International Journal of Engineering & Technology, 7 (4.10) (2018) 949-955

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Planar Characterization – Graph Domination Graphs

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Abstract

In this paper, we characterize planarity and outer planarity of complement of graph domination graphs and provide a MATLAB program for identifying graph domination graphs.

Keywords: *Complement; Graph domination; Non planar; Non outer planar; Planar;*

1. Introduction

In [1], [2], M. Yamuna et al introduced γ - uniquely colorablegraphs and also provided the constructive characterization of γ -uniquely colorable trees. In [3], [4], they have introduced Non domination subdivision stable graphs (NDSS) and characterized planarity of complement of NDSS graphs .In [5], M. Yamuna et al have introduced graph domination graph.

2. Terminology

We consider simple connected graphs G with n vertices and m edges. K_n is a complete graphwith n vertices..Results related to graph theorywe refer to [6].

D is a dominating set if every vertex of V – D is adjacent to some vertex of D. Minimum cardinality of D, is said to be a minimum dominating set (MDS). The cardinality of any MDS for G is said to be domination number of G, represented by γ(G). Results related to domination we refer to [7].

3. Result and discussion

A γ -set D that covers all the vertices and edges of G is said to be graph domination set.

R1:A γ - set D is a graph domination set iff V − D is independent

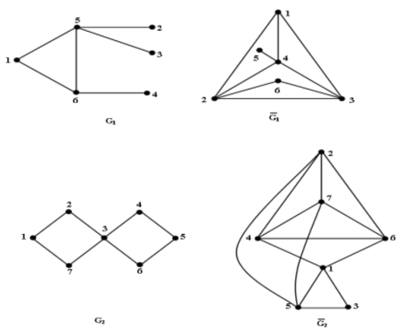


Fig.1: Planar and non planar graph domination graphs

In Fig.1, G_1 and G_2 are graph domination graphs,$\overline{G}_1$is planar while, $\overline{G}_2$ is non planar. So we observe that, when G is a graph domination graph,$\overline{G}$ need not to be planar. In this section, we characterize the planarity of $\overline{G}$, when G is a graph domination graph.

If γ (G) = 1,then $\overline{G}$ is disconnected. By R1, we know that every vertex in V- D is independent. If | V – D | ≥ 5, then as ⟨ V – D ⟩ contains K_5 implies $\overline{G}$ is non planar. So we restrict our discussion to case where | V – D | ≤ 4. We continue our discussion to graphs where γ(G) ≥ 2 and | V – D | ≤ 4 . By Ore's, we know that γ(G) ≤ $\lfloor n/2 \rfloor$so it would be sufficient to consider graphs such that 2 ≤γ (G) ≤ $\lfloor n/2 \rfloor$ such that V – D is independent | V – D | ≤ 4.

Case i $\gamma(G) = \lfloor n/2 \rfloor$

A connected graph G satisfies $\gamma(G) = \lfloor n/2 \rfloor$ iff $G \in \zeta = \cup_{i=1}^{6} \zeta_i$.

The six family of graphs ζ_i ,i = 1 to 6 are defined based on two family of graphs A, B seen in Fig.2.

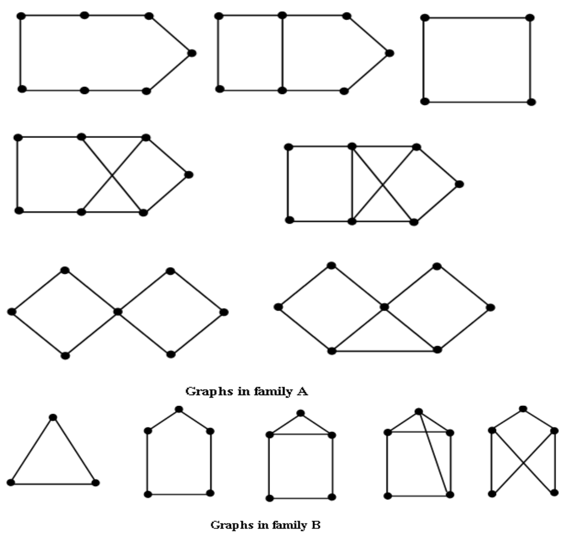


Fig.2

Since | V – D | ≤ 4, can have a maximum of 8 vertices.

Let us list out the graph domination graphs belonging to the six classes of graphs.

$\zeta_1 = \{ C_4 \} \cup \{ G : G = H \circ K_1$, where H is connected $\}$

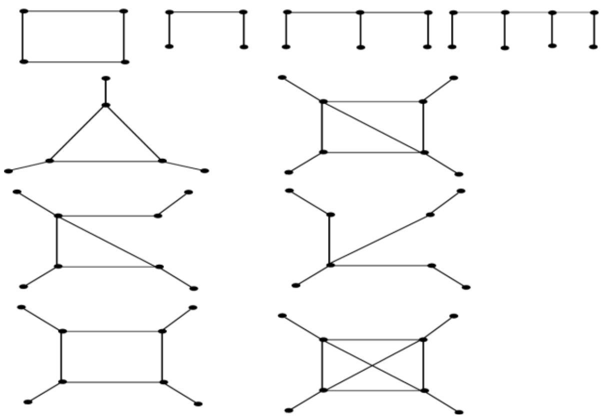


Fig.3: ζ_1

$\zeta_2 = A \cup B - \{ C_4 \}$.

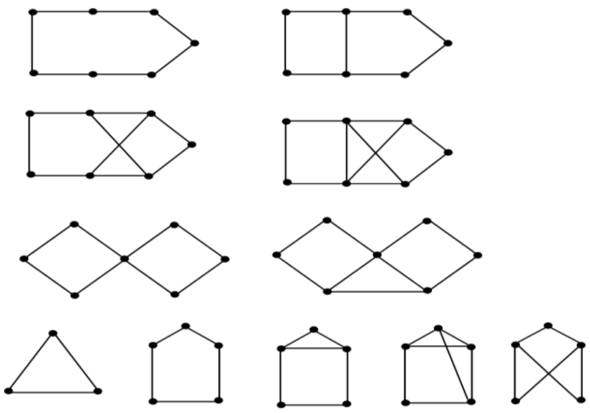


Fig.4: ζ_2

$\zeta_3 = \cup_H S(H)$ where the union is taken over all graphs H, where S (H) is the set of connected graph formed from $H \circ K_1$.

$\zeta_4 = \{ \theta(G) : G \in \zeta_3 \}$, where $\theta(G)$ is generated by joining G to C_4.

From Fig. 5 , we know that any graph belonging to ζ_3 has a minimum of five vertices, implies any graph in ζ_4 should have a minimum of nine vertices implies if any graph in family ζ_4 is a graph domination graph, then $|V - D| \geq 5$.

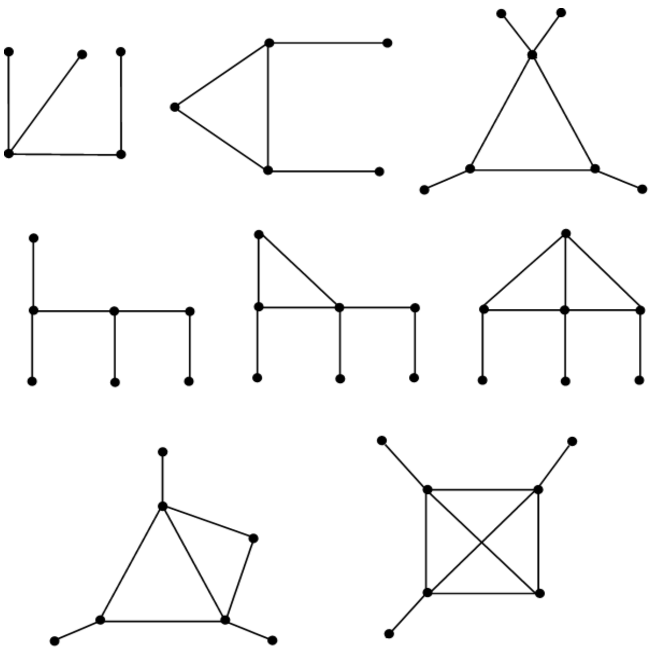


Fig.5: ζ_3

$\zeta_5 = \cup_H \wp(H)$, where $\wp(H)$ be the set of connected graph formed from $H \circ K_1$ by joining each of u and w to one or more vertices of H and u, v, w be a vertex sequences of a path P_3.

Let H be a graph and $X \in B$. Let R(H, X)be the set of connected graphs which may be formed from $H \circ K_1$ by joining each vertex of $U \subseteq V(X)$ to one or more vertices of H such that no set with fewer than $\gamma(X)$ vertices of X dominates $V(X) - U$. The define $\zeta_6 = \cup_{H, X} R(H, X)$.

Case ii $\gamma(G) \lhd \lfloor n/2 \rfloor$

From case i, we know that $|V(G)| \leq 8$ and $\gamma(G) \leq 3$.

$\gamma(G) = 3$

When $\gamma(G) = \lfloor n/2 \rfloor$ and $\gamma(G) = 3$,the graph domination graphs are listed in case 1, so $|V(G)| \leq 6$, when $\gamma(G) = 3$, all possible graphs, when n = 1,2, …, 6 are listed in [7]. We pick the graphs that are graph domination graphs and list them as follows

n = 6, $\gamma(G) = 3$

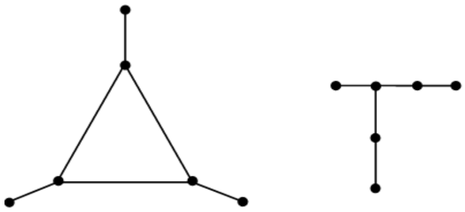


Fig.6: n = 6, $\gamma(G) = 3$

$\gamma(G) = 2$

Since $|V - D| \leq 4$, $|V(G)| \leq 6$. The list of graph domination graphs is given in Figure

n = 4, $\gamma(G) = 2$

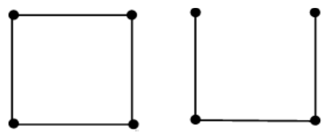


Fig.7.: n = 4, $\gamma(G) = 2$

n = 5, $\gamma(G) = 2$

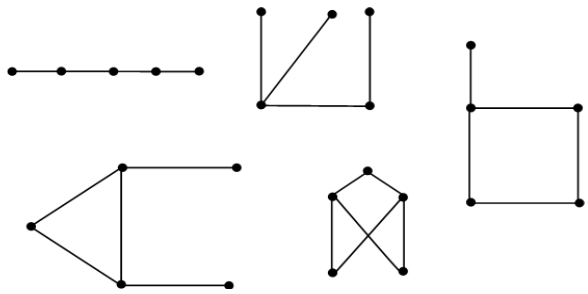


Fig.8.: n = 5, $\gamma(G) = 2$

n = 6, $\gamma(G) = 2$

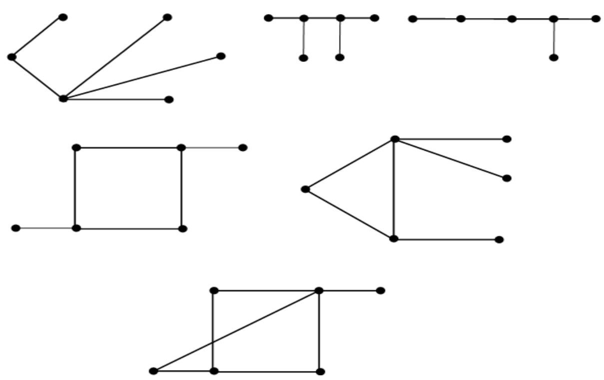
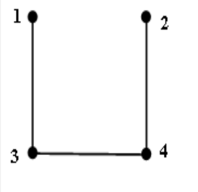
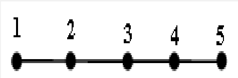
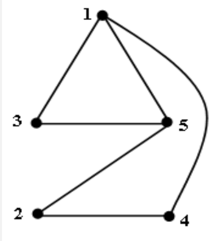
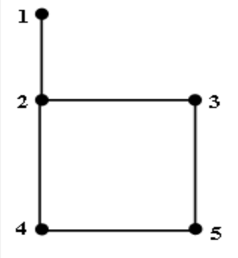
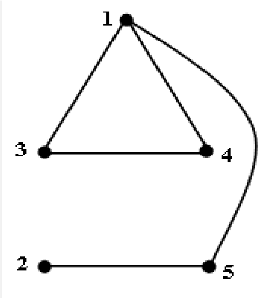
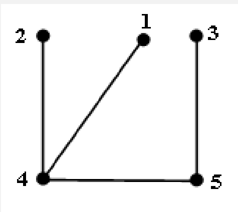
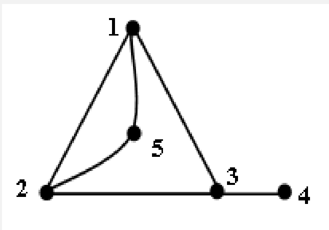
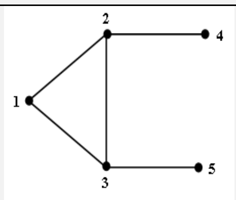
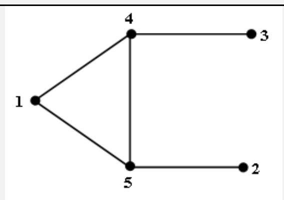
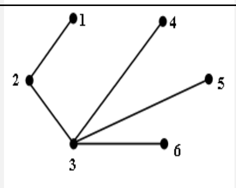
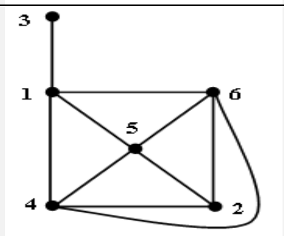
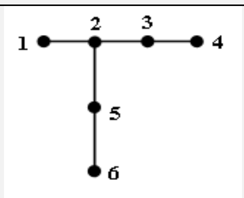
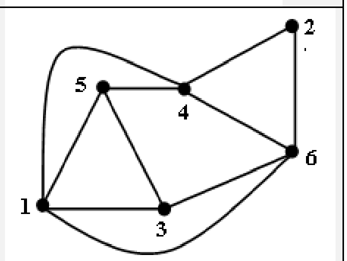
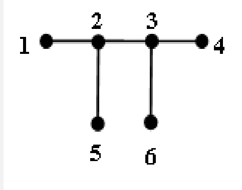
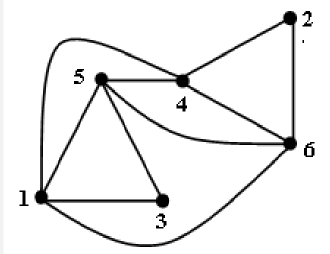
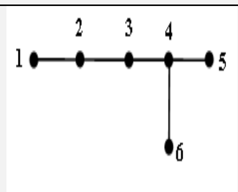
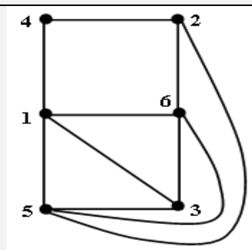
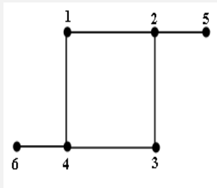
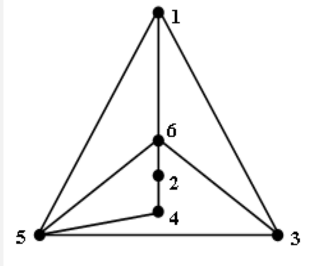
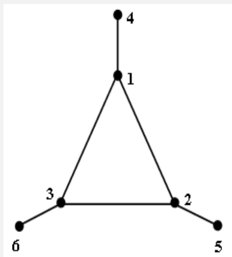
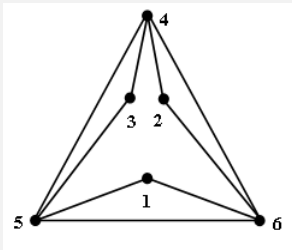
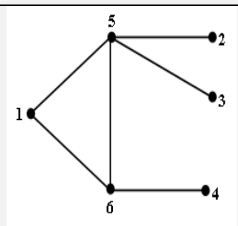
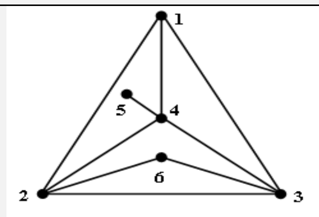
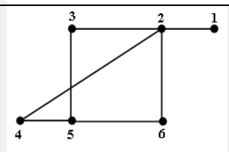
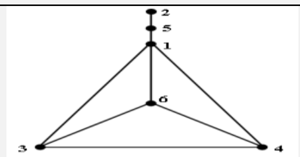
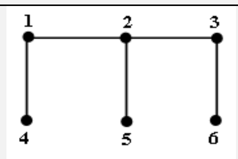
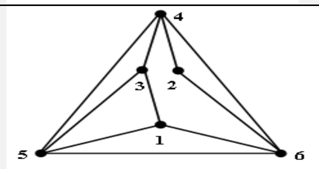
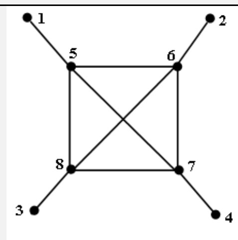
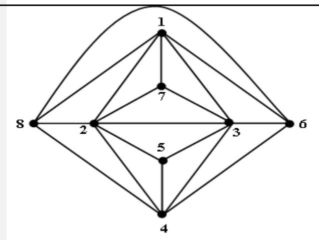
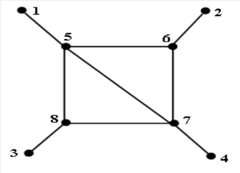
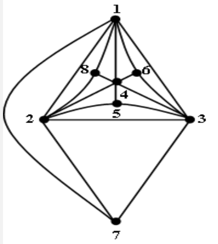
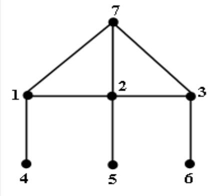
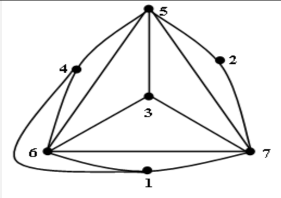
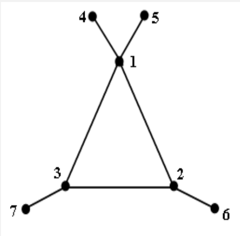
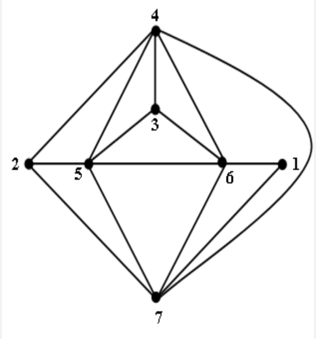
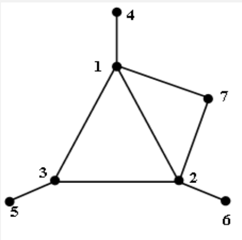
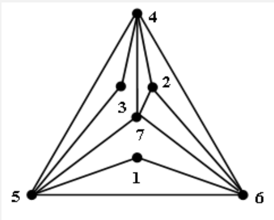
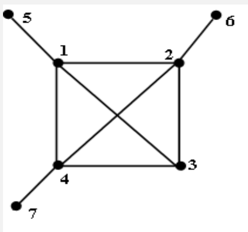
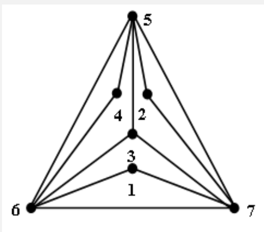


Fig.9: n = 6, $\gamma(G) = 2$

The graphs in Fig. 2,3,…, 8 gives the complete list of graphs that are graph domination graphs such that $| V - D | \leq 4$. Table – 1 provides the complements of these graphs.

Table 1: Graph G and its complement

S. no	G	Complement of G
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

16		
17		
18		
19		
20		

Graphs for which the complements are planar are represented as planar graphs in Table – 1. We prove that the remaining graphs have a non – planar complement. We know that,

i. if $e \nleq 3n - 6$, then G is non planar.
ii. A necessary and sufficient condition for a graph G to be planar is that G does not contain either of K_5 or $K_{3,3}$ or any graph homeomorphic to either of them.

We show that $\overline{G}$ is non – planar in the remaining cases by using i or ii.

Case i

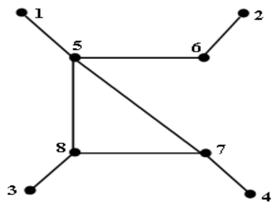


Fig.10

Number of edges in G + Number of edges in $\overline{G}$ = n (n -1) / 2
8 + Number of edges in $\overline{G}$ = 28
Number of edges in $\overline{G}$ = 20
$20 \nleq 3(8) - 6$, ⇒ $\overline{G}$ is non planar .

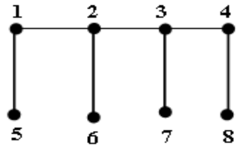


Fig.11

Number of edges in $\overline{G}$ = 21
$21 \nleq 3(8) - 6$, ⇒ $\overline{G}$ is non planar .

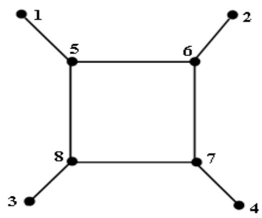


Fig.12

$20 \nleq 3(8) - 6$, ⇒ $\overline{G}$ is non planar .

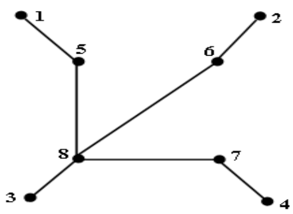
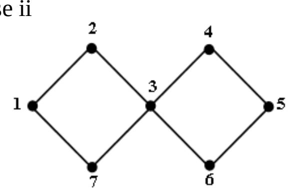
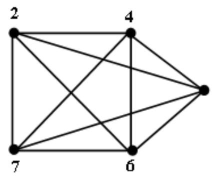


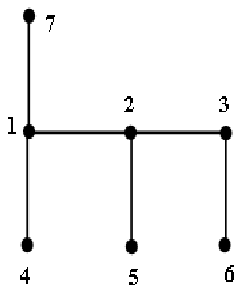
Fig.13

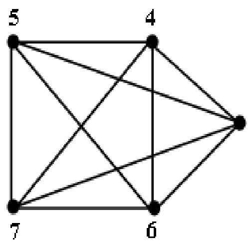
$21 \nleq 3(8) - 6$, ⇒ $\overline{G}$ is non planar.

Case ii









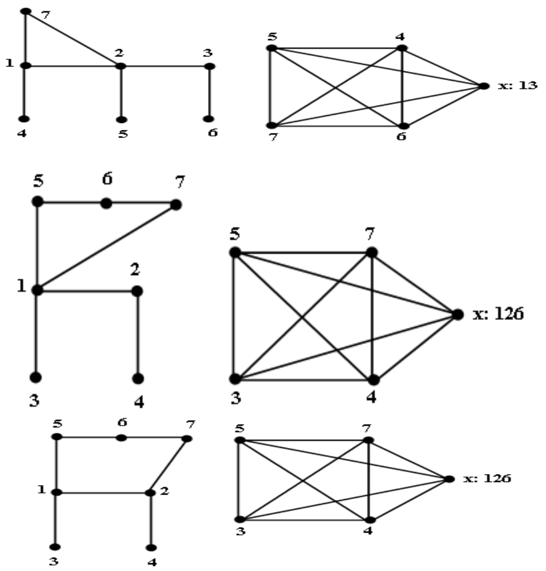


Fig.14

Outer planar graph

We know that, G is outer planar iff it does not contain K_4 or $K_{2,3}$ or any graph homeomorphic to K_4 or $K_{2,3}$ as a subgraph. Let us consider the set of graphs G for which $\overline{G}$ is planar from Table – 1.

If G has atleast four independent vertices, then $\overline{G}$ is non – outer planar. The only graphs for which the independence number is atleast three are the one's in Fig.15

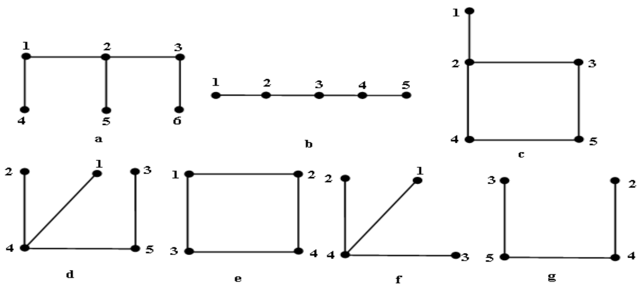


Fig.15

The complement of these graphs are shown in Table – 1. The complements of the graphs in Fig a and b. K_4 can be generated for their complements by edge contraction as seen in Fig.16

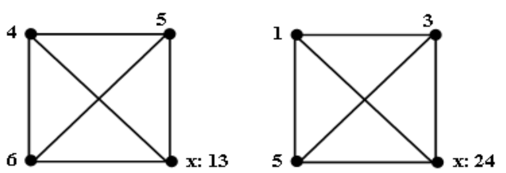


Fig.16

The complements of the graph in Fig c,d,e,f,g is seen in Fig. 17

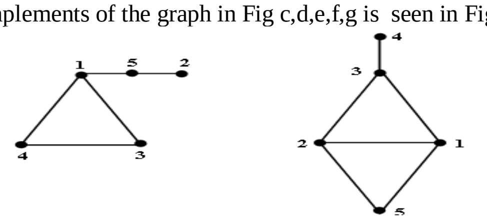


Fig.17

Since all the complements have a representation to that all their vertices lies in the same region, they are outerplanar.

From this discussion, we conclude that the only graphs for which $\overline{G}$ is outer planar are the one's seen in Fig. 18

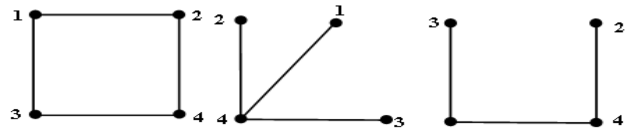


Fig. 18

4. Matrix representation for graph domination graphs

Let G be any graph with n – vertices. Let A represents the adjacency matrix of G. Let N denote a n ×n matrix, where

$$N = [n_{ij}]_{n\times n} = \begin{cases} 1 & \text{if } i = j \\ a_{ij} & \text{the } (i,j)^{th} \text{ entry in the adjacency matrix} \end{cases}$$

Let $x = \langle x(v_1), x(v_2), \ldots, x(v_n) \rangle^T$ be a { 0, 1 } vectors . We know that, if x represents the any dominating set, then $Nx \geq 1$, that is in a resulting matrix Nx, all the entry values are non zero.

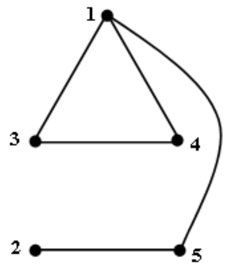


Fig.19

$$\begin{array}{cccc} N & x & & Nx \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = & \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} \geq & \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \end{array}$$

That is { v_1, v_5} is a dominating set for G.
A γ -set D that covers all the vertices and edges of G is said to be graph domination set.
Consider the non – zero columns of NV. Let X be the set of all vectors such that $Nx_i \geq 1$, i.e., $Ns \geq 1$. Let $|s| = q$, $q \leq p$.
By [8], we know that if a vector v is a dominating set, then $Nv \geq 1$. We know that whenever D is dominating set, V – D is an independent dominating set for some dominating set D. Let S_1 be a n × q matrix obtained by replacing 0 by 1 and 1 by 0 in matrix S. In NS1 , if for atleast one x_j for all non zero entry in x_j, the nx_jentry in the corresponding positionisalso 1, then no two vertices in V – D are adjacent, that is V – D is independent, implies ∃ one γ - set D ∍ V – D is independent ⇒ G is a graph domination graph [5] .

Example

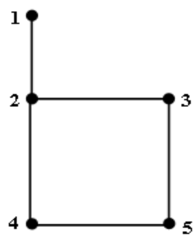


Fig. 20

For the graph in Fig. 20, γ (G) = 2. We consider all possible subsets with 2 vertices and label them as { S_1, S_2, S_3, …, S_{10}} = {{ v_1, v_2}, { v_1, v_3}, { v_1, v_4}, { v_1, v_5}, { v_2, v_3 }, { v_2, v_4 }, { v_2, v_5 }, { v_3, v_4 }, { v_3, v_5 }, { v_4, v_5 } }.

NV =

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 00 & 0 & 0 & 11 & 1 & 1 \\ 0 & 0 & 01 & 1 & 1 & 00 & 0 & 1 \\ 0 & 1 & 10 & 0 & 1 & 00 & 1 & 0 \\ 1 & 0 & 10 & 1 & 0 & 01 & 0 & 0 \\ 1 & 1 & 01 & 0 & 0 & 10 & 0 & 0 \end{pmatrix}$$

$$NV = \begin{pmatrix} 0 & 0 & 01 & 1 & 1 & 22 & 2 & 3 \\ 1 & 1 & 22 & 3 & 3 & 12 & 2 & 3 \\ 1 & 3 & 22 & 1 & 3 & 10 & 2 & 1 \\ 3 & 1 & 22 & 3 & 1 & 12 & 0 & 1 \\ 3 & 3 & 22 & 1 & 1 & 21 & 1 & 0 \end{pmatrix}$$

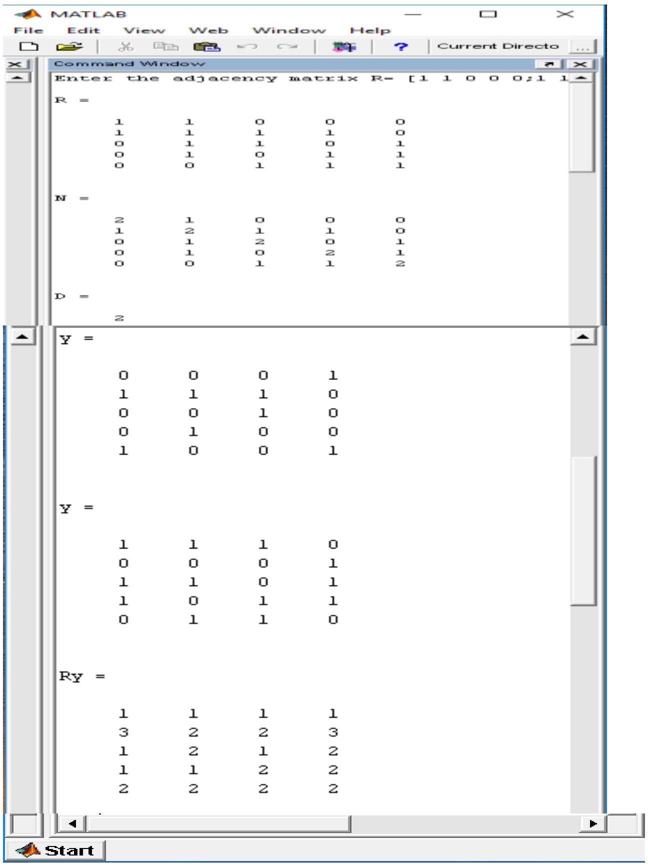
$$S = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} y = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

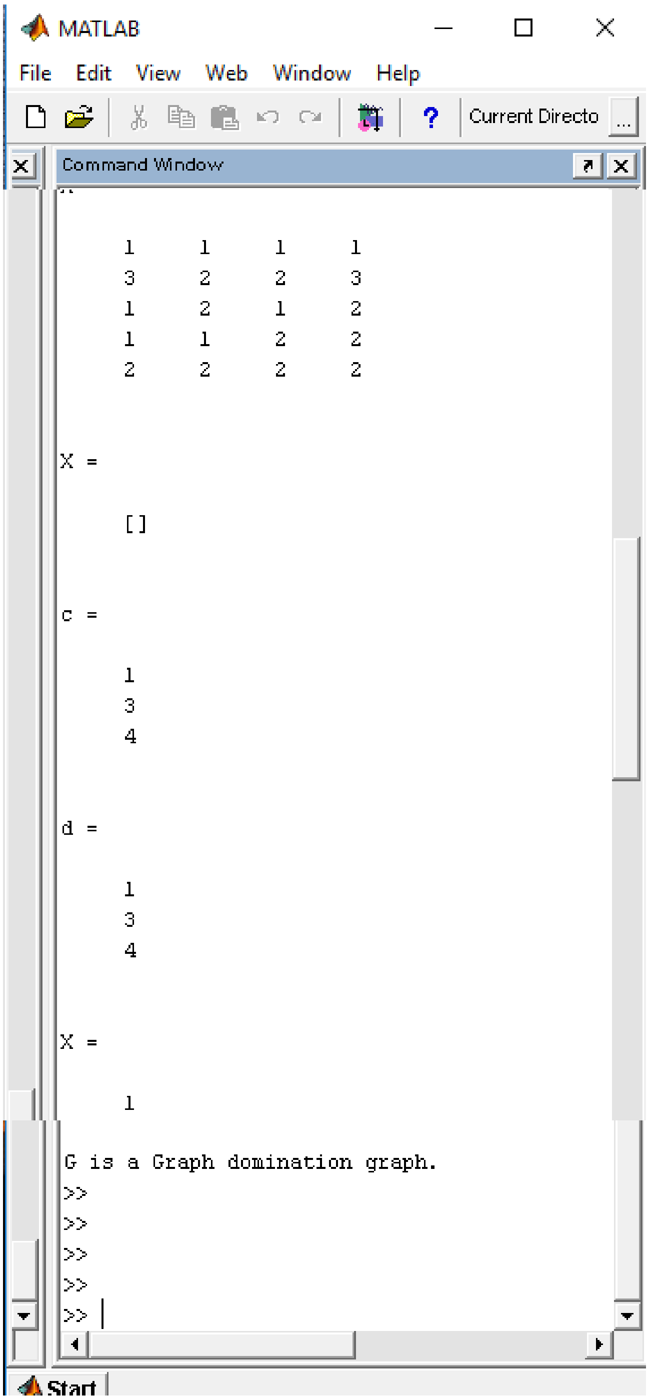
$$Ny = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 2 & 2 & 3 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix}$$

For any column, the non zero entry in y, the entry in the corresponding position of Ny is also 1, $\Rightarrow$ V – D is independent for atleast one γ - set.

MAT Lab program for graph domination graphs

Snapshot – 1 provides the output for the graph in Fig. 20.





Snapshot - 1

5. Conclusion

We conclude that $\overline{G}$ isplanaronlyif G is one of the graph domination graphs listed in Table – 1 for which $\overline{G}$ has a planar representation. In other cases $\overline{G}$is nonplanar that is only 20 possible complements of G are planar when G is graph domination graph and the only graphs for which $\overline{G}$ is outer planar are the one's seen in Fig. 17.

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