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Solving Intuitionistic Fuzzy Multi-Objective Nonlinear Programming Problem

D Anuradha, V E Sobana

Department of Mathematics, VIT University, Vellore-632014, TN, India

Email: anuradhadhanapal1981@gmail.com

Abstract. This paper presents intuitionistic fuzzy multi-objective nonlinear programming problem (IFMONLPP). All the coefficients of the multi-objective nonlinear programming problem (MONLPP) and the constraints are taken to be intuitionistic fuzzy numbers (IFN).The IFMONLPP has been transformed into crisp one and solved by using Kuhn-Tucker condition. A numerical example is provided to illustrate the approach.

1. Introduction

Nonlinear programming problem (NLPP) is the problem of optimization in which either the objective function is nonlinear, or one or more constraints have a nonlinear relationship or both. Fuzzy NLPP is helpful in solving problems which are challenging and it is impractical to solve the uncertain, subjective nature of the formation of the problem or have an exact solution. Zimmermann [20] introduced the concept of fuzzy NLPP. Bellman and Zadeh [3] discussed the concept of fuzzy decision making and also they discussed fuzzy quantities. Maleki et al. [11] applied the ranking technique for solving LPP with imprecise data in constraints. Tanaka et al. [18] used the technique of ranking for solving mathematical programming problems. Pandian and Jayalakshmi [14] discussed a new procedure for solving fully FLPP with fuzzy variables. Zimmermann [19] proposed a fuzzy programming technique to solve crisp multi-objective LPP. Kiruthiga and Loganathan [5] used the ranking function for solving a fuzzy multi-objective nonlinear fractional programming problem. Loganathan and Kiruthiga [7] proposed a solution procedure for solving fuzzy nonlinear programming problem using the ranking function. Loganathan and Lalitha [8] presented a procedure for finding the solution of fuzzy multi-objective NLPP using alpha-cut. Kirtiwant Ghadle et al. [6] discussed a new algorithm for finding an optimal solution to FNLPP. An IFS is one of the generalizations of fuzzy set theory. Out of various higher order fuzzy sets, IFS [1, 2] have been found to be helpful to deal with imprecise information. There are cases where due to lack of available information, the assessment of membership values is not also always applicable and consequently there remains an indeterministic part on which hesitation remains. Certainly, fuzzy sets theory is not applicable to deal with such problems; rather intuitionistic fuzzy sets theory is more appropriate**.** Atanassov [1] in 1986 introduced the concept of IFS. In the uncertain multi-objective decision-making problems, the degree of satisfiability and the degree of unsatisfiability of each alternative with respect to a set of criteria is often expressed as an IFN. By means of a literature survey, we have found out that many authors [10, 12, 13, 17, 9, 15] have discussed LPP under intuitionistic fuzzy environment. Using Taylor series, Irene Hepzibah et al. [4] proposed a procedure for solving IFMONLPP. In this paper, we propose a new method using the concept of a ranking technique for finding an optimal solution to an

IFMONLPP. The problem is solved by using the Kuhn-Tucker condition and the same is explained with a numerical example.

2. Preliminaries

We need the following basics of IFS, triangular IFN, membership and non-membership function of an intuitionistic fuzzy set/number which can be found in Atanassov [1].

2.1. Definition

Let Y is a nonempty set. An IFN B in Y is given by a set of ordered triples, $\tilde{B}^I = \{ < y, \mu_B(y), \nu_B(y) >; y \in Y\}$ where $\mu_B(y), \nu_B(y) : Y \to [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $y \in Y$ to the set B, which is a subset of Y, and for every element $0 \le \mu_B(y) + \nu_B(y) \le 1$.

2.2. Definition

A triangular IFN $\tilde{B}^I$ is denoted by $\tilde{B}^I = (b_1, b_2, b_3)(b_1', b_2, b_3')$ where $b_1' \le b_1 \le b_2 \le b_3 \le b_3'$ with the following membership function $\mu_{\tilde{B}^I}(y)$ and non-membership function $\nu_{\tilde{B}^I}(y)$ is given as:

$$\mu_{\tilde{B}}(y) = \begin{cases} \dfrac{y - b_1}{b_2 - b_1} & \text{if} \quad b_1 \le y \le b_2 \\ \dfrac{b_3 - y}{b_3 - b_2} & \text{if} \quad b_2 \le y \le b_3 \\ 0 & \text{otherwise} \end{cases} \quad \nu_{\tilde{B}}(y) = \begin{cases} \dfrac{b_2 - y}{b_2 - b_1'} & \text{if} \quad b_1' \le y \le b_2 \\ \dfrac{y - b_2}{b_3' - b_2} & \text{if} \quad b_2 \le y \le b_3' \\ 1 & \text{otherwise} \end{cases}$$

2.3. Definition

Let $\tilde{C}^I = (c_1, c_2, c_3)(c_1', c_2, c_3')$ and $\tilde{D}^I = (d_1, d_2, d_3)(d_1', d_2, d_3')$ be any two triangular IFNs then the arithmetic operations as follows:

(i) $\tilde{C}^I + \tilde{D}^I = (c_1 + d_1, c_2 + d_2, c_3 + d_3)(c_1' + d_1', c_2 + d_2, c_3' + d_3')$

(ii) $\tilde{C}^I - \tilde{D}^I = (c_1 - d_3, c_2 - d_2, c_3 - d_1)(c_1' - d_3', c_2 + d_2, c_3' + d_1')$

(iii) $\tilde{C}^I \otimes \tilde{D}^I = (c_1 d_1, c_2 d_2, c_3 d_3)(c_1' d_1', c_2 d_2, c_3' d_3')$

(iv) $k\tilde{C}^I = (kc_1, kc_2, kc_3)(kc_1', kc_2, kc_3')$ for $k \ge 0$

(v) $k\tilde{C}^I = (kc_3, kc_2, kc_1)(kc_3', kc_2, kc_1')$ for $k < 0$

(vi) $\dfrac{\tilde{C}^I}{\tilde{D}^I} = \left(\dfrac{c_1}{c_3}, \dfrac{c_2}{c_2}, \dfrac{c_3}{c_1}\right)\left(\dfrac{c_1'}{c_3'}, \dfrac{c_2}{c_2}, \dfrac{c_3'}{c_1'}\right)$

2.4. Definition

The ranking of a triangular IFNs $\tilde{B}^I = (b_1, b_2, b_3)(b_1', b_2, b_3')$ is defined as

$$R(\tilde{B}^I) = \frac{1}{3}\left[\frac{(b_3^{'}-b_1^{'})(b_2-2b_3^{'}-2b_1^{'})+(b_3-b_1)(b_1+b_2+b_3)+3(b_3^{'2}-b_1^{'2})}{b_3^{'}-b_1^{'}+b_3-b_1}\right]$$

2.5. Definition

Let $\tilde{C}^I$ and $\tilde{D}^I$ be two triangular IFNs. The ranking of $\tilde{C}^I$ and $\tilde{D}^I$ by the R(.) on E, the set of triangular IFNs is defined as follows:

(i) $R(\tilde{C}^I) > R(\tilde{D}^I)$ if and only if $\tilde{C}^I \succ \tilde{D}^I$

(ii) $R(\tilde{C}^I) < R(\tilde{D}^I)$ if and only if $\tilde{C}^I \prec \tilde{D}^I$

(iii) $R(\tilde{C}^I) = R(\tilde{D}^I)$ if and only if $\tilde{C}^I \approx \tilde{D}^I$

(iv) $R(\tilde{C}^I + \tilde{D}^I) = R(\tilde{C}^I) + R(\tilde{D}^I)$

(v) $R(\tilde{C}^I - \tilde{D}^I) = R(\tilde{C}^I) - R(\tilde{D}^I)$

2.6. Definition

The Kuhn-Tucker (KT) conditions are both necessary and sufficient if the objective is concave and each constraint is linear or each constraint function is concave, that is, the problems called as convex programming problems.

2.6.1 .General formula of Kuhn-Tucker Condition for Minimization case with ≤ type constraints

Minimize g(x)

Subject to

$h_j(x) \le 0$

(i) $\frac{\partial g}{\partial x_i} + \sum_{j=1}^{m} \lambda_j \frac{\partial h_j}{\partial x_i} = 0$

(ii) $\lambda_j h_j = 0,$

(iii) $h_j \le 0,$

(iv) $\lambda_j \ge 0$

2.6.2. General formula of Kuhn-Tucker Condition for Maximization case with ≤ type constraints

Maximize g(x)

Subject to

$h_j(x) \le 0$

(i) $\frac{\partial g}{\partial x_i} + \sum_{j=1}^{m} \lambda_j \frac{\partial h_j}{\partial x_i} = 0$

(ii) $\lambda_j h_j = 0,$

(iii) $h_j \le 0,$

(iv) $\lambda_j \le 0$

3. Intuitionistic Fuzzy Multi-Objective Nonlinear Programming Problem (IFMONLPP)

Now, the mathematical model of an IFMONLPP is given as follows

(P) Maximize (or) Minimize $\tilde{Z}^I = \sum_j \tilde{c}_j^I \tilde{x}_j^{I\alpha_j}$

Subject to the constraints

$$\sum_{j=1}^{n} \tilde{a}_{ij}^I \tilde{x}_j^I (\leq, \geq) \tilde{b}_i^I \ , \ i = 1,2,...,m$$

$$\tilde{x}_j^I \geq 0 \ , \ j = 1,2,...,n$$

where $\tilde{A}^I = \tilde{a}_{ij}^I, \tilde{c}^I, \tilde{b}^I, \tilde{x}^I$ are triangular IFNs.

The proposed algorithm for IFMONLPP proceeds as follows:

Step 1: Compute the ranking index for each parameter of the given problem (P) by the definition 2.4.

Step 2: Replace the intuitionistic fuzzy parameters by their respective ranking indices obtained from Step 1.

Step 3: Solve the reduced problem obtained from Step 2 using the Kuhn-Tucker condition to find the optimal solution.

The solution procedure for obtaining an optimal solution to IFMONLPP using the proposed algorithm is illustrated by the following example.

3.1. Example

Consider the following IFMONLPP problem

$$\text{Minimum } \tilde{z} = (26,28,30)(24,28,32)\tilde{x}_1 + (10,12,14)(8,12,16)\tilde{x}_2^2 \quad (1)$$

$$\text{Maximum } \tilde{z} = (30,32,34)(28,32,36)\tilde{x}_1 + (20,24,28)(18,24,30)\tilde{x}_2^2 \quad (2)$$

Subject to the constraints

$$\begin{aligned} &(6,8,10)(4,8,12)\tilde{x}_1 + (8,12,16)(6,12,18)\tilde{x}_2 \leq (38,40,42)(36,40,44) \\ &(4,8,12)(2,8,14)\tilde{x}_1 + (14,16,18)(12,16,20)\tilde{x}_2 \leq (28,32,36)(24,32,40) \end{aligned} \quad (3)$$

Now using step1 and step 2 the ranking indices for the parameters corresponding to the given IFMONLPP is given below

$$Min\, z = 28x_1 + 12x_2^2 \quad (4)$$

$$Max\, z = 32x_1 + 24x_2^2 \quad (5)$$

Subject to the constraints

$$\begin{aligned} &8x_1 + 12x_2 \leq 40 \\ &8x_1 + 16x_2 \leq 32 \\ &x_1, x_2 \geq 0 \end{aligned} \quad (6)$$

Now, by solving the reduced problem (4) with (6) and (5) with (6) using Kuhn-Tucker conditions (2.6.1 and 2.6.2), we obtain the following optimal solutions:

$x_1 = 0.4, x_2 = 1.8$ and Minimize Z =48

$x_1 = 1, x_2 = 2.67$ and Maximize Z =114

4. Conclusion

In this paper, we introduced a new method for solving IFMONLPP. In the proposed algorithm, IFMONLPP is reduced to MONLPP using the ranking function. The problem is solved by using the Kuhn-Tucker condition and the same is explained with a numerical example.

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