Prediction of Inelastic Mechanisms Leading to Seismic Failure of Interior Reinforced Concrete Beam–Column Connections

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Abstract: Inelastic mechanisms leading to failure in interior reinforced concrete beam–column (RCBC) connections, designed on the concept of strong column–weak beam philosophy, primarily result from failure of the joint region and yielding of longitudinal reinforcement in beams. In this manuscript, two novel easy-to-use probabilistic methodologies have been developed that can determine with sufficient accuracy the occurrence of either of these inelastic mechanisms leading to failure, given the geometric, material and loading parameters of an experimental investigation. One model was developed by using the relevance vector machine method, a machine learning methodology that uses a Bayesian formulation and results in a sparse representation. Another model was binomial logistic regression, which can relate the qualitative event of inelastic mechanism resulting in failure initiation with several experimentally obtained independent parameters. It can also quantify the relative importance of each of these independent parameters. Both methods show good predictive efficiency and can be utilized by a designer, engineer, or researcher to obtain a preliminary probabilistic estimate of inelastic mechanisms that lead to failure of interior RCBC connections. This manuscript also presents comparative evaluations of utilizing these two models. **DOI: 10.1061/(ASCE)SC.1943-5576.0000115.** © *2012 American Society of Civil Engineers.*

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Introduction

Reinforced concrete beam–column (RCBC) joints are one of the most complex, least studied, and important structural components of a building or bridge structure that is subjected to seismic loading. Experimental investigations have determined that stiffness and strength reduction of a joint component results in stiffness and strength reduction of the structure. Postearthquake reconnaissance (Hall 1994) has also revealed that failure of RCBC components may even lead to catastrophic collapse of the entire structure. Therefore, it is absolutely necessary for engineers designing new connections to identify the failure initiation mechanism and make design changes to prevent a catastrophic failure. It is similarly necessary for engineers who specialize in rehabilitating and/or retrofitting damaged structures to understand the failure initiation mechanism and to take proper steps to retrofit the structure suitably in an already damaged structure.

There are primarily two established inelastic behavioral mechanisms leading to failure within the connection region: (1) failure within the joint (which can either result from the bond of the reinforcement steel with concrete or from shear failure or from a combination of these two mechanisms) and (2) yielding of the beam reinforcing bar adjacent to the joint (Paulay 1989). Yielding of the column reinforcing bar has not been considered because nearly all structures are built on the concept of "strong column–weak beam." Shear failure and bond failure in beam and columns further away from the joint has also not been considered as a failure mode since it is beyond the scope of this manuscript, which deals with interior connection regions. According to ACI 352R-02 [American Concrete Institute (ACI) 2002], "a connection is the joint plus the columns, beams and slabs adjacent to the joint." A closer inspection reveals that after failure has occurred, it is difficult to identify which inelastic mechanism was primarily responsible for the failure. The two inelastic mechanisms are coupled, and typically one mechanism induces another mechanism of failure. For example, yielding of the beam reinforcing bars adjacent to the joint results in a weaker bond between the reinforcing steel and concrete. This eventually leads to the development of failure within the joint. Therefore, rather than classifying the connections on the basis of the two failure mechanisms of joint failure and beam reinforcement bar yielding, it is perhaps better to classify them on the basis of *initiation* of these two mechanisms.

The initiation mechanism of these two failure mechanisms can be easily and distinctly identified. The failure initiation process may also determine how the connection region would behave. For example, a connection in which joint failure initiates before beam bar yielding are typically expected to witness a more nonductile method of failure in comparison with a connection in which beam reinforcing bar yields before joint failure initiation.

If the initiation of these failure mechanisms can be properly identified, then univariate or multivariate probabilistic capacity models can be developed. From a design perspective, steps can be taken to prevent failure and to initiate a much favored ductile

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method of failure (in an overload), rather than a nonductile failure mechanism. Structural codes (ACI 2002; ACI 2005) usually specify rules for better performance of connection regions. These requirements include a minimum volume of transverse reinforcement, a minimum anchorage length for beam longitudinal reinforcement ratio, a minimum column-to-beam flexural strength ratio, and a limit on the joint shear stress demand. However, these rules are far from being complete and do not account for the effect of coupling these parameters and several other parameters that affect joint response. They also do not provide any probabilistic estimate of the failure mechanism. On the basis of recommendations from the structural codes, a designer is unable to make judgments such as what happens if the aspect ratio of the joint or the yield strength of reinforcing bar or the concrete compressive strength continues increasing while all other parameters remain constant in accordance with ACI standards. With current ACI recommendations, a designer is unable to decide if any shift in probabilistic estimate of the failure initiation mechanisms would be associated with changes in the geometric and/or the material parameters in a structural construction.

Research Objective and Methodology

The objective of this unique research is to develop an easy-to-use, yet robust and effective means of predicting the probabilistic failure initiation mechanism of RCBC connections. The methodology is intended to be used by practicing engineers, researchers, and academicians. The process does not require complex computational time-intensive methods that use state-of-art component-based or continuum-based finite element analysis of joints. To accomplish the objective, a data set from the experimental investigation of interior reinforced concrete beam–column connections was assembled and utilized to develop two types of probabilistic models. The models were then compared.

Research Significance

The simple cost-effective proposed model in this research study is a first step in providing practicing engineers the ability to make better decisions in designing new RCBC joints in challenging situations for which ACI recommendations are not explicit. The developed methodologies help a designing engineer to

- Determine the probabilistic estimation of the failure initiation mechanism of RCBC connections that are subjected to seismic loads. The outcome of this research is to help a practicing engineer develop probabilistic estimation. No methodology prescribed by ACI exists by which geometric and material parameters can be varied in a simple, cost-effective manner to obtain a probabilistic estimate of the failure initiation of the joint and therefore obtain the desired mechanism of ductile failure.
- Provide recommendations for structures designed with high-strength concrete. High-strength concrete with a compressive strength of concrete greater than 8.56 ksi (59 MPa), as a report by ACI Committee 363 (ACI 1997) specifies, has recently been successfully utilized for cast-in-place concrete buildings and high-rise structures. This enables structural designers to design more slender reinforced concrete members (Please provide citation for (Sanada and Maruta 2004). Research studies by Noguchi and Kashiwazaki (1992) report that smaller column sections with high-strength concrete increase the potential for joint failure before beam yielding. Because ACI recommendations provide no guidelines, the outcome of this research is intended to help practicing engineers who need to make decisions in designing joints for structures with high-strength concrete.
- Provide recommendations for structures designed with high-strength reinforcing steel. High-strength reinforcing steel has also been used in recent constructions to prevent congested detailing of reinforcing bars in slender members. Fujii and Morita (1991) conclude that high-strength reinforcing steel in beams passing through joints prevents flexural yielding of the bars, and therefore may result in the development of joint failure before flexural yielding of longitudinal beam bars. Because ACI recommendations provide no guidelines, the outcome of this research is intended to help practicing engineers who need to make decisions in designing joints for structures with high-strength steel.

Apart from the design of new RCBC connections, the proposed research models help design engineers make suitable measures in the repair and retrofitting of damaged joints. Before retrofitting, it is necessary to assess the design parameters that lead to failure initiation and eventual collapse. The current model provides a first step in the quantifying the design parameters that result in the development of inelastic mechanisms leading to the initiation of failure (i.e., strength degradation) in RCBC connections.

With the exception of the previously noted significance to practicing engineers, the research also provides researchers with innovative probabilistic techniques that can be further used to assess different types of failure mechanisms and failure initiations for different structural components that are subjected to different loading conditions.

Experimental Data Set

The data set consists of 110 laboratory tests of two-dimensional interior beam–column joint subassemblages. Twenty research teams from around the world conducted the tests during the last 40 years (Durrani and Wight 1982; Otani et al. 1984; Meinheit and Jirsa 1977; Alire 2002; Lehmann et al. 2004; Park and Ruitong 1988; Noguchi and Kashiwazaki 1992; Oka and Shiohara 1992; Kitayama et al. 1987; Park and Milburn 1983; Endoh et al. 1991; Higashi and Ohwada 1969; Beckingsale 1980; Attaalla and Agbabian 2004; Birss 1978; Teraoka et al. 1997; Hayashi et al. 1994; Teraoka et al. 1994; Zaid 2001; Joh et al. 1991; Fujii and Morita 1991). The two-dimensional interior beam–column joint subassemblages utilized for the data set are representative of actual building frame structures. Table 1 provides a detailed presentation of the data set and includes material, geometric, and design parameters for each test specimen. The specimens in the data set span a wide range of joint design parameters. The data set is limited to two-dimensional joint subassemblages in which a continuous column intersects a continuous beam. Specimen response is determined by flexural yielding of beams and/or joint failure. Joint subassemblages with slabs, eccentric beams (i.e., the axis of the beam and column are not aligned), or out-of-plane beams were not included in the data set. Test specimens with plain round (i.e., smooth) reinforcing steel bars were not included in the data set. The data set does not include specimens in which the beam-hinging effect has been shifted from the beam–column joint interface to a site away from the interface. All specimens were subjected to quasi-static cyclic load distributions in the laboratory. These were that are representative of loads that develop in a frame in an earthquake.

The independent parameters or covariates for developing the model were the aspect ratio of the joint (ASP), defined as ratio of the height of the beam to the width of the column; the total yield strength of the top longitudinal beam reinforcement (ASFYTP), defined as the product of the beam top reinforcement yield stress and the total area of the top beam longitudinal reinforcements; the total yield strength of the bottom longitudinal beam reinforcement

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Table 1. Experimental Dataset

Specimen number	Reference number	Specimen	Fail mech.	ASP	ASFYTP (lb)	ASFYBT (lb)	CFYAS (lb)	HFY (psi)	HREINF	PFC (psi)	TYLD (psi)
1[a]	Durrani-Wight	X1	0	1.16	119,399	91,377	48,734	51,000	0.798	270.25	506.85
2[a]	(1982)	X2	0	1.16	119,399	91,377	48,734	51,000	1.597	270.25	506.91
3		X3	0	1.16	89,549	68,532	29,850	51,000	0.798	236.84	386.67
4[a]	Otani et al.	J1	0	1.00	94,124	47,062	11,765	53,338	0.281	283.69	341.33
5[a]	(1984)	J2	0	1.00	94,124	47,062	11,765	53,338	0.562	283.69	341.04
6[a]		J3	0	1.00	94,124	47,062	11,765	53,338	1.693	283.69	341.04
7		J4	0	1.00	94,124	47,062	11,765	53,338	0.281	1,134.74	341.33
8[a]		J5	0	1.00	94,124	47,062	11,765	53,338	0.281	283.69	343.98
9[a,b]		J6	0	1.00	40,607	30,440	5,993	53,338	0.422	851.06	220.85
10[a]	Meinheit-Jirsa	1	1	1.00	25,5854	143,278	41,230	59,300	0.503	1,522.26	972.94
11	(1977)	2	1	1.00	255,854	143,278	85,285	59,300	0.503	1,535.05	978.53
12[a,b]		3	1	1.00	255,854	143,278	94,143	59,300	0.503	1,517.99	973.09
13		5	1	1.00	255,854	143,278	85,285	59,300	0.503	204.67	976.20
14[a]		6	1	1.00	255,854	143,278	85,285	59,300	0.503	2,571.21	976.55
15		12	0	1.00	255,854	143,278	85,285	61,300	2.359	1,547.84	975.94
16[a]		13	1	1.00	255,854	143,278	85,285	59,300	1.510	1,505.20	978.34
17[a]	Alire (2002)	PEER14	0	1.11	152,482	92,646	38,121	0	0.000	459.57	343.18
18[a]	Lehmann et al. (2004)	PEER22	0	1.11	285,438	190,292	80,611	0	0.000	555.78	707.40
19		PEER0850	0	1.11	90,917	90,917	15,798	0	0.000	505.89	409.56
20[a]		PEER0995	0	1.11	227,293	136,376	33,444	0	0.000	874.76	612.96
21		PEER4150	1	1.11	484,182	487,266	81,211	0	0.000	477.24	1,592.83
22[a]	Park and Ruitong	1	0	1.13	68,535	27,414	23,218	43,713	1.298	128.81	184.45
23	(1988)	2	0	1.13	89,667	43,709	34,676	41,035	1.580	156.91	325.63
24[a]		3	0	1.13	68,535	27,414	23,218	47,200	0.557	128.81	183.45
25[a]		4	0	1.13	89,667	43,709	34,676	45,278	0.807	156.91	327.87
26[a]	Noguchi and	OKJ1	0	1.00	198,889	154,692	22,099	138,475	0.754	1,218.00	1262.95
27[a]	Kashiwazaki	OKJ3	1	1.00	220,988	220,988	22,099	138,475	0.754	1,861.80	1,710.89
28	(1992)	OKJ4	0	1.00	198,889	154,692	22,099	138,475	0.754	1,218.00	1,262.95
29[a]		OKJ5	1	1.00	220,988	220,988	22,099	138,475	0.754	1,218.00	1,687.52
30[a]		OKJ6	1	1.00	176,790	154,692	22,099	138,475	0.754	930.90	1,243.12
31	Oka and Shiohara	J1	0	1.00	168,666	131,185	18,741	199,160	0.377	1,343.67	1,055.32
32[a]	(1992)	J2	1	1.00	358,505	358,505	44,813	199,160	0.377	1,343.67	2,692.17
33[a]		J4	0	1.00	151,277	151,277	15,128	199,160	0.377	1,343.67	1,174.68
34[a]		J5	0	1.00	221,804	172,514	24,645	199,160	0.377	1,343.67	1,366.95
35		J6	0	1.00	178,712	138,998	19,857	112,349	0.188	1,343.67	1,106.78
36[a]		J7	0	1.00	138,998	99,285	19,857	124,208	0.377	1,343.67	871.17
37[a]		J8	0	1.00	221,243	172,078	24,583	112,349	0.377	1,343.67	1,349.50
38[a]		J10	1	1.00	185,057	143,933	20,562	86,697	0.377	671.83	1,120.49
39		J11	1	1.00	222,438	173,008	24,715	58,115	0.377	671.83	1,298.55
40[a]	Kitayama et al.	J1	0	1.00	98,759	49,379	12,345	53,324	0.269	284.39	422.03
41[a]	(1987)	J6	0	1.00	42,618	31,964	5,822	46,925	0.377	284.39	265.03
42[a]		C1	0	1.00	69,869	34,934	12,979	46,925	0.269	284.39	446.15
43		C3	0	1.00	69,869	34,934	12,979	46,925	2.326	284.39	446.15
44[a]	Park and Milburn	1	0	1.13	117,489	117,489	49,618	46,400	3.801	598.85	787.99
45	(1983)	2	0	1.13	89,457	89,457	49,618	46,400	3.041	680.05	694.38
46[a]	Endoh et al.	HC	0	1.00	74,161	74,161	17,191	40,952	0.314	284.39	581.52
47	(1991)	HLC	0	1.00	72,992	72,992	16,780	42,091	0.314	284.39	571.79
48[a]		LA1	1	1.00	188,276	94,138	25,649	41,450	0.462	284.39	781.90
49		A1	1	1.00	183,208	91,604	25,147	46,357	0.419	284.39	755.23
50[a,b]	Higashi and Ohwada	SD35Aa4	1	1.50	22,878	22,878	7,626	50,750	0.220	284.39	466.07
51[a]	(1969)	SD35Aa7	1	1.50	21,860	21,860	7,287	50,750	0.220	284.39	448.64
52		SD35Aa8	1	1.50	21,860	21,860	7,287	50,750	0.220	568.79	448.64
53		LSD35Aa1	1	1.50	21,860	21,860	7,287	50,750	0.220	284.39	449.00
54[a]		LSD35Aa2	1	1.50	21,860	21,860	7,287	50,750	0.220	568.79	449.39
55		LSD35Ab1	1	1.50	21,860	21,860	7,287	50,750	0.220	284.39	449.39
56		LSD35Ab2	1	1.50	21,860	21,860	7,287	50,750	0.220	568.79	449.39

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Table 1. (*Continued*.)

Specimen number	Reference number	Specimen	Fail mech.	ASP	ASFYTP (lb)	ASFYBT (lb)	CFYAS (lb)	HFY (psi)	HREINF	PFC (psi)	TYLD (psi)
57[a]	Beckingsale	B11	0	1.33	156,526	78,368	37,251	48,749	1.635	215.92	323.50
58[a]	(1980)	B12	0	1.33	117,395	117,395	37,224	48,749	1.635	215.92	457.83
59	Attalla and Agbabian	SHC1	1	1.14	20,536	20,536	12,170	79,895	0.506	372.03	706.31
60[a]	(2004)	SHC2	1	1.14	20,536	20,536	12,170	79,895	1.013	372.03	688.26
61[a]		SOC3	1	1.14	20,536	20,536	12,170	79,895	1.013	372.03	688.56
62[a,b]	Birss (1978)	1	0	1.33	168,365	168,365	44,835	50,098	0.727	215.92	657.31
63		2	0	1.33	168,365	168,365	44,835	57,681	0.190	2006.47	658.28
64[a]	Teraoka et al.	HJ1	0	1.00	100,580	100,580	25,164	50,338	0.710	1,564.16	503.93
65[a]	(1997)	HJ2	0	1.00	116,404	116,404	25,164	50,338	0.710	1,564.16	592.50
66[a]		HJ3	0	1.00	112,829	112,829	25,164	50,338	0.710	1,564.16	575.69
67		HJ4	0	1.00	150,870	150,870	42,456	50,338	0.710	1,564.16	712.12
68[a]		HJ5	0	1.00	169,697	169,697	42,456	50,338	0.710	1,564.16	859.07
69[a]		HJ6	0	1.00	169,244	169,244	42,456	50,338	0.710	1,564.16	855.02
70[a,b]		HJ7	0	1.00	223,022	223,022	42,456	98,684	0.852	2,559.54	1,062.03
71		HJ8	0	1.00	211,265	211,265	42,456	98,684	0.852	2,559.54	1,071.13
72[a]		HJ9	0	1.00	225,659	225,659	42,456	98,684	0.852	2,559.54	1,149.38
73[a]		HJ10	0	1.00	227,873	227,873	49,331	98,684	0.852	2,559.54	1,075.90
74[a]		HJ11	0	1.00	311,191	311,191	35,469	98,684	0.852	2,559.54	1,420.75
75		HJ12	0	1.00	425,985	425,985	49,331	98,684	0.852	2,559.54	1,938.51
76[a]		HJ13	0	1.00	277,854	277,854	49,331	98,684	0.852	3,412.71	1,321.51
77[a]		HJ14	0	1.00	425,985	425,985	49,331	98,684	0.852	3,412.71	2,030.24
78[a]	Hayashi et al.	NO43	0	1.00	100,656	100,656	25,148	50,338	0.785	1,450.00	489.79
79	(1994)	NO44	0	1.00	116,314	116,314	25,148	50,338	0.785	1,450.00	568.55
80[a]		NO45	1	1.00	211,265	211,265	25,148	50,338	0.785	1,450.00	1,001.76
81[a]		NO46	0	1.00	112,829	112,829	25,148	50,338	0.785	1,450.00	555.94
82[a]		NO47	0	1.00	150,870	150,870	42,424	50,338	0.785	1,450.00	682.20
83[a]		NO48	0	1.00	169,697	169,697	42,424	50,338	0.785	1,450.00	820.82
84		NO49	0	1.00	264,081	264,081	42,424	50,338	0.785	1,450.00	1,152.40
85[a]		NO50	0	1.00	169,244	169,244	42,424	50,338	0.785	1,450.00	822.69
86[a]		HNO8	0	1.00	223,022	223,022	42,424	115,606	0.857	2,559.54	1,013.23
87		HNO9	0	1.00	211,265	211,265	42,424	115,606	0.857	2,559.54	1,025.58
88[a]		HNO10	0	1.00	225,659	225,659	42,424	115,606	0.857	2,559.54	1,098.74
89[a]	Teraoka et al.	HNO1	0	1.00	228,587	228,587	53,896	98,684	0.857	2,132.95	999.65
90[a]	(1994)	HNO2	0	1.00	228,587	228,587	53,896	98,684	0.857	2,132.95	999.65
91		HNO3	0	1.00	312,162	312,162	39,020	98,684	0.857	2,132.95	1,299.65
92[a]		HNO4	0	1.00	427,316	427,316	53,896	98,684	0.857	2,132.95	1,762.21
93[a]		HNO5	0	1.00	277,854	277,854	53,896	98,684	0.857	2,132.95	1,247.41
94[a]		HNO6	0	1.00	427,316	427,316	53,896	98,684	0.857	2,132.95	1,805.73
95	Zaid	ZS1	1	1.00	35,513	35,513	29,585	56,550	0.355	161.11	257.30
96[a]	(2001)	ZS2	1	1.00	107,113	107,113	29,585	56,550	0.355	161.11	667.09
97		ZS3	1	1.00	108,398	108,398	29,585	56,550	0.355	161.11	709.78
98[a]	Joh et al.	B1	0	1.17	34,228	34,228	11,409	44,507	0.215	497.69	300.93
99	(1991)	B2	0	1.17	34,228	34,228	11,409	44,507	0.431	497.69	310.39
100[a]		B8HH	0	1.17	37,303	37,303	12,434	191,400	0.609	568.72	333.81
101[a]		B8HL	0	1.17	37,303	37,303	12,434	191,400	0.609	568.72	334.82
102[a]		B8LH	0	1.17	37,303	37,303	12,434	54,665	0.215	568.72	334.53
103		B8MH	0	1.17	37,303	37,303	12,434	54,665	0.419	568.72	335.21
104[a]		B9	0	1.17	37,303	37,303	12,434	191,400	0.609	568.72	333.81
105[a]		B10	0	1.17	37,303	37,303	12,434	191,400	0.609	568.72	333.42
106[a]		B11	0	1.17	37,303	37,303	12,434	191,400	0.609	568.72	333.97
107	Fujii and Morita	A1	1	1.14	141,492	141,492	18,903	42,195	0.428	440.84	2,108.15
108[a]	(1991)	A2	1	1.14	54,129	54,129	11,382	42,195	0.428	440.84	826.22
109[a]		A3	1	1.14	141,492	141,492	18,903	42,195	0.428	1,322.53	2,108.15
110[a,b]		A4	1	1.14	141,492	141,492	18,903	42,195	1.142	1,322.53	2,108.15

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Table 1. (*Continued.*)

Specimen number	Reference number	Specimen	Fail mech.	ASP	ASFYTP (lb)	ASFYBT (lb)	CFYAS (lb)	HFY (psi)	HREINF	PFC (psi)	TYLD (psi)
Minimum				1.00	20,536	20,536	5,822	0	0.000	128.81	183.45
Maximum				1.50	484,182	487,266	94,143	199,160	3.801	3,412.71	2,692.17

[a]Specimens used in the training data set for Model III in the RVM approach.
[b]Specimens used as relevance vectors in the RVM approach.

(ASFYBT), defined as the product of beam bottom reinforcement yield stress and the total area of the beam bottom longitudinal reinforcement; the total yield strength of the column reinforcement (CFYAS), defined as the product of column reinforcement yield stress and the total area of the column longitudinal reinforcement; the yield stress of transverse or hoop reinforcement within the joint (HFY); the percentage of transverse or hoop reinforcement within the joint (HREINF); axial load stress (PFC), defined as total axial load on the specimen divided by the cross-sectional area of the column; and the joint shear stress at first yielding of the beam longitudinal reinforcement (TYLD). The TYLD is represented by

$$\mathrm{TYLD} = \frac{1}{h_c b_j}\left(\frac{M_L + M_R}{h_b} - V_c\right) \tag{1}$$

where h_c = the height of the column; b_j = the maximum out-of-plane dimension of the beam or column; h_b = the height of the beam; M_L and M_R are the flexural strengths of the beam on the right and left of the joint and computed in accordance with ACI 318-R05 (ACI 2005); and V_c = the lateral load applied to the top of the column at the nominal strength of the beams. The definition of joint shear stress is similar to that recommended by ACI Committee 352 (ACI 2002) with the exception that it (1) employs a slightly larger joint volume, with the result that horizontal and vertical shear stresses are equal and (2) defines demand on the basis of frame member flexural strength, rather than longitudinal steel area, with the result that determining the frame member moments and column shear is consistent.

By utilizing the data set in Table 1, the authors propose probabilistic methods that would determine the probability of inelastic mechanisms leading to failure initiation of RCBC interior connections.

Relevance Vector Machine Methodology

Probabilistic identification of failure initiation mode in a reinforced concrete interior beam–column connection is an example of a classification problem that can be solved by using a recently introduced machine learning methodology. The methodology uses Bayesian formulation of a linear model with an appropriate prior that results in sparse representation and a probabilistic output through Bayesian inference [i.e., relevance vector machine (RVM)]. Tipping (2000, 2001) introduced the concept of the RVM, which allows computation of the prediction intervals by measuring the uncertainties of the parameters and the data. Tipping (2000) provides details about the RVM methodology. The following paragraph describes the method briefly.

The entire data set in Table 1 is to be divided into training and a testing data set. The training data set (represented by “a” in Table 1) is utilized to develop the model, whereas the model is evaluated by using the testing data set. To develop the model for the previously mentioned classification problem, a set of targets, Y_i, represent the occurrence of either Event 0 (defined as beam yielding before joint failure) or Event 1 (defined as joint failure before beam yielding). The input parameters, x_i, are the geometric, material, and loading parameters of a particular experimental specimen, as Table 1 shows. A basis function, $\Phi j(x)$, represents the input parameters for each experimental specimen test. Therefore, the total number of basis functions is the total number of samples in the training data set. Each basis function is defined as the function of the kernel, K, by

$$\Phi(x_n) = [1, K(x_n, x_1), K(x_n, x_2), \ldots, K(x_n, x_N)] \tag{2}$$

The kernel K, is a Gaussian function with a mean of 0 and a variance of 1. In RVM methodology, usually the set of targets are correlated with a set of input vectors by using the following equation:

$$Y = w_1\Phi_1(x) + w_2\Phi_2(x) + w_3\Phi_3(x) + \cdots + w_k\Phi_k(x) \tag{3}$$

where $\Phi_i(x)$ = the previously defined basis functions and w_i = the weight parameters. Gamma priors are introduced to the model weight, w_i, which is governed by a set of hyperparameters (i.e., maximum likelihood estimations). One gamma prior is one associated with each weight for which the most probable value is iteratively estimated from the data. To obtain the probabilistic estimates for the binomial classification problem in this manuscript, a statistical convention is followed and the linear model in Eq. (3) is generalized by applying a logistic sigmoid function $\sigma\{Y(x)\} = 1/(1 + 1/e^Y)$ to $Y(x)$.

Discussion of Results

In performing the formulation, weight parameters were determined from randomly selected training data set by using the Gaussian kernel. The design value of the width of the Gaussian kernel was chosen by a trial-and-error approach to get the best performance within a range of 0.1 to 10. The input parameters of the model were also normalized within 0 to 1 by the ratio of the difference between the actual value and the minimum value of the data set to the difference between the maximum and minimum value. The performance of the training data set is expressed as a percentage and is the ratio of the number of data predicted accurately by RVM to the total number of data in the training set. The obtained weight parameters are utilized to validate the results in the testing data set. The performance of the testing data set is expressed as a percentage and is the ratio of the number of data predicted accurately by RVM to the total number of data in the testing set.

Different combinations of training and the testing data set were analyzed to obtain the best result. Table 2 shows different percentage choices of training and testing and their performances, and the Gaussian kernel widths and the number of relevance vectors. By contrast, Fig. 1 shows the variations of the training and testing performance with different percentage choices for the training and the testing data set. On the basis of observations in Table 2 and Fig. 1, the difference between training and testing performance is clearly lowest for Model III. Therefore, Model III (training data, 70%; testing data, 30%) gives the best result. The difference between training performance and testing performance is marginal for Model III; therefore, the developed RVM model has the capability to avoid and therefore has good generalization capability.

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Table 2. RVM Models with Different Percentages for the Training Data Set

Model number	Training data (%)	Testing data (%)	Gaussian kernel width	Number of relevance vectors	Training performance (%)	Testing performance (%)
I	50	50	1.5	10	92.7	61.8
II	60	40	0.3	14	90.9	81.8
III	70	30	1.2	6	94.8	93.9
IV	80	20	0.1	50	88.6	81.8
V	90	10	1.3	7	80.8	100

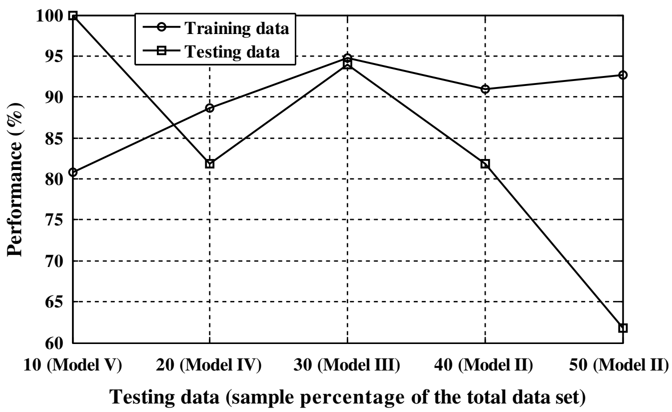

Fig. 1. Performance variation with different percentage choices of training and testing data set in the RVM model

RVM models employ approximately 7 to 57% of the training data as relevance vectors (Model I=18.18%; Model II=21.21%; Model III=7.79%; Model IV=56.81%; and Model V=7.07%). (This is represented by "b" in Table 1.) The results from the compact, computationally efficient, sparse and simple model consist of weights assigned only to relevance vectors in the training data set (which are shown in Table 1 with "a" and "b"). Table 3 provides these weights. The model also produces smooth functions and probabilistic estimates of specified targets.

Binomial Logistic Regression Model

In a logistic regression model (Hosmer and Lemeshow 2000; Greene 2000), the likelihood of the occurrence of a discrete qualitative event (Y), is linearly related to several discrete quantitative material and geometric parameters (X) of the specimen, as Eq. (4) shows. Mitra et al. (2011) have described the details of the methodology and the authors of this paper briefly describe it for the sake of completeness. The discrete qualitative event, Y, refers either to joint shear failure before beam yielding (marked by Event 1) or to beam yielding before joint failure (marked by Event 0). It is represented by

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \cdots + \beta_k X_k \tag{4}$$

The independent covariates, X_i, are the geometric, material and loading parameters of an experimental investigations. These covariates are given in Table 1 and discussed in the "Experimental Data Set" section. In Eq. (4), the likelihood of observing a discrete event of brittle joint shear failure is defined by the log of the odds ratio for that event. The odds ratio for Event 1 is the ratio of the probability of the occurrence of Event 1 ($P_{E=1}$) to the probability of the occurrence of Event 0 ($P_{E=0}$). Thus,

$$Y = \log\left(\frac{P_{E=1}}{1 - P_{E=1}}\right) = \log\left(\frac{P_{E=1}}{P_{E=0}}\right) = \beta_0 + \sum_{k=1}^{K} \beta_k X_k \tag{5}$$

As Ben-Akiva and Lerman (1985) described, Eq. (5) may be manipulated to define the probability of the occurrence the of Events 1 and 0, respectively, by

$$P_{E=1} = \frac{e^Y}{1 + e^Y} \tag{6a}$$

$$P_{E=1} = \frac{1}{1 + e^Y} \tag{6b}$$

The method of maximum likelihood (Greene 2000), which provides a means of choosing an asymptotically efficient estimator for a set of parameters, is typically used to compute logistic regression parameters, β_i.

The current model differs from the model by Mitra et al. (2011) in the selection of the independent covariates. The current model utilizes basic geometric and material parameters as independent covariates, whereas the Mitra et al. (2011) model uses demand parameters (which are complex functions of geometric and material parameters) as independent covariates. Therefore, the current model is more user-friendly and developed primarily for use by practicing engineers.

Table 3. RVM Model III Weights

Specimen number (Table 1)	9	12	50	62	70	110
RVM weights	−8.22	26.61	19.6	−45.32	−37.61	54.96

Discussion of Results

Table 4 presents the computed regression parameters, which were obtained from the previously mentioned binomial logit model. By using the definition of Y in Eq. (5), the sign of a regression parameter indicates whether an increase in the associated design parameter increases or decreases the likelihood of joint failure before beam yielding. A positive (negative) regression parameter indicates that increasing the associated design parameter increases (decreases) the likelihood of a joint failure before beam yielding. A negative (positive) regression parameter similarly indicates that increasing the associated design parameter increases (decreases) the likelihood of beam yielding before joint failure. On the basis of the signs of the parameters in Table 4, an increase in aspect ratio (ASP), column yield strength (CFYAS), and joint shear strength at the yielding of beam longitudinal reinforcements (TYLD) would increase the likelihood of joint shear failure before beam yielding. An increase in the other parameters would increase the likelihood of beam yielding before joint failure or otherwise would decrease the likelihood of joint failure before beam yielding.

The magnitude of a regression parameter multiplied by the mean of its corresponding design variables (the "influence factor" column in Table 4) indicates the relative importance of the design variables

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Table 4. Regression Parameters of the Binomial Logit Model

Covariate	Estimated β	Influence factor
ASP	5.103	5.53
ASFYTP	$-5.77E-06$	-3.84
ASFYBT	$-1.03E-05$	-5.92
CFYAS	$2.03E-05$	2.76
HFY	$-9.00E-03$	-4.58
HREINF	-2.102	-1.47
PFC	-0.266	-1.96
TYLD	2.895	16.59
Constant	-7.673	

in determining connection failure initiation response. The sign of the influence factor is similar to the sign of the regression parameter. The conclusion is similar to that of the regression parameter. On the basis of the results in Table 4, the joint shear strength at yielding of the longitudinal reinforcement bars in the beam (TYLD) is the most influential parameter that determines the likelihood of failure initiation mechanism; by contrast, the least influential parameter is the percentage of transverse reinforcement within the connection region (HREINF).

Predictive Efficiency of the Models

The predictive efficiency of the models are determined by the number of samples they correctly predict and the probability associated with their ability to predict the occurrence of Event 1. Fig. 2 shows the predictive efficiency of the probabilistic models. Fig. 2(a) shows the results from the RVM, whereas Fig. 2(b) shows the results from the binomial logit regression model. In Fig. 2, specimens from the data set that exhibit beam yielding before joint failure (i.e., Event 0) are plotted as circles and specimens that exhibit joint failure before beam yielding (i.e., Event 1) are plotted as squares. The ordinate of Fig. 2 is the probability of the occurrence of Event 1. If the model were perfect, all specimens exhibiting Event 0 would have a computed probability of occurrence of 0.00 for Event 1, whereas all specimens exhibiting Event 1 would have a computed probability of occurrence of 1.0. The data in Fig. 2 show that neither model is perfect. On the basis of the RVM model plot in Fig. 2 and by using a probability of 50% as indicative of a response, the RVM model correctly predicts joint failure before beam yielding for 96% of the specimens exhibiting the response mode of Event 1 and it correctly predicts beam yielding before joint failure for 91% of specimens exhibiting the response mode of Event 0. The model overall correctly predicts (by 95%) the failure initiation mechanism within the connection. On the other hand, on the basis of the binomial logit plot in Fig. 2 and by using a probability of 50% as indicative of a response, the binomial logit model correctly predicts joint failure before beam yielding for 88% of the specimens exhibiting the response mode of Event 1 and it correctly predicts beam yielding before joint failure for 95% of specimens exhibiting the response mode of Event 0. The model overall correctly predicts (by 93%) the failure initiation mechanism within the connection. The data in Fig. 2 also reveal that the observed data points are much closer to 0 and to 1 for the binomial logit model than for to the RVM model. Therefore, the probability associated with predicting the correct response is much higher in the binomial logit model than in the RVM model. The authors postulate that a different prior utilization in the RVM model may account for similarly high probabilities, as observed in the binomial logit model. A Bayesian-based model does not always give the best prediction, unless the prior is known and defined appropriately.

Model Application by User

The section describes in detail the steps a user needs to perform to evaluate the probability of occurrence of either failure initiation mechanism, given a set of material, geometric, and axial load parameters of a specific interior reinforced concrete beam–column joint. The first step for both models involves computing the independent material, geometric parameters x_i (i.e., ASP, HREINF, ASFYTP, ASFYBT, CFYAS, HFY, PFC and TYLD), as discussed in the "Experimental Data Set" section, for a specific interior beam–column connection. The parameters ASP and HREINF are dimensionless. Parameters ASFYTP, ASFYBT, and CFYAS are measured in Newtons or pounds, whereas parameters HFY, PFC, and TYLD are measured in Mpa or psi.

Binomial Logit Regression Model

1. These parameters are used with the logistic regression coefficients, β_i in Eq. (5) (obtained from Table 4) to determine the probability of the occurrence of either of the inelastic events that lead to failure initiation within an interior connection. If the parameters are specified in pounds and psi, then the following equation can determine Y:

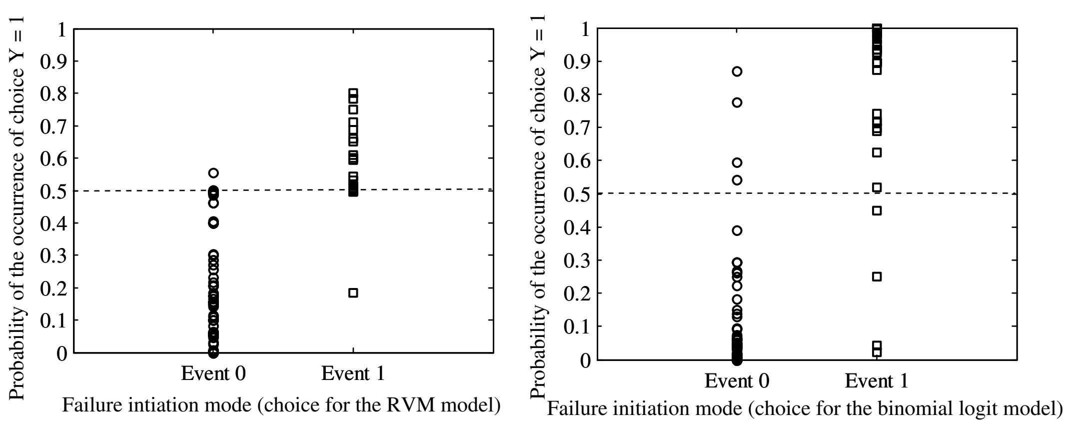

Fig. 2. Predictive efficiency of the two probabilistic models

$$
\begin{aligned}
Y = &\ 5.103 \times \mathrm{ASP} - (5.77E-6) \\
&\times \mathrm{ASFYTP}/(145 \times 0.04 \times 0.04) - (1.03E-5) \\
&\times \mathrm{ASFYBT}/(145 \times 0.04 \times 0.04) + (2.03E-5) \\
&\times \mathrm{CFYAS}/(145 \times 0.04 \times 0.04) - 0.009 \times \mathrm{HFY}/(145) \\
&- 2.102 \times \mathrm{HREINF} - 0.266 \times \mathrm{PFC}/(145) + 2.895 \\
&\times \mathrm{TYLD}/(145) - 7.673
\end{aligned} \tag{7a}
$$

However, if the parameters specified are in Newtons and Mpa, then the following equation can determine Y:

$$
\begin{aligned}
Y = &\ 5.103 \times \mathrm{ASP} - (5.77E-6) \times \mathrm{ASFYTP} - (1.03E-5) \\
&\times \mathrm{ASFYBT} + (2.03E-5) \times \mathrm{CFYAS} - 0.009 \\
&\times \mathrm{HFY} - 2.102 \times \mathrm{HREINF} - 0.266 \\
&\times \mathrm{PFC} + 2.895 \times \mathrm{TYLD} - 7.673
\end{aligned} \tag{7b}
$$

2. Once the value of Y has been determined from the previous equations, the value of Y is replaced in Eqs. (6a) and (6b) to obtain the probabilities for Event 1. If the value of the probability in Eq. (6a) is greater than 0.5, then the joint exhibits Event 1 (i.e., joint failure before beam yielding); otherwise, it exhibits Event 0 (i.e., beam yielding before joint failure).

Relevance Vector Machine Model

1. Compute the ratios for each independent parameter (xi) by

$$
z_i = \frac{x_i - x_{i,\min}}{x_{i,\max} - x_{i,\min}} \tag{8}
$$

The value of $x_{i,\min}$ represents the minimum value and $x_{i,\max}$ represents the maximum value of the specific independent parameter. The end of Table 1 provides the values of $x_{i,\min}$ and $x_{i,\max}$ for each independent parameter. A user should be careful when using these values because the values in Table 1 are in U.S. customary units. These values should be changed appropriately if a user wants to use SI units. The value for the ratios of each independent parameter (z_i) should be between 0 and 1. Therefore, the model cannot be utilized if the value of any independent parameters is more than the maximum value or less than the minimum value.

2. Compute similar ratios for the relevance vectors. As Tables 1 and 3 indicate, the relevance vectors correspond to specimen number 9, 12, 50, 62, 70, and 110. The ratios for each independent parameter are denoted as $z_i^9, z_i^{12}, z_i^{50}, z_i^{62}, z_i^{70}$, and z_i^{110}. As noted previously, users should be careful with regards to the units.
3. To determine the target value y, the parameters are used with the relevance vector weights (Table 3) and with a design value of 1.2 for the Gaussian kernel width. If the calculated value of y is negative, then it usually corresponds to Event 0; if the value of y is positive, then it corresponds to Event 1. The following equation is utilized to determine y:

$$
\begin{aligned}
y = &-8.22 \exp\left\{-\frac{(z_i^9 - z_i)^2}{2.88}\right\} + 26.61 \exp\left\{-\frac{(z_i^{12} - z_i)^2}{2.88}\right\} \\
&+ 19.6 \exp\left\{-\frac{(z_i^{50} - z_i)^2}{2.88}\right\} - 45.32 \exp\left\{-\frac{(z_i^{62} - z_i)^2}{2.88}\right\} \\
&- 37.61 \exp\left\{-\frac{(z_i^{70} - z_i)^2}{2.88}\right\} + 54.96 \exp\left\{-\frac{(z_i^{110} - z_i)^2}{2.88}\right\}
\end{aligned} \tag{9}
$$

4. On determining the value of y, it is entered into the following equation to determine the probability of the occurrence of Event 1:

$$
P_{E=1} = \frac{1}{1 + (1/e^y)} \tag{10}
$$

If the obtained value of the probability is greater than 0.5, then the joint exhibits Event 1 (i.e., joint failure before beam yielding); otherwise, it exhibits Event 0 (i.e., beam yielding before joint failure).

Summary and Conclusions

This research has presented two probabilistic easy-to-use models for practicing engineers. This represents a first attempt in which the qualitative nature of an inelastic mechanism leading to failure or the failure initiation mechanism of RCBC connections (either joint failure or beam reinforcement yielding) has been related to several independent, discrete, quantitative parameters. These parameters can be easily obtained from the geometric, material, and loading characteristics of an experimental specimen. Both models can provide a preliminary understanding of the behavioral mechanism within the connection and thereby help an engineer design a new connection or retrofit a damaged connection.

The binomial logistic regression and the relevance vector machine methodologies exhibit good predictive efficiency. The relevance vector machine methodology, which uses a Bayesian method and a Gauss prior, very accurately predicts the occurrence of either of the events. However, the binomial logistic regression model provides better certainty to the obtained results. The binomial logistic regression model can also obtain a quantitative estimate relative effect of each independent parameter on the failure initiation mechanism response. The relevance vector machine methodology represents a limitation in that it cannot be used if the values of the independent parameters are beyond the maximum and minimum bounds of each independent parameter in the data set.

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