Robust Stability Analysis of Delayed Stochastic Neural Networks via Wirtinger-Based Integral Inequality

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We discuss stability analysis for uncertain stochastic neural networks (SNNs) with time delay in this letter. By constructing a suitable Lyapunov-Krasovskii functional (LKF) and utilizing Wirtinger inequalities for estimating the integral inequalities, the delay-dependent stochastic stability conditions are derived in terms of linear matrix inequalities (LMIs). We discuss the parameter uncertainties in terms of norm-bounded conditions in the given interval with constant delay. The derived conditions ensure that the global, asymptotic stability of the states for the proposed SNNs. We verify the effectiveness and applicability of the proposed criteria with numerical examples.

1 Introduction

The significance of neural networks (NNs) cannot be limited to being a class of mathematical models and information processing systems. Their application is far-reaching in many areas, among them automatic control, signal processing, pattern recognition, and quadric recognition (Haykin, 2007). The stability of NNs has been discussed by many researchers (Anbuvithya, Mathiyalagan, Sakthivel, & Prakash, 2016; Cichocki & Unbehauen, 1993; Lakshmanan, Prakash, Rakkiyappan, & Joo, 2020; Liu, Zeng, & Wang, 2017; Liu, Wang, & Liu, 2006; Li, Zheng, & Lin, 2011; Lv et al., 2017; Zhang, Liu, & Zhou, 2012; Wong & Selvi, 1998; Zheng, Zhang, & Wang, 2009). However, in many practical NNs, time delays are unavoidable, and they lead to NN instability, oscillation, and poor performance. Due to this, stability investigation of NNs with time delays has become an important area for research, and many levant reports have been published (Chen & Rong, 2003; Chen & Wu, 2009; Chen, Sun, Liu, & Rees, 2010; Fu & Li, 2011; Lakshmanan et al., 2018; Li, Wang, Yang, Zhang, & Wang, 2008; Li & Chen,

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2009; Qiu, Cui, & Wu, 2009; Shao, Huang, & Zhou, 2009; Yu, Zhang, & Quan, 2015; Zhang, Cao, Wu, Chen, & Alsaadi, 2018; Zhang & Quan, 2015). A global exponential stability condition and inequality based on a linear matrix inequality (LMI) that forms an global exponential stability condition for inertial Cohen-Grossberg NNs with time delays is discussed in Yu et al. (2015). Projective synchronization of fractional-order NNs with multiple time delays was studied in Zhang et al. (2018), Zhang and Yu (2016), and Zhang and Quan (2015). Zhang and Quan (2015) sought to obtain sufficient LMI-based conditions for the existence and global exponential stability of inertial bidirectional associative memory NNs with time delays. Therefore, it becomes imperative to include the factor of time delays in the dynamical analysis of NNs.

Stochastic disturbance generally is affected by network models. Thus, when the stability of NNs is analyzed, stochastic disturbance becomes unavoidable. This happens due to the common factor that synaptic transmission is a noisy process, and the neurons' connection weights rely on certain values of resistance and capacitance where there are uncertainties. In this regard, a great deal of work has been conducted on stability analysis for delayed SNNs and robust stability for uncertain stochastic neural networks (SNNs). As a result, scientific results have been published in relation to the stability of NNs with stochastic disturbance (Balasubramaniam & Lakshmanan, 2011; Blythe, Mao, & Liao, 2001; Chen & Wu, 2009; Liao & Mao, 1996; Mao, 1997; Muralisankar, Manivannan, & Balasubramaniam, 2015; Xia, Yu, Li, & Zheng, 2012; Zhao, Gao, & Mou, 2008; Zhu & Cao, 2010a, 2010b, 2014). Stability analysis for NNs by using specific stochastic inputs was discussed in Blythe et al. (2001) and Liao and Mao (1996). For Markovian jump impulsive stochastic Cohen-Grossberg NNs with mixed time delays, Zhu and Cao (2010b) used the Lyapunov-Krasovskii functional (LKF) method for structuring a novel robust exponential stability criterion and known or unknown parameters to be achieved. Zhu and Cao (2014) investigated the stability of stochastic delayed recurrent NNs with the use of an augmented LKF method. This leads to the need for increased attention to the issue of stability investigation for SNNs with time delays.

However, there are also inevitable uncertainties in modeling NNs due to errors in modeling and fluctuating parameters at the time of execution, resulting in instability and poor performance. There have also been many interesting results recently (Chen & Qin, 2010; Deng, Hua, Liu, Peng, & Fei, 2011; Hua, Liu, Deng, & Fei, 2010; Huang & Cao, 2007; Li, Chen, Zhou, & Fang, 2008; Wang, Shu, Fang, & Liu, 2006; Wu, Su, Chu, & Zhou, 2009; Zhang, Shi, & Qiu, 2007; Zhang, Shi, Qiu, & Yang, 2008) on the stability of uncertain SNNs with delay. Chen and Qin (2010), Hua et al. (2010), Huang and Cao (2007), Li et al. (2008), and Zhang et al. (2008) investigated uncertain SNNs with robust stability and time-varying delays in terms of LKF and stochastic analysis approaches. The robust stability in terms of stochastic Hopfield NNs with time delays was examined by using the LKF functional and conducting stochastic analysis by, Wang, Shu, Fang, and Liu

(2006) and Zhang et al. (2007). Deng et al. (2011) studied delay-dependent exponential stability of uncertain where SNNs with mixed delays, based on the LKF method. Wu, Su, Chu, and Zhou (2009) discussed some novel delay-dependent conditions, sufficient to ensure the global exponential stability of discrete, recurrent NNs with time-varying delays. Thus, it is evident that many researchers have contributed to the analysis of the stability of time-delayed NNs. A number of methods have been developed to minimize the conservatism of stability criteria: the multiple integral approach (Fang & Park, 2013), model transformation (Kwon & Park, 2004), free-weighting matrix techniques (He, Liu, Rees, & Wu, 2007; Liu, Wu, Martin, & Tang, 2007), park inequality (Park, 1999), the convex combination technique (Park & Ko, 2007), and reciprocally convex optimization (Park, Ko, & Jeong, 2011). Most important, since estimating a lower bound of the quadratic integral term such as $\int_{t-\vartheta}^{t} x^T(s)Dx(s)ds$, $(D > 0)$ is one of the major research topics on time-delay systems, Jensen's inequality has been used widely as a key lemma in obtaining delay-dependent stability criteria. The Wirtinger-based integral inequality, introduced recently in Seuret and Gouaisbaut (2013), also reduced the conservatism of Jensen's inequality, and its advantage was reflected in the comparisons of delay bounds for numerous systems, such as systems with constant, known, and time-varying delay. However, some new LKFs were not considered, and use of the Wirtinger-based integral inequality was concentrated only in Seuret and Gouaisbaut (2013). Therefore, further improvement on the reduction of conservatism in stability analysis for a system with time delays can be achieved, the motivation behind the research we present in this letter.

This letter discusses robust stability analysis for SNNs with time delay. We also consider parameter uncertainties in the system matrices of delayed SNNs. Based on suitable LKF, we derive the delay stability conditions in line with LMIs.

This letter focuses on the following points:

- Parameter uncertainties and stochastic disturbance are taken into account.
- Integral terms are estimated based on Wirtinger's integral inequalities. With appropriate LKF and stochastic stability theory, the delay-dependent stability conditions are attained to ensure the global asymptotic stability of the proposed system. We have employed well-known software to identify the effectiveness of the intended LMIs. Finally, we provide a number of figures to check the effectiveness of our intended method.

We use the following notations:

$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}^{n\times n}$	$n \times n$ real matrices

$\lvert\cdot\rvert$	Euclidean norm in $\mathbb{R}^n$
$(\Omega, \mathcal{F}, \mathcal{P})$	Complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$
A^T	Transpose of a matrix A
$*$	Symmetric block in a symmetric matrix

2 Problem Formulation and Preliminaries

We consider the following Hopfield NNs with time delays,

$$\frac{d\eta_i(t)}{dt} = -d_i(\eta_i(t)) + \sum_{j=1}^{n} b_{ij}^0 \sigma_j(\eta_j(t)) + \sum_{j=1}^{n} c_{ij}^1 \sigma_j(\eta_j(t-\vartheta) + J_i,$$
$$i = 1, 2, \ldots, n, \tag{2.1}$$

or, equivalently, the vector form,

$$\dot{\eta}_i(t) = -G_0\eta(t) + G_1\sigma(\eta(t)) + G_2\sigma(\eta(t-\vartheta) + J, \tag{2.2}$$

where $\eta(t) = [\eta_1(t), \eta_2(t), \ldots, \eta_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector; $J = [J_1, J_2, \ldots, J_n]$ denotes the external input; $\sigma(\eta) = [\sigma_1(\eta_1(t)), \sigma_2(\eta_2(t)), \ldots, \sigma_n(\eta_n(t))]^T$ denotes the neuron activation function; $G_0 = diag(d_1, d_2, \ldots, d_n)$, $G_1 = (b_{ij}^0)_{n\times n}$, $G_2 = (c_{ij}^1)_{n\times n}$ are the connection weight matrix; and $\vartheta > 0$ denotes the discrete time delay.

We make following assumptions throughout this letter.

Assumption 1. *For any $j = 1, 2, \ldots, n$, $\sigma_j(\cdot)$ satisfies the following inequality:*

$$0 \leq \frac{\sigma_j(\beta_1) - \sigma_j(\beta_2)}{\beta_1 - \beta_2} \leq p_j, \quad \forall \beta_1, \beta_2 \in \mathbb{R}, \quad \beta_1 \neq \beta_2,$$

where $P = diag(p_1, p_2, \ldots, p_n) > 0$.

Assuming that $\eta^* = (\eta_1^*, \eta_2^*, \ldots, \eta_n^*)^T$ is an equilibrium point of system 2.2, one can derive from that system $\xi(t) = \eta(t) - \eta^*$, which transforms system 2.2 as follows:

$$\dot{\xi}(t) = -G_0\xi(t) + G_1 f(\xi(t)) + G_2 f(\xi(t-\vartheta)), \tag{2.3}$$

where $\xi(t)$ is the state vector of the transformed system, $f_j(\xi_j(t)) = \sigma_j(\xi_j(t) + \eta_j^*) - \sigma_j(\eta_j^*)$. Consider that the function $f_j(\cdot)$, $j = 1, 2, \ldots, n$, satisfies the following condition:

$$0 \leq \frac{f_j(\xi_j)}{\xi_j} \leq p_j, \quad f_j(0) = 0, \quad \forall \xi_j \neq 0, \quad j = 1, 2, \ldots, n. \tag{2.4}$$

We consider parameter uncertainties and stochastic perturbations as follows:

$$\begin{aligned} d\xi(t) &= \big[-G_0(t)\xi(t) + G_1(t)f(\xi(t)) + G_2(t)f(\xi(t-\vartheta))\big]dt \\ &\quad + \big[G_3(t)\xi(t) + G_4(t)\xi(t-\vartheta)\big]dw(t), \\ \xi(t) &= \Psi(t), \quad \forall t \in [-\vartheta, 0], \end{aligned} \tag{2.5}$$

where $w(t)$ indicates a one-dimensional Brownian motion satisfying $E\{dw(t)\} = 0$ and $E\{dw(t)^2\} = dt$. $G_0(t) = G_0 + \Delta G_0(t)$, $G_1(t) = G_1 + \Delta G_1(t)$, $G_2(t) = G_2 + \Delta G_2(t)$, $G_3(t) = G_3 + \Delta G_3(t)$, and $G_4(t) = G_4 + \Delta G_4(t)$, where G_3 and G_4 are connection weight matrices with appropriate dimensions. In equation 2.5, the parametric uncertainties are assumed to have the form

$$\big[\Delta G_0(t) \;\; \Delta G_1(t) \;\; \Delta G_2(t) \;\; \Delta G_3(t) \;\; \Delta G_4(t)\big] = EF(t)[H_1 \;\; H_2 \;\; H_3 \;\; H_4 \;\; H_5], \tag{2.6}$$

where E and $H_i (i = 1, \dots, 5)$ are known, real, constant matrices:

$$F^T(t)F(t) \le I. \tag{2.7}$$

It is assumed that all elements of $F(t)$ are Lebesque measurable. The matrices $\Delta G_0(t)$, $\Delta G_1(t)$, $\Delta G_2(t)$, $\Delta G_3(t)$, and $\Delta G_4(t)$ are said to be admissible if equations 2.5 to 2.7 hold. The initial condition of equation 2.5 is given as $\xi(t) = \Psi(t), t \in [-\vartheta, 0]$.

Remark 1. The structure of the parameter uncertainty as in equations 2.6 and 2.7 was extensively exploited in the analysis of robust control and filtering of uncertain systems (Wang, Xie, & De Souza, 1992; Wang & Qiao, 2002). Many practical systems have unknown parameters that can either be modeled exactly or overbound by equation 2.7.

The following lemmas are useful in deriving the stability results for SNNs, equation 2.5:

Lemma 1 *(Boyd, El Ghaoui, Feron, & Balakrishnan, 1994). Given constant matrices μ_2, μ_3, and μ_4 with appropriate dimensions, where $\mu_2^T = \mu_2$ and $\mu_3^T = \mu_3$, $\mu_2 + \mu_4^T \mu_3^{-1} \mu_4 < 0$, if and only if*

$$\begin{bmatrix} \mu_2 & \mu_4^T \\ \mu_4 & -\mu_3 \end{bmatrix} < 0.$$

Lemma 2 *(Yue, Tian, Zhang, & Peng, 2009). Let B, F, N_0, N_1 and M be real matrices of appropriate dimensions with $M > 0$, $F^T(t)F(t) \le I$. Then for any scalar $\epsilon > 0$ satisfying $M^{-1} - \epsilon^{-1} N_1 N_1^T > 0$, we have*

1. $N_1F(t)N_0 + N_0^TF^T(t)N_1^T \leq \epsilon^{-1}N_1N_1^T + \epsilon N_0^TN_0$
2. $(B+N_1F(t)N_0)^TP(B+N_1F(t)N_0) \leq B^T(M^{-1} - \epsilon^{-1}N_1N_1^T)^{-1}B+\epsilon N_0^TN_0$.

Lemma 3 *(Seuret & Gouaisbaut, 2013). For any constant matrix $M_1 > 0$, the following inequality holds for all continuously differentiable function φ in $[b, c] \to \mathbb{R}^n$:*

$$(c-b)\int_b^c \varphi^T(s)M_1\varphi(s)ds \geq \left(\int_b^c \varphi(s)ds\right)^T M_1\left(\int_b^c \varphi(s)ds\right) + 3\Theta^TM_1\Theta,$$

$$\textit{where} \quad \Theta = \int_b^c \varphi(s)ds - \frac{2}{c-b}\int_b^c\int_b^s \varphi(u)duds.$$

3 Main Results

In this section, we derive a delay-dependent stochastic stability condition based on suitable LKF and LMI approaches.

We introduce two new state variables for the SNNs, equation 2.5,

$$\gamma(t) = -G_0(t)\xi(t) + G_1(t)f(\xi(t)) + G_2(t)f(\xi(t-\vartheta)) \tag{3.1}$$

and

$$\zeta(t) = G_3(t)\xi(t) + G_4(t)\xi(t-\vartheta), \tag{3.2}$$

and have

$$d\xi(t) = \gamma(t)dt + \zeta(t)dw(t). \tag{3.3}$$

Moreover, the following equality holds . . .

$$\xi(t) - \xi(t-\vartheta) = \int_{t-\vartheta}^t d\xi(s) = \int_{t-\vartheta}^t \gamma(s)ds + \int_{t-\vartheta}^t \zeta(s)dw(s). \tag{3.4}$$

The following theorem provides the mean-square asymptotic stability results for SNNs, equation 2.5.

Theorem 1. *SNNs, equation 2.5, are globally asymptotically stable in the mean square if there exist positive-definite matrices $Q = Q^T > 0$, $Z_1 = Z_1^T > 0$, $Z_2 = Z_2^T > 0$, and $R_l = R_l^T > 0$, $l = 1, 2$, and diagonal matrices $U_0 > 0$ and $U_1 > 0$,* *such that the following LMIs hold:*

$$\Pi=\begin{bmatrix}\Pi_{1,1} & 0 & \Pi_{1,3} & \Pi_{1,4} & 0 & 0 & 0 & 0 & \Pi_{1,9} & \Pi_{1,10} & \Pi_{1,11}\\ * & -R_1 & 0 & \Pi_{2,4} & 0 & 0 & 0 & 0 & \Pi_{2,9} & 0 & \Pi_{2,11}\\ * & * & \Pi_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & \Pi_{3,10} & 0\\ * & * & * & \Pi_{4,4} & 0 & 0 & 0 & 0 & 0 & \Pi_{4,10} & 0\\ * & * & * & * & \Pi_{5,5} & \Pi_{5,6} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & \Pi_{6,6} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & \Pi_{7,7} & \Pi_{7,8} & 0 & 0 & 0\\ * & * & * & * & * & * & * & \Pi_{8,8} & 0 & 0 & 0\\ * & * & * & * & * & * & * & * & -Q & 0 & 0\\ * & * & * & * & * & * & * & * & * & -\vartheta Z_1 & 0\\ * & * & * & * & * & * & * & * & * & * & -\vartheta Z_2\end{bmatrix}<0, \tag{3.5}$$

where

$$\begin{aligned}\Pi_{1,1} &= -2QG_0+R_1,\ \Pi_{1,3}=QG_1+U_0P,\ \Pi_{1,4}=QG_2,\ \Pi_{1,9}=G_3^TQ,\\ \Pi_{1,10} &= -G_0^T\vartheta Z_1,\ \Pi_{1,11}=G_3^T\vartheta Z_2,\ \Pi_{2,4}=U_1P,\ \Pi_{2,9}=G_4^TQ,\\ \Pi_{2,11} &= G_4^T\vartheta Z_2,\ \Pi_{3,3}=R_2-2U_0,\ \Pi_{3,10}=G_1^T\vartheta Z_1,\ \Pi_{4,4}=-R_2-2U_1,\\ \Pi_{4,10} &= G_2^T\vartheta Z_1,\ \Pi_{5,5}=-\frac{4}{\vartheta}Z_1,\ \Pi_{5,6}=\frac{6Z_1}{\vartheta^2},\ \Pi_{6,6}=\frac{-12Z_1}{\vartheta^3},\\ \Pi_{7,7} &= -\frac{4}{\vartheta}Z_2,\ \Pi_{7,8}=\frac{6Z_2}{\vartheta^2},\ \Pi_{8,8}=\frac{-12Z_2}{\vartheta^3}.\end{aligned}$$

Proof. In order to prove the asymptotically stable criteria, we consider the following LKF,

$$V(t)=\sum_{i=1}^{3}V_i(t), \tag{3.6}$$

where

$$\begin{aligned}V_1(t) &= \xi^T(t)Q\xi(t),\\ V_2(t) &= \int_{t-\vartheta}^{t}\xi^T(s)R_1\xi(s)ds+\int_{t-\vartheta}^{t}f^T(\xi(s))R_2f(\xi(s))ds,\\ V_3(t) &= \int_{-\vartheta}^{0}\int_{t+\theta}^{t}\gamma^T(s)Z_1\gamma(s)dsd\theta+\int_{-\vartheta}^{0}\int_{t+\theta}^{t}\zeta^T(s)Z_2\zeta(s)dsd\theta.\end{aligned}$$

Then it can be obtained by Ito's differential formula (Mao, 1997) that

$$dV(t) = LV(t)dt + 2\xi^T(t)Q\zeta(t)dw(t), \tag{3.7}$$

where

$$LV_1(t) = 2\xi^T(t)Q\gamma(t) + \zeta^T(t)Q\zeta(t), \tag{3.8}$$

$$\begin{aligned} LV_2(t) = {} & \xi^T(t)R_1\xi(t) - \xi^T(t-\vartheta)R_1\xi(t-\vartheta) + f^T(\xi(t))R_2 f(\xi(t)) \\ & - f^T(\xi(t-\vartheta))R_2 f(\xi(t-\vartheta)), \end{aligned} \tag{3.9}$$

$$\begin{aligned} LV_3(t) \le {} & \vartheta\gamma^T(t)Z_1\gamma(t) - \int_{t-\vartheta}^{t} \gamma^T(s)Z_1\gamma(s)ds + \vartheta\zeta^T(t)Z_2\zeta(t) \\ & - \int_{t-\vartheta}^{t} \zeta^T(s)Z_2\zeta(s)ds. \end{aligned} \tag{3.10}$$

By lemma 3,

$$\begin{aligned} -\int_{t-\vartheta}^{t} \gamma^T(s)Z_1\gamma(s)ds \le {} & -\frac{1}{\vartheta}\left\{\int_{t-\vartheta}^{t} \gamma(s)ds\right\}^T Z_1 \left\{\int_{t-\vartheta}^{t} \gamma(s)ds\right\} \\ & - \frac{3}{\vartheta}\left\{\int_{t-\vartheta}^{t} \gamma(s)ds - \frac{2}{\vartheta}\int_{t-\vartheta}^{t}\int_{s}^{t} \gamma(u)duds\right\}^T Z_1 \\ & \times \left\{\int_{t-\vartheta}^{t} \gamma(s)ds - \frac{2}{\vartheta}\int_{t-\vartheta}^{t}\int_{s}^{t} \gamma(u)duds\right\} \end{aligned} \tag{3.11}$$

$$\begin{aligned} -\int_{t-\vartheta}^{t} \zeta^T(s)Z_2\zeta(s)ds \le {} & -\frac{1}{\vartheta}\left\{\int_{t-\vartheta}^{t} \zeta(s)ds\right\}^T Z_2 \left\{\int_{t-\vartheta}^{t} \zeta(s)ds\right\} \\ & - \frac{3}{\vartheta}\left\{\int_{t-\vartheta}^{t} \zeta(s)ds - \frac{2}{\vartheta}\int_{t-\vartheta}^{t}\int_{s}^{t} \zeta(u)duds\right\}^T Z_2 \\ & \times \left\{\int_{t-\vartheta}^{t} \zeta(s)ds - \frac{2}{\vartheta}\int_{t-\vartheta}^{t}\int_{s}^{t} \zeta(u)duds\right\}. \end{aligned} \tag{3.12}$$

From condition 2.4, for any

$$U_0 = \text{diag}\{e_{11}, e_{21}, \ldots, e_{n1}\} > 0 \text{ and } U_1 = \text{diag}\{e_{12}, e_{22}, \ldots, e_{n2}\} > 0,$$

it may be noted that

$$0 \le -2\sum_{j=1}^{n} e_{j1} f_j\big(\xi_j(t)\big)\big[f_j(\xi_j(t)) - p_j\xi_j(t)\big]$$

$$
\begin{aligned}
&- 2\sum_{j=1}^{n} e_{j2} f_j\big(\xi_j(t-\vartheta)\big) \times \big[f_j(\xi_j(t-\vartheta)) - p_j\xi_j(t-\vartheta)\big] \\
&= 2\xi^T(t)U_0Pf(\xi(t)) - 2f^T(\xi(t))U_0f(\xi(t)) + 2\xi^T(t-\vartheta)U_1Pf(\xi(t-\vartheta)) \\
&\quad - 2f^T(\xi(t-\vartheta))U_1f(\xi(t-\vartheta)). \qquad (3.13)
\end{aligned}
$$

Substituting equations 3.8 to 3.13 into 3.7, we have

$$dV(t) \leq \chi^T(t)\Pi\,\chi(t)dt + 2\xi^T(t)Q\zeta(t)dw(t). \qquad (3.14)$$

Taking the mathematical expectation of both sides of equation 3.14, there exists a positive scalar $\alpha_1 > 0$ satisfying

$$E\big[dV(t)\big] \leq E\big(\chi^T(t)\Pi\,\chi(t)\big) \leq -\alpha_1 E\,\|\xi(t)\|^2. \qquad (3.15)$$

Π is defined in theorem 1 with

$$
\begin{aligned}
\chi^T(t) = \Bigg[&\xi^T(t), \xi^T(t-\vartheta), f^T(\xi(t)), f^T(\xi(t-\vartheta)), \\
&\left(\int_{t-\vartheta}^{t} \gamma(s)ds\right)^T, \left(\int_{t-\vartheta}^{t}\int_{s}^{t} \gamma(u)duds\right)^T, \left(\int_{t-\vartheta}^{t} \zeta(s)ds\right)^T, \\
&\left(\int_{t-\vartheta}^{t}\int_{s}^{t} \zeta(u)duds\right)^T\Bigg].
\end{aligned}
$$

Thus, if $\Pi < 0$, the SNNs, equation 2.5, are globally asymptotically stable in the mean square. □

Now we can study the robust stability analysis for SNNs, equation 2.5, with parameter uncertainties. Based on theorem 1, we provide a delay-dependent criterion:

Theorem 2. *SNNs, equation 2.5, are globally robustly asymptotically stable in the mean square if there exist positive-definite matrices* $Q = Q^T > 0, Z_1 = Z_1^T > 0, Z_2 = Z_2^T > 0, R_l = R_l^T > 0, l = 1, 2$; *diagonal matrices* $U_0 > 0$ *and* $U_1 > 0$;

and scalars $\epsilon_i > 0$, $(i = 1, 2, 3)$ *such that the following LMIs hold:*

$$\begin{bmatrix} \Xi & \hat{Q}E & \Gamma_1\vartheta Z_1 & 0 & \epsilon_1\Gamma_2 & \Gamma_3 Q & 0 & \epsilon_2\Gamma_4 & \Gamma_5\vartheta Z_2 & 0 & \epsilon_3\Gamma_6 \\ * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\vartheta Z_1 & \vartheta Z_1 E & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Q & QE & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\epsilon_2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\epsilon_2 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\vartheta Z_2 & \vartheta Z_2 E & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon_3 I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\epsilon_3 I \end{bmatrix} < 0, \tag{3.16}$$

where

$$\Xi = \begin{bmatrix} \hat{\Pi}_{1,1} & 0 & \Pi_{1,3} & \Pi_{1,4} & 0 & 0 & 0 & 0 \\ * & -R_1 & 0 & \Pi_{2,4} & 0 & 0 & 0 & 0 \\ * & * & \hat{\Pi}_{3,3} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Pi}_{4,4} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{5,5} & \Pi_{5,6} & 0 & 0 \\ * & * & * & * & * & \Pi_{6,6} & 0 & 0 \\ * & * & * & * & * & * & \Pi_{7,7} & \Pi_{7,8} \\ * & * & * & * & * & * & * & \Pi_{8,8} \end{bmatrix},$$

$$\hat{Q} = [Q\,0\,0\,0\,0\,0\,0\,0]^T,\ \Gamma_1 = [-G_0^T\ 0\ G_1^T\ G_2^T\ 0\,0\,0\,0]^T,$$

$$\Gamma_2 = [-H_1\ 0\ H_2\ H_3\ 0\,0\,0\,0]^T,\ \Gamma_3 = [G_3{}^T\ G_4{}^T\ 0\,0\,0\,0\,0\,0]^T,$$

$$\Gamma_4 = [H_4\ H_5\ 0\,0\,0\,0\,0\,0]^T,\ \Gamma_5 = [G_3{}^T\ G_4{}^T\ 0\,0\,0\,0\,0\,0]^T,$$

$$\Gamma_6 = [H_4\ H_5\ 0\,0\,0\,0\,0\,0]^T,\ \hat{\Pi}_{1,1} = \Pi_{1,1} + \epsilon_1 H_1{}^T H_1,$$

$$\hat{\Pi}_{3,3} = \Pi_{3,3} + \epsilon_1 H_2{}^T H_2,\ \hat{\Pi}_{4,4} = \Pi_{4,4} + \epsilon_1 H_3{}^T H_3.$$

Proof. Replacing G_0, G_1, G_2, G_3, G_4 in LMI, equation 3.5, with $G_0 + EF(t)H_1, G_1 + EF(t)H_2, G_2 + EF(t)H_3, G_3 + EF(t)H_4, G_4 + EF(t)H_5$ and using lemmas 1 and 2, we obtain the LMI, equation 3.16. □

Remark 2. Theorem 2 presents a sufficient condition to test the global robust stability for uncertain SNNs with time delay. Therefore, it is straightforward to test the feasibility of equation 3.16 without tuning any parameters using the Matlab LMI toolbox.

To show that our major results are sufficiently general to cover certain cases that have been discussed in the literature, we give a few corollaries.

Case 1. In the case that there are no stochastic disturbances in (2.5), we can get the following deterministic system,

$$\dot{\xi}(t) = -(G_0 + \Delta G_0(t))\xi(t) + (G_1 + \Delta G_1(t))f(\xi(t)) + (G_2 + \Delta G_2(t))f(\xi(t-\vartheta)), \tag{3.17}$$

then we have the given corollary.

Corollary 1. *If there exist positive-definite matrices $Q = Q^T > 0, Z_1 = Z_1^T > 0, R_l = R_l^T > 0, l = 1, 2$, diagonal matrices $U_0 > 0$ and $U_1 > 0$, scalar $\epsilon_1 > 0$, such that the below LMIs*

$$\begin{bmatrix} \Xi & \hat{Q}E & \Gamma_1\vartheta Z_1 & 0 & \epsilon_1\Gamma_2 \\ * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & -\vartheta Z_1 & \vartheta Z_1 E & 0 \\ * & * & * & -\epsilon_1 I & 0 \\ * & * & * & * & -\epsilon_1 I \end{bmatrix} < 0 \tag{3.18}$$

where

$$\Xi = \begin{bmatrix} \hat{\Pi}_{1,1} & 0 & QG_1 + U_0P & QG_2 & 0 & 0 \\ * & -R_1 & 0 & U_1P & 0 & 0 \\ * & * & R_2 - 2U_0 + \epsilon_1 {H_2}^T H_2 & 0 & 0 & 0 \\ * & * & * & -R_2 - 2U_1 + \epsilon_1 {H_3}^T H_3 & 0 & 0 \\ * & * & * & * & -\frac{4}{\vartheta}Z_1 & \frac{6Z_1}{\vartheta^2} \\ * & * & * & * & * & \frac{-12Z_1}{\vartheta^3} \end{bmatrix}$$

$\hat{\Pi}_{1,1} = \Pi_{1,1} + \epsilon_1 {H_1}^T H_1,\ \hat{Q} = [Q\ 0\ 0\ 0\ 0\ 0]^T,\ \Gamma_1 = [-G_0^T\ 0\ G_1^T\ G_2^T\ 0\ 0]^T,$

$\Gamma_2 = [-H_1\ 0\ H_2\ H_3\ 0\ 0]^T$

hold, then the system (3.17) is globally robustly asymptotically stable. Corollary 1 provide the stability condition of delayed NNs without stochastic disturbance in terms of LMI.

Case 2. In the absence of uncertainties in equation 3.10, we can get the following systems:

$$\dot{\xi}(t) = -G_0\xi(t) + G_1 f(\xi(t)) + G_2 f(\xi(t-\vartheta)). \tag{3.19}$$

The corresponding stability condition is derived in the following corollary:

Corollary 2. *If there exist positive-definite matrices* $Q = Q^T > 0$, $Z_1 = Z_1^T > 0$, $R_l = R_l^T > 0$, $l = 1, 2$, *and diagonal matrices* $U_0 > 0$ *and* $U_1 > 0$, *such that the following LMIs hold,*

$$\begin{bmatrix} -2QG_0 + R_1 & 0 & QG_1 + U_0P & QG_2 & 0 & 0 & -G_0^T\vartheta Z_1 \\ * & -R_1 & 0 & U_1P & 0 & 0 & 0 \\ * & * & R_2 - 2U_0 & 0 & 0 & 0 & G_1^T\vartheta Z_1 \\ * & * & * & -R_2 - 2U_1 & 0 & 0 & G_2^T\vartheta Z_1 \\ * & * & * & * & -\frac{4}{\vartheta}Z_1 & \frac{6Z_1}{\vartheta^2} & 0 \\ * & * & * & * & * & \frac{-12Z_1}{\vartheta^3} & 0 \\ * & * & * & * & * & * & -\vartheta Z_1 \end{bmatrix} < 0. \tag{3.20}$$

hold, then the system, equation 3.19, is globally asymptotically stable.

4 Numerical Examples

4.1 Example 1. System 2.5 without uncertainties, may be considered with the given matrices:

$$G_0 = \begin{bmatrix} 4.5 & 0 & 0 \\ 0 & 5.2 & 0 \\ 0 & 0 & 3.6 \end{bmatrix}, \quad G_1 = \begin{bmatrix} -1 & 0.4 & -0.5 \\ 0 & -0.7 & 0.7 \\ 0.2 & 0.6 & 0.8 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 0.5 & 0.7 & 1.1 \\ -0.1 & 0.4 & 0 \\ 0 & -0.2 & -0.8 \end{bmatrix}, \quad G_3 = \begin{bmatrix} 1.2 & 0.4 & -0.8 \\ -1.5 & -1.8 & 0.9 \\ 0.5 & 1.1 & 2.1 \end{bmatrix},$$

$$G_4 = \begin{bmatrix} 0.2 & 0.1 & -0.4 \\ 0 & 0.2 & 0.5 \\ 0.6 & 0 & 0 \end{bmatrix}, \quad P = 0.4I, \quad f(\xi(t)) = 0.4\tanh(\xi(t)).$$

By using the Matlab LMI toolbox, setting $\vartheta = 1.07$, and solving the LMI condition in theorem 1, the following feasible solutions may be obtained:

$$Q = \begin{bmatrix} 63.5169 & 28.5110 & 12.2574 \\ 28.5110 & 32.0008 & 10.1113 \\ 12.2574 & 10.1113 & 41.8480 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 247.3935 & 39.1445 & 52.7761 \\ 39.1445 & 42.9861 & 4.6154 \\ 52.7761 & 4.6154 & 30.6158 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 67.8405 & 9.3455 & -6.3186 \\ 9.3455 & 52.0187 & -16.1782 \\ -6.3186 & -16.1782 & 12.7751 \end{bmatrix}, \quad Z_1 = \begin{bmatrix} 3.4487 & 1.6577 & 0.7620 \\ 1.6577 & 1.6485 & 0.0579 \\ 0.7620 & 0.0579 & 0.4856 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 9.4027 & 5.5021 & 1.9470 \\ 5.5021 & 5.5867 & 0.4425 \\ 1.9470 & 0.4425 & 0.9293 \end{bmatrix}, \quad U_0 = \begin{bmatrix} 90.1657 & 0 & 0 \\ 0 & 90.1657 & 0 \\ 0 & 0 & 90.1657 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 70.2397 & 0 & 0 \\ 0 & 70.2397 & 0 \\ 0 & 0 & 70.2397 \end{bmatrix}.$$

Therefore, it follows from theorem 1 that the delayed stochastic neural network, equation 2.5, is globally asymptotically stable in the mean square.

4.2 Example 2. Consider the following uncertain stochastic NNs,

$$\begin{aligned} d\xi(t) = \big[&- (G_0 + \Delta G_0(t))\xi(t) + (G_1 + \Delta G_1(t)) f(\xi(t)) \\ &+ (G_2 + \Delta G_2(t)) f(\xi(t - \vartheta))\big] dt \\ &+ \big[(G_3 + \Delta G_3(t))\xi(t) + (G_4 + \Delta G_4(t))\xi(t - \vartheta)\big] dw(t), \end{aligned} \tag{4.1}$$

where

$$G_0 = \begin{bmatrix} 2 & 1 \\ 1.2 & 3 \end{bmatrix}, \quad G_1 = \begin{bmatrix} -1.5 & 0.6 \\ 0.6 & -1.5 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.5 & 1 \\ 1.2 & 0.6 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad G_4 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad E = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$P = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad H_i = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, \; i = 1, 2, \ldots, 5.$$

By using the Matlab LMI toolbox, setting $\vartheta = 1.07$ and solving the LMI condition in theorem 2, the following feasible solutions may be obtained:

$$Q = \begin{bmatrix} 0.8138 & -0.1295 \\ -0.1295 & 0.7336 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 0.8319 & 0.3215 \\ 0.3215 & 1.0335 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 1.1506 & 0.1763 \\ 0.1763 & 0.8954 \end{bmatrix}, \quad Z_1 = \begin{bmatrix} 0.1333 & -0.0283 \\ -0.0283 & 0.1264 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 0.2124 & 0.0077 \\ 0.0077 & 0.2186 \end{bmatrix}, \quad U_0 = \begin{bmatrix} 2.0680 & 0 \\ 0 & 2.0680 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 0.7096 & 0 \\ 0 & 0.7096 \end{bmatrix}, \quad \epsilon_1 = 0.8241, \quad \epsilon_2 = 0.6937, \quad \epsilon_3 = 0.5805.$$

Feng, Zhang, and Wu (2008) showed that the uncertain SNNs are globally, robustly, and asymptotically stable in mean square for the maximum time delay allowed, 0.6. However, using theorem 2, the maximum allowable bound can be obtained as $\vartheta = 1.07$. Hence, the results provided in this example are less conservative compared to those of Feng et al. (2008), and it follows from theorem 2 that the delayed SNNs, equation 4.1, are globally, robustly, and asymptotically stable in the mean square.

5 Conclusion

This letter has discussed robust, asymptotic stability analysis for uncertain, stochastic-delayed NNs. In theorem 1, by constructing a suitable LKF and utilizing Wirtinger-based inequality, we derived the sufficient condition for asymptotic stability of the system with time delay. An LMI approach has been proposed to check the mean square stability of stochastic uncertain neural networks, which can be tested easily using Matlab's LMI toolbox. We provided examples to illustrate the effectiveness of our main results.

References

Anbuvithya, R., Mathiyalagan, K., Sakthivel, R., & Prakash, P. (2016). Passivity of memristor-based BAM neural networks with different memductance and uncertain delays. *Cognitive Neurodynamics*, *10*(4), 339–351.

Balasubramaniam, P., & Lakshmanan, S. (2011). LMI conditions for robust stability analysis of stochastic Hopfield neural networks with interval time-varying delays and linear fractional uncertainties. *Circuits, Systems, and Signal Processing*, *30*(5), 1011–1028.

Blythe, S., Mao, X., & Liao, X. (2001). Stability of stochastic delay neural networks. *Journal of the Franklin Institute*, *338*(4), 481–495.

Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory*. Philadelphia: Society for Industrial and Applied Mathematics.

Chen, J., Sun, J., Liu, G. P., & Rees, D. (2010). New delay-dependent stability criteria for neural networks with time-varying interval delay. *Physics Letters A*, *374*(43), 4397–4405.

Chen, T., & Rong, L. (2003). Delay-independent stability analysis of Cohen–Grossberg neural networks. *Physics Letters A*, *317*(5–6), 436–449.

Chen, Y., & Qin, T. (2010). Robust stability analysis for uncertain stochastic neural networks with mixed time-varying delays. *International Journal of Systems, Control and Communications*, *2*(4), 364–378.

Chen, Y., & Wu, Y. (2009). Novel delay-dependent stability criteria of neural networks with time-varying delay. *Neurocomputing*, *72*(4–6), 1065–1070.

Cichocki, A., & Unbehauen, R. (1993). *Neural networks for optimization and signal processing*. New York: Wiley.

Deng, F., Hua, M., Liu, X., Peng, Y., & Fei, J. (2011). Robust delay-dependent exponential stability for uncertain stochastic neural networks with mixed delays. *Neurocomputing*, *74*(10), 1503–1509.

Fang, M., & Park, J. H. (2013). A multiple integral approach to stability of neutral time-delay systems. *Applied Mathematics and Computation*, *224*, 714–718.

Feng, W., Zhang, W., & Wu, H. (2008). Robust stability analysis of uncertain stochastic neural networks with time-varying delays. In *Proceedings of the 2008 Fourth International Conference on Natural Computation* (vol. 2, pp. 522–526). Piscataway, NJ: IEEE.

Fu, X., & Li, X. (2011). LMI conditions for stability of impulsive stochastic Cohen–Grossberg neural networks with mixed delays. *Communications in Nonlinear Science and Numerical Simulation*, *16*(1), 435–454.

Haykin, S. (2007). *Neural networks: A comprehensive foundation*. Upper Saddle River, NJ: Prentice Hall.

He, Y., Liu, G. P., Rees, D., & Wu, M. (2007). Stability analysis for neural networks with time-varying interval delay. *IEEE Transactions on Neural Networks*, *18*(6), 1850–1854.

Hua, M., Liu, X., Deng, F., & Fei, J. (2010). New results on robust exponential stability of uncertain stochastic neural networks with mixed time-varying delays. *Neural Processing Letters*, *32*(3), 219–233.

Huang, H., & Cao, J. (2007). Exponential stability analysis of uncertain stochastic neural networks with multiple delays. *Nonlinear Analysis: Real World Applications*, *8*(2), 646–653.

Kwon, O. M., & Park, J. H. (2004). On improved delay-dependent robust control for uncertain time-delay systems. *IEEE Transactions on Automatic Control*, *49*(11), 1991–1995.

Lakshmanan, S., Prakash, M., Lim, C. P., Rakkiyappan, R., Balasubramaniam, P., & Nahavandi, S. (2018). Synchronization of an inertial neural network with time-varying delays and its application to secure communication. *IEEE Transactions on Neural Networks and Learning Systems*, *29*(1), 195–207.

Lakshmanan, S., Prakash, M., Rakkiyappan, R., & Joo, Y. H. (2020). Adaptive synchronization of reaction–diffusion neural networks and its application to secure communication. *IEEE Transactions on Cybernetics*, *50*(3), 911–922.

Li, D., Wang, H., Yang, D., Zhang, X. H., & Wang, S. L. (2008). Exponential stability of cellular neural networks with multiple time delays and impulsive effects. *Chinese Physics B*, *17*(11), 4091–4099.

Li, H., Chen, B., Zhou, Q., & Fang, S. (2008). Robust exponential stability for uncertain stochastic neural networks with discrete and distributed time-varying delays. *Physics Letters A*, *372*(19), 3385–3394.

Li, X., & Chen, Z. (2009). Stability properties for Hopfield neural networks with delays and impulsive perturbations. *Nonlinear Analysis: Real World Applications*, *10*(5), 3253–3265.

Li, T., Zheng, W. X., & Lin, C. (2011). Delay-slope-dependent stability results of recurrent neural networks. *IEEE Transactions on Neural Networks*, *22*(12), 2138– 2143.

Liao, X., & Mao, X. (1996). Exponential stability and instability of stochastic neural networks. *Stochastic Analysis and Applications*, *14*(2), 165–185.

Liu, P., Zeng, Z., & Wang, J. (2017). Multistability of delayed recurrent neural networks with Mexican hat activation functions. *Neural Computation*, *29*(2), 423–457.

Liu, X. G., Wu, M., Martin, R., & Tang, M. L. (2007). Stability analysis for neutral systems with mixed delays. *Journal of Computational and Applied Mathematics*, *202*(2), 478–497.

Liu, Y., Wang, Z., & Liu, X. (2006). Global exponential stability of generalized recurrent neural networks with discrete and distributed delays. *Neural Networks*, *19*(5), 667–675.

Lv, Z. S., Zhu, C. P., Nie, P., Zhao, Z., Yang, H. J., Wang, Y. J., & Hu, C. K. (2017). Exponential distance distribution of connected neurons in simulations of two- dimensional in vitro neural network development. *Frontiers of Physics*, *12*(3), 128902.

Mao, X. (1997). *Stochastic differential equations and their applications.* Chichester: Horwood.

Muralisankar, S., Manivannan, A., & Balasubramaniam, P. (2015). Mean square delay dependent-probability-distribution stability analysis of neutral type stochastic neural networks. *ISA Transactions*, *58*, 11–19.

Park, P. (1999). A delay-dependent stability criterion for systems with uncertain time-invariant delays. *IEEE Transactions on Automatic Control*, *44*(4), 876–877.

Park, P., & Ko, J. W. (2007). Stability and robust stability for systems with a time-varying delay. *Automatica*, *43*(10), 1855–1858.

Park, P. G., Ko, J. W., & Jeong, C. (2011). Reciprocally convex approach to stability of systems with time-varying delays. *Automatica*, *47*(1), 235–238.

Qiu, F., Cui, B., & Wu, W. (2009). Global exponential stability of high order recurrent neural network with time-varying delays. *Applied Mathematical Modelling*, *33*(1), 198–210.

Seuret, A., & Gouaisbaut, F. (2013). Wirtinger-based integral inequality: Application to time-delay systems. *Automatica*, *49*(9), 2860–2866.

Shao, J. L., Huang, T. Z., & Zhou, S. (2009). An analysis on global robust exponential stability of neural networks with time-varying delays. *Neurocomputing*, *72*(7–9), 1993–1998.

Wang, Y., Xie, L., & De Souza, C. E. (1992). Robust control of a class of uncertain nonlinear systems. *Systems & Control Letters*, *19*(2), 139–149.

Wang, Z., & Qiao, H. (2002). Robust filtering for bilinear uncertain stochastic discrete-time systems. *IEEE Transactions on Signal Processing*, *50*(3), 560–567.

Wang, Z., Shu, H., Fang, J., & Liu, X. (2006). Robust stability for stochastic Hopfield neural networks with time delays. *Nonlinear Analysis: Real World Applications*, *7*(5), 1119–1128.

Wong, B. K., & Selvi, Y. (1998). Neural network applications in finance: A review and analysis of literature. *Information and Management*, *34*(3), 129–139.

Wu, Z., Su, H., Chu, J., & Zhou, W. (2009). New results on robust exponential stability for discrete recurrent neural networks with time-varying delays. *Neurocomputing*, *72*(13–15), 3337–3342.

Xia, J., Yu, J., Li, Y., & Zheng, H. (2012). New delay-interval-dependent exponential stability for stochastic neural networks with interval time-varying delay and distributed delay. *Circuits, Systems, and Signal Processing*, *31*, 1535–1557.

Yu, S., Zhang, Z., & Quan, Z. (2015). New global exponential stability conditions for inertial Cohen–Grossberg neural networks with time delays. *Neurocomputing*, *151*(3), 1446–1454.

Yue, D., Tian, E., Zhang, Y., & Peng, C. (2009). Delay-distribution-dependent robust stability of uncertain systems with time-varying delay. *International Journal of Robust and Nonlinear Control*, *19*(4), 377–393.

Zhang, J., Shi, P., & Qiu, J. (2007). Novel robust stability criteria for uncertain stochastic Hopfield neural networks with time-varying delays. *Nonlinear Analysis: Real World Applications*, *8*(4), 1349–1357.

Zhang, J., Shi, P., Qiu, J., & Yang, H. (2008). A new criterion for exponential stability of uncertain stochastic neural networks with mixed delays. *Mathematical and Computer Modelling*, *47*(9–10), 1042–1051.

Zhang, W., Cao, J., Wu, R., Chen, D., & Alsaadi, F. E. (2018). Novel results on projective synchronization of fractional-order neural networks with multiple time delays. *Chaos, Solitons and Fractals*, *117*, 76–83.

Zhang, Z., Liu, W., & Zhou, D. (2012). Global asymptotic stability to a generalized Cohen-Grossberg BAM neural networks of neutral type delays. *Neural Networks*, *25*, 94–105.

Zhang, Z., & Quan, Z. (2015). Global exponential stability via inequality technique for inertial BAM neural networks with time delays. *Neurocomputing*, *151*(3), 1316–1326.

Zhang, Z., & Yu, S. (2016). Global asymptotic stability for a class of complex-valued Cohen-Grossberg neural networks with time delays. *Neurocomputing*, *171*, 1158–1166.

Zhao, Y., Gao, H., & Mou, S. (2008). Asymptotic stability analysis of neural networks with successive time delay components. *Neurocomputing*, *71*(13–15), 2848–2856.

Zheng, C. D., Zhang, H., & Wang, Z. (2009). New delay-dependent global exponential stability criterion for cellular-type neural networks with time-varying delays. *IEEE Transactions on Circuits and Systems II: Express Briefs*, *56*(3), 250–254.

Zhu, Q., & Cao, J. (2010a). Stability analysis for stochastic neural networks of neutral type with both Markovian jump parameters and mixed time delays. *Neurocomputing*, *73*(13–15), 2671–2680.

Zhu, Q., & Cao, J. (2010b). Robust exponential stability of Markovian jump impulsive stochastic Cohen-Grossberg neural networks with mixed time delays. *IEEE Transactions on Neural Networks*, *21*(8), 1314–1325.

Zhu, Q., & Cao, J. (2014). Mean-square exponential input-to-state stability of stochastic delayed neural networks. *Neurocomputing*, *131*, 157–163.

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