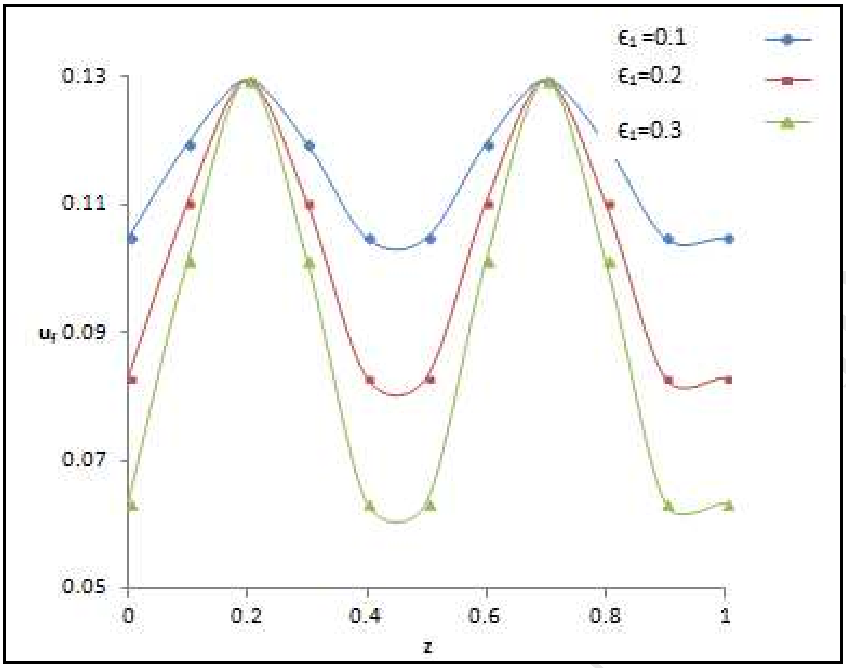

Fig. 4: Peripheral velocity u_f versus z ε_1 for values θ when $\mu_2 = \mu_1/10$

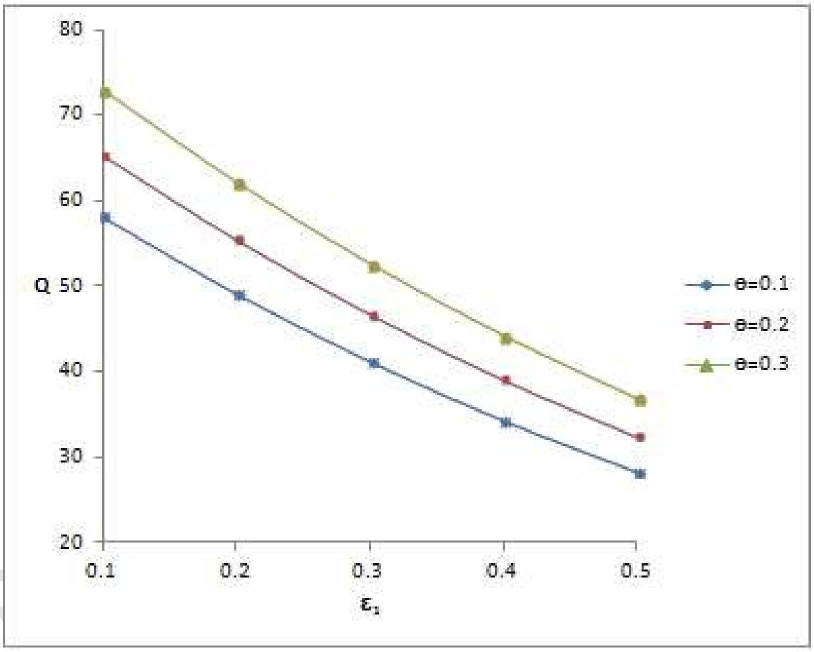

Fig. 5: Mass-flux Q variation vs. aspect ratio profiles for values ε_1

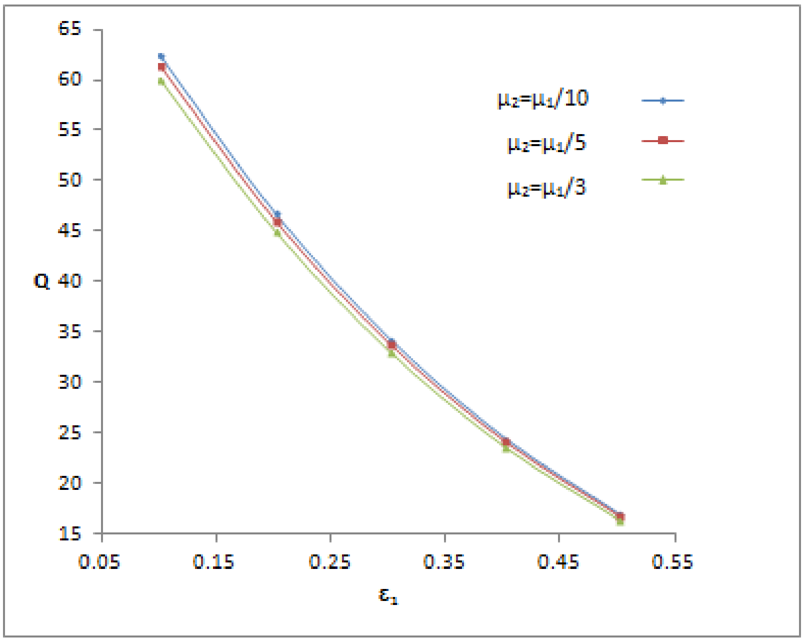

Fig. 6: Mass-flux Q variation versus aspect ratio ε_1 for values viscosity ratio when $\theta = 0.3$

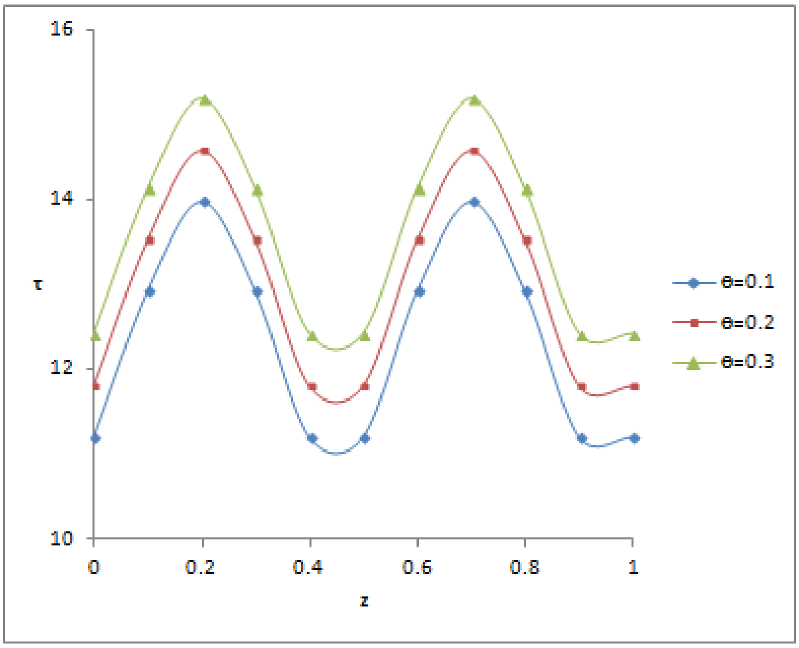

Fig.7: Shear stress variation τ versus z for values θ when $\mu_2 = \mu_1/10$, $\varepsilon_1 = 0.2$

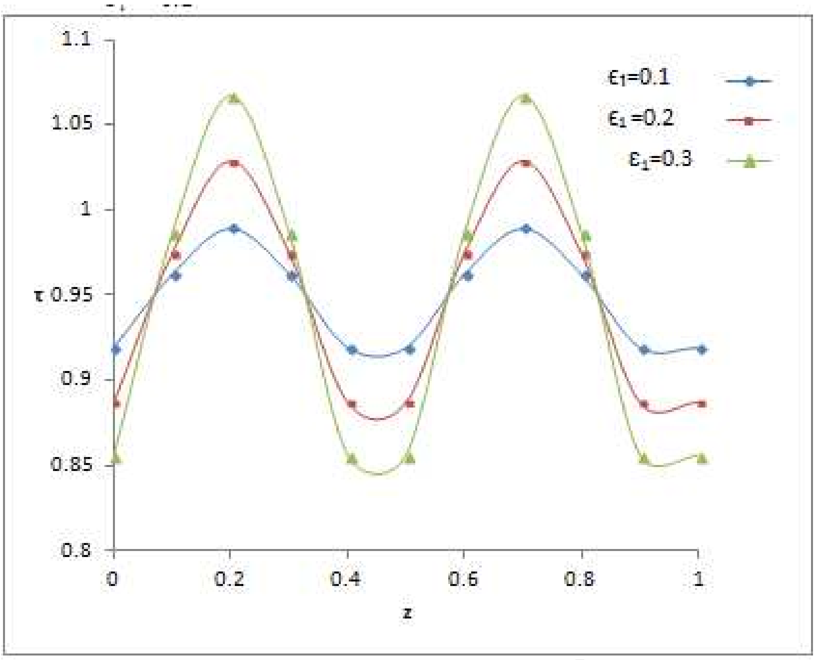

Fig. 8: Shear stress variation versus z axis for values ε_1 when $\mu_2 = \mu_1/10$, $\theta = 0.3$

Table 1: Pressure-flow rate relationship for different values of μ_2 and θ when $\varepsilon_1 = 0.2$

P	Q					
	$\mu_2 = \mu_1/10$	$\mu_2 = \mu_1/5$	$\mu_2 = \mu_1/3$	θ=0.1	θ=0.2	θ=0.3
5	59.48176	58.47899	57.14197	49.72296	56.1249	62.94163
15	144.1209	141.6	138.2387	135.3945	141.179	147.0958
25	226.5221	222.5288	217.2044	218.113	223.6997	229.3638
35	307.4018	301.9643	294.7143	299.1739	304.6456	310.1716
45	387.1841	380.3223	371.1734	379.0814	384.4727	389.9059
55	466.1133	457.8428	446.8154	458.1056	463.4356	468.7995

Table 2: Volumetric flow rate Q distribution in various blood vessels

Blood vessels Radius R_0 cms	Two layered H-B fluid model Q		Single layered H-B fluid model Q	
	$\varepsilon_1 = 0.1$	$\varepsilon_1 = 0.15$	$\varepsilon_1 = 0.1$	$\varepsilon_1 = 0.15$
Arteriole 0.008	0.023933	0.024468	0.023987	0.02452
Coronary 0.15	0.027472	0.024271	0.027868	0.024354
Carotid 0.4	0.253161	0.185929	0.283987	0.207249
Femoral 0.5	0.56206	0.433316	0.638	0.490247
Aorta 1.7	59.26	54.72643	69.01058	63.71778

Highlights

1. The existence of peripheral layer will be useful for understanding the functioning of the diseased arterial system

2. The present study reveals that the physiological model can be applied to stenosis affected small vessels where the non-Newtonian behavior is more prominent such as carotid artery, femoral arteries, coronaries and arterioles.