A review on Computational Electromagnetics Methods

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Abstract

Computational electromagnetics (CEM) is applied to model the interaction of electromagnetic fields with the objects like antenna, waveguides, aircraft and their environment using Maxwell equations. In this paper the strength and weakness of various computational electromagnetic techniques are deliberated in detail. Performance of various techniques in terms accuracy, memory and computation time for application specific tasks such as modeling RCS (Radar Cross Section), space Applications, thin wires and antenna arrays are presented in this paper. Commercial software codes has certain limitations, Agilent ADS could not model 3D structures, HFSS is accurate but execution time is high, WIPL-D® does not support modeling Inhomogeneous dielectrics embedded metal objects and periodic structures. IE3D® is not suited for geometry with finite details. However for regular shapes like rectangular patch MOM based IE3D provides accurate results than FEM based IE3D. The complicated structures are dealt with accuracy by using CST Microwave Studio® and HFSS®. Although CST and HFSS has similar interface in dealing with geometry with fine details, CST had edge over HFSS software as it starts in time domain and ends in frequency domain. HFSS uses Finite Element Method (FEM) to arrive at frequency domain solution. FEKO® has two main solvers MOM based and GTD based. GTD based FEKO is good in handling large structures like reflector antennas.

1. Introduction

The Widespread use of antennas has spurred considerable attention to the computational analysis of electromagnetics The CEM techniques came into limelight after the introduction of three pillars of numerical analysis viz. FDTD (Finite Difference Time Domain) , FEM (Finite Element Method) and MOM (Method of moments). Most EM problems ultimately involve solving only one or two partial differential equations subject to boundary constraints but a very few practical problems like modeling homogeneous, inhomogeneous problems and boundary value problem can be solved without the aid of a computer. Computational Electromagnetics techniques are flexible. CEM finds its application in fields like design and analysis of RCS (Radar Cross Section), antenna geometry, bio medical applications, space borne radar and satellite applications, hand held devices, nano photonic devices and other communication devices.

CEM is used in solving EM compatibility problems and issues associated with them. Few issues like Multiscale model, macro-models, time-domain and frequency-domain models, the use of structured meshes, un-structured meshes and stochastic models in EM compatibility are discussed in [1]. CEM is broadly classified into numerical methods, high frequency methods and other methods. The Numerical methods includes integral equation based MOM method, differential equation based FEM and FDTD. High frequency methods include current based Physical optics (PO) and field based Geometric optics (GO). Other methods include Generalized Multipole Technique (GTM), Transmission line matrix method (TLM) and modal methods (MM) to name a few. The computational hierarchy of these methods is depicted in Figure.1.

CEM practitioners in the recent days are aiming to use existing software packages to solve a particular problem as early as possible. Present CEM researchers have limitation in learning the CEM methods, due to the readily available software packages. It is more important that experienced CEM researchers impart the knowledge and share the experience with young researchers. An attempt has been made in this paper to report on the growth and advancements in the field of computational electromagnetics.

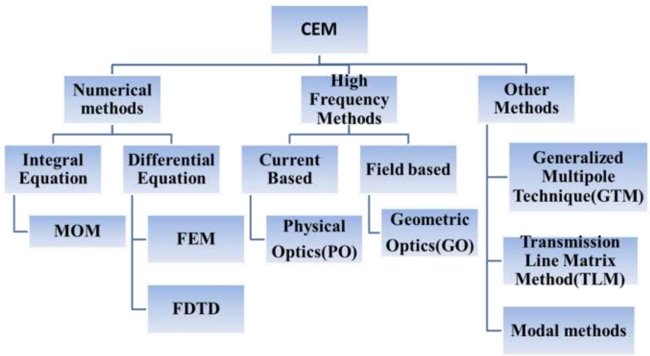

Figure 1: Computational hierarchy

2. Survey of CEM

The growing field of CEM research has sprouted various divisions of research. CEM research is carried out mainly in three ways viz. (i) analytical techniques, (ii) numerical analysis technique, (iii) expert systems. Analytical techniques make some assumptions (for example geometry like ground plane is considered as infinite ground plane) to get closed form solution (look up table). Analytical techniques are used not only in simple computer programs but also in elaborate IEMCAP (Intra system Electromagnetic Compatibility program) provided they have anticipation of EM interactions [2]. Analysis of numerical techniques plays a vital role in selecting suitable method for various geometries. Expert systems estimate values for the parameters of interest through their knowledge on EM interactions that cause EMI sources to radiate [3, 4]. Expert system is unsuitable for difficult EM problems. Prominent methods used in this domain of research are discussed to provide insight in to computational electromagnetics.

2.1. FDTD (Finite Difference Time Domain)

Yee et al defined FDTD scheme in the year 1966. He provided the solution to Maxwell's curl equation involving centred finite difference approximations to find partial space and time derivatives. Consider one component of Maxwell's equation:

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma_z E_z\right) \quad (1)$$

Using the central difference scheme, the above equation is discretized. FDTD algorithm comprises a grid of points containing computational domain, a stencil to approximate PDE, a boundary condition to approximate points on boundary of computational domain, an excitation source and solution method to solve PDE. Post processing is done to find physical quantities from field equations [5]. In 1980, Taflove created an acronym to refer to finite difference time domain schemes (FDTD).

The computational domain of structure under analysis is discretised using two techniques viz.(i) Bergner's PML (Perfectly Matched Layer) and (ii) Modified PML. The PML is illustrated as follows.

2.1.1. Perfectly Matched Layer (PML)

Mesh termination remains as problem in modelling the computational domain using FDTD. Reflection from boundary that arises due to coarse meshing affects accuracy of computation. Absorbing Boundary Conditions (ABC) demands larger computational domain to be meshed. Domain is large in ABC as they need adequate distance between radiating body and boundary. The ABC can achieve a return loss of -20 dB to -30 dB. However ABC need to battle to achieve return loss less than -50 dB [6] because ABC shows good performance in absorbing reflection only for normal angles and performance is poor for angles other than normal incidence [7]. This reflection is greatly reduced by making computational domain finite through domain truncation techniques. Truncation techniques experiences the truncation error in cases wherever they are not properly implemented, therefore accuracy of computed results is affected. Computational resource requirement such as time and memory for high order ABC is high as computational domain is larger. The computational effort can be reduced and accuracy is enhanced by introducing an absorbing layer called Perfectly Matched Layer (PML). Berenger [8] introduced this method for 2D cases in 1994 by placing boundary close to radiating body. The E or H field is split into two and different E and H loss is assigned to each field component [7]. FDTD update equations are used by PML to accommodate these split fields. The layout of Perfectly Matched Layer is depicted in Figure 2. X-PML is X – oriented PML and Y- PML is Y oriented PML and X-Y PML is X-Y oriented PML. σ_x, σ_y are conductivities in X and Y direction. Corner regions are handled well by overlapping X and Y PML resulting in X-Y PML. Absorption of reflection with PML is good at all frequencies and angle of incidence regardless of polarization of angle of incidence [7]. Perfect matching is achieved by PML using absorbing materials with electric and magnetic losses at termination of mesh. PML uses two approaches namely stretched approach (mathematical in nature) and anisotropic media with Uniaxial PML (popular method) [6]. PML Techniques allows Electromagnetic waves to be absorbed with minimal reflection and further this reflection magnitude is decreased by fine tuning of parameters like thickness of layer [8] thereby achieving return loss greater than -100 dB [6] better than anechoic chamber (-70 dB). The PML requires more computational domain and CPU time due to the splitting of the fields which makes it unreliable.

4 X-Y PML σ_x, $\sigma_y \neq 0$	3 Y PML $\sigma_x = 0$, $\sigma_y \neq 0$	2 X-Y PML σ_x, $\sigma_y \neq 0$
5 X PML $\sigma_x \neq 0$, $\sigma_y = 0$	Object for Computation	1 X PML $\sigma_x \neq 0$, $\sigma_y = 0$
6 X-Y PML σ_x, $\sigma_y \neq 0$	7 Y PML $\sigma_x = 0$, $\sigma_y \neq 0$	8 X-Y PML σ_x, $\sigma_y \neq 0$

Perfect Conductor

Figure 2: Layout of Perfectly Matched Layer

Further in FDTD discretization of computational domain poses error due to truncation. Staircase approximation also contributes error accumulation at each time step for surface with sharp and fine edges [9]. Also reflection error increases with increase in time step affecting accuracy of method.

The reflection error could be reduced by placing non-resonant absorber adjacent resonating object (i.e.) Resonating Perfectly Matched Layer (RPML).

Periodic structures could be analysed using this technique due to its capability, reduced computation domain of repetitive structures up to 70% for 1D objects and 90% for 2D objects [10]. Also, this method has good convergence. Courant, Friedrichs, Lewy discovered and named it as CFL limit in order to ease the solution of second order wave equation. Desired accuracy is obtained when time step is within CFL limit making the system unconditionally stable at the expense of computational time. The FDTD is solved by Zheng Yu Huang in [11] by a new unconditionally stable method using Associate Hermite (AH) function. In [11] Orthonormal basis function is used to expand the E field and time derivatives. Time variables associated with it is removed by Galerkin testing procedure. Here in [11] the CPU time is reduced by 0.59% than traditional FDTD without any impact on the accuracy at the cost of memory consumption. Spheroidal FDTD uses CFL limit and field formulas to deal with singularity at the center and edges of spherical cavities and patch antennas with additional computational cost than conventional FDTD [12].

An efficient 3D FDTD gives solution to Maxwell's equation using two different time step increments. Applications like space weather effects, satellite communications operating at high altitudes, high collisional regimes are analysed by this method. Easy implementation, less memory and time makes this method more attractive than anisotropic approach. However, stability is largely affected for strong electric field as error increases with increase in time step [13].

Semi Implicit Schemes (SIS) depend on current step alone, over ruling the dependence of CFL limit for applications involving larger time step. This method computes E and H field at each step, forcing the time step be within CFL limit. For computational domain problems requiring larger time step, this method is an ideal choice. This method is used when time step (> 10 times) is larger than FDTD stability limit at the cost of additional memory consumption. Memory storage and computational cost is proportional to the electrical size of geometry and grid resolution. Electrical properties of the scatterers are varied at each step by varying the values of μ, σ, and ϵ assigned to each field component.
SIS performs certain modification to CPML-FDTD enabling implementation of CFS-PML to resolve synchronisation issues in CPML-FDTD, RIPML that affects accuracy in cases like dispersive media. Further this CFS-PML requires less computation and memory in treating unbounded 2D region and thin bounded PEC [14].

An algorithm called sub-cell algorithm is used to model flat electrode using coarse grids of FDTD preserves 12% computational memory and 3% of CPU time usage than traditional FDTD without compromising efficiency [15]. Long and short apertures without depth or with finite depth could be modelled using uniform two step method in FDTD analysis with good accuracy, high resolution in [16]. The two step uniform method is processed in steps like: (i) estimate aperture-field singularity using standard FDTD simulation at the edge of the receiver to yield aperture coefficients (ii) coefficients thus obtained are used in contour path to define FDTD update equations for fields near aperture. The challenges faced by this method are computational memory and accuracy.

Finite Difference Time Domain – Alternating Direction Implicit Method (FDTD-ADI) discussed in [17] is an unconditionally stable method that operates in one step leap frog fashion to solve open region (isotropic lossless) problems using Sherman Morrison formula to solve tridiagonal equations efficiently. Also this method is efficient in terms of computation time, memory and similar accuracy in comparison with two step schemes. The main issues associated with this method are increase in error with increase in time.

Locally One-Dimensional-Finite-Difference Time-Domain Method (LOD-FDTD) is found to be faster than parallel (FDTD-ADI) method, where parallelization is achieved using message passing interface. Debye-dispersive media, complex bio-electromagnetic problems like deep brain stimulation could be handled with good scalability and performance. LOD-FDTD achieves lower communications between cores (up to 40 cores) than FDTD-ADI, however suffers efficiency issues if number of cores goes beyond 40 cores [18]. This method uses less CPU time at the cost of memory consumption.

In order to improve performance of conventional FDTD in terms of CPU time and memory, Zhi-Hong proposed in [19] a DD (Domain Decomposition)-Laguerre-FDTD method to solve very large rough surface, PEC, lossy dielectric media, large scatterers. The computational domain is discretised into sub domains, radiated and scattered fields. DD-FDTD can model vast rough region than conventional FDTD. Characteristic basis functions (CBFs) are employed to mitigate interpolation errors between the boundaries thereby increasing accuracy of this method.

Min Zhu proposed in [20] a Novel RK-HO-FDTD (Runge Kutta Higher Order FDTD) used in computation of EM regions which is in terms of accuracy, speed, convergence and has reduced dispersion. This method combines SSP-RK and HO-FDTD. RK-HO-FDTD is pretty much attractive than HO-FDTD, MRTD and RK-MRTD methods due to its better scattering properties and quick convergence.

2.2. FEM (Finite Element Method)

FEM code meshes computational domain problem into small portions and forms linear equations using weighted residual method and solves the same by reducing the energy of geometry viz. inhomogeneous material resonant cavities. Thin wires, large radiation problem like Eigen value problem and 3D problems are difficult to be modelled using

FEM due to its unstructured mesh. FEM for 3D EM problems faces certain issues than 2D problems like need for excessive computation and vector parasites that results in false solutions. FEM is used in modelling Yagi-Uda antennas, horn antennas, waveguides, vehicular and conformal antennas [21].

Bangda Zhou proposed Direct FE solver [22] that uses the following algorithm: (i) Dividing of unknown nested dissection (ii) Constructing elimination tree, (iii) Symbolic factorization (iv) H matrix creation for frontal matrix for every node of elimination tree (v) H matrix based algorithm to perform numerical factorization to arrive at the solution and (vi) post processing. Although this method is far better than the available commercial solvers for applications like 3D structures, Patch antennas, it is not ideal in terms of CPU time and memory storage.

Dual Prime (FETI-DP) method in [23] incorporates Hierarchical-Lower Upper (H-LU) and Nested Dissection (ND) is a domain decomposition method. This solver with sub-domain finite element systems has faster convergence and numerical scalability without sacrificing efficiency. The applications of this method include 3D structure problem, periodic array problem, Jerusalem type array, Vivaldi array. These applications use LU for maintaining computation resources and accuracy while ND conforms less memory and CPU time. Though this method facilitate parameter choice based on applications, accuracy and computational cost for unconditionally stable system has a trade-off [24].

Ivan Voznyuk [25] discussed Finite - Element Tearing and interconnecting Full-Dual-Primal (FETI-FDP2) (FETI-FDP2) method an extended version of FETI-DPEM2 is used in analysis of 3D large-scale electromagnetic problems using robin type boundary condition at corners and interface. Electrically small problems are solved by using Krylov solver and ASP/AMG(Auxiallary Space Preconditioners / Algebraic Multigrid) preconditioners as they facilitate convergence with lesser iterations. However the implementation could be extended not only to geometry that is tedious to solve but also to electrically large problems using DDM [26]. The incapability of iterative solver to converge due to presence of PML is over ruled by Dirichlet-to-Neumann (DtN) approximation [25]. Further, algorithm's parallelization improves convergence. The use of Robin type boundary condition at interface cause the Interface issue, that is solved by an effective code based on mesh partitioning without affecting accuracy. This method requires performance enhancement in terms of memory and CPU time and is not suitable for hierarchical elements that are higher in order.

Zhi-Qing Lü [27] proposed Non-Conforming FETI (NC-FETI). Few steps involves are (i) Computational domain is discretised to subdomain (ii) Data transfer between subdomains is done by imposing Robin type boundary condition (iii) Lagrange multiplier scheme and Schur complement approach are used to solve interface issue and (iv) Iterative algorithm is to find unknown electric field. NC-FETI is far better than DDM and is capable of modeling 3D large-scale slot array, complicated electromagnetic problems such as photonic band gap and antenna arrays with efficiency and accuracy at the cost of memory. Memory consumption can be greatly reduced for periodic structures due to their repetitive nature [27].

Jian Guan proposed an accurate and efficient Finite Element-Boundary Integral-Multilevel Fast Multipole Algorithm method (FE-BI-MLFMA) [28]. The formulation of FE-BI is first approximated by FEM with absorbing boundary condition. Further it is solved using FETI with numerical complexity and considerable scalability. The efficiency of this method is increased by Graphics Processing Unit (GPU) accelerated MLFMA. Testing schemes involved in this method are enhancing accuracy and ABC preconditioner improves speed by faster convergence of iterative solution. However ABC preconditioners like algebraic preconditioners demands bulky factorization which is solved by DDM. In spite of parallelization difficulty, this method is suitable in real applications like biomedical application involving human body, RCS, space applications involving missiles, antenna arrays due to its accuracy and efficiency.

Ming Lin in [29] suggested that Domain Decomposition based Preconditioner - Finite Element-Boundary Integral-Multilevel Fast Multipole Algorithm (DDP-FE-BI-MLFMA) is best for large lossy objects; it finds difficulty in dealing with 3D objects that are lossless with high μ_r and ε_r. CFL limit is having an important role in unconditional stability. Also semi implicit schemes could be used to attain memory and CPU time and cost reduction. GPU with Compute Unified Device Architecture (CUDA) quickens solution to mitigate parallelization blocks.

GPU (Graphics Processing Unit) accelerated parallelization is done using ILU (Incomplete LU) preconditioner, SSOR preconditioner, Pardiso, MKL solver provided DDA speeds up large scale problem [30, 31]. This method holds good for finite element structures and single array with large structures due to the increased speed within less time, but complicated antenna and microwave devices are having difficulty in convergence.

2.3. MOM (Method of Moments)

Antenna structure is analyzed using MOM by splitting the computational area into various segments viz. meshing and evaluating each segment using basis functions. The selection of basis functions plays a vital role in arriving at the solution with accuracy, efficiency. Current distribution is decomposed with this basis function. Green's functions help in studying current on each segment and strength of each moment. Accuracy of calculating integral of Green's function is enhanced by employing singularity extraction. In addition with accuracy in calculating self-interaction and near interaction matrix, the problem converges to finite solution with less number of iterations using singularity extraction. MOM discretization leads to very large and dense matrix, consumes more time making it not suited for large problems, this could be solved by using preconditioners and iterative methods [32]. MOM is capable of modeling thin wire structures with speed, accuracy, convergence and versatility using triangular basis functions.

MOM, a projection based process that solves linear equations to find unknown current distribution requires the solution to be well conditioned and error controllable. To attain these targets we emphasis on an equation to have single solution conforming to discretization and well-conditioned MOM matrix. However various issues associated with MOM like singularity, low-frequency breakdown and charge cancellation, non-smooth surfaces and physical resonances, composite structures, inhomogeneous and anisotropic medium, multi-scale problems poses great difficulty in realizing these targets. Low-frequency breakdown and charge cancellation related problems are rectified by splitting the problem potentially and by the use of preconditioners [33].

Charge cancellation is restricted by derivative recovery of charges based on RWG basis functions proposed by Willem J. Strydom [34] uses Galerkin testing procedure to maintain current continuity between subdomains and nodal ZZ (Zhimin Zhang) patch to calculate charge density on various non-overlapping RWG basis functions.

The performance of MOM in computational electromagnetics is based on its speed, accuracy and memory occupancy. Accurate error controllable MOM could be achieved by suppressing error measures from current, boundary conditions and scattering amplitudes. Testing and basis functions contribute to error. Also MOM is a projection based method where projection error acts as reference error [35]. The Pocklington integral equation (PIE) using pulse basis and weight functions fail to address singularity. However error controllable and good conditioned MOM elliptic formulation finds unknown current with less number of basis functions. Low-Frequency Fast Multipole Method Based on Multiple-Precision Arithmetic method (FMM-MPA) discussed in [36] produces consistent field values compared to the MOM with lesser relative error.

Multiresolution approach enhances spectrum of MOM matrix. This enhancement is achieved by using regularizing diagonal basis functions obtained by using near field part of MOM in standard basis functions that has low computational cost and memory occupation [37]. Electrically large structures like Vivaldi array [4, 8 element array] are analyzed and numerical results shows that MLFMA, acceleration of MOM is better than other methods (FDTD, FEM, conventional MOM) when electrical size of structure increases [38]. For Electrically small problems in which preconditioning could not be achieved, near field and self-interactions matrix must be focused. Multisolver DDM method is used in air platform modeling where touching sub regions is inevitable. MS-DDM method uses different techniques for different subregions to achieve antenna isolation in the air platform. An example discussed in [39] shows EMC interaction effects on air platform using MS-DDM at 10 GHz uses DD-FEM-BEM for region 1 (overview) and FEM-DDM for region 2 (mid plane). MS-DDM solves ill-conditioned MOM matrix with non-smooth edges, tips and multiscale structures.

Geometrical continuity and current continuity in PEC scatterers is maintained by using Non-Uniform Rational B-Spline (NURBS) and approximate functions respectively developed by modified GMM. Contrasting DDM, GMM unites different basis and geometry descriptions to form MOM and solves it after breaking it in to sub problem [40]. Preconditioners discussed in [33] remove ill conditioning effects and fasten convergence to a finite solution with less number of iterations. This operates in two phases (i) generation of Z by filtering the NFI matrix and (ii) approximating the inverse of Z. Former operation is widely used due to its capability to handle non uniform meshing [41]. Many preconditioners like block preconditioners, Calderon preconditioner, Algebraic preconditioners, MR Preconditioners, Block diagonal preconditioners, incomplete LU, Sparse Approximate Inverse (SAI) preconditioners mitigate ill conditioning in various scenarios [40].

Block diagonal preconditioners finds its use in preconditioning MLFMA accelerated form of MOM using new basis function that arises by linear combination of subdomain and entire domain basis functions. Calderon preconditioners are used in both open and closed structures. This preconditioner promotes convergences without affecting discretization density [42]. Calderon preconditioner utilized in antenna array treat ill condition in open problems employing testing basis function for faster convergence of solution. MR preconditioners along with diagonal preconditioner handles ill conditioning in low frequency MOM employing incremental multilevel filling and sparsification thereby enhancing memory requirements, CPU time and accuracy than standard MOM, MLFMM, ACA [43].

Preconditioning with overlapping triangular basis functions, PMCHWT (Poggio, Miller, Chang, Harrington, Wu, Tsai) formulation can address scattering that occurs from penetrable bodies like Yagi – Uda nano antennas [44]. The MOM technique is used in modeling conformal antenna structures. Besides this conformal microstrip patch antenna, windscreen antennas are also modeled using MOM method [21].

2.4. FDFD (Finite Difference Frequency Domain)

FDFD is similar to FDTD and related to FEM in the method of arriving at solutions. FDFD is obtained from finite difference approximation of time harmonic Maxwell's curl equations to find partial space and time derivatives. This method is not a time stepping procedure, so the FDFD meshes are similar to FEM meshes. Moreover FDFD creates linear equations that produce sparse matrix like MOM and FEM to compensate time stepping procedure. Although FDFD is on par with FEM, it has not drawn considerable attention like the later owing to fast growth of FEM in mechanics. FDFD finds its application in 2D Eigen formulation and scattering problems at optical frequencies

2.5. TLM (Transmission Line Matrix Method)

Although TLM method has similarities to the FDTD, it is unique. Modeling nonlinear materials and absorbing boundary conditions is simple in TLM. In this method E-field and H-field grids are interleaved as single grid and the nodes of this grid are connected by virtual transmission lines. At each time step excitations at source nodes are transmitted to adjacent nodes through these transmission lines. Requirement of excessive computation time for large problem and increased memory per node than FDTD makes this method less appealing. As they are similar to FDTD, designer can chose between the two, based on their applications. The applications of TLM method include inhomogeneous media, lossy media, and nonlinear device.

2.6. GMT (Generalized Multipole Techniques)

GMT is based on method of weighted residuals similar to MOM. Field based GMT is advantageous over current based MOM due to the fact that it is not necessary to do further computation to obtain fields. However MOM yields field only after integrating the charge over the surface. GMT finds its applications in waveguides and thin-wire modeling.

2.7. CGM (Conjugate Gradient Method)

CGM is a frequency domain method that differs from MOM in two aspects viz. the way the weighted residual functions are employed and by the method of solving linear equations. Hilbert inner product is used for inner product of weighing functions instead of symmetric products used in MOM. Sparse matrix is solved by iterative solution procedure instead of Gaussian Jacobian method used in case of MOM.

2.8. AEM (Asymptotic - Expansion Methods)

AEM method includes High frequency CEM [45] and Low Frequency CEM (Quasistatic approximation) reviewed in [46]. The various high frequency techniques like GO, UTD and PO are discussed in [45].

2.8.1 UTD (Uniform Theory of Diffraction (UTD)

UTD falls under the category of High frequency methods. This is an extension of GTD. This method produces more accurate results relative to the field wavelength. Fields at the point of excitation where wavelength is zero can be evaluated using geometric optics. The effects of diffraction are included in UTD and GTD [46]. Small and complex geometries that require accurate surface and wire currents could not be modeled using GMT and UTD codes. The UTD is used in the applications involving modern naval vessel.

2.8.2 PO (Physical Optics)

Physical optics is a current based method used to find current density induced on a surface. This finds application in large scatterers. This method could be used for smooth and curved surface since they ignore edge diffractions. However, integration with reflector is complicated and time consuming. In ray launchers contribution of rays in finding fields is done by using PO integration [47]. In some cases that demand efficiency and accuracy PO could not be used. High frequency (HF) methods like GO-PO combination are extensively used in electrically large objects for spline approximations prediction and coarse oceanic surface [48]. NSDP (Numerical Steepest Descent Path) method is a used widely to find solution for PO scattered field [49]. High-resolution RCS matrices, generated using Physical Optics (PO) were used in an investigation of RCS matrix resolution. In [50] accuracy dependent problems use RCS interpolation that is obtained from spline approximations. Computational cost dependent PO Problems use bilinear interpolation with compromise in accuracy. Multilevel method based on PO in [51] is used in analysis of near field single bounce back scattering. Computational time is minimized by data from far fields and areas around near field. Further Near field (NF) scattering analysis makes it viable for realistic applications with less CPU timers.

2.8.3 GO (Geometric Optics):

This is a field based method in which equivalent currents on geometric plane is set up using ray tracing. Integration of GO with Aperture Integration (AI) is performed with ease. This method is advantageous as it ignores edge diffractions. The applications like ray-launching uses Multi-core CPUs or cluster computers to solve ray launching problems using geometric optics. The entire process of RL-GO is controlled by angular spacing or transverse spacing. GO meshes the region in same way as MOM and PO. However, GO triangles are larger than for MOM as mesh is implemented in surface geometry thereby reducing mesh storage efficiently. Finite dielectric materials, dielectric coatings of metallic surfaces, anisotropic materials are modeled using GO.

2.9. EEM (Eigen Expansion Method)

A challenge is posed on CEM to provide efficient solutions to Generalized Eigenvalue Problems (GEP) in order to aid CM analysis. The computational time and memory requirement for matrix-vector product is greatly reduced with the use of Multilevel Fast Multipole Algorithm (MLFMA). MLFMA could be combined into the implicit restarted Arnoldi (IRA) method for the estimation of Characteristic modes. MLFMA is combined with the sparse approximate inverse (SAI) to accelerate the creation of Arnoldi vectors.

Large-scale and complicated 3D objects with restricted computational resources can be modeled by this method for CM analysis [52]. Developing MLFMA based CM analysis is a challenge to conventional MOM approach. MLFMA can be implemented in parallel processing using iterative Eigen

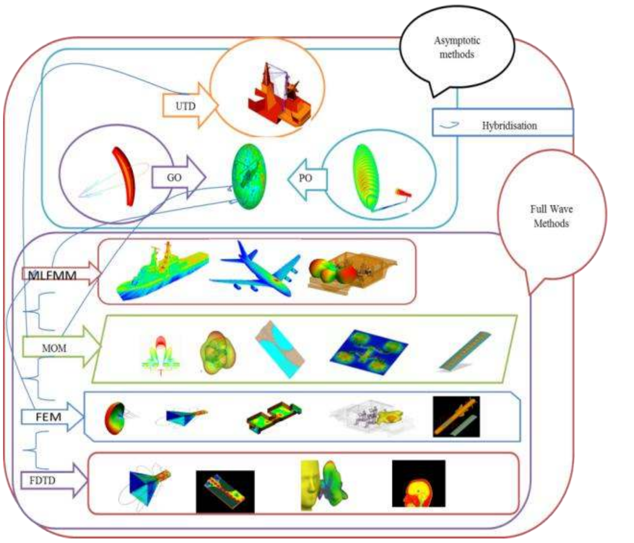

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Figure : 3. Applications of computational Electromagnetics

solvers, yield desired performance in CM analysis. The applications of various computational electromagnetic techniques are depicted in Figure: 3.

The commercial codes like HFSS®, CST®, FEKO®, XFDTD®, Empire®, SEMCAD® uses differential methods like finite difference method and finite element method. FEKO, WIPL-D® (IE3D, Sonnet, Designer etc. – 2D) are using integral methods like method of moments.

The Survey of CEM method is compared in Table 1: Survey of these methods based on issues associated with each method like MOM, FEM, FDTD and Hybridization of Techniques is discussed. Correspondingly solutions to these issues are also addressed in this Table.

3. Meshless and Meshfree methods

Mesh less method are used in arriving at numerical solution for problems where meshes are not necessary however they use mesh only once in final stage to solve linear equations. In order to remove meshes, we require designing of fitting policies for scattered data in multi-dimensional spaces that results in definition of mesh less shape functions [53]. Point interpolation method, radial point interpolation method and Shepard approximation are few natural neighboring methods used to define shape function. The Shepard approximation help in achieving good accuracy and considerable computing time [54]. These problems are solved by expanding unknown field variable over such shape function and also reduce number of unknowns. This method has few drawbacks in terms of accuracy and computational time. Sharp discontinuity and difficult simulation are dealt in mesh less method. The various advantages of mesh less method are increased support domain, change in basis functions, adopting nodal densities. The meshless methods like Meshless Local Petrov- Galerkin (MLPG) , Local Boundary Integral Equation (LBIE) and Meshless Local Boundary Equation (MLBE) are used in the analysis of microstrip antennas [53].

Table 1: Survey on CEM methods

Categories	Issues	Solution
MOM Based Techniques	Singularity, Low frequency break down, Charge cancellation	Singularity extraction, MOM elliptic formulation, Preconditioners, Charge recovery
FEM based Techniques	Internal interface issue, Spurious solution, Difficult in solving electrically large problem	HLU, DDA-ABC preconditioners. DDA- parallelization.
FDTD based Techniques	Truncation error at each step, synchronization issues, interpolation error	CFL limit- step size small, Semi implicit schemes- step size large, RK-HO-FDTD.
Hybridization of Techniques	Ill conditioning effects Multiscale problems	DDA-ABC preconditioner MLFMA with some FFT-based algorithm

All these methods decrease computational time by changing domain integrals to boundary integrals. Amongst these three methods it is expected that MLBE is fastest. The comparison of all these methods for thin microstrip with coax fed and line fed shows same convergence rate. LBIE has least condition number and MLBE needs low CPU time.

Mesh free method usually bypasses mesh generation. Mesh free methods are used in solving computationally difficult problems at the cost of computational time and programming effort. The mesh free method has several benefits over FEM and FDM such as Overlapping domain gives much support and gives good approximation.

4. GPU acceleration

In spite of development in GPGPU computing and speeding of parts of programs or using easy problems many factors that make problem robust are considered before implementing this technique to commercial software. The expected solution needs increased computational power and therefore time required for solution is reduced. Successful GPU implementation needs code implementation and optimization depending on problem time regardless of whether it is my memory bound or compound bound. GPGPU has both hardware capability like c/c++ and software programming. This method is used in FEKO and choice of solution depends on electrical size of problem and intricacy of materials that are simulated. The Challenges in GPU acceleration are versatility, reliability and reproducibility, variety of CEM methods and software and design decisions. Multi GPU and heterogeneous system increase computational performance are discussed in [55].

Challenges faced by GPU acceleration are discussed in [56].

1. **Flexibility, consistency and reproducibility**

   Commercial software settings demands accurate results without exceeding allocated memory for GPU acceleration. They require additional computational resources when they run out of GPU memory, in such cases the switching algorithms transfer control to CPU.
2. **Wide variety of computational electromagnetics methods**

   GPU faces several challenges in parallelization techniques like MPI, Open MP and GPU computation. GPU acceleration has similar approaches for MOM and FEM, In spite of their differences (Linear system is dense matrix for MOM and sparse for FEM). However realized speed will differ for dense and sparse GPU computation.
3. **Software used and design decisions**

   GPU acceleration on commercial CEM software are based on decisions such as language for software implementation and low-level program flow such that it maps well to GPU architecture

Open CL, Open MP and CUDA are few programming languages preferred and partially used in commercial soft wares like FEKO. GPU programming is C/C++ based and acceleration of this is FORTRAN based. Any modification of codes may result in bugs; further introduce need for testing, tuning and software verification

GPU processing for MOM matrix experiences run time that is quadratic and solution that is cubic. In case of larger problem matrix solution will dominate the run time. Speed up for MOM code needs resources to be invested due to complex nature of code.

In case of FEM GPU acceleration must be overlooked. The total simulation time of FEM include construction of preconditioner and solution to sparse matrix. Use of right preconditioner decreases the solution time in comparison to direct sparse solvers. Single GPU is used for small problems and GPU cluster is used for large problem. Iterative solvers show performance improvement in cases where GPU memory is a limitation and they provide a speed up of 50%.

FDTD based GPU acceleration is no hybridization is involved so computational resources are reduced. Optimization of GPU based FDTD is done by exploiting shared memory, achieves global memory coalesced accesses, employing texture caches, use of build in arrays, properly arrange computation in third domain. GPU provides speed up of ten times compared to CPU. Parallelization of 75% is achieved by overlapping computation and communication. Ray Launching-Geometric optics used UTD, PO, RL-GO collectively resulting in Shooting and Bouncing Ray (SBR) for dealing with large objects and is well suited for GPU acceleration. CUDA code is run through GPU compiler to obtain accelerated GPU implementation. Stack size must be addressed well to deal with GPU limitation in terms of difficulty and recursive code.

Further acceleration of these methods is done by using numerical algorithms through hybridization for all methods except FDTD.

5. Hybridization

MOM faces difficulty in modeling inhomogeneous, interior of conducting enclosures and dielectrics with non-linearity. FEM is not suited for efficient modeling of thin wires, large radiation problems and Eigen value problem due to its unstructured mesh. FDTD is difficult in modeling structure with sharp edges. GTM and UTD techniques are not suitable for problems that need accurate measurement of surface currents. It is obvious from the above discussion that none of the numerical techniques can solve all EM problems. All these techniques fail to cater the needs of printed radiation models that have all these structures. The most appropriate solution found by the researchers is to club two or three techniques and produce one code i.e. Hybridization code. Hybridization techniques involve combing two or more techniques into a single code. Various hybridization techniques are discussed in the forthcoming section.

5.1. Hybrid MOM - FDTD

FDTD method accomplishes propagation simulations excellently but not suited for modeling complex metallic structures like antennas. On the other hand the Method of Moments (MOM) is ideal for modeling complex metallic structures and is not very well suited for penetrating into such structures, hybrid MOM /FDTD method is used in application that require penetration into these structures eg. human tissue [57]. Antenna and scattering problems could be resolved using hybrid MOM and FDTD based on IMR (Iterative Multi-Region) technique [58]. IMR divides the domain problem into separate sub regions. In a problem with thin wire and scatterer MOM is used to solve thin wire antenna while the other region can be solved using FDTD solutions. Iterative algorithm helps in achieving the solution for combined sub regions. Radiated fields arising from MOM region due to current distribution on the antenna helps in interaction of two sub regions. Since the FDTD is a time domain solver, fields emanating from the MOM region that excite the FDTD region needs to be changed into time domain waveforms. This method helps in achieving reduction in the memory storage requirements and computation time.

Hybrid MOM-FDTD method employing the Asymptotic Waveform Evaluation (AWE) technique [59] involves swapping back and forth information between the MOM (DFT) and FDTD (IDFT). The AWE technique in the MOM domain is implemented to reduce the computational time needed for wide-band analysis. Frequency hopping technique is suggested for choice of expansion points in AWE technique. The computational time is reduced from 3 hours 18 min to 1 hour 4 min by using AWE technique.

5.2. Hybrid FDTD - PO

Radiating planar antennas in the existence of large conductive structures are analyzed using Hybrid FDTD and PO [60]. Surface equivalence theorem is used to combine FDTD and PO and spatial interpolation technique is used to enhance computational efficiency of the proposed approach. The idea of using this technique is to calculate samples of the electric and magnetic fields on a comparatively coarse spatial grid over the Huygens surface and then to use interpolation for obtaining field values on the required fine grid. FDTD regions enclose the antennas and inhomogeneous dielectric objects surrounded by Huygens Surface. Conducting bodies in frequency domain are analyzed using PO. Memory storage and CPU time are saved by using this technique.

5.3. Hybrid FEM - FDTD

Hybrid FEM - FDTD is aids to present modified equivalent surface current [61] by means of equivalent principle theorem to extend the field transformation. FDTD gained popularity due to its simplicity and efficiency. However compromise is made in terms of accuracy. FEM permits good estimates of complicated boundaries and with edge elements it performs well for Maxwell's equations but needs further memory hybrid that applies FDTD in large volumes. FEM is difficult to be applied for problems with large dimensions but FDTD can handle this even for penetrable structures. 70% of the required memory locations of the field points between the two domains are saved along with increase in speed for updating boundary equations inside the FDTD method [62].

5.4. Hybrid MOM - PO

MOM could be hybridized with FEM, FDTD, TLM, AEM, and PO. In case of MOM - PO hybrid MOM method is used in small, resonating structures near edge while PO is used for large and smooth regions. MOM /PO hybrid is preferred for modeling large reflector antenna instead of MLFMM. Reflector is modeled using PO and feed of the reflector is accurately modeled using MOM.

5.5. Hybrid FEM - MOM

In spite of the advantage of FEM like mesh adaption and mesh refinement that improves accuracy, there is disadvantage in this technique in case of mesh termination. Mesh termination issue arises when radiation condition are enforced in open region. Various methodologies have been used for mesh termination but FEM-MOM hybrid is the best method. FEM uses curl and field based Whitney element while MOM uses divergence and current based RWG element.

5.6. Hybrid FDTD with two level atomic systems

Hybrid of atomic systems emerged from coupling rate equations and FDTD. EM fields and atoms population density are updated and leads to the significant reduction of computational effort for the analysis of absorbing materials [63]. Short FDTD simulations allow updating the population density of the system at a much longer time scale than a single cycle of the wave

5.7. Hybrid FDTD with DGF formulation

Hybridization of FDTD method with formulation of DGF (Discrete Green Function) [64, 65] has limited utility due to computation overhead. It anticipates new fast methods for DGF generation. However this hybrid formulation finds its application in antenna and disjoints domains. The antenna is modeled by DGF formulation of FDTD, while scatterer is modeled with FDTD. Implementation of this method in parallel processing using iterative Eigen solvers yield desired performance in CM analysis.

5.8. Advanced Hybridization Schemes

FE-BI-MLFMA which is derived from FE-BI uses absorbing boundary condition to do first approximation by FEM. The solution based on FETI faces issues in terms of numerical scalability and computational difficulty. Although, preconditioners are utilized to reduce computational time enabling fast convergence towards desired solution with less number of iterations, algebraic preconditioners need complex factorization making them unsuitable in this regard. However DDA-ABC preconditioner employing iterative solver using DDM satisfies the need. This method finds its application in antenna arrays and large lossless objects. Computational time can be reduced by incorporating ID algorithm and the solution is obtained by combining direct solvers with iterative solvers. The proposed method with ID exhibits more accurate, efficient and robust result. Further it decreases the peak memory requirement while it maintains the number of the final skeleton directions the same as or less is applicable in 3-D composite objects [66].

Multiscale problems are analyzed using various hybrid techniques like MLFMA, LFFIPWA effective in solving sub wavelength breakdown with high accuracy and low efficiency compromising the speed. MLIPFFT-MLFMA (Multilevel Interpolatory Fast Fourier Transform –MLFMA) is a broadband method with higher efficiency. ID-MLFMA, MLFMA-ACA and MLFMA with some Fast Fourier Transform based method solves multiscale problem in large structures. On the other hand in terms of memory storage and computing resources ID-MLFMA and MLFMA-ACA techniques are capable of treating multiscale problem [67].
MLFMA-PO is a hybrid technique used in scenarios where geometry with fine details needs to be modeled with

accuracy and efficiency. MLFMA is an acceleration of MOM that is responsible for modeling finite parts with accuracy. PO can model large structures with efficiency. Hybrid MLFMA-PO can be used in applications involving large scattering and large radiation problems like certain critical parts of antenna [68].

Parallelization improve the speed and efficiency of a process, also this is implemented in hybrid parallel open MP -VALU (Vector Arithmetic Logic Unit) MLFMA to achieve the same. In contrast to GPU acceleration the above cited technique does not demand extra device due to the presence of essential VALU inside CPU. The hybrid SPMD-SIMD parallel scheme is not supported by hybrid parallel Open MP-VALU MLFMA method [69]. Hybrid MPI and Open MP parallel programming technique is one such technique where in BOR-MOD-CFIE code is subjected to parallelization to enhance performance of MOD method in terms of memory, accuracy and CPU time [70].

MLFMA with PO to form hybrid finds its application in analyzing scattering and radiation problems for electrically large structures. The proposed hybrid uses MLFMA, accelerated version of MOM for analyzing certain structures where complex linear equation are solved by iterative solvers explicitly and PO, on the other hand to enhance the accuracy dealing with scattered fields. The application involving fast RCS prediction of electrically large target are accomplished using hybridization of GO/PO accelerated by open graphics library enabling accuracy in capturing creeping effect and wave diffraction effects [71].

High frequency approximation method called shooting and bouncing ray is used in applications involving PEC and dielectric using CUDA. This method uses physical approximation, current based method in SBR owing to its capability to penetrating into objects for applications like RADAR and ISAR [72]

Juan Chen [73] proposed a hybridization method based on WCS-PSTD (Weakly Conditionally Stable – Pseudo Spectral Time Domain).This method is formulated to deal with electrically large object with continuity between subdomains implies less memory and less computational time. For photonic crystal this method is better than FDTD. However, accuracy of the proposed method that largely depends on step size and is not applicable to surfaces with discontinuity.

Time domain application of Hybridization like DGTD and TDBI incorporates calculation of truncated boundary while truncation is brought about by ABC or EAC, PML. Flux produced by truncated boundary is responsible for communication between the domains and determines shape of scatterer by imposing physical radiation. DGTD truncated by EAC makes radiation condition elastic than all DGTD systems to address EM scattering problems [74].

TABLE 2: Techniques for performance up gradation of CEM methods

Category	**Techniques for performance upgradation of CEM**			
	MOM®	**FEM®**	**FDTD®**	**Hybridization ®**
Accuracy	Singularity extraction, MOM elliptic formulation	1.H-LU - computation cost and accuracy	CFL limit- step size small Semi implicit schemes- step size large RK-HO-FDTD.	Hybrid techniques accelerated by open graphics library enabling accuracy DDP-FE-BI-MLFMA
Memory	Incremental multilevel filling and sparsification	Nested Dissection- DDM. Semi Implicit Schemes . -	Sub cell algorithm, RK-HO-FDTD.	ID-MLFMA and MLFMA-ACA, Open MP parallel programming
CPU time	Singularity extraction, Parallelization	DDA-parallelization-increases speed	ADI-FDTD, LOD-FDTD – parallelization, Sub cell algorithm, RK-HO-FDTD.	Hybrid MPI and Open MP parallel programming technique, WCS-PSTD, Preconditioner, ID algorithm. AWE technique in MOM-FDTD hybrid reduce resource

The strengths and weakness of different CEM techniques were discussed. In summary, the following were observed: FDTD is a time stepping phenomena used in the analysis of domains like Debye dispersive media. Discretization of this media imposes error due to truncation, thereby increasing dependency of accuracy to Courant, Friedrichs, Lewy (CFL) limit [8, 10].

This dependency is mitigated by using Semi implicit schemes (SIS) [12]. Novel Runge-Kutta algorithm makes FDTD stable, dispersive and convergent [18]. FEM schemes faced problems in modeling 3D objects, Eigen value problem and thin wires. Domain decomposition method (DDM) based FETI-DPEM solves FE system and makes good in terms of convergence and scalability with aid of direct FEM solvers. Nested Dissection assures less memory and CPU time, while H-LU provides accuracy and fewer resources when used along with FEM. Table 2 depicts techniques for performance upgradation of CEM methods.

Iterative solvers face difficulty in convergence due to existence of PML. However DtN approximation, preconditioners promotes convergence [25]. DDM makes it applicable to large domain problems. MLFMA acceleration incorporates accuracy as depicted DDP-FE-BI-MLFMA technique, is suitable for all domains [29]. MOM needs the domain to be well conditioned and error controllable. Ill conditioning effects could be reduced by potentially splitting the domain or by the use of preconditioners. Error could be controlled by using semi implicit schemes and formulation like MOM elliptic approach [36]. Various hybrid combinations and advanced hybridization like FE-BI-MLFMA with limited scalability and computational complications [28,29], ID-MLFMA [67], MLFMA-ACA that treat multiscale problems [67], Open MP VALU[69], MLFMA, WCS-PSTD[73], DGTD–TDBI with truncated by EAC shows better performance in terms of accuracy, convergence and CPU time [74]. Most widely used softwares are CST®[75] based on Finite Integration Technique, HFSS® [76] based on FEM and FEKO ® [77] based on MOM and FEM.

6. Future of CEM

CEM Research has wide scope of evolution in next few decades. A few techniques have been incorporated to improve the performance of this stream of research. The hybrid GPU–CPU parallelization may acquire considerable attention in few years. The computational time is expected to be reduced by implementing fast convergence methods and implementing efficient preconditioners. Multi scaling methods that are fast and accurate use efficient DDM methods like discontinuous Galerkin methods and generalized transition matrix. Methods with higher order modeling capabilities with high order basis function, characteristic basis functions are expected to have considerable growth. Novel integral and differential methods can be used for several realistic applications and these applications may also require choice of different material in cases like nonlinear sensing, Spectroscopy, frequency generation. The collaboration of CEM with other field of research like Multiphysics aids in dealing with realistic applications and has potential to grow tremendously.

7. Conclusion

A review on different computational electromagnetics shows that these methods are application specific. Selecting a correct method that will afford fast and accurate solutions may be a difficult task. Comparing the performance of FDTD, FEM, MLFMA accelerated MOM will give better insight on CEM. Although FDTD uses SIS for enhancing accuracy, parallelization for reducing CPU time and Novel RK-HO FDTD to enhance both, is time consuming. On the other hand FEM is producing spurious solution that makes it not ideal for Eigen value problem. However MOM accelerated MLFMA is found to be suitable for well-conditioned and error controllable solver. Preconditioners speed the convergence, parallelization makes CPU time usage less, incremental multilevel filling and sparsification reduces memory and MOM elliptic formulation promotes the accuracy, thereby proving efficiency of MOM.

To cater the needs of emerging technology CEM incorporates the technique called hybridisation. Introduction of this technique helps in achieving the efficiency in terms of computational time, memory storage and accuracy. ID algorithm reduces computational issues, ID-MLFMA and MLFMA-ACA reduces multiscale problem, open MP-VALU MLFMA method enhances the CPU time, in turn making MOM hybrid more reliable. On analysing pros and cons of CEM techniques, it could be noticed that in application specific tasks, hybridisation of techniques involving MOM accelerated MLFMA are exhibiting better results in terms of memory, accuracy and computational time.

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