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# Sensitizing The Multiple Damage Identification In Beam Structure Using Hoelder Exponent And Wavelets

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**Abstract.** In case of damage identification using continuous wavelet transform, the damage is identified by sensing the local perturbations in the spatial signal. First, a damage identification algorithm based on continuous wavelet transform is presented in this paper and next the algorithm is verified using a numerical beam model with different simulated damage cases. Multiple location damage cases are studied for damage identification. The effect of using only damaged data (mode shape from damaged beam) is investigated and compared. Another method of simultaneously detecting, locating and quantifying damage in a single graph is to plot the Hoelder exponent along the beam length. At each discrete point (node number) of beam, the Hoelder exponent is estimated and the resulting values are plotted along the length of beam. Any sudden variation of the exponent at a particular region, provide information on possibility of damage and the minimum value of exponent at that region gives the damage severity.

**Keywords:** Continuous Wavelet Transform, Spatial Data, Composite, Hoelder Exponent, Structural Health Monitoring And Damage Detection

## 1. Introduction

The process of implementing condition/damage detection and monitoring strategy for aerospace, civil and mechanical engineering infrastructure is commonly referred to as structural health monitoring (SHM). Here damage is defined as changes to the material and/or geometric properties of these systems, including changes to the boundary conditions and system connectivity, which adversely affect the system's performance (Doebbling and Farrar, 1996). The SHM process involves the observation of a system over time using periodically sampled dynamic response measurements from an array of sensors, the extraction of damage-sensitive features from these measurements and statistical analysis of these features to determine the current state of system health (Farrar and Sohn, 2001). The need for new structural damage detection methods that can be applied to complex structures has led to the development of methodologies that examine changes in the vibration characteristics of the structure (Farrar et al., 2001). In view of the above, it is important to examine some of the global damage detection methodologies for the structural damage identification currently in use. Ren and De Roeck (2002) proposed a damage identification method based on changes in frequencies and mode shapes of vibration for predicting damage location and severity. Pandey et al. (1991) proposed a parameter called curvature mode shape to detect and locate damage (Level II) in a structure. By using a cantilever and a simply supported analytical beam models, they showed that the absolute changes in the curvature mode shapes are localized in the region of damage and hence they can be used to detect damage in a structure. Salawu and Williams (1994), Chance, et al. (1994), Sampaio et al. (1999), Jaun (2005) proposed modal data based damage detection in structures. Reviews of these papers indicate that methods based on modal curvature, modal strain energy, and FRF curvature require undamaged (baseline) data and numerical differentiation for effective damage identification. Unfortunately, such an approach often is not feasible for a class of structures where undamaged state is unknown and cannot be acquired experimentally. This class of structures primarily includes old structures where damage state is unknown (Edward et al., 2002). Also these methods had inherent errors due to numerical



differentiations and integrations.

Liew and Wang (1998) was the first to satisfactorily report the application of the wavelet theory for crack identification in structures. The crack identification makes use of the wavelet theory applied to a simply supported beam with a transverse on-edge open crack. Quek et al., (2001) examined the sensitivity of spatial wavelet technique in the detection of cracks in beam structures. The effects of different crack characteristics (length, orientation and width of slit) and boundary conditions are investigated. The results show that the wavelet transform is a useful tool in detection of crack in beam structures. Hong et al. (2002) presented a new method to estimate the damage location and extent in a beam using a fundamental mode shapes. The mode shape is wavelet transformed for damage diagnostics, and beam damage is investigated in terms of the Hoelder (Lipschitz) exponent representing the order of the singularity. Reddy et.al.(2008) showed that through modelling of a bridge with damage, the effectiveness of using wavelet transform by means of its capability to detect and localise damage. Reddy et.al.(2012) show the effectiveness of using wavelet transform for detection and localization of small damages. The spatial data used here are the mode shapes and strain energy data of the damaged plate.

Reviews of papers on damage identification methods indicate that no paper clearly addresses the factors affecting the damage identification process. These factors include the study of locate and quantify multiple damages using only the fundamental mode shape obtained from damaged beam using different spatial inputs to wavelet transform. Another important observation is that no paper addresses the problem of wavelet transform based damage identification in beam structures by processes the modal data using the wavelet transform and Hoelder exponent so as to even detect very small damage cases.

## 2. Theory of Wavelet Analysis

### *Continuous Wavelet Transform (CWT)*

The Continuous Wavelet Transform (CWT) is defined as the sum over all time of the signal function of time or space (infinite) multiplied by a scaled, shifted version of a wavelet function  $\psi$ . For a spatial signal,

$$Wf(u, s) = \int_{-\infty}^{\infty} f(x) \psi(u, s, x) dx \quad (1)$$

Where,  $f(x)$  is the spatial input signal, and  $x$  being the spatial coordinate. The results of the CWT are wavelet coefficients  $Wf(u, s)$  that are a function of the scale  $s$  and position  $u$ . Since the input is spatial signal the wavelet transform is called Spatial Wavelet Transform. In case of damage identification in beam structures, the input signal may be mode shape, modal strain energy or the forced vibration data, where  $x$  is length of beam or node (element) number.

To perform the CWT, a basic wavelet function must be selected from the existing wavelet families. The basic wavelet function, known as the “mother wavelet”  $\psi(x)$  is dilated by a value  $s$  and translated by the parameter  $u$ .

The dilation (expansion or compression) and the translation yield a set of basis functions defined as

$$\psi(s, u, x) = \frac{1}{\sqrt{s}} \psi\left(\frac{x-u}{s}\right) \quad (2)$$

The translation parameter,  $u$ , indicates the space (or time) position of the relocated wavelet window in the wavelet transform. Shifting the wavelet window along the space (or time) axis implies examining the signal  $f(x)$  in the neighbourhood of the current window location. The scale parameter,  $s$ , indicates the width of the wavelet window.

The wavelets coefficients defined in equation indicate how similar is the function  $f(x)$  being analyzed to the wavelet function as  $\psi(s, u, x)$

In terms of a selected mother wavelet function  $\psi(x)$ , the continuous wavelet transform of a signal  $f(x)$  is defined as (Daubechies, 1992)

$$Wf(s, u) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(x) \psi\left(\frac{x-u}{s}\right) dx \quad (3)$$

The wavelet transform can detect and characterize transients (caused due to damage) in a spatial signal with zooming procedure across scales. The wavelet coefficients measures the variation of  $f(x)$  in a neighborhood of  $u$  whose size is proportional to  $s$ . Sharp transients create large amplitude wavelet coefficients. Thus high wavelet coefficients  $Wf(u,s)$  at a particular point on the spatial signal detects and locates the damage (Level I and II identification)

*Local Regularity And Hoelder (Lipschitz) Exponent*

Continuous wavelet transform provides a way to estimate the local smoothness of functions, which can be characterized by Hoelder exponents. A signal is regular if it can be locally approximated by a polynomial. The Hoelder exponent often measures this local regularity (Mallat and Hwang, 1992). A function  $f(x)$  has a Hoelder exponent  $\alpha \geq 0$  at  $x=v$  if there exists constant  $C > 0$  and a polynomial  $p_v$ , of degree  $m$  (where  $m$  is the largest integer satisfying  $m \leq \alpha$ ) such that

$$|f(x) - p_v(x)| \leq C|x - v|^\alpha \quad (4)$$

The polynomial  $p(x)$  is associated with Taylor expansion of  $f(x)$  at  $v$ .

By examining the decay of wavelet maxima  $|Wf(u, s)|$  coefficients as scale ( $s$ ) tends to zero, it can be proved (Mallat and Hwang, 1992) , for isolated singularities, the wavelet maxima obey an exponential law with an exponent equal to Hoelder exponent given by

$$|Wf(u, s)| \leq Cs^{\alpha+(1/2)} \quad (5)$$

The points of sharp variations of a signal cause local maxima at fixed scale of the wavelet transform modulus. The decay of modulus maxima along the modulus maximum line is characterized by Hoelder exponent. By rewriting above equation in logarithmic form, one gets

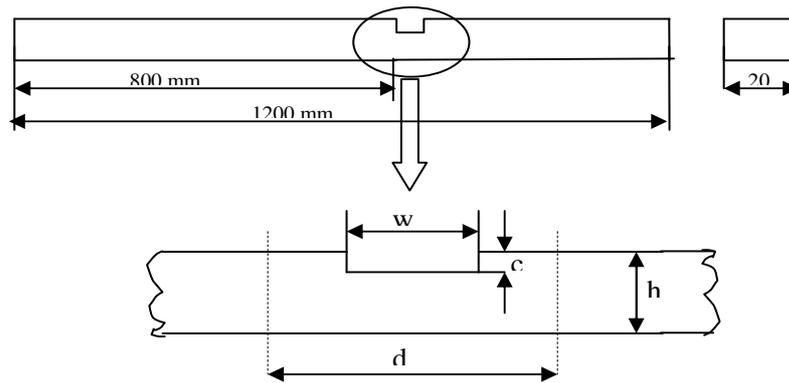
$$\log_2 |Wf(u, s)| \leq \log_2 C + \left(\alpha + \frac{1}{2}\right) \log_2 s \quad (6)$$

Where  $\alpha$  is the Hoelder exponent and  $C$  is a constant called intensity factor. This equation resembles a straight line equation with slope  $(\alpha+1/2)$  and y-intercept  $\log_2 C$ . The Hoelder exponent gives information about differentiability of a function more precisely. For example, if the exponent value is 1.5 at a point  $v$ , then the function  $f(x)$  is said to be differentiable one time but not twice. The greater the value of Hoelder exponent at a point, the more regular is the function at that point. Hence, value of Hoelder exponent decreases as the damage severity increases.

**3. Geometrical Model**

*Model Description*

For numerical simulations, aluminum beam with square cross section of dimensions 1200 x 20 x 20 mm with young's modulus of 70 GPa and density of 2700 kg/m<sup>3</sup> is used (Hong et al., 2002). Modeling and modal analysis is performed in ANSYS 16.0. The length of beam is divided into 2400 one-dimensional elements which fix width of each element equal to 0.5 mm, approximates width of actual crack. Damage is simulated at the 1600<sup>th</sup> element which is at 800 mm from left end as shown in Figure 1. The damage ( $c/h$ ) was varied from 0.5 to 0.9 to give four different damage cases. The beam is free at both the ends.



**Figure 1.** Beam model and Damage geometry

The parameter  $d$  represents the spatial sampling distance which is the distance between successive elements to obtain mode shape and  $w$  is the width of cut which is equal to the width of the element. In the present study damage is simulated by reducing the area moment of inertia of a desired element.

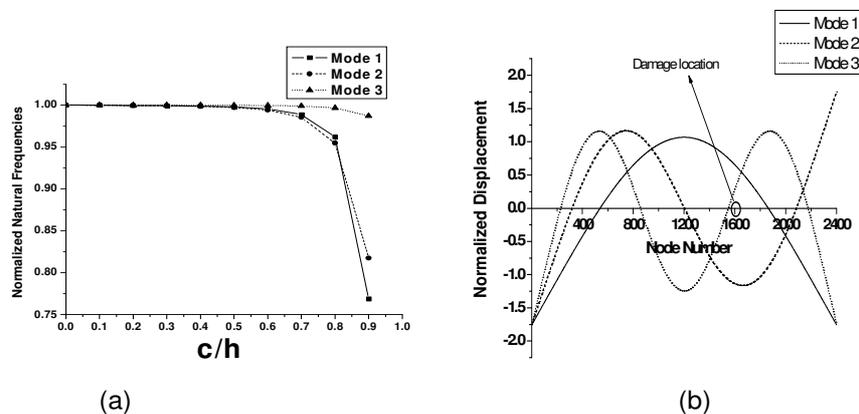
*Modal Analysis*

Numerical modal analysis is carried out in ANSYS 16.0 to get first three natural frequencies and displacement mode shape for all damage case ( $c/h=0.5$  to  $0.9$ ) and tabulated in Table 1.

**Table 1.** First three natural frequencies for undamaged and all damage case.

Damage Case ( $c/h$ )	Natural Frequencies (Hz)		
	Mode 1	Mode 2	Mode 3
Undamaged	72.562	199.88	391.49
0.5	72.404	199.32	391.41
0.6	72.238	198.71	391.33
0.7	71.745	196.99	391.08
0.8	69.824	190.8	390.21
0.9	55.789	163.38	386.45

To better visualize the shift in natural frequency due to damage from the natural frequency of undamaged beam, a plot of normalized natural frequency for first three modes with different damage ( $c/h$ ) is shown in Figure 2(a).



**Figure 2.** (a) normalized natural frequencies different damage cases for first three modes shapes (b) First three mode shapes

The natural frequency for different damage cases corresponding to particular mode is normalized with respect to undamaged natural frequency. It is observed that the shift for second mode is more compared to first and third. This is due to anti-node of second mode very close to damage location. The shift is minimum for third mode due to presence of node of mode near the damaged 800<sup>th</sup> and 1600<sup>th</sup> element as shown in Figure 2(b). Further wavelet analysis is carried out by using first fundamental mode shape as input parameter for damage detection.

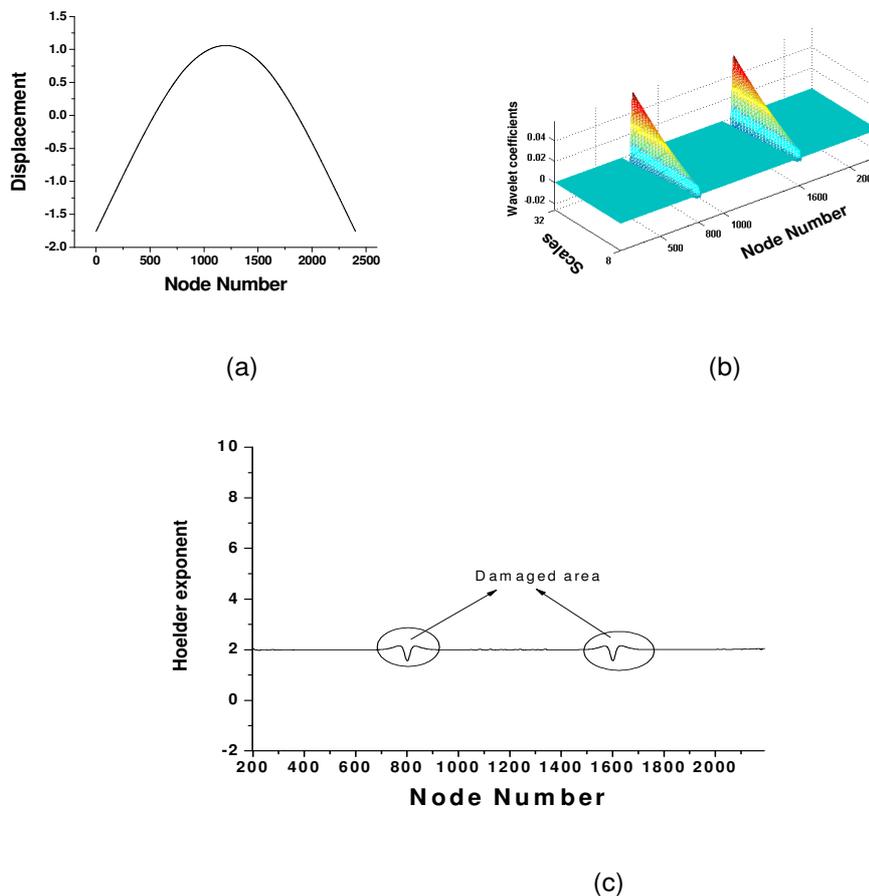
*Wavelet Analysis and results*

The Spatial signal (fundamental mode shapes/ modal strain energy) from damaged beam is wavelet transformed using Wavelet Toolbox available in MATLAB 12. After some experimentation it is found that scales of 8 to 32 provided better results. The mother wavelet selected is Gaussian wavelet with four vanishing moments. The resulting wavelet coefficients are used in damage identification.

To investigate the effectiveness of the proposed method to locate and quantify multiple damages using only the fundamental mode shape obtained from damaged beam the same beam previously considered is used with varying damages at 800 and 1600<sup>th</sup> element.

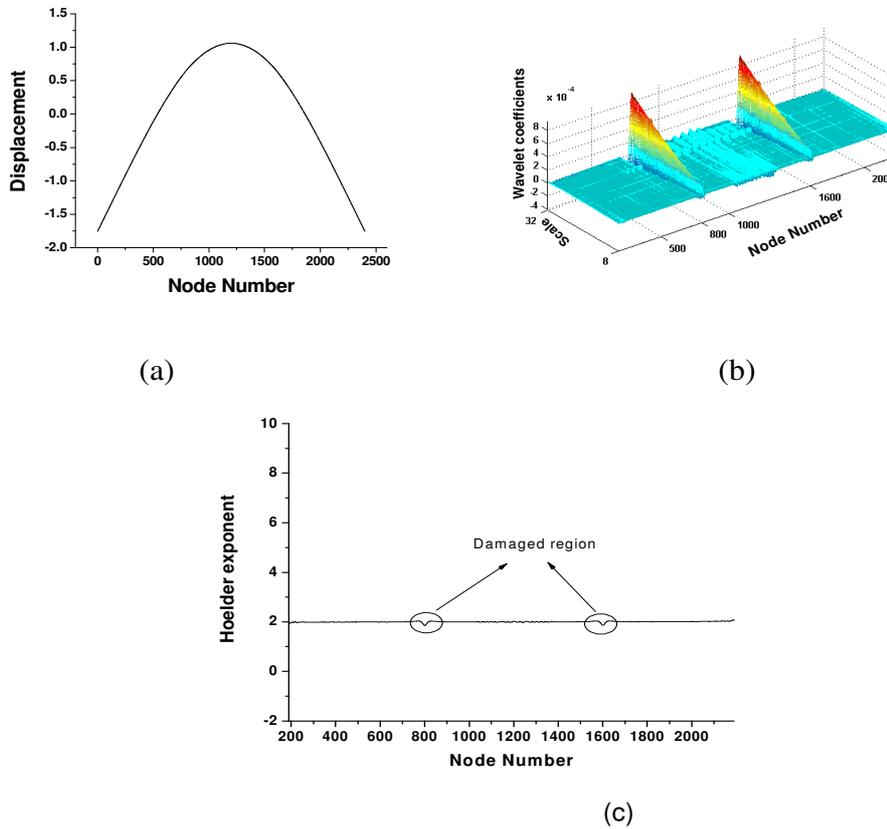
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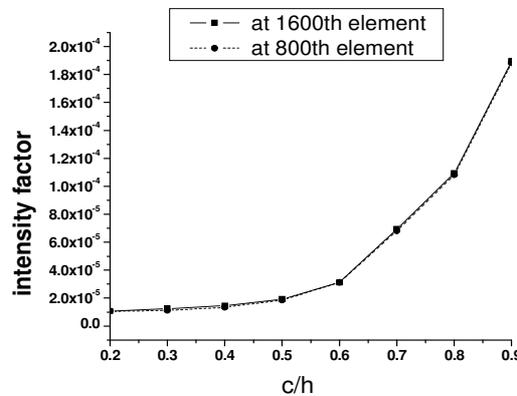
**Figure 3.** (a) Fundamental mode shape for damage case  $c/h=0.7$  (b) 3-D Wavelet plot in Scale-translation plane (c) Hoelder exponent along the length.

Figure 3(a) shows the fundamental mode shape obtained from beam with damage  $c/h=0.7$  at 800 and 1600th element. Again, for this case it is practically impossible to locate damage just by observing the mode shapes. Figure 3(b) shows the corresponding wavelet plot which clearly identifies the two damages by high wavelet coefficients at that site. Figure 3(c) gives the Hoelder exponent which is same for both the damages because of the same curvature (symmetric points) at both the damaged locations.



**Figure 4.** (a) Fundamental mode shape for damage case  $c/h=0.5$  (b) 3-D Wavelet plot in Scale-translation plane (c) Hoelder exponent along the length

Similarly, corresponding plots for damage case  $c/h=0.5$  are shown in Figure 4(a). Damages are clearly identified by high wavelet coefficients as seen in Figure 4(b) or from sudden variation of Holder exponent as in Figure 4(c).



**Figure. 5** Damage quantification using Intensity factor for different damage cases ( $c/h=0.2$  to  $0.9$ )

Figure 5 shows damage quantification plot for damages located at 800 and 1600<sup>th</sup> element. It is observed that the value intensity factor value increase with increase in  $c/h$ . It is seen that the intensity factor

becomes more sensitive to quantify the severity of damage.

#### 4. Conclusion

The proposed method processes the modal data so as to even detect the damage in the structures. The results obtained show that the process of wavelet transformed mode shape is much more sensitive to multiple damages compared to other conventional methods. Damage extent is investigated in terms of Hoelder exponent, which is estimated by plotting the decay behavior of maximum absolute wavelet coefficients at different scales. It is shown that the magnitude of Hoelder exponent is linked to damage extent. One of the main advantage of proposed methods is that there is no need to have undamaged (baseline) data as required by existing methods of on-line structural health monitoring of complex structures or structures with pre-existing damage. Results show the great promise of the wavelet approach for damage detection and structural health monitoring.

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