The Effect of Wall Properties on the Convective Peristaltic Transport of a Conducting Bingham Fluid through Porous Medium

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Abstract

Objective: In the present paper the influence of heat transfer, wall slip conditions, and wall properties on the peristaltic transport of an incompressible conducting Bingham fluid in a non-uniform porous channel is studied. **Methods/Statistical Analysis:** Exact analytical solutions have been obtained for the axial velocity and the temperature by using the assumptions of long wavelength and low-Reynolds number. **Findings:** The effects of the essential parameters on the velocity and temperature distributions are demonstrated through graphs. It is noticed that the presence of porous medium reduces the velocity and temperature in the peristaltic channel. Further, the size of the trapped bolus gets reduced due to the presence of porous medium/magnetic field whereas opposite behaviour is noticed with the increasing slip at the walls. **Application/Improvements:** The results reveal that the presence of magnetic field/porous medium has remarkable effect on the peristaltic transport of yield stress fluids (such as blood) which may lead to possible technological applications in designing bio- medical instruments.

Keywords: Conducting Bingham Fluid, Convective Peristaltic Transport, Porous Medium, Trapping Phenomena, Wall Properties

1. Introduction

Peristalsis is a well known mechanism of fluid transport which occurs due to the progressive wave of area contraction or expansion along the distensible tube. Peristaltic flows have attracted several researchers because of extensive applications in physiology and industry. In particular, the existence of such flows are quite prevalent in physiological organs such as urine transport from kidney to urinary bladder, the movement of ovum in the fallopian tubes, the swallowing of food through oesophagus and many others. The mechanism of peristalsis is also used in blood pumps, heart -lung machine, sanitary fluid transport and transport of corrosive fluids etc. Due to the aforesaid applications, various studies on the peristaltic transport of Newtonian/non-Newtonian fluids have been reported in the literature^{1–10}.

Magneto hydrodynamics flows have many applications in the biomedical sciences such as cancer tumour treatment, targeted transport of drug using magnetic particles as drug carrier, bleeding reduction during surgeries, hyperthermia, design of heat exchangers, radar systems, power generation development of magnetic devices etc. Therefore several researchers having in mind such importance broadly discussed the peristaltic transport with magnetic field effects^{11–15.}

Several authors have considerable interest on the flow through porous media with heat transfer $^{16-25}$ due to its wide

range of claims in fluid mechanics. In²⁶ have investigated the peristaltic flow of a Maxwell fluid including the Hall Effect through porous medium. In²⁷ studied the influence of heat and mass transfer on MHD peristaltic flow through porous space with compliant walls. In²⁸⁻³¹ examined the peristaltic flow and heat transfer with porous medium. The effect of slip and heat transfer on peristaltic transport of a Jeffrey fluid in a vertical asymmetric porous channel is discussed by³². A few theoretical models to describe the peristaltic transport with wall properties are available in literature³³⁻³⁹.

In the present paper the effects of both wall slip conditions and wall properties on peristaltic flow of a conducting Bingham fluid in a non-uniform porous channel with heat transfer have been investigated under the assumptions of long wavelength and low-Reynolds number. The expressions for velocity, temperature, and stream function and heat transfer coefficient are obtained. The effects of various important parameters on velocity and temperature are explained through graphs. The trapping phenomenon is discussed in detail

2. Mathematical Formulation

We consider the peristaltic flow of a conducting Bingham fluid through a two-dimensional porous channel of uniform thickness. The flow is considered to be induced by sinusoidal wave trains propagating along the channel walls with a constant speed C. The channel walls are supposed to be flexible and are taken as stretched membranes⁴⁰, such that

$$\overline{\eta}(\overline{x},\overline{t}) = d(x) + a \sin \frac{2\pi}{\lambda} (\overline{x} - c\overline{t}) \text{ where } d(x) = d + \overline{m}x, \ \overline{m} <<1$$
(1)



Geometry of the problem

 $\frac{\partial \overline{u}}{\partial \overline{u}} \frac{\partial \overline{v}}{\partial \overline{v}e} = 0$ (2)

$$\rho \left[\frac{\partial}{\partial t} + \overline{u} \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right] \overline{u} = -\frac{\partial \overline{p}}{\partial \overline{x}} + \mu \frac{\partial^2 \overline{u}}{\partial \overline{x}^2} - \frac{\partial}{\partial \overline{y}} (\tau_0 - \mu \frac{\partial \overline{u}}{\partial \overline{y}}) - \sigma_0 B_0^2 \overline{u} - \frac{\mu}{k} \overline{u}$$
(3)

$$\rho \left[\frac{\partial}{\partial t} + \overline{u} \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right] \overline{v} = -\frac{\partial \overline{p}}{\partial \overline{y}} + \mu \left(\frac{\partial^2 \overline{v}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{v}}{\partial \overline{y}^2} \right) - \frac{\mu}{k} \overline{v}$$
(4)
$$\xi \left[\frac{\partial}{\partial t} + \overline{u} \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right] T = \frac{k_0}{\rho} (\frac{\partial^2}{\partial \overline{x}^2} + \frac{\partial^2}{\partial \overline{y}^2}) T + 2v \left[\left(\frac{\partial \overline{u}}{\partial \overline{x}} \right)^2 + \left(\frac{\partial \overline{v}}{\partial \overline{y}} \right)^2 \right] + \left(\frac{\partial \overline{v}}{\partial \overline{x}} + \frac{\partial \overline{u}}{\partial \overline{y}} \right)^2$$

where $u, v, \rho, \mu, p, d, a, \lambda, c, m, \xi, v, k_0, k, T, \sigma_0, B_0$ and τ_0 are the axial velocity, transverse velocity, density, viscosity of the fluid, pressure, mean width of the channel, amplitude, wavelength, wave speed, dimensional nonuniformity of the channel, specific heat at constant volume, kinematic viscosity, thermal conductivity, permeability, temperature, electrical conductivity, applied magnetic field and yield stress.

The equation of motion of the stretched membrane is given by

$$L^*(\overline{\eta}) = \overline{p} - \overline{p_0} \tag{6}$$

where L^* is an operator, which is used to describe the motion of flexible wall with viscosity damping forces such that

$$L^* = -\tau \frac{\partial^2}{\partial \overline{x}^2} + m_1 \frac{\partial^2}{\partial \overline{t}^2} + C \frac{\partial}{\partial \overline{t}}$$
(7)

where τ is the elastic tension in the membrane, m_1 is the mass per unit area, *C* is the coefficient of viscous damping forces, p_0 is the pressure on the outside surface of the wall due to the tension in the muscles and *h* is the slip parameter. We assumed $p_0 = 0$.

Continuity of stress at $y = \pm \eta$ and using Equation (2), we obtain

$$\frac{\partial}{\partial \overline{x}}L^{*}(\overline{\eta}) = \frac{\partial\overline{p}}{\partial \overline{x}} = \mu \frac{\partial^{2}\overline{u}}{\partial \overline{x}^{2}} - \frac{\partial}{\partial \overline{y}}(\tau_{0} - \mu \frac{\partial\overline{u}}{\partial \overline{y}}) - \sigma_{0}B_{0}^{2}\overline{u} - \frac{\mu}{k_{1}}\overline{u} - \rho \left[\frac{\partial}{\partial \overline{t}} + \overline{u}\frac{\partial}{\partial \overline{x}} + \overline{v}\frac{\partial}{\partial \overline{y}}\right]\overline{u}$$

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(9)

$$\frac{\partial T}{\partial y} = 0 \quad \text{on } y = y_0, T = T_1 \quad \text{on } y = \eta \tag{10}$$

Introducing ψ such that

$$u = \frac{\partial \psi}{\partial y}$$
 and $v = -\frac{\partial \psi}{\partial x}$

and the following non-dimensional parameters are given by

$$x = \frac{\overline{x}}{\lambda}, \quad y = \frac{\overline{y}}{d}, \quad \psi = \frac{\overline{\psi}}{cd}, \quad p = \frac{d^2\overline{p}}{\mu c\lambda}, \quad \tau_0 = \frac{d\overline{\tau_0}}{\mu c}, \quad \tau_{xy} = \frac{d\overline{\tau_{xy}}}{\mu c}, \quad t = \frac{c\overline{t}}{\lambda}, \quad m = \frac{\lambda \overline{m}}{d}, \quad \delta = \frac{d}{\lambda}, \quad \varepsilon = \frac{a}{d}$$

$$R = \frac{\rho cd}{\mu}, \quad \theta = \frac{(T - T_0)}{(T_1 - T_0)}, \quad M = \sqrt{\frac{\sigma_0}{\mu}} B_0, \quad \sigma = \frac{d}{\sqrt{k}} \Pr = \frac{\rho v \xi}{k_0}, \quad Ec = \frac{c^2}{\xi (T_1 - T_0)},$$

$$E_1 = \frac{-\tau d^3}{\lambda^3 \mu c}, \quad E_2 = \frac{m_c d^3}{\lambda^3 \mu}, \quad E_3 = \frac{cd^3}{\lambda^2 \mu}, \quad \beta = \frac{h}{d}, \quad \eta = \frac{\overline{\eta}}{d} = 1 + mx + \varepsilon \sin 2\pi (x - t),$$

$$(11)$$

where R is the Reynolds number, δ and ε are the non-dimensional geometric parameters, M is the Hartmann number, Pr is the Prandtl number, Ec is the Eckert number, E_1, E_2 and E_3 are the dimensionless elasticity parameters, m is the non-uniform parameter σ is the permeability parameter and β is the Knudsen number (slip parameter).

3. Solution of the Problem

Using the above dimensionless quantities and applying the long wavelength and low Reynolds number approximation, the governing Equations (3) - (10) reduce to

$$0 = -\frac{\partial p}{\partial x} - \frac{\partial}{\partial y} \left(\tau_0 - \frac{\partial^2 \psi}{\partial y^2} \right) - \left(M^2 + \sigma^2 \right) \frac{\partial \psi}{\partial y}$$
(12)

$$0 = \frac{\partial p}{\partial y} \tag{13}$$

$$0 = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} + E(\frac{\partial^2 \psi}{\partial y^2})^2$$
(14)

$$\frac{\partial}{\partial y}(-\tau_0 + \frac{\partial^2 \psi}{\partial y^2}) - \left(M^2 + \sigma^2\right)\frac{\partial \psi}{\partial y} = \left[E_1\frac{\partial^3 \eta}{\partial x^3} + E_2\frac{\partial^3 \eta}{\partial x \partial t^2} + E_3\frac{\partial^2 \eta}{\partial x \partial t}\right]$$
(15)

$$\frac{\partial \psi}{\partial y} = -\beta \frac{\partial^2 \psi}{\partial y^2} \quad \text{at } y = \eta = [1 + m x + \varepsilon \sin 2\pi (x - t)]$$
(16)

Further, it is assumed that [30] $\psi_p(0) = 0,$ $\psi_{yy}(0) = \tau_0 \text{ at } y = 0$ $\psi = \psi_p \text{ at } y = y_0$ (17) ≥ 0

$$\frac{\partial \theta}{\partial y} = 0 \text{ for } 0 \le y \le y_0, \theta = 1 \text{ at } y = \eta$$
 (18)

By differentiating Equation (12) with respect to $y_{(19)}$ obtain

$$\frac{\partial^2}{\partial y^2} \left(-\tau_0 + \frac{\partial^2 \psi}{\partial y^2} \right) - \left(M^2 + \sigma^2 \right) \frac{\partial^2 \psi}{\partial y^2} = 0$$

By solving Equation (19) with boundary Conditions (15), (16) and (17) we obtain the stream function in the plug flow region as

$$\psi_{p} = \left[\frac{\tau_{0}}{\sqrt{M^{2} + \sigma^{2}}} \left(\sinh \sqrt{M^{2} + \sigma^{2}} y_{0} - L_{1} \cosh \sqrt{M^{2} + \sigma^{2}} y_{0} \right) - \frac{A}{\left(M^{2} + \sigma^{2}\right)} \right] y$$
(20)

and corresponding plug flow velocity is given by

$$u_{p} = \frac{\tau_{0}}{\sqrt{M^{2} + \sigma^{2}}} \left(\sinh \sqrt{M^{2} + \sigma^{2}} y_{0} - L_{1} \cosh \sqrt{M^{2} + \sigma^{2}} y_{0} \right) - \frac{A}{\left(M^{2} + \sigma^{2}\right)}$$
(21)

where
$$y_0 = \frac{1}{\sqrt{M^2 + \sigma^2}} \tanh^{-1}(1/L_1)$$
,
 $4 = -8\varepsilon \pi^3 \left[(E_1 + E_2)\cos 2\pi (x - t) - \frac{E_3}{2\pi} \sin 2\pi (x - t) \right]$

and

E.

$$L_{1} = \frac{\sinh\sqrt{M^{2} + \sigma^{2}\eta} + \beta\sqrt{M^{2} + \sigma^{2}}\cosh\sqrt{M^{2} + \sigma^{2}\eta} - \frac{A}{\left(M^{2} + \sigma^{2}\right)}}{\cosh\sqrt{M^{2} + \sigma^{2}\eta} + \beta\sqrt{M^{2} + \sigma^{2}}\sinh\sqrt{M^{2} + \sigma^{2}\eta}}$$

and the stream function in the non-plug flow region as

$$\psi = \frac{\tau_0}{\left(M^2 + \sigma^2\right)} \left(\cosh\sqrt{M^2 + \sigma^2}y - L_1 \sinh\sqrt{M^2 + \sigma^2}y\right) - \frac{\left(Ay + B + \tau_0\right)}{\left(M^2 + \sigma^2\right)}$$
(22)

where

$$B = \tau_0 \left((My_0 L_1 - 1) \cosh My_0 - (My_0 - L_1) \sinh My_0 - 1 \right)$$

(24)

The corresponding velocity in the non-plug flow region is given by

$$u = \frac{\tau_0}{\sqrt{M^2 + \sigma^2}} \left(\sinh \sqrt{M^2 + \sigma^2} y - L_1 \cosh \sqrt{M^2 + \sigma^2} y \right) - \frac{A}{\left(M^2 + \sigma^2\right)}$$
(23)

Using the Equations (22) and (18) in Equation (14) we obtain the temperature as

$$\theta = -Br\tau_0^2 \left[\frac{Cosh2\sqrt{M^2 + \sigma^2}y}{4\sqrt{M^2 + \sigma^2}} (1 + L_1^2) + \frac{y^2}{4} (1 - L_1^2) - L_1 \frac{Sinh2\sqrt{M^2 + \sigma^2}y}{4\sqrt{M^2 + \sigma^2}} \right] + C_1 y + C_2$$

where

$$C_{1} = Br\tau_{0}^{2} \left[\frac{Sinh2\sqrt{M^{2} + \sigma^{2}}y_{0}}{4\sqrt{M^{2} + \sigma^{2}}} (1 + L_{1}^{2}) + \frac{y_{0}}{2} (1 - L_{1}^{2}) - L_{1} \frac{Cosh2\sqrt{M^{2} + \sigma^{2}}y_{0}}{2\sqrt{M^{2} + \sigma^{2}}} \right]$$

$$C_{2} = 1 + Br\tau_{0}^{2} \left[\frac{Cosh2\sqrt{M^{2} + \sigma^{2}\eta}}{8(M^{2} + \sigma^{2})} (1 + L_{1}^{2}) + \frac{\eta^{2}}{4} (1 - L_{1}^{2}) - L_{1} \frac{Sinh2\sqrt{M^{2} + \sigma^{2}\eta}}{4(M^{2} + \sigma^{2})} \right] - C_{1}\eta^{2}$$

 $Br = Ec \operatorname{Pr}$ is the Brinkman number

when the permeability parameter σ tends to zero the Results (20) - (24) reduce to the corresponding ones of²⁹.

The skin friction at the wall is given by

$$\tau_{xy} = \left(\frac{du}{dy}\right)_{at \, y = \eta} \tag{25}$$

The nusselt number at the wall is given by

$$Nu = -\left(\frac{d\theta}{dy}\right)_{at \, y = \eta} \tag{26}$$



Fig.1 Velocity distribution for fixed values: $x=0.2, t=0.1, m=0.1, \epsilon=0.3, \beta=0.1, \tau_0=0.4, M=1.5, \sigma=1, E_1=0.4, E_2=0.3, E_3=0.2$. Figure 1. Velocity distribution for fixed values:



Fig.2 Temperature distribution for fixed values: x = 0.2, t = 0.1, Br = 2, $\tau_0 = 0.2$, $\varepsilon = 0.3$, $\beta = 0.1$, M = 1.5, m = 0.1, $\sigma = 1$, $E_1 = 0.4$, $E_2 = 0.3$, $E_3 = 0.2$

Figure 2. Temperature distribution for fixed values:

Table 1. Velocity and Temperature for fixed $x = 0.2, t = 1, m = 0.1, \beta = 0.1, \tau_0 = 0.4, \varepsilon = 0.3$,

 $E_1 = 0.4, E_2 = 0.3, E_3 = 0.2, y = 1$

	Present study with $M = 1, \sigma = 1$	In ²⁹ with M = 1, σ = 0
Velocity	16.3867	27.8258
Temperature	1.4785	3.0151



Fig. 3 Variation of Nusselt number for fixed values $x = 0.2, t = 0.1, m = 0.1, M = 1.5, \sigma = 1, \tau_0 = 0.2, \beta = 0.1, Br = 2, \varepsilon = 0.3, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1$

Figure 3. Variation of Nusselt number for fixed values.

4. Results and Discussions

Equation (23) gives the expression for velocity as a function of y. Velocity profiles are plotted from Figure 1 to study the effects of different parameters such as slip parameter β , non-uniform parameter m, magnetic parameter M, permeability parameter σ and yield stress τ_0 on the velocity distribution. Figure 1(a) is plotted for different values of slip parameter β . It is observed that the velocity profiles are parabolic and the velocity increases with increasing β . Figure 1(b) describes that the velocity for a divergent channel (m > 0) is greater compared with uniform channel (m < 0). From Figures 1(c), 1(d), and 1 (e) we observe that the velocity decreases

with increasing yield stress τ_0 , magnetic parameter Mand permeability parameter σ . Figure 1(f) depicts that the velocity rises with increasing E_1 and E_2 whereas it falls with increasing E_3 .

The expression for the temperature in terms of y is given by Equation (24). The effect of heat transfer on peristalsis is described in Figure 2. Figure 2(a) 2(b) are drawn to study the variation of temperature for different values of Br and τ_0 . It is observed that that the temperature field increases with the increase of Brinkman number Br and yield stress τ_0 . From Figure 2(c) we noticed that the temperature is higher for diverging channel (m > 0) compared with uniform (m = 0) and convergent (m < 0) channels. Figure 2(d) and 2(e) shows that the temperature reduces with increasing magnetic parameter M and permeability parameter σ . Figure 2(f) depicts that the temperature enhances with increasing E_1 and E_2 whereas it decreases with increasing E_3 . The variation in nusselt number Nu for various values of the interesting parameters can be analyzed through Figure 2. It is noticed that due to peristalsis, the heat transfer coefficient is in oscillatory behaviour. The absolute value of heat transfer coefficient (Nu) rises with increase of Br and τ_0 while it reduces with increasing β , M and σ .

In order to find the effect of permeability on the flow, we have presented the values of velocity and temperature of the present study and compared with free flow case (absence of porous material) in Table 1. It is clear that the porous medium reduces both the velocity and temperature in the peristaltic channel.

4.1 Trapping Phenomenon

Trapping is an important phenomenon which refers to closed circulating streamlines that exist at every high flow rates and when occlusions are very large. Streamlines are plotted to study the effect of slip parameter β magnetic parameter M and permeability parameter σ in Figures 4, 5 and 6. We observe that the size of trapped bolus enhances with increasing slip parameter where as it decreases with increasing magnetic parameter M and permeability parameter σ .

5. Conclusions

In this study, we examine the effects of wall slip, wall properties, yield stress and heat transfer on the peristaltic flow of conducting Bingham fluid in a non-uniform



Fig. 4Stream lines for x = 0.2, t=1, m=0.1, M=5, σ = 1, τ_0 = 0.8, ε = 0.3, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1(a) β = 0.05(b) β = 0.1(c) β = 0.15(b) β = 0.1(c) β



Fig.5Stream lines for $x = 0.2, t = 1, m = 0.1, \beta = 0.2, \sigma = 1, \tau_0 = 0.8, \varepsilon = 0.3, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1(a)M = 5(b)M = 5.5(c)M = 6$ Figure 5. Stream lines for.

Figure 4. Stream lines for.



Fig. 6Stream lines for x = 0.2,t=1,m=0.1, $\beta = 0.2,M=6,\tau_0 = 0.8, \varepsilon = 0.3, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1(a)\sigma = 1(b)\sigma = 1.5(c)\sigma = 1.8$

Figure 5. Stream lines for.

porous channel under long wavelength and low-Reynolds number approximations. The slip phenomenon has been attributed as the cause of spurt in polymers. In view of this, the study of slip on certain non-Newtonian porous flows finds applications in chemical engineering and medicine. Some of the interesting observations are presented below:

- The velocity increases with increasing slip parameter β where as it decreases with increasing yield stress τ_0 , magnetic parameter M and permeability parameter σ .
- The temperature field decreases with the increase of magnetic parameter M or permeability parameter σ and it increases with increasing Brinkman number Br.
- The velocity and temperature increase with increasing E_1 and E_2 while they decrease with increasing E_3 .
- The absolute value of nusselt number enhances with increase of yield stress while it decreases with increasing permeability parameter.
- The size of trapped bolus increases with increasing slip parameter where as it decreases with increasing permeability parameter.

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7. References

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