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# Process Identification with Autoregressive Linear Regression Method using Experimental Data: Review

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## Abstract

**Objective:** To obtain mathematical model and parameters of poorly understood and imprecisely known plant/process. **Methods:** One solution to this problem is to obtain these using identification techniques. Process identification is a technique where a mathematical model of the process under study is build from process input-output data. Several autoregressive models are used to estimate the present value of the model using its past values of the process. Initially, the data set is generated for the given system and the auto regressive model is fitted to it, for the estimation of the model parameters. Residual error for a system is calculated using auto regression model parameters. **Findings:** A mathematical model for plant under study can be formulated with different system identification using Linear Regression methods like Auto Regressive eXogenous variable (ARX), Auto Regressive Moving Average with an eXogenous variable (ARMAX), Output Error (OE) and Box-Jenkins (BJ). For high model order ARX model is preferred and takes low computations but only suitable for white noise. The ARMAX model considers disturbance affecting process and provides higher performance index i.e. fitness which reveals percentage variation in output estimated by respective model. The Output-Error (OE) model estimates process model but cannot model disturbance features. The residual analysis of Box-Jenkins model shows that the prediction error is not auto-correlated, correlated and is uncorrelated with the input applied to process, thus showing Box-Jenkins model ability to capture noise dynamics of process. **Applications/Improvements:** The process model can be identified for the unknown, poorly known or partially known system and formulated model can be used in model Predictive controller design and Adaptive control techniques.

**Keywords:** ARX, ARMAX, BJ, Parametric, Plant Modelling, System Identification OE

## 1. Introduction

The dynamic process or plant modelling becomes difficult for Partial or completely unknown process, process order, etc. as present output depends on the instantaneous values of its input and previous process output. A dynamical system model can be found mathematically by relating output, input and noise/disturbance signals of a plant. Process or plant identification is the process of constructing a model of dynamic system and estimates the process parameters from the given input and output data. Regression is used to predict output of the system. An autoregressive model is a stochastic model which uses the present and past values of the process to predict model present values. Autoregressive steps involves proper model selection based on criterion, estimation of model parameters, checking model fitness, model order selection, simulation and model validation. If the results are unsatisfactory and not within constraints, then revise the parameters and iterate through the process[1–3].

### 1.1 Autoregressive Linear Regression Method

The developed models using Autoregressive Linear Regression Method ARMAX were good representatives of

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the process with the lower mean of squared error value and the higher % fit value than the developed ARX model[1]. A MIMO adaptive dual MPC also can be developed using ARMAX models that can estimate parameters on line[2]. A black box mathematical ARX and ARMAX models is developed for thermoelectric refrigerator systems by Recursive Least Square (RLS) method to predict parameters[3]. A state space ARX model can be developed in Model Predictive Controller for a first order system with three different dead times that gives offset-free control for type sudden disturbances to the plant and to track set point[4]. Steady-state identification with ARX for nonlinear Continuous Stirred-Tank Reactor (CSTR) can be done with regression method. Out of the ARX, ARMAX and BJ models developed for steam generation plant using linear identification techniques, it is found that the fitness level of the ARMAX model data was the best as ARMAX model structure includes disturbance dynamics[5]. The Box-Jenkins (BJ) model describes disturbance properties in a process dynamics and showed that the ARX model may not be as good if disturbance dynamics are considered[6]. The performance of different models are performed equally well[7]. The predicted OE model has less % peak overshoot and lesser setting time than the ARX, ARMAX and BJ models[8]. For tracking the time variant parameters, several recursive estimation methods and their properties were demonstrated used along with forgetting factor by modifying basic algorithm[9]. A new approach to recursive identification for ARMAX systems was implemented that assumes independent and identically distributed input signals and also handles a wider class of input signals[10]. The advantage of the multi-time scale algorithm is verified with numerical examples. It was found that the accuracy of ARMAX model is better than that of ARX model as ARMAX model estimates characteristics of the colour noise can also be predicted with ARMAX model and the predicted model output almost matches actual output[11]. An offset-free MPC based controller can be designed with ARX models[12]. Accurate and matched PID response can be obtained with an ARX Model Predictive control model using for the pH Process in a CSTR[13]. A multi-time-scale scheme for different class of input signals that can be extended to MIMO systems[14]. Acceptable performance was achieved in an adaptive controller design for a pressure tank using nonparametric and parametric identification recursive least squares to obtain an ARMAX model of the system[18]. A predicted mathematical model of a complex system show slighter delay, less oscillations and faster response than the model predicted with first principles method[19]. OE model gives the better closed loop response with less settling time and small peak overshoot[20]. The parameters of a multi machine power system were accurately computed by using Recursive Least Square (RLS) technique and the identified linear model was validated as the response of identified linear model is very near to nonlinear model[21].

## 2. Theoretical Analysis

A black box linear process model can be mathematically represented as shown in Equation (1).

$$\left[A(q-1).y(t)\right] = u(t).\left[\frac{B(q-1)}{F(q-1)}\right] + e(t).\left[\frac{C(q-1)}{D(q-1)}\right] \quad (1)$$

Where, y (t), u (t) and e (t) are the pl output, process input and noise affecting the process. All A, B, C, D and F are polynomials. The parametric model structures differ by presence of few or all these polynomials. Thus, various model structures with flexibility of modelling the process dynamics and noise occurred in process[9,15–17].

## 3. (A) ARX Model

AR is Auto-regressive nature of the noise model and X is eXogenous input 'e'. The input-output relationship of an ARX model[16] is given by a linear differential equation as Equation (2).

$$\begin{aligned} &Y(t) + a_1 y(t-1) + \ldots + a_n y(t-n_a) \\ &= b_0 u(t-d) + \ldots + b_n bu(t-d-n_b) + e(t) \end{aligned} \quad (2)$$

The input-output ARAX model structure is as given by Equation (3).

$$A\ (q\text{-}1).y\ (t) = u\ (t).\ B\ (q\text{-}1) + e\ (t) \quad (3)$$

The Equation (4) gives the output of this model structure in which the dynamic noise model is 1/A and [B (q-1)/A (q-1)] extracts the input-output model.

$$y(t) = u(t).\left[\frac{B(q^{-1})}{A(q^{-1})}\right] + e(t).\left[\frac{1}{A(q^{-1})}\right] \quad (4)$$

The ARX model as shown Figure 1 analytical form makes it the simplest and efficient estimation methods with unique solution, hence preferred for higher order models. The drawback of the ARX model therefore is preferable, especially when the model order is high. The drawback of ARX model structure is its system dynamics which consists of that the disturbances.

### 3.1 (B) ARMAX Model

ARMAX model as shown Figure 2 provides more flexibility to model a noise using a moving average of white noise called C parameters and also the present and past values of the disturbances acting on the process[16]. Also this model considers load disturbances entering in a process as a error term.

The ARMAX model can be described by the Equation (5).

$$y(t) = u(t).\left[\frac{B(q^{-1})}{A(q^{-1})}\right] + e(t).\left[\frac{C(q^{-1})}{A(q^{-1})}\right] \quad (5)$$

### 3.2 (C) Output-Error (OE) Model

The OE model as shown Figure 3 gives description of the process dynamics separately without disturbance characteristics consideration[16].

The output is give as Equation (6).

$$y(t) = u(t).\left[\frac{B(q^{-1})}{F(q^{-1})}\right] + e(t) \quad (6)$$

### 3.3 (D) Box-Jenkins Model

The Box-Jenkins (BJ) structure models disturbance properties separately from system dynamics[16].

The output of this model which is as shown Figure 4 is given as Equation (7).

$$y(t) = u(t).\left[\frac{B(q^{-1})}{F(q^{-1})}\right] + e(t).\left[\frac{C(q^{-1})}{D(q^{-1})}\right] \quad (7)$$

## 4. Experimental Analysis of Different Model Estimation using Prediction Error Method and their Comparison

Following revisited example[16] shows how ARX, ARMAX, OE and BJ models are used for parameter estimation and

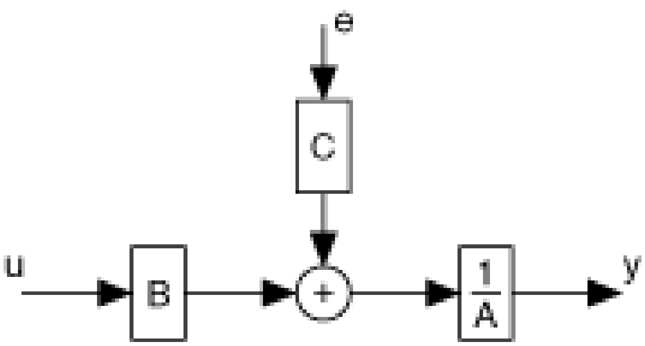


**Figure 1.** Signal flow of an ARX model.

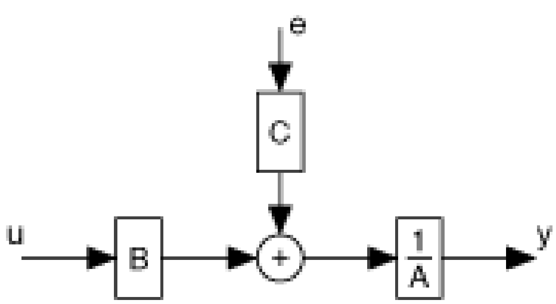


**Figure 2.** Signal flow of an ARMAX model.

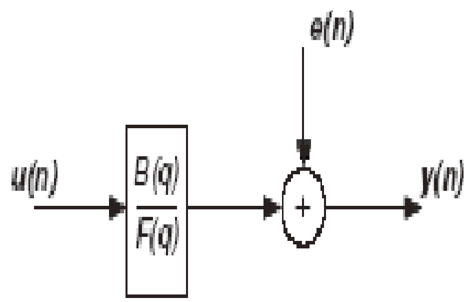


**Figure 3.** Signal flow of an OE model.

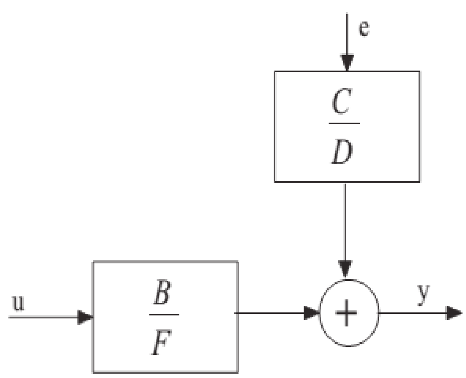


**Figure 4.** Signal flow of BJ model.

their performance analysis is compared with state space and transfer function models[9,15–17].

The estimated model details are as shown in Table 1 which includes Predicted Model, coefficients , polynomial order, FPE (Final Prediction Error), MSE (Mean Square Error) and fitness which reveals percentage variation in output estimated by respective model.

**Table 1.**

| Estimated Model | PARAMETERS | | | |
|---|---|---|---|---|
| | **Estimated Model , Coefficients and Polynomial Orders** | **Model Fit %** | **FPE** | **MSE** |
| Transfer function | $\frac{(-0.05428\,s^2 - 0.02386\,s + 29.6)}{(s^2 + 1.361\,s + 4.102)}$ | 71.69 | 1.279 | 1.195 |
| State Space | A =[0.007167,1.743,–2.181, –1.337] B =[ –0.09388 –0.2408] C =[–47.34 14.4] D = 0 , K =[0.04108 0.03751] | 75.08 | 1.019 | 0.9262 |
| ARX | A(z) = 1 – 1.32z–1 + 0.5393 z–2 B(z) = 0.9817 z–1+ 0.4049 z–2 ; na = 2 , nb = 2, nk = 1 | 71.46 | 1.297 | 1.214 |
| OE | $B(z) = 0.8383z^{-1} + 0.7199\,z^{-2}$ $F(z) = 1\text{-}1.497\,z^{-1} + 0.7099\,z^{-2}$ nb=2 ,nf=2 , nk=1 | 71.46 | 1.297 | 1.214 |
| ARMAX | A(z) = 1 –1.516 $z^{-1}$+ 0.7145 $z^{-2}$ B(z) = 0.982 $z^{-1}$+ 0.5091 $z^{-2}$ C(z) = 1–0.9762 $z^{-1}$+ 0.218 $z^{-2}$ na=2; nb=2 ;nc=2; nk=1 | 75.08 | 1.02 | 0.9264 |
| Box Jenkins | B(z) = 0.992 z–1+ 0.470 z–2 C(z) = 1–0.628 z–1– 0.122 z–2 D(z) = 1– 1.22 z–1+ 0.379 z–2 F(z) = 1– 1.52 z–1+ 0.724 z–2 | 75.47 | 1.024 | 0.8974 |

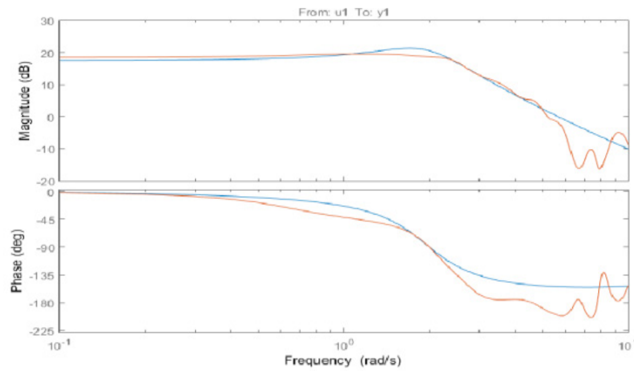


**Figure 5.** Frequency responses of state space model with original model.

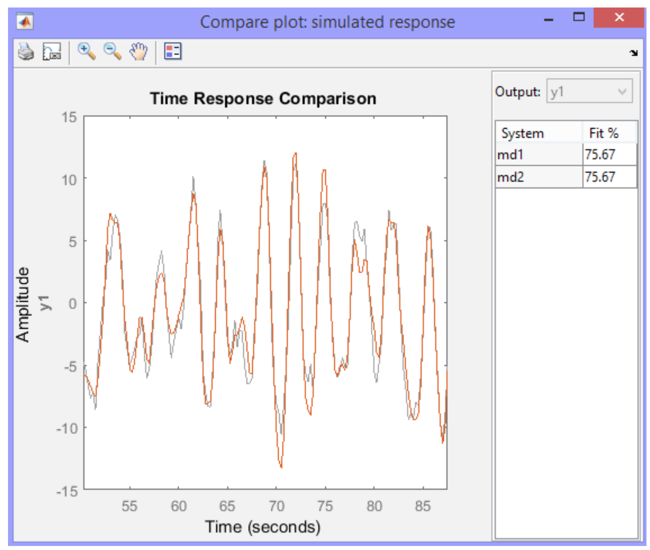


**Figure 6.** Response of estimated OE model.

As shown in Figure 5, the frequency response from state space model is very close to the transfer function model frequency response.

The response of OE model is as shown in Figure. 6. Quality of a model is to compute the residuals disturbance that could not be explained by the OE model. The residuals are calculated to find residuals disturbance with OE model and is as shown in Figure.7

From Figure 7, the cross correlation between residuals and input lies in the confidence region shows that that there is no significant correlation and gives adequate estimation index. But the correlation of noise is significant as it cannot be seen as a noise which indicates that the estimated noise model in inadequate.The predicted behaviour of estimated models is as shown in Figure 8.

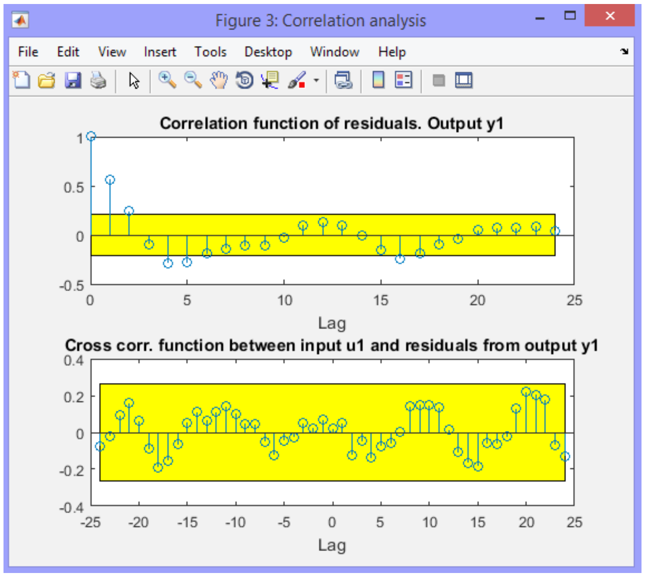


Figure 7. Residual analysis of estimated OE model.

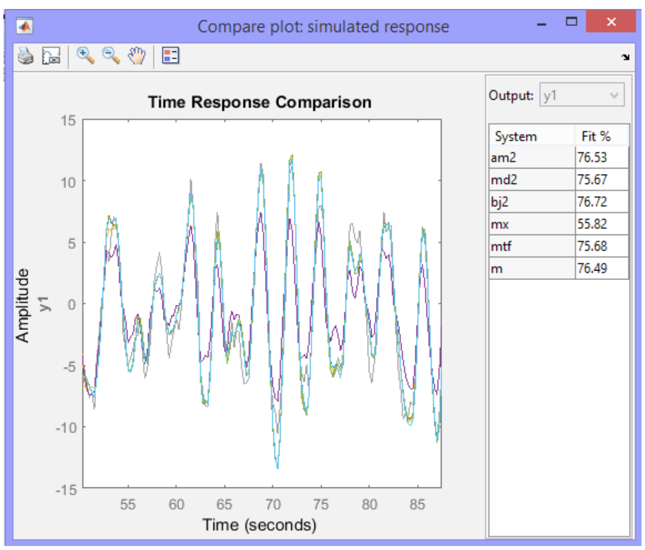


Figure 8. Comparison of estimated models.

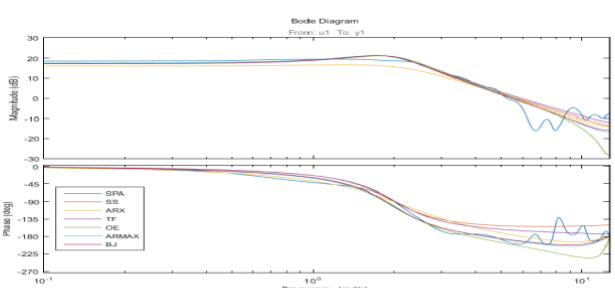


Figure 9. Comparison of frequency response of estimated models.

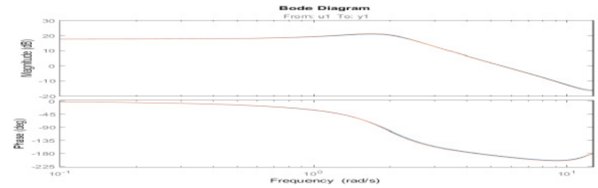


Figure 10. Comparison of frequency responses of estimated ARMAX model with actual system.

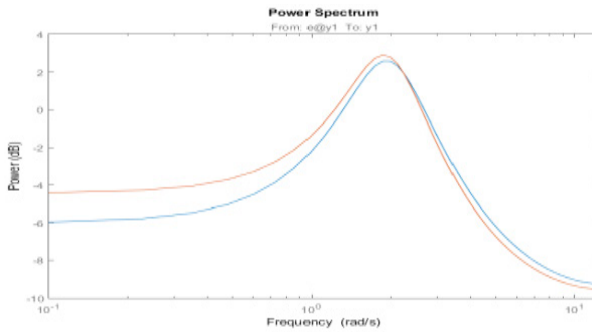


Figure 11. Noise spectrum of ARMAX and actual system.

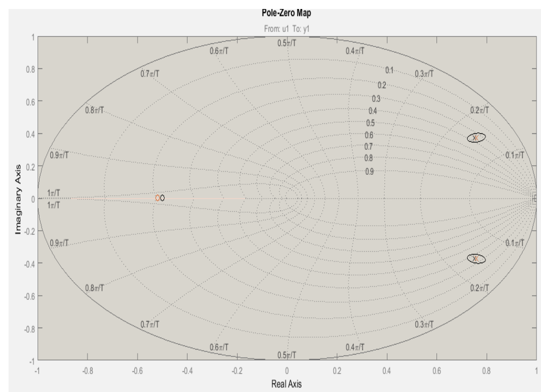


Figure 12. Pole zero plot of all estimated and actual model.

The residual analysis of Box-Jenkins model shows that the prediction error is not auto-correlated, correlated and is uncorrelated with the input applied to process. Thus showing Box-Jenkins model ability to capture noise dynamics of process with the C and D polynomials estimation. The frequency functions of all the estimated models are compared and are shown in Figure 9.

Comparison of frequency responses of estimated ARMAX model with actual system is done and is shown in Figure 10.

As shown in Figure 11, the noise spectra for ARMAX is compared with actual system. Figure 12 shows pole zero plot for all the estimated models which shows that like actual system pole zeros, pole zeros of estimated models are well inside the unit circle/uncertainty region[9,15–17].

## 5. Conclusion

This paper focuses on review of four parametric models ARX, ARMAX, OE and BJ. After revisiting the example explained[17], following features of above four models can be revised. For high model order ARX model is preferred and takes low computations but only suitable for white noise. The ARMAX model considers disturbance affecting process and provides higher performance index i.e. fitness which reveals percentage variation in output estimated by respective model. The Output-Error (OE) model estimates process model but cannot model disturbance features. The residual analysis of Box-Jenkins model shows that the prediction error is not auto-correlated, correlated and is uncorrelated with the input applied to process, thus showing Box-Jenkins model ability to capture noise dynamics of process with the C and D polynomials estimation.

The analysis done in this paper with the revisited examples and literature shows that the process identification is reliably possible from the process input, output and noise data to implement linear process model using ARX, ARMAX, BJ and OE models.

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