Solving fully interval assignment problems

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Solving fully interval assignment problems

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Abstract. A new method namely, center point method is proposed to find an optimal interval assignment solution for fully interval assignment problems (FIAP). In the proposed method, the given FIAP is decomposed into a crisp assignment problem (AP) with the help of midpoint technique, solving it with the existing technique and by using its optimal solutions; an optimal interval assignment solution to the given FIAP is obtained. Comparison of intervals and partial ordering techniques were not used and there is no restriction on the elements of coefficient interval matrix in the proposed method. The proposed method is easier and also, simply because of the CA techniques. Using numerical examples the above method is illustrated.

1. Introduction

The assignment problem (AP) is a distinct form of a transportation problem where the key objective is to find an assignment schedule in a job where n jobs are assigned to n workers and each worker accepts exactly just one job so that the entire assignment cost must be minimum. Kuhn [3] introduced a specially designed algorithm so called Hungarian method to solve AP in the crisp environment.
In certainty, the entries of the cost matrix are not constantly crisp. In numerous application these limits are ambiguous and these uncertain parameters are signified by the interval. The theory of fuzzy set announced by Zadeh [7] in 1965 has attained effective applications in many fields. Chen [2] proved some theorems and proved a fuzzy assignment model that studies all individuals to have similar skills. More recently, Sarangam Majumdar [6] solved interval linear assignment problems using a new method named interval Hungarian method. Ramesh and Ganesan [5] have proposed a new computational technique to solve AP with generalized interval Hungarian method. Ramesh Kumar and Deepa [4] have introduced a matrix ones interval linear assignment method and compared with the existing method. Amutha et al. [1] has introduced a method of solved extension of the interval in assignment problem.

2. Preliminaries

Here we list the basic definitions and properties of interval numbers (or interval) and their arithmetic properties which can be found in [4, 5, 6].
Definition 2.1. A closed real interval $[x_I, x_S]$ denoted by $\underline{x}$, is a real interval number, defined as

$$\underline{x} = [x_I, x_S] = \left\{ x \in \Re \setminus x_I \leq x \leq x_S;\, x_I, x_S \in R \right\}$$

Where x_I and x_S are named infimum and supremum, respectively.

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Definition 2.2. A real interval vector $\underline{X} \in I(R^n)$ is a set of the form $\underline{X} = (\underline{x}_i)_{n\times 1}$, where $i = 1, 2..., n$ and $\underline{x}_i = [\, x_{iI}, x_{iS} \,] \in I(R)$.

Definition 2.3. A real interval matrix $\underline{A} \in I(M(R^n))$ is a set of the form $\underline{A} = (\underline{a}_{ij})_{n\times n}$, where $i = 1, 2..., n$ and $\underline{a}_{ij} = [a_{ij\,I}, a_{ij\,S}] \in I(R)$.

i) $\underline{x} + \underline{y} = [x_I + y_I, x_S + y_S]$ (addition)

ii) $\underline{x} - \underline{y} = [x_I - y_S, x_S + y_I]$ (subtraction)

Definition 2.4. Let $\underline{x} = [x_I, x_S]$ be an interval number, then the midpoint (or) center is defined as $m = \dfrac{x_I + x_S}{2}$, satisfying the relation $x_I \le x_m \le x_S$ where $x_m = \dfrac{x_I + x_S}{2}$.

3. Interval assignment problem

Assume there are n works to be completed and n persons are offered for doing the works. Assume that each person can do each work at a time, yet with a variable grade of efficiency. Let $\underline{c}_{ij}$ be the interval cost if the i[th] person is allocated the j[th] work, the problem is to discover a minimum interval cost through optimal interval assignment. The mathematical interval assignment problem with interval cost can be represented in table I.

Table 1: Interval assignment with interval cost works

	1	**2**	**. . .**	**j**	**. . .**	**n**
1	$\underline{c}_{11}$	$\underline{c}_{12}$	$\cdots$	$\underline{c}_{1j}$	$\cdots$	$\underline{c}_{1n}$
2	$\underline{c}_{21}$	$\underline{c}_{22}$	$\cdots$	$\underline{c}_{2j}$	$\cdots$	$\underline{c}_{2n}$
Persons	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
i	$\underline{c}_{i1}$	$\underline{c}_{i2}$	$\cdots$	$\underline{c}_{ij}$	$\cdots$	$\underline{c}_{in}$
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
n	$\underline{c}_{n1}$	$\underline{c}_{n2}$	$\cdots$	$\underline{c}_{nj}$	$\cdots$	$\underline{c}_{nn}$

3.1. *Mathematical formulation of an interval assignment problem*

Mathematically, the interval assignment problem is given in the above table I can be written as:

(FIAP) Minimize $\underline{z} \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \underline{c}_{ij}\, x_{ij}$, $\mathrm{i} = 1, 2, ..., \mathrm{n}$

subject to

$$x_{ij} = \begin{cases} 1, & \text{if } \mathrm{i}^{\text{th}} \text{ person is assigned } \mathrm{j}^{\text{th}} \text{ work} \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{x}_{\mathrm{ij}} = 1 \text{ (one work is done by the } \mathrm{i}^{\text{th}} \text{ person, } \mathrm{i} = 1,2,...,\mathrm{n}) \text{ and}$$

$$\sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{x}_{\mathrm{ij}} = 1 \text{ (only one person should be assigned the } \mathrm{j}^{\text{th}} \text{work, } \mathrm{j} = 1,2,\ldots,\mathrm{n})$$

where x_{ij} indicates that j^{th} work is assigned to the i^{th} person, $\underline{c}_{ij} = [c_{ij\,I}, c_{ij\,S}]$ is an interval cost to i^{th} person for doing j^{th} work and $\sum_{i-1}^{n}\sum_{j=1}^{n} \underline{c}_{ij}\, x_{ij}$ is the total interval cost for carrying out all the works.

Let the parameters $\underline{c}_{ij}$ be an interval number as $[c_{ij\,I}, c_{ij\,m}, c_{ij\,S}]$ respectively. Then the above FIAP can be given as follows:

$$\text{Minimize}\quad [z_I, z_m, z_S] = \sum_{i=1}^{n}\sum_{j=1}^{n} [c_{ij\,I}, c_{ij\,m}, c_{ij\,S}]\, x_{ij}\ ,\ i = 1, 2, \ldots, n$$

subject to

$$x_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ (one work is done by the } i^{th} \text{ person}, i = 1,2,\ldots,n) \text{ and}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ (only one person should be assigned the } j^{th} \text{ work}, j = 1,2,\ldots,n)$$

In this section, we proposed a new method namely center point method to find an optimal interval assignment solution for FIAP.

Algorithm:
Step 1: First test whether the given FIAP is a balanced or not. If it is balanced (i.e, number of persons is the same to the number of works) then go to step 3. If it is an unbalanced (i.e, number of persons are not equal to the number of works) then go to step 2.
Step 2: Introduce dummy rows and/or column with zero interval costs so as to bring a balanced one.
Step 3: Decompose the interval cost objective function $\underline{z} = [z_I, z_S]$ into three linear cost objectives namely lower, middle and upper as $\underline{z} = [z_I, z_m, z_S]$:

$$z_I = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij\,I}\, x_{ij}\ ;\ z_m = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij\,m}\, x_{ij}\ ;\ z_S = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij\,S}\, x_{ij}$$

Step 4: Using existing assignment technique, solve the middle-cost assignment problem. Let x_{ij}° represent the optimal assignment and c_{ij}° represent the optimal assignment cost.
Step 5: To acquire the total optimal interval assignment cost substitute the optimal assignment solution found in step 4 in the corresponding objective of step 3.

Example 3.1. Let us study an interval assignment problem with 4 rows indicating 4 machines M_1, M_2, M_3, M_4 and columns representing the 4 jobs J_1, J_2, J_3, J_4. Find the optimal interval assignment so that the total interval cost matrix $\underline{c}_{ij}$ given in the below table becomes minimum.

	J_1	J_2	J_3	J_3
M_1	[9,11]	[4,6]	[12,14]	[14,16]
M_2	[2,4]	[8,10]	[17,19]	[2,4]
M_3	[9,11]	[6,8]	[2,4]	[1,3]
M_4	[4,6]	[10,12]	[8,10]	[6,8]

Solution: The given FIAP is a balanced one.
Using step 3, we decompose the interval cost objective function as $\underline{z} = [z_I, z_m, z_S]$:

	J_1	J_2	J_3	J_3
M_1	[9,10,11]	[4,5,6]	[12,13,14]	[14,15,16]
M_2	[2,3,4]	[8,9,10]	[17,18,19]	[2,3,4]
M_3	[9,10,11]	[6,7,8]	[2,3,4]	[1,2,3]
M_4	[4,5,6]	[10,11,12]	[8,9,10]	[6,7,8]

Using step 4, solve the middle cost assignment problem:

	J_1	J_2	J_3	J_3
M_1	10	**5**	13	15
M_2	3	9	18	**3**
M_3	10	7	**3**	2
M_4	**5**	11	9	7

Solving with the existing technique, its optimal assignment solution is $M_1 \to J_2$, $M_2 \to J_4$, $M_3 \to J_3$, $M_4 \to J_1$ and Minimum $z_m = 16$

The minimum total interval assignment cost is attained by replacing the above optimal assignment solution in the interval cost objective function of step 3 as, [4,6]+[2,4]+[2,4]+[4,6]=[12,20].

Note3.1. The solution of Example 3.1, obtained by the center point method is same as in [1, 5, 6], but without using the arithmetic operation of intervals throughout the entire procedure.

4. Conclusion

In this present work, a new method to solve FIAP without using arithmetic operations on intervals is proposed. In the existing method [3,4 and 5] throughout the entire procedure they applied Hungarian method based on the computation of intervals. In this proposed method we decomposed FIAP into a crisp AP using midpoint, based on the optimal assignment of AP, we obtained an optimal interval assignment cost for FIAP. Meanwhile, the proposed method is based only on crisp AP and there is no calculation of intervals, it is very easy and simple to solve FIAP.

References

[1] Amutha S, Lakshmi S, Narmatha S 2017 *Int. J. Eng. Sci. Comp.* **7**(4) 10484-10488
[2] Chen M S 1985 *Tamkang J* **22** 407-411
[3] Kuhn H W 1955 *Naval Research Logistics Quarterly* 283-97
[4] Ramesh Kumar A, and Deepa, S 2016 *Int. J. Eng. Res. App.* **6**(10) 69-75
[5] Ramesh G and Ganesan K 2015 *Int. J. Scientific & Engineering Research* **6**(3)81-85
[6] Sarangam Majumdar 2013 *Universal Journal of Applied Mathematics* **1**(1)14-16
[7] Zadeh L A 1965 *Information and Computation* **8** 338-353