

Research Article

Sensor and Actuator Fault Detection and Isolation in Nonlinear System using Multi Model Adaptive Linear Kalman Filter

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Abstract: Fault Detection and Isolation (FDI) using Linear Kalman Filter (LKF) is not sufficient for effective monitoring of nonlinear processes. Most of the chemical plants are nonlinear in nature while operating the plant in a wide range of process variables. In this study we present an approach for designing of Multi Model Adaptive Linear Kalman Filter (MMALKF) for Fault Detection and Isolation (FDI) of a nonlinear system. The uses a bank of adaptive Kalman filter, with each model based on different fault hypothesis. In this study the effectiveness of the MMALKF has been demonstrated on a spherical tank system. The proposed method is detecting and isolating the sensor and actuator soft faults which occur sequentially or simultaneously.

Keywords: Fault detection and isolation, multi model adaptive linear kalman filter, nonlinear, residual generation, spherical tank, state estimation

INTRODUCTION

Sensors and actuators are playing major role in generating controller output and implementing the control action. Malfunction may occur either in plant or in the sensors or in actuators. The controllers are developed by assuming all the sensing and actuating elements are reliable and there is no fault in the system. If bias is present either in the actuator or in the sensor even though control algorithm is advanced one the product quality would not be good. It will affect the economy, safety of the plant and also affects the atmosphere. The sensor or actuator output is not the true value if bias is present either in the sensor or in the actuator. So detecting and isolating the soft failure is essential. The Fault Detection and Isolation (FDI) algorithm consists of making binary decision whether a fault has occurred or not, if fault has occurred isolating the faulty component. Fault Tolerant Control (FTC) will ensure the continual safe operation of the plant till the next scheduled maintenance.

Most of the FDI approaches use analytical redundancy. Faults are detected and isolated by comparing actual plant output and expected output based on model (Isermann, 1984; Frank, 1990; Patton and Chen, 1997). The system considered here is a stochastic process and the expected output is generated by statistical filter. The difference between the process and the estimator output is error and called residuals, which are used to detect and isolated different kinds of faults. This residual is also used to find the time of occurrence of faults.

This study uses Linear Kalman Filter to nonlinear system estimation. Most of the chemical processes are highly nonlinear in nature while operating the process in wide range of process variables. Accurate estimation of states is important for fault detection and control purposes. The widely used estimation technique for nonlinear system is Extended Kalman Filter (EKF). EKF linearizes all nonlinear transformations and substitutes Jacobian matrices in the KF equations. Linearization is reliable only if the error propagation is well approximated by linear transformation. For some nonlinear systems Jacobian matrix may not exists. Nonlinear estimation methods are computationally complex. Most of the existing algorithms are designed for sequential faults not for simultaneous faults.

The aim of the present study is to develop a MMALKF, which uses multiple ALKFs each with different hypothesis (Willsky, 1976). First the nonlinear model is linearized around different operating points and the local linear models are fused to get a global linear model at current operating point (Danielle and Cooper, 2003; Anjali and Patwardhan, 2008; Vinodha *et al.*, 2010). The LKF (estimator) is designed for each local linear model and the LKFs are fused using gain scheduling technique to get the Adaptive Linear Kalman Filter (ALKF). The ALKFs has multiple models because each of which is designed for detecting specific sensor and/or actuator faults. The proposed technique will detect the faults which occur sequentially as well as simultaneously and the time of occurrence of fault.

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METHODOLOGY

F used linear model: Let us consider a nonlinear stochastic system represented by the following state and output equations:

$$x_k = f(x_{k-1}, u_k, w_{k-1}) \tag{1}$$

$$y_k = h(x_k, v_k) \tag{2}$$

The nonlinear system is linearized around different operating points using Taylor series expansion. The linear system around operating points (\bar{x}_i, \bar{u}_i) is given as follows:

$$x_i(k) = \Phi_i(x(k-1) - \bar{x}_i) + \Gamma_{ui}(u(k-1) - \bar{u}_i) + \Gamma_n w(k) \tag{3}$$

$$y_i(k) = C_i x_i(k) + v_i(k) \tag{4}$$

where, $x \in R^n$ represents state variables, $u \in R^m$ represents inputs, $y \in R^r$ represents measured output and $w \in R^q$ and $v \in R^r$ represents state and measurement noise respectively. $w(k)$ and $v(k)$ are assumed to be Gaussian noises with covariance matrices Q and R, respectively. Φ_i, Γ_{ui} and Γ_n and C_i are known time invariant matrices of appropriate size. The nonlinear system is represented by a fused linear model using gain scheduling technique at a given operating point. For a given input vector $u(k)$, the state and output of fused linear model is represented as follows:

$$x(k) = \sum_{i=1}^N g_i \left\{ \begin{array}{l} \Phi_i(x(k-1) - \bar{x}_i) \\ + \Gamma_i(u(k-1) - \bar{u}_i) + \bar{x}_i \end{array} \right\} \tag{5}$$

$$y(k) = Cx(k) \tag{6}$$

To cover the entire operating horizon, five operating points has been selected ($i = 1$ to 5). Let y_m is the actual value of the measured process variable at current sampling instant and g_i is the weighting factor (Danielle and Cooper, 2003; Anjali and Patwardhan, 2008; Manimozhi *et al.*, 2013; Vinodha *et al.*, 2010):

$$\begin{aligned} \text{If } (y_m \geq y_5), \text{ then} \\ g_1 = g_2 = g_3 = g_4 = 0, g_5 = 1 \end{aligned} \tag{7}$$

$$\begin{aligned} \text{If } (y_4 < y_m \leq y_5), \text{ then} \\ g_1 = g_2 = g_3 = 0, g_4 = \frac{y_m - y_4}{y_5 - y_4}, g_5 = 1 - g_4 \end{aligned} \tag{8}$$

$$\begin{aligned} \text{If } (y_3 < y_m \leq y_4), \text{ then} \\ g_1 = g_2 = 0, g_3 = \frac{y_m - y_3}{y_4 - y_3}, g_4 = 1 - g_3, g_5 = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} \text{If } (y_2 < y_m \leq y_3), \text{ then} \\ g_1 = 0, g_2 = \frac{y_m - y_2}{y_3 - y_2}, g_3 = 1 - g_2, g_4 = g_5 = 0 \end{aligned} \tag{10}$$

$$\begin{aligned} \text{If } (y_1 < y_m \leq y_2), \text{ then} \\ g_1 = \frac{y_m - y_1}{y_2 - y_1}, g_2 = 1 - g_1, g_3 = g_4 = g_5 = 0 \end{aligned} \tag{11}$$

$$\begin{aligned} \text{If } (y_m \leq y_1), \text{ then} \\ g_1 = 1, g_2 = g_3 = g_4 = g_5 = 0 \end{aligned} \tag{12}$$

The weighting factors are in the range of [0 1].

Adaptive linear kalman filter: For the nonlinear model a ALKF can be designed to estimate the system states. This approach consists of family of local linear estimators and a scheduler. At each sampling instant the scheduler will assign weights (gain scheduling) for each local linear estimator and the weighted sum of the outputs will be the estimate of the current state. The scheduler assigns weight based on scheduling variable. The scheduling variable may be input variable or state variable or some auxiliary variable, the scheduling variable considered here is level of the process.

The LKF is designed for each local linear model using kalman filter theory as follows:

$$\hat{x}(k|k-1) = \Phi_i(x(k-1|k-1) - \bar{x}_i) + \Gamma_i(u(k-1) - \bar{u}_i) \tag{13}$$

$$\hat{y}_i(k|k-1) = C_i \hat{x}_i(k-1) \tag{14}$$

$$\hat{x}_i(k|k) = \hat{x}_i(k|k-1) + K_i(k) [(y(k) - \bar{y}_i) - \hat{y}_i(k|k-1)] \tag{15}$$

- where,
- $k_i(k)$ = Kalman gain matrix
- $\hat{x}_i(k|k-1)$ = Predicted state estimates
- $\hat{x}_i(k|k)$ = Corrected state estimates

The Kalman gain matrix can be calculated from the following equations:

$$P_i(k|k-1) = \Phi_i P_i(k-1|k-1) + \Phi_i^T \Gamma_{ni} Q \Gamma_{ni}^T \tag{16}$$

$$V_i(k) = C_i(k) P_i(k|k-1) C_i^T + R \tag{17}$$

$$K_i(k) = P_i(k|k-1) C_i^T V_i^{-1}(k) \tag{18}$$

$$P_i(k|k) = (I - K_i(k) C_i) P_i(k|k-1) \tag{19}$$

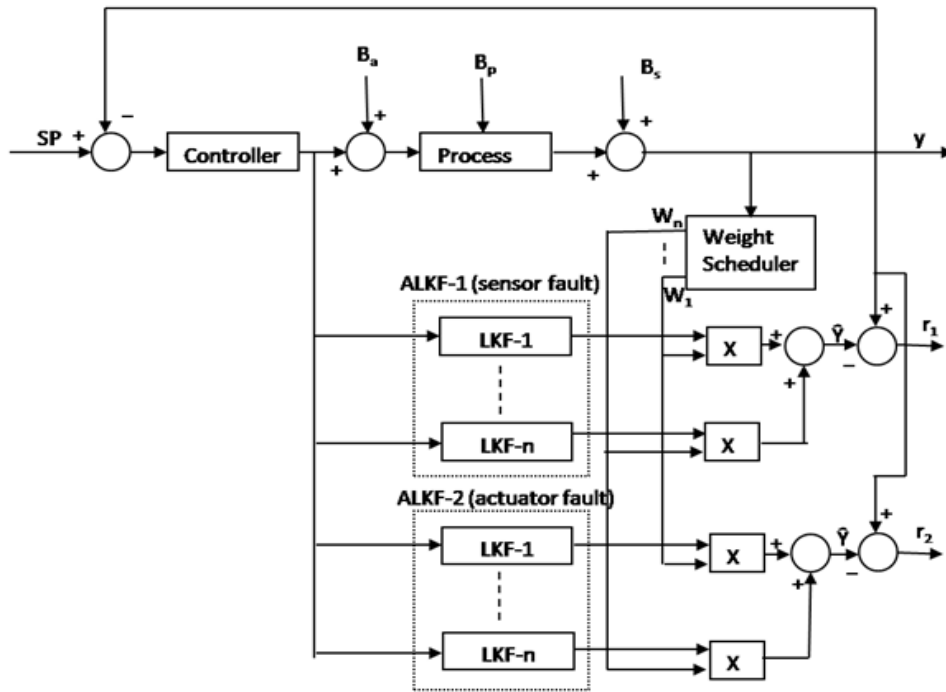


Fig. 1: Structure of the proposed MMALKF

where, $P_i(k|k-1)$ and $P_i(k|k)$ are the covariance matrices of errors in predicted and corrected state estimates of i^{th} local estimator, respectively.

The ALKF (global estimator) dynamics will be weighted sum of individual LKF and it is given below:

$$\hat{x}(k|k) = \sum_{i=1}^N g_i \left\{ \begin{array}{l} \hat{x}_i(k|k-1) + k_i(k) \\ [(\hat{y}(k) - \bar{y}_i) - \hat{y}_i(k|k-1)] \end{array} \right\} \quad (20)$$

Multi model adaptive linear kalman filter: This approach uses multiple ALKF. Each ALKF is designed based on specific hypothesis to detect a specific fault. The fault considered here is soft fault of fixed bias. The same approach can be used to detect drift like faults. This approach is capable of detecting multiple sequential as well as multiple simultaneous faults which occur either in sensors or in actuators.

If a bias of magnitude $B_{s,j}$ occurs at time t in the j^{th} sensor, then the measurement equation is given by:

$$y(k) = Cx(k) + v(k) + B_{s,j} F_{y,j} \varphi(k-t) \quad (21)$$

where, $F_{y,j}$ is a sensor fault vector with j^{th} element equal to unity and other elements equal to zero:

$$\varphi(k-t) = \begin{cases} 0 & \text{if } k < t \\ 1 & \text{if } k > t \end{cases} \quad (22)$$

If a bias of magnitude $B_{a,j}$ occurs in the j^{th} actuator at time t then the state equation is given by:

$$x(k+1) = \Phi x(k) + \Gamma_{ui} (u(k) + B_{a,j} F_{u,j} \varphi(k-t)) + \Gamma_n w(k) \quad (23)$$

where, $F_{u,j}$ is an actuator fault vector with j^{th} element equal to one and other elements equal to zero (Basseville and Nikiforov, 1993; Gertler, 1998; Prakash et al., 2002).

All the ALKF except the one using correct hypothesis will produce large estimation error. By monitoring the residuals of each ALKF, the faulty element (sensor or actuator) can be detected and isolated. Similarly we can model faults due to unmeasured disturbances (input variables other than manipulating variable are considered as disturbances) and parameter changes. We can model these because the process dynamics are derived using first principles.

The proposed MMALKF scheme is shown in Fig. 1. Each ALKF consists of five LKFs developed at different operating points. The weights are calculated by using level of the process as scheduling variable. The LKF outputs are weighted and added to get the global output estimate (\hat{y}). The process output is compared with the ALKF output to generate residuals. Under fault free condition the magnitude of the residuals are maximum. If fault occurs in any of the sensor or actuator, the estimators except the one using the correct hypothesis will remain same (produces large estimation error).

If the ALKF is designed for 1% error and the error occurred is less than or above 1%, then the residual

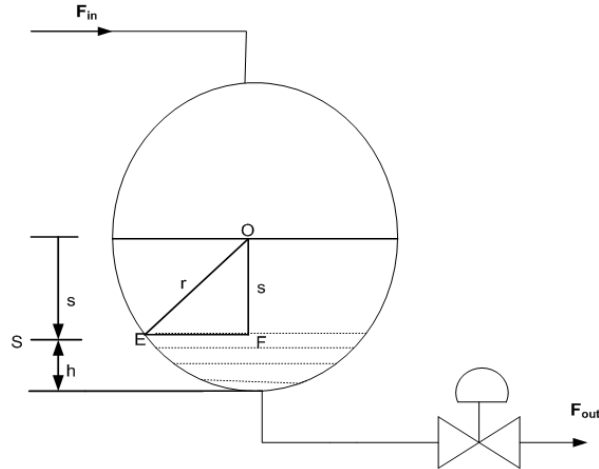


Fig. 2: Schematic of spherical tank process

generated will be different from the one during the normal operating condition. By closely observing the innovations, the faults which occurs either sequentially or simultaneously can be isolated and the time of occurrence can also be detected.

Spherical tank level process: A spherical tank level process shown in Fig. 2 was considered to test the effectiveness of the proposed method. The process is modeled by the following equation:

$$\frac{dh(t)}{dt} = \frac{F_{in}(t)}{\pi(2rh(t) - h(t)^2)} - \frac{C_s X_s \sqrt{2gh(t)}}{\pi(2rh(t) - h(t)^2)} \quad (24)$$

The steady state operating point data and the state space model of the system at five different operating points (selected based on gain and time constant variation with respect to level) were given in Table 1 and 2 respectively. The spherical process has one state variable level 'h' and one input variable i.e., flow rate f_{in} . The continuous linear state space model is obtained by linearizing the differential Eq. (20) at different operating points (level). The sampling time selected is 0.5 sec for all simulation studies.

SIMULATION RESULTS

The spherical tank process is simulated using first principles model given in Eq. (20) and the true state variable level is computed by solving the nonlinear differential equation using MATLAB 7.1. The dynamic behavior of the spherical tank process is not same at different operating points and the process is nonlinear. This can be verified from process gain and time constant at different operating points given in Table 3.

Fused linear model of the process: Figure 3 shows the open loop response of rigorous nonlinear model (process) and the fused linear model developed at five

Table 1: Steady state operating data of spherical tank process

Process variable	Nominal operating conditions
Level (h)	1 m
Flow rate (F_{in})	0.2215 m ³ /sec
Radius of the tank (r)	1 m
Constant of the outlet valve (C_s)	0.05 m ²
Outlet valve stem position (x_s)	1
Gravitational acceleration (g)	9.8/ms ²
Maximum level	2 m

Table 2: State space process model at different operating points for spherical tank

Model number	Flow $\bar{F}_{in,i}$ (m ³ /sec)	Level \bar{h}_i (m)	State space model
Model 1 i = 1	0.0990	0.2	A = 0.8963; B = 0.4763; C = 0.8842; D = 0
Model 2 i = 2	0.1716	0.6	A = 0.9733; B = 0.4933; C = 0.3789; D = 0
Model 3 i = 3	0.2100	0.9	A = 0.9814; B = 0.4953; C = 0.3215; D = 0
Model 4 i = 4	0.2425	1.2	A = 0.9834; B = 0.4958; C = 0.3316; D = 0
Model 5 i = 5	0.2971	1.8	A = 0.9642; B = 0.4910; C = 0.8842; D = 0

Table 3: Process gain and time constant at different operating points

Model number	Level \bar{h}_i (m)	System gain k_p	System time constant τ_p
Model 1 i = 1	0.2	4.038	4.567
Model 2 i = 2	0.6	6.995	18.459
Model 3 i = 3	0.9	8.571	26.658
Model 4 i = 4	1.2	9.658	29.850
Model 5 i = 5	1.8	12.115	13.702

different operating points for the given flow rate variations. The inlet flow rate has been varied to operate the system at different levels viz., 0.2, 0.4, 0.6,

0.8, 1.0, 1, 2, 1.5 and 1.8, respectively. It is evident from the response that the fused model is capturing the dynamics of the spherical tank level process exactly, at the same time slight steady state error is found in the open loop response when the system is operated away from the operating points.

State estimation using adaptive linear kalman filter for the spherical process: The performance of the ALKF is validated for the same flow rate variations as given in Fig. 3a and level has been estimated. The process and measurement noise covariance are assumed to be 0.25% of flow rate and 0.5% of level, respectively. Figure 4a and b shows the estimated process output using ALKF and rigorous nonlinear model output. It has been observed that the ALKF exactly estimates the process output without dynamic and steady state error.

Sequential sensor fault detection using MMALKF:

The sensor fault was detected and isolated using MMALKF developed in the previous section. The ALKF for detecting level sensor fault was hypothesized with 5% sensor fault and the residuals were generated by introducing 0, 2 and 5% faults in level sensor at 1001st sampling instant. The fault detected is confirmed if the mean value of the residual exceeds the threshold value. By averaging the residual over a period of time the false alarm due to spike like disturbances can be avoided. To reduce the false alarm due to modeling uncertainties the threshold level may be set high. Fig. 5a and b shows the true and estimated system output and the residual generated respectively. When there is no fault in the level sensor, the residual generated is more negative, as the fault magnitude increases the residual generated will become more and more positive.

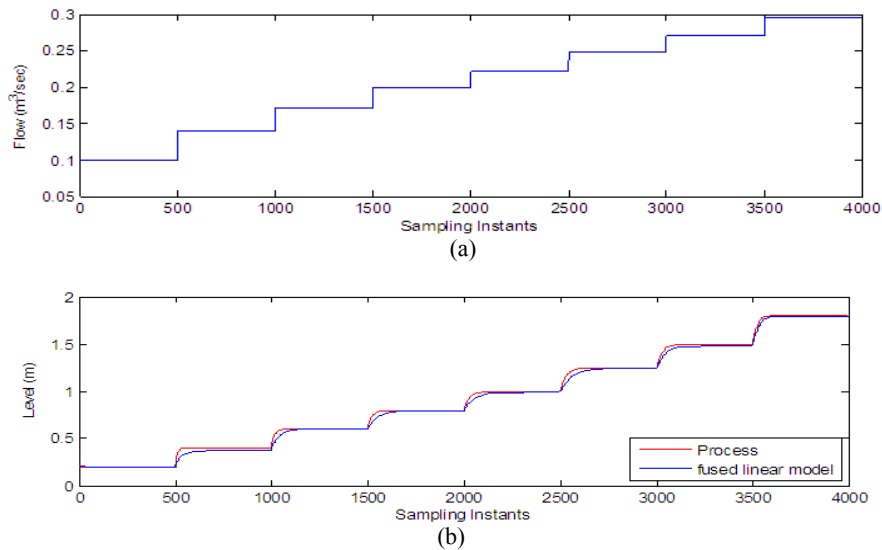


Fig. 3: Open loop response of process and fused linear model, (a) flow rate input and (b) level output

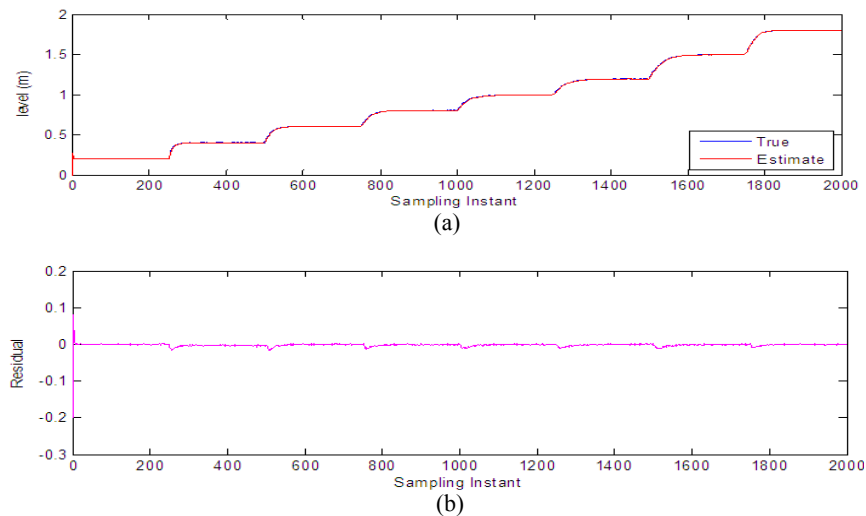


Fig. 4: (a) Estimate of level output using ALKF, (b) residual generated at different operating points

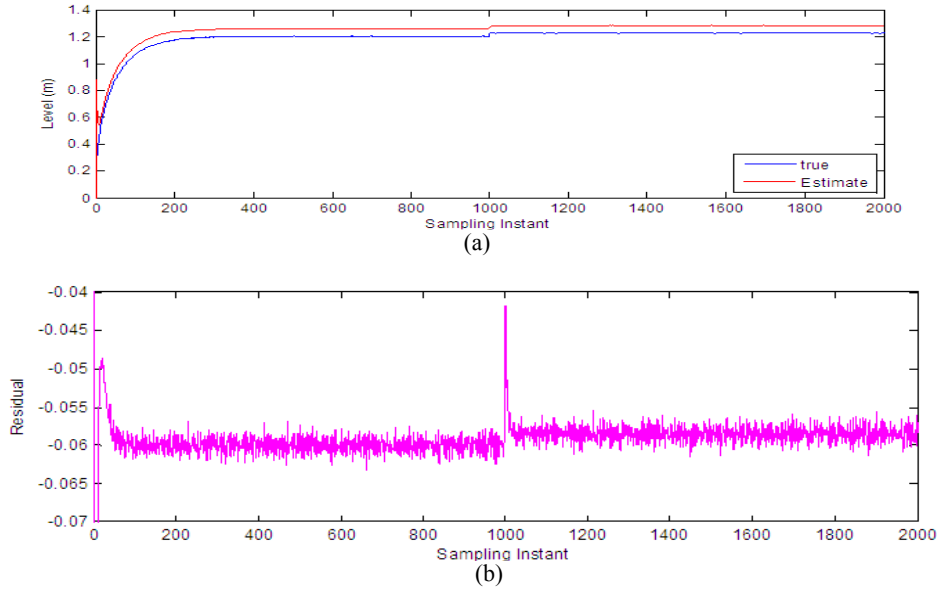


Fig. 5: (a) True and estimated values of level in the presence of sensor fault of 2% introduced at 1001th sampling instant, (b) generated residual

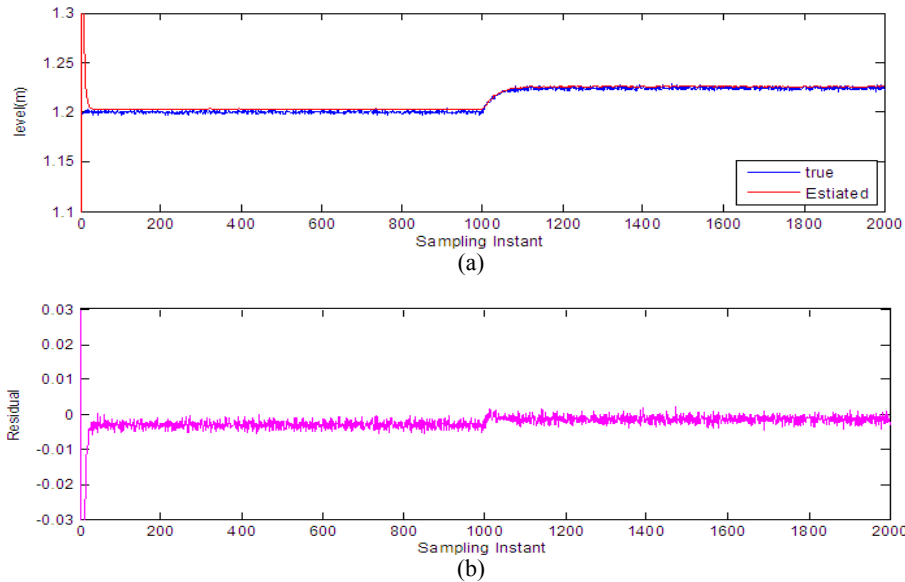


Fig. 6: (a) True and estimated values of level in the presence of actuator fault of 1% introduced at 1001th sampling instant, (b) generated residual

Sequential actuator fault detection using MMALKF: To detect and isolate the actuator fault the ALKF was hypothesized with 2% actuator fault and the residuals were generated for 0, 1 and 2%, respectively of actuator faults introduced at 1001st sampling instant. True and the estimated output and the residual generated are shown in Fig. 6a and b, respectively.

Simultaneous sensor and actuator fault detection using MMLAKF: To detect simultaneous sensor and actuator fault, in the MMALKF approach two ALKF were used. First ALKF is hypothesized with 5% sensor fault and the second ALKF with 2% actuator fault. If it

is important to monitor the health of two sensors and two actuators, four estimators are designed with four different hypothesis. Fault can be easily isolated by checking the trends of all four residuals. From the magnitude of the residual itself the magnitude of fault occurred can also be calculated. The time of occurrence of fault is the time at which the residual changes its trend and the fault is confirmed by comparing the mean of the residual over a period of time with the threshold value. The sensor and actuator faults of 0&0, 2&1 and 5 and 2%, respectively were introduced simultaneously at 1001st sampling instant. The estimated output and residual generated are shown in Fig. 7.

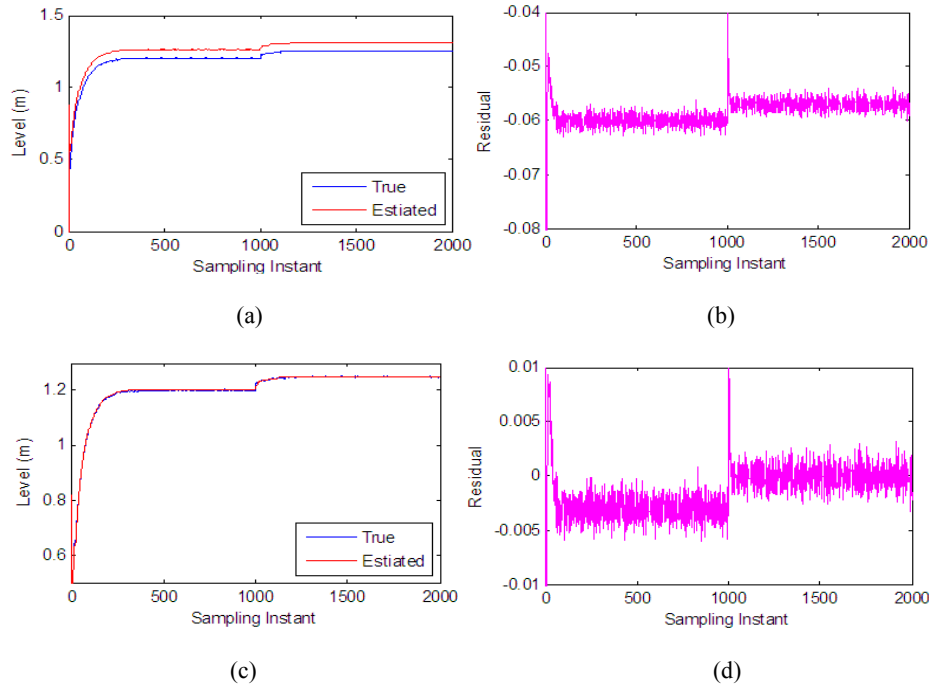


Fig. 7: (a) True and estimated values of level in the presence of sensor fault of 2% introduced at 1001th sampling instant, (b) generated residual, (c) true and estimated values of level in the presence of actuator fault of 1% introduced 1001th sampling instant, (d) generated residual

Table 4: Sensor bias alone

Bias magnitude (% bias)			
Estimator hypothesis	System bias magnitude	Mean value of residual generated	S.D.
0.06 (5%)	0 (0%)	-0.0600	0.0013
0.06 (5%)	0.024 (2%)	-0.0582	0.0014
0.06 (5%)	0.06 (5%)	-0.0560	0.0025

S.D.: Standard deviation

Table 5: Actuator bias alone

Bias magnitude (% bias)			
Estimator hypothesis	System bias magnitude	Mean value of residual generated	S.D.
4.85e-3 (2%)	0 (0%)	-0.0030	0.0010
4.85e-3 (2%)	2.425e-3 (1%)	-0.0014	0.0010
4.85e-3 (2%)	4.85e-3 (2%)	1.0485e-4	0.0011

S.D.: Standard deviation

Table 6: Simultaneous sensor and actuator faults estimator hypothesis: SF-0.06 (5%), AF-4.85e-3 (2%)

System bias magnitude (% bias)				
Sensor fault	Actuator fault	Mean value of SF residual generated	Mean value of AF residual generated	S.D.
0 (0%)	0 (0%)	-0.0600	-0.003	9.537e-4
0.024 (2%)	2.425e-3 (1%)	-0.0569	8.656e-5	0.0013
0.06 (5%)	4.85e-3 (2%)	-0.0530	0.004	0.0023

S: Sensor; A: Actuator; SF: Sensor fault; AF: Actuator fault; S.D.: Standard deviation

Estimator hypothesis, percentage of fault introduced in the system, the mean value of the residual generated and the standard deviation for different types of faults are given in Table 4 to 6.

CONCLUSION

In this study we have proposed MMALKF approach that includes adaptive gain scheduling

algorithm along with the multiple linear kalman filters to detect and isolate multiple sensor and actuator faults which occurs sequentially and simultaneously. The efficiency of the proposed approach was tested through extensive simulation on spherical process. The MMALKF can be used to develop a nonlinear model based FDI scheme for faults which occurs sequentially and simultaneously and fault tolerant control schemes. The proposed MMALKF performs better even in the

presence of considerable amount of plant-model mismatch.

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