ARTICLE IN PRESS
Ain Shams Engineering Journal (2015) **xxx**, xxx–xxx

Ain Shams University
Ain Shams Engineering Journal
www.elsevier.com/locate/asej
www.sciencedirect.com

MECHANICAL ENGINEERING

Peristaltic transport of a Jeffrey fluid in contact with a Newtonian fluid in an inclined channel

A. Kavitha [a], R. Hemadri Reddy [a,*], R. Saravana [b], S. Sreenadh [c]

[a] *School of Advanced Sciences, VIT University, Vellore 632014, India*
[b] *Department of Mathematics, Madanapalle Institute of Technology and Science, Madanapalle 517 325, India*
[c] *Department of Mathematics, Sri Venkateswara University, Tirupati 517 502, India*

Received 2 June 2015; revised 3 September 2015; accepted 18 October 2015

KEYWORDS
Jeffrey fluid;
Newtonian fluid;
Interface;
Inclined channel

Abstract The peristaltic flow of a Jeffrey fluid in contact with a Newtonian fluid in an inclined symmetric channel is analyzed under the assumptions of long wavelength and low Reynolds number. The channel is inclined at angle of β with the horizontal. This model is useful to understand the two fluid flow behaviors in physiological systems. The velocity field, stream function, interface shape, pressure rise and frictional force at the wall over one cycle of wavelength are obtained and the results are shown graphically. It is observed that the variation of the interface shape gives rise to thinner peripheral region with increasing Jeffrey parameter λ_1.

1. Introduction

Peristalsis is well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. This mechanism occurs in food transport through esophagus, movement of chyme in the gastrointestinal tract, transport of lymph in the lymphatic vessels, vasomotion of small blood vessels and urine transport from kidney to bladder through the ureter. In practical peristaltic pumps are designed by engineers for pumping corrosive fluids without contact with the walls of the pumping machinery. Applying a wave frame of reference, Jaffrin and Shapiro [1] made a detailed analysis on the peristaltic pumping of a viscous fluid under long wavelength and low Reynolds number assumptions. The fixed frame analysis of peristaltic flow of generalized Newtonian fluid with the influence of heat and mass transfer has been presented by Vajravelu et al. [2].

Among several non-Newtonian fluid models proposed for biofluids, Jeffrey model is one of the simplest non-Newtonian fluid models accepted by the researchers. Hayat et al. [3] analyzed the peristaltic transport of a compressible Jeffrey fluid in a circular tube. Kothandapani and Srinivas [4] studied the effect of magnetic field on the peristaltic transport of a Jeffrey fluid in an asymmetric channel. Nadeem and Akbar [5] studied the peristaltic flow of an incompressible Jeffrey fluid with variable viscosity in an asymmetric channel. Pandey and Tripathi [6] discussed the peristaltic transport of a Jeffrey fluid in a circular tube as well as in a channel. Kavitha et al. [7] presented the peristaltic transport of a Jeffrey fluid in

* Corresponding author. Mobile: +91 9791452220.
E-mail addresses: drrhreddy@gmail.com, drrhreddy@vit.ac.in (R. Hemadri Reddy).
Peer review under responsibility of Ain Shams University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.asej.2015.10.014
2090-4479 © 2015 Faculty of Engineering, Ain Shams University. Production and hosting by Elsevier B.V.

a porous channel with suction and injection. Many studies have been carried out for the peristaltic transport of a Jeffrey fluid [8–12].

It is observed in some physiological systems such as esophagus, small blood vessels and ureter, the wall structure doing the pumping is typically coated with a fluid of different properties from those of the fluid being pumped. In order to understand the effect of fluid coating on the transport, the single fluid analysis of peristaltic pumping is extended to two fluid analysis by including peripheral layer of different viscosity. Such an analysis was first done by Shukla et al. [13] for channel and axisymmetric geometries. For non-uniform axisymmetric tubes, Srivastava et al. [14] made an important contribution in peristaltic pumping. Brasseur et al. [15] analyzed the influence of a peripheral layer of different viscosity on peristaltic pumping with Newtonian fluids. Ramachandra Rao and Usha [16] discussed the pumping of two immiscible viscous fluids in a circular tube. The interface between the two layers is determined from a transcendental equation in the core radius. Vajravelu et al. [17] studied peristaltic pumping of a Herschel–Bulkley fluid in contact with a Newtonian fluid. Vajravelu et al. [18] studied Peristaltic transport of a Casson fluid in contact with a Newtonian fluid in a circular tube with permeable wall. Narahari and Sreenadh [19] presented the peristaltic transport of a Bingham fluid in contact with a Newtonian fluid. Hari Prabakaran et al. [20] analyzed the peristaltic pumping of a Bingham fluid in contact with Newtonian fluid in an inclined channel. All these authors have specified the interface shape.

Motivated by the above studies, we propose to study the peristaltic pumping of a Jeffrey fluid in contact with a Newtonian fluid in an inclined channel. The velocity field, the stream function and the pressure rise over one cycle of wavelength are obtained.

2. Mathematical formulation and solution

Consider the peristaltic transport of a biofluid consisting of two immiscible and incompressible fluids of different viscosities μ_1 and μ_2 occupying the core by a Jeffrey fluid and peripheral layer by a Newtonian fluid in an inclined channel. The channel is inclined at angle of β with the horizontal. The half width of the channel is a. The wall deformation due to the propagation of an infinite train of peristaltic waves is given by

$$Y = H(X,t) = a + b\sin\frac{2\pi}{\lambda}(X - ct) \tag{1}$$

where λ is the wavelength, b is the amplitude and c is the wave speed.

The subsequent deformation of the interface separating the core and the peripheral layers is denoted by $Y = H_1(X,t)$ (Fig. 1) which is not known a priori.

2.1. Equations of motion

Under the assumptions that the channel length is an integral multiple of the wavelength λ, the pressure difference across the ends of the channel is a constant and the periodicity of the interface is same as that of the peristaltic wave. The flow becomes steady in the wave frame (x, y) moving with the velocity c away from the fixed frame (X, Y) called laboratory frame. The transformation between these two frames is given by

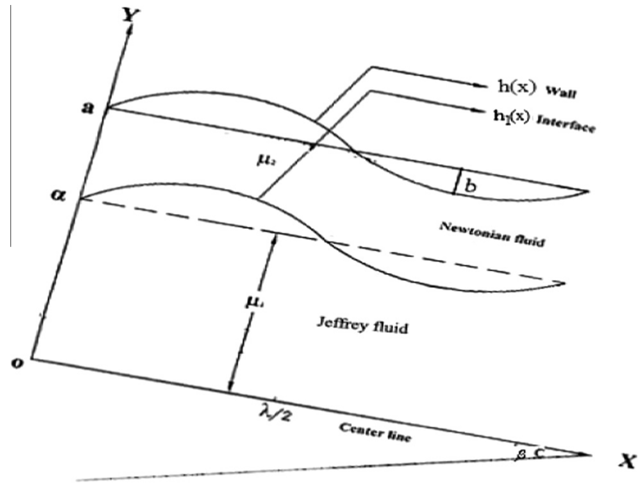

Figure 1 Physical model.

$$x = X - ct,\ y = Y,\ u(x,y) = U(X - ct, Y) - c,\ v(x,y) = V(X - ct, Y),\ p(x) = P(X,t),\ \psi = \Psi - Y \tag{2}$$

where ψ and Ψ are the stream functions in the wave and laboratory frames respectively. Using the non-dimensional quantities,

$$\bar{x} = \frac{x}{\lambda},\quad \bar{y} = \frac{y}{a},\quad \bar{h} = \frac{h}{a},\quad \bar{h}_1 = \frac{h_1}{a},\quad \bar{t} = \frac{ct}{\lambda},$$
$$\overline{P} = \frac{Pa^2}{\mu_1\lambda c},\quad \phi = \frac{b}{a},\quad \overline{S} = \frac{a}{\mu_1 c}S,\quad \bar{\psi}^{(i)} = \frac{\psi^{(i)}}{ac},\quad \bar{q} = \frac{q}{ac},$$
$$\overline{F} = \frac{Fa}{\mu_1\lambda c},\quad \bar{u}^{(i)} = \frac{u^{(i)}}{c} = \frac{\partial\psi^{(i)}}{\partial\bar{y}},\quad \eta = \frac{\rho g a^2}{c\mu_1},$$
$$\bar{v}^{(i)} = \frac{v^{(i)}\lambda}{ac} = \frac{-\partial\psi^{(i)}}{\partial\bar{x}}(i = 1,2),$$
$$\bar{\mu} = \begin{cases} 1, & 0 \leqslant \bar{y} \leqslant \bar{h}_1 \\ \mu\left(=\frac{\mu_2}{\mu_1}\right), & \bar{h}_1 \leqslant \bar{y} \leqslant \bar{h} \end{cases} \tag{3}$$

where $\bar{u}^{(i)}$ and $\bar{v}^{(i)}$ are the $\bar{x}$ and $\bar{y}$ components of velocities in the wave frame.

The equations governing the flow of Jeffrey fluid in core region (for details see Kavitha et al. [7]) and Newtonian fluid in peripheral region in wave frame analysis under the long wavelength and low Reynolds number assumptions are (dropping the bars)

$$\frac{\partial}{\partial y}\left[\frac{1}{(1+\lambda_1)}\frac{\partial^2\psi^{(1)}}{\partial y^2}\right] + \eta\sin\beta = \frac{\partial P}{\partial x} \tag{4}$$

$$0 = \frac{\partial p}{\partial y} \tag{5}$$

and

$$\frac{\partial^2}{\partial y^2}\left[\mu\frac{\partial^2\psi^{(2)}}{\partial y^2}\right] + \eta\sin\beta = \frac{\partial P}{\partial x} \tag{6}$$

Please cite this article in press as: Kavitha A et al., Peristaltic transport of a Jeffrey fluid in contact with a Newtonian fluid in an inclined channel, Ain Shams Eng J (2015), http://dx.doi.org/10.1016/j.asej.2015.10.014

The dimensionless boundary conditions are

$$\psi^{(1)} = 0 \quad \text{at} \quad y = 0 \tag{7}$$

$$\psi^{(1)}_{yy} = 0 \quad \text{at} \quad y = 0 \tag{8}$$

$$\psi^{(2)} = q = \text{Constant at } y = h \tag{9}$$

$$\psi^{(1)} = \psi^{(2)} = q_1 = \text{Constant at } y = h_1 \tag{10}$$

$$\psi^{(2)}_{y} = -1 \quad \text{at} \quad y = h \tag{11}$$

where q and q_1 are the total and the core fluxes respectively across any cross section in the wave frame. Further the velocity and the shear stress are continuous across the interface. The peripheral layer flux is given by $q_2 = q - q_1$. It follows from the incompressibility of the fluids that q, q_1 and q_2 are independent of x. The average non-dimensional volume flow rate $\overline{Q}$ over one period $T(=\frac{\lambda}{c})$ of the peristaltic wave is defined as

$$\overline{Q} = \frac{1}{T}\int_0^T \int_0^h (u+1)\,dy\,dt = q + 1 \tag{12}$$

The stream function is obtained by applying the boundary conditions (7) to (11) together with the boundary conditions at the ends of the channel given by specifying $\overline{Q}$ or the pressure difference ΔP across one wavelength.

2.2. Solution

Solving Eqs. (4–6) together with the boundary conditions (7) to (11) we obtain the stream function in the core and peripheral layers as

$$\psi^{(1)} = -y + \left[\frac{3y(q+h)F_2 - \mu(1+\lambda_1)(q+h)y^3}{2F_3}\right] \quad \text{for} \quad 0 \leqslant y \leqslant h_1 \tag{13}$$

$$\psi^{(2)} = -y + (q+h) + \left[\frac{9(q+h)h^2y - 3(q+h)y^3 - 6(q+h)h^3}{6F_3}\right] \quad \text{for} \quad h_1 \leqslant y \leqslant h \tag{14}$$

The axial pressure gradient is obtained from (4) or (6) as

$$\frac{dp}{dx} = \frac{-3\mu(q+h)}{F_3} + \eta \sin\beta \tag{15}$$

2.3. The equation of the interface

The interface is also a streamline as seen from the boundary condition (10). For a given geometry of the wave and the time averaged flux $\overline{Q}$, the unknown interface $h_1(x)$ is solved from (14) using the boundary condition (10). Substituting (10) in (14) we get the algebraic equation governing the interface $h_1(x)$ as

$$2[(1+\lambda_1)\mu - 1]h_1^4 - [(q+h)[2\mu(1+\lambda_1) - 3] + 2(1-\mu(1+\lambda_1))q_1]h_1^3 - [h^3 + 3qh^2]h_1 + 2q_1h^3 = 0 \tag{16}$$

where q and q_1 are independent of x.

We use the following condition to obtain q_1.
$h_1 = \alpha$ at $x = 0$ in Eq. (16), we get

$$q_1 = \frac{\overline{Q}(2\mu(1+\lambda_1) - 3)\alpha^3 + (3\overline{Q} - 2)\alpha - 2(\mu(1+\lambda_1) - 1)\alpha^4}{2((1-(1+\lambda_1)\mu)\alpha^3 + 1)} \tag{17}$$

2.4. The pumping characteristics

Integrating Eq. (15) with respect to x over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p = -3\mu(\overline{Q} - 1)I_1 - 3\mu I_2 + \eta \sin\beta\, I_3 \tag{18}$$

where

$$I_1 = \int_0^1 \frac{dx}{F_3}, \quad I_2 = \int_0^1 \frac{h}{F_3}dx, \quad I_3 = \int_0^1 dx$$

The dimensionless frictional force F at the wall across one wavelength is given by

$$F = \int_0^1 -h\frac{dp}{dx}dx \tag{19}$$

3. Results and discussions

The shape of the interface for different λ_1 with $\overline{Q} = 0.1, \alpha = 0.7, \phi = 0.4$ and $\mu = 0.1$ is shown in Fig. 2. We observe that the variation of the interface shape gives rise to thinner peripheral region with increasing λ_1. The shape of the interface for different μ with $\overline{Q} = 0.1, \alpha = 0.5, \phi = 0.4$ and $\lambda_1 = 0.1$ is depicted in Fig. 3. The variation of the interface shape for low viscosity ratios gives rise to a thicker peripheral – layer in the dilated region.

The shape of the interface for different ϕ with $\overline{Q} = 0.1, \alpha = 0.5, \lambda_1 = 0.1$ and $\mu = 0.1$ is shown in Fig. 4. We observe that the variation of the interface shape gives rise to thinner peripheral region with increasing ϕ.

The variation of pressure rise with time averaged flux is calculated from Eq. (18) for different values of λ_1 with $\phi = 0.4, \alpha = 0.5, \beta = \frac{\pi}{6}, \eta = 1$ and $\mu = 0.1$ is shown in Fig. 5. For $0 \leqslant \overline{Q} \leqslant 0.2$ we observe that ΔP decreases with increase in λ_1 and increases in the rest of the region.

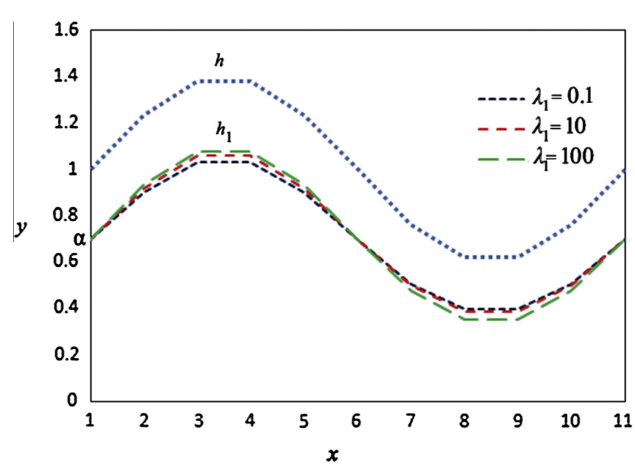

Figure 2 The variation of shape of interface for different values of λ_1 with $\overline{Q} = 0.1, \alpha = 0.7, \phi = 0.4$ and $\mu = 0.1$.

Please cite this article in press as: Kavitha A et al., Peristaltic transport of a Jeffrey fluid in contact with a Newtonian fluid in an inclined channel, Ain Shams Eng J (2015), http://dx.doi.org/10.1016/j.asej.2015.10.014

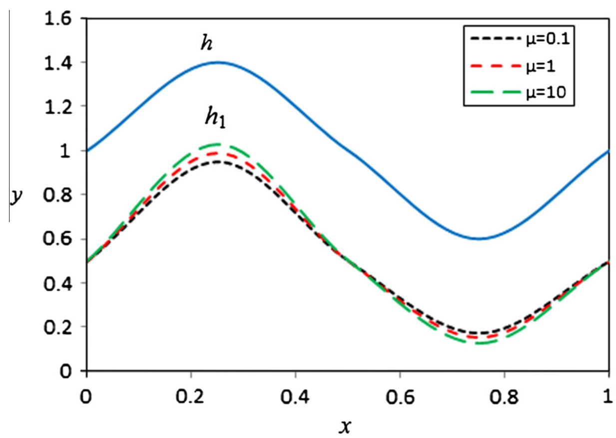

Figure 3 The variation of shape of interface for different values of μ with $\overline{Q} = 0.1, \alpha = 0.5, \phi = 0.4$ and $\lambda_1 = 0.1$.

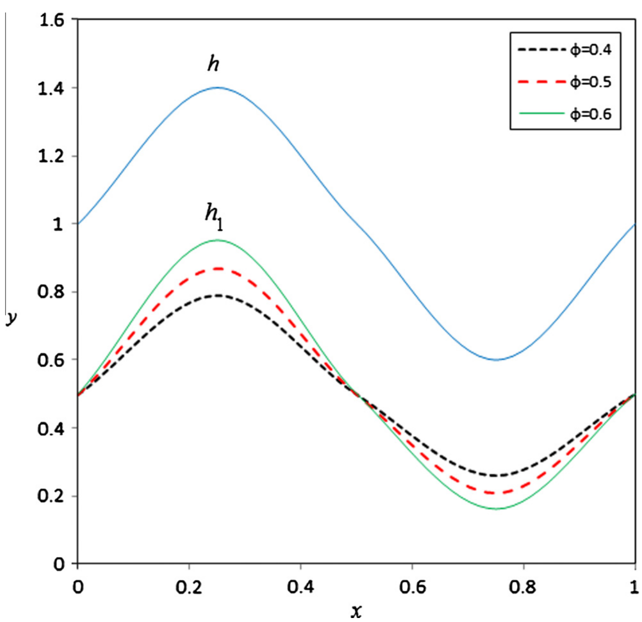

Figure 4 The variation of shape of interface for different values of ϕ with $\overline{Q} = 0.1, \alpha = 0.5, \mu = 0.1$ and $\lambda_1 = 0.1$.

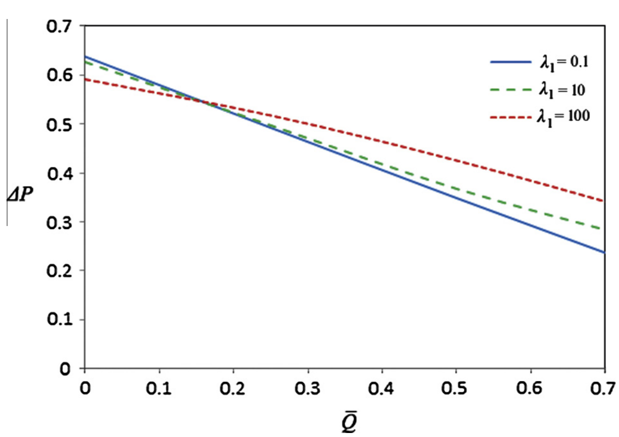

Figure 5 The variation of ΔP with $\overline{Q}$ for different values of λ_1 with $\alpha = 0.5, \phi = 0.4, \beta = \frac{\pi}{6}, \eta = 1$ and $\mu = 0.1$.

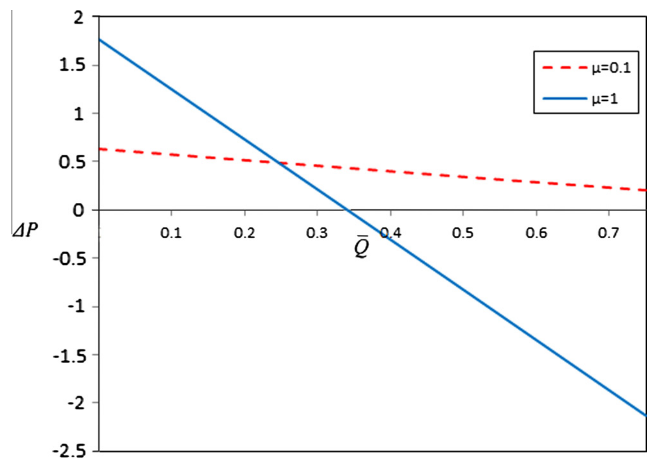

Figure 6 The variation of ΔP with $\overline{Q}$ for different values of μ with $\alpha = 0.5, \phi = 0.4, \beta = \frac{\pi}{6}, \eta = 1$ and $\lambda_1 = 0.1$.

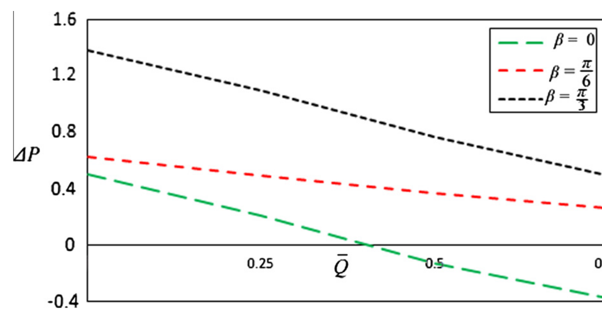

Figure 7 The variation of ΔP with $\overline{Q}$ for different values of β with $\alpha = 0.5, \phi = 0.4\mu = 0.1, \eta = 1$ and $\lambda_1 = 10$.

The variation of pressure rise with time averaged flux is calculated from Eq. (18) for different values μ with $\alpha = 0.5, \phi = 0.4, \beta = \frac{\pi}{6}, \eta = 1$ and $\lambda_1 = 0.1$ and is shown in Fig. 6. We observe that ΔP increases with increase in μ for $0 \leqslant \overline{Q} \leqslant 0.25$ and decreases in the rest of the region. Further, the variation of pressure rise with time averaged flux is shown in Fig. 7. It is found that as the angle of inclination β increases the ΔP also increases.

4. Conclusions

In this paper, we study the peristaltic pumping of a Jeffrey fluid in contact with a Newtonian fluid in an inclined symmetric channel under the assumptions of long wavelength and low Reynolds number. The variation of time-averaged flux with pressure rise and the interface shape is obtained. Some of the interesting findings in this analysis are

1. The variation of the interface shape gives rise to thinner peripheral region with increasing the Jeffrey fluid parameter λ_1.
2. The interface shape for low viscosity ratios gives rise to a thicker peripheral – layer in the dilated region.
3. For time-averaged flux $0 \leqslant \overline{Q} \leqslant 0.2$, the pressure rise ΔP decreases with increasing Jeffrey fluid parameter λ_1 and increases in the rest of the region.

4. The pressure rise ΔP increases with the increase in angle of inclination β.

References

[1] Jaffrin MY, Shapiro AH. Peristaltic pumping. Ann Rev Fluid Mech 1971;3:13–36.
[2] Vajravelu K, Sreenadh S, Saravana R. Combined influence of velocity slip, temperature and concentration jump conditions on MHD peristaltic transport of a Carreau fluid in a non-uniform channel. Appl Math Comput 2013;225:656–76.
[3] Hayat T, Ali N, Asghar S. An analysis of peristaltic transport for flow of a Jeffrey fluid. Acta Mech 2007;193(1):101–12. http://dx.doi.org/10.1007/s00707-007-0468-2.
[4] Kothandapani M, Srinivas S. Peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. Int J Non-Linear Mech 2008;43(9):915–24.
[5] Nadeem Sohail, Akbar Noreen Sher. Peristaltic flow of a Jeffrey fluid with variable viscosity in an asymmetric channel. Z Naturforsch 2009;64a:713–22.
[6] Pandey SK, Tripathi D. Unsteady model of transport of Jeffrey fluid by Peristalsis. Int J Biomath 2010;03(4):473.
[7] Kavitha A, Reddy RH, Srinivas ANS, Sreenadh S, Saravana R. Peristaltic transport of a Jeffrey fluid in a porous channel with suction and injection. Int J Mech Mater Eng 2012;7(2):152–7.
[8] Hayat T, Ahmad N, Ali N. Effects of an endoscope and magnetic field on the peristalsis involving Jeffrey fluid. Commun Nonlinear Sci Numer Simul 2008;13(8):1581–91.
[9] Nadeem S, Akram Safia. Peristaltic flow of a Jeffrey fluid in a rectangular duct. Nonlinear Anal: Real World Appl 2010;11(5):4238–47.
[10] Hayat T, Ali N. Peristaltic motion of a Jeffrey fluid under the effect of magnetic field in a tube. Commun Nonlinear Sci Numer Simul 2008;13(7):1343–52.
[11] Nadeem S, Akram Safia. Slip effects on the peristaltic flow of a Jeffrey fluid in an asymmetric channel under the effect of induced magnetic field. Int J Numer Meth Fluids 2010;63(3):374–94.
[12] Saravana R, Sreenadh S, Venkataramana S, Hemadri Reddy R, Kavitha A. Influence of slip conditions, wall properties and heat transfer on MHD peristaltic transport of a Jeffrey fluid in a non-uniform porous channel. Int J Innovat Technol Creat Eng 2011;1(11):10–24.
[13] Shukla JB, Parihar RS, Rao BRP, Gupta SP. Effects of peripheral – layer viscosity on peristaltic transport of a bio-fluid. J Fluid Mech 1980;97:225–37.
[14] Srivastava LM, Srivastava VP, Sinha SN. Peristaltic transport of a physiological fluid Part – I: flow in non-uniform geometry. Biorheology 1983;20:153–66.
[15] Brasseur JG, Corrsin S, Lu Nan Q. The influence of a peripheral layer of different viscosity on peristaltic pumping with Newtonian fluids. J Fluid Mech 1987;174:495–519.
[16] Ramachandra Rao A, Usha S. Peristaltic transport of two immiscible viscous fluid in a circular tube. J Fluid Mech 1995;298:271–85.
[17] Vajravelu K, Sreenadh S, Ramesh Babu V. Peristaltic pumping of a Herschel–Bulkley fluid in contact with a Newtonian fluid. Quart Appl Math 2006;64(4):593–604.
[18] Vajravelu K, Sreenadh S, Hemadri Reddy R, Murugeshan K. Peristaltic transport of a Casson fluid in contact with a Newtonian fluid in a circular tube with permeable wall. Int J Fluid Mech Res 2009;36(3):244–54.
[19] Narahari M, Sreenadh S. Peristalatic transport of a Bingham fluid in contact with a Newtonian fluid. Int J Appl Math Mech 2010;6(11):41–54.
[20] Hari Prabakaran P, Hemadri Reddy R, Sreenadh S, Saravana R, Kavitha A. Peristaltic pumping of a Bingham fluid in contact with a Newtonian fluid in an inclined channel under long wave length approximation. Adv Appl Fluid Mech 2013;13(2):127–39.

Dr. A. Kavitha obtained Ph.D. in the year 2011. She is having 7 years of teaching and research experience. To her credit, she published 15 research articles in reputed International Journals. Her current research interest is Bio Fluid Dynamics.

Dr. R. Hemadri Reddy completed his Ph.D. (Mathematics) in the year 2007. He is having 10 years of teaching and research experience. He published more than 25 research papers in various reputed International Journals. His area of Specialization is Bio fluid Dynamics.

Dr. R. Saravana received his Ph.D. (Mathematics) in 2013. He is having 7 years of teaching and research experience. His research area interest is Peristaltic transport, Heat and Mass transfer. He published 20 research articles in reputed International Journals.

Dr. S. Sreenadh is a professor and Head of the department in Mathematics at Sri Venkateswara University, Tirupathi, India. He obtained his Ph.D in the year 1988 (in the area of fluid Mechanics) and he is having 27 years of experience in teaching and research. 28 Ph.Ds and 20 M.Phils are awarded under his guidance. He is supervising 7 Ph.D Students at present. He has 155 publications in reputed international and national journals. He published 4 text books. He worked as Associate Professor in Mechanical Engineering at KUTKM University, Melaka, Malaysia for one year (2005–2006). He has three completed major research projects in his credit. He organised 10 workshops/conferences. He is an elected fellow of Andhra Pradesh Academy Sciences, Hyderabad, India. His areas of research specializations are Fluid Dynamics, Peristaltic Transport (Bio Fluid Dynamics), Neuro Science.

Please cite this article in press as: Kavitha A et al., Peristaltic transport of a Jeffrey fluid in contact with a Newtonian fluid in an inclined channel, Ain Shams Eng J (2015), http://dx.doi.org/10.1016/j.asej.2015.10.014