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Thermal Radiation Effects on MHD Boundary Layer Slip Flow Past a permeable Exponential Stretching Sheet in the Presence of Joule Heating and Viscous Dissipation

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ABSTRACT

An analysis of the thermal radiation effects on MHD boundary layer flow past a permeable exponential stretching surface in the presence of Joule heating and viscous dissipation is presented. Velocity and thermal slips are considered instead of no-slip conditions at the boundary. Stretching velocity and wall temperature are assumed to have specific exponential function forms. The governing system of partial differential equations is transformed into a system of ordinary differential equations using similarity transformations and then solved numerically using the Runge-Kutta fourth order method along with shooting technique. The effects of the various parameters on the velocity, shear stress, temperature and temperature gradient profiles are illustrated graphically and discussed in detail. The influence of the slip parameters causes significant fluctuations in velocity of the flow field. Viscous dissipation characterized by Eckert number enhances the temperature of the fluid, as the heat gets transferred from the sheet to the fluid.

Keywords: MHD; Thermal radiation; Viscous dissipation; Boundary layer flow; Joule heating; Exponentially stretching surface.

NOMENCLATURE

B	magnetic induction	$T_w(x)$	surface temperature
D	thermal slip factor	$T_\infty(x)$	variable ambient temperature
Ec	Eckert Number	$U(x)$	stretching velocity
k	thermal conductivity of the fluid	U_0	reference velocity
k^*	mean absorption coefficient	V_0	strength of suction
L	characteristic length	u, v	velocities along x and y directions
M	magnetic parameter		
N	velocity slip factor	ν	kinematic viscosity
Nr	radiation parameter	ρ	fluid density
Pr	Prandtl number	μ	coefficient of fluid viscosity
q_r	radiative heat flux	σ^*	stephen Boltzmann constant
S	suction parameter	η	similarity variable
T	fluid temperature.	λ	velocity slip parameter
T_0	reference temperature	δ	thermal slip parameter

1. INTRODUCTION

The problem of viscous flow and heat transfer over a stretching sheet has important industrial applications, for example, in metallurgical processes, such as drawing of continuous filaments through quiescent fluids, annealing and tinning of copper wires, glass blowing, manufacturing of plastic and rubber sheets, crystal growing, and continuous cooling and fiber spinning, in addition to wide-ranging applications in many engineering processes, such as polymer extrusion, wire drawing, continuous casting, manufacturing of foods and paper, glass fiber production, stretching of plastic films, and many others. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The final product with the desired characteristics strictly depends upon the stretching rate, the rate of cooling in the process, and the process of stretching. Crane (1970) investigated the flow caused by the stretching of a sheet. Several researchers viz. Gupta and Gupta (1977), Dutta *et al*. (1985), Chen and Char (1988) extended the work of Crane (1970) by including the effects of heat and mass transfer under different situations. Gupta and Gupta (1977) reported that stretching surface is not necessarily continuous. Most of the earlier investigations deal with the study of boundary layer flow past a stretching surface in which the velocity of the stretching surface is assumed linearly proportional to the distance from the fixed origin. Sheikholeslami *et al.* (2015, January) analyzed the Lattice Boltzmann simulation of magnetohydrodynamic natural convection heat transfer of Al_2O_3–water nanofluid in a horizontal cylindrical enclosure with an inner triangular cylinder.Sakiadis (1961) investigated the boundary-layer flow of a viscous fluid past a moving solid surface. Recently, various aspects of such problem have been investigated either analytically or numerically by many authors such as Xu and Liao (2005), Cortell (2005;2006), Hayat *et al*. (2006), Ariel *et al*. (2006; 2006) and Hayat and Sajid (2007), Ariel (2007), Chamkha and Aly (2011).

However, all these studies are restricted to linear stretching of the sheet. It is worth mentioning that the stretching is not necessarily linear. In view of this, Ali (1995) has investigated the thermal boundary layer. The heat and mass transfer on boundary layer flow due to an exponentially continuous stretching sheet was considered by Magyari and Keller (2000). Elbashbeshy (2001) added a new dimension to the study of Ali (1995) by considering exponentially continuous stretching surface. Sharma *et al.*(2014) studied the boundary layer flow and heat transfer over a permeable exponentially shrinking sheet in the presence of thermal radiation and partial slip. Vajravelu (2002), Vajravelu and Cannon (2006) also considered the flow over a nonlinear stretching sheet. The viscous-elastic boundary layer flow and heat transfer due to an exponentially stretching sheet was investigated by Khan (2006) and Sanjayanand and Khan (2006). Akyildiz *et al*. (2010) assumed a power law stretching velocity. Following the modification provided by Van Gorder and Vajravelu (2010), an analysis can be presented for any values of the power law exponent. Recently, El-Aziz (2009), Ishak (2011) described the flow and heat transfer past an exponentially stretching sheet.

In physics and engineering, the radiative effects have important applications. In space technology and high temperature processes, the radiation heat transfer effects on different flows are very important. But about the effects of radiation on the boundary layer, very little is known. Sajid and Hayat (2008) considered the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet by solving the problem analytically via homotopy analysis method (HAM). Bidin and Nazar (2009) studied the boundary layer flow over an exponential stretching sheet with thermal radiation, using Keller-box method. Sheikholeslami *et al.* (2015, Jaunary) analyzed the effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model. Loganthan and Vimala (2015) investigated the MHD flow of nanofluids over an exponentially stretching sheet embedded in a stratified medium with suction and radiation effects. Nadeem *et al.* (2011) studied the Effects of thermal radiation on the boundary layer flow of a Jeffrey fluid over an exponentially stretching surface by solving the problem analytically via homotopy analysis method (HAM). Sheikholeslami and Domiri Ganji (2015, Jaunary). Entropy generation of nanofluid in presence of magnetic field using Lattice Boltzmann Method.

There has been a renewed interest is studying magnetohydrodynamic flows and heat transfer due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems involving electrically conductive flows. In addition, this type of flow finds applications in many engineering problems such as MHD generators, Plasma studies, Nuclear reactors, and Geothermal energy extractions. Raptis *et al*. (2004) studied the effect of thermal radiation on the magnetohydrodynamic flow of a viscous fluid past semi-infinite stationary plate and Hayat *et al*. (2007) extended the analysis for a second grade fluid. Later Aliakbar *et al*. (2008) analyzed the influence of thermal radiation on MHD flow of Maxwellian fluids above stretching sheets. Sheikholeslami *et al.* (2014, October) examined the Ferrofluid flow and heat transfer in a semi annulus enclosure in the presence of magnetic source considering thermal radiation. Al-Odat *et al*. (2006) analyzed the thermal boundary layer on an exponentially stretching continuous surface in the presence of magnetic field effect.

Viscous dissipation in the natural convection flow, characterized by the Eckert number, plays an important role, when the flow field is of extreme size or in high gravitational field. The effect of Joule heating is usually characterized by the product of the Eckert number and the magnetic parameter, and it is important in nuclear engineering (Alim *et al*., (2007)). Gebhart and Mollendorf (1969) considered the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar (1972) analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate. Israel-Cookey *et al*. (2003) investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Aissa and Mohammadein (2005) analyzed the effects of the magnetic parameter, Joule heating, viscous dissipation and heat generation on the MHD micropolar fluids that past a stretching sheet. Duwairi (2005) presented the effects of Joule heating and viscous dissipation on the forced convection flow in the presence of thermal radiation. Partha *et al.* (2005) studied the effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. Cortell (2008) considered the Effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. Anki Reddy and Bhaskar Reddy (2011) studied the thermal radiation and viscous dissipation effects on hydro-magnetic flow due to an exponentially stretching sheet. Recently Gnaneswara Reddy (2014) investigated the effects of thermophoresis, viscous dissipation and joule heating on steady MHD flow over an inclined radiative isothermal permeable surface with variable thermal conductivity. Bhuiyan *et al.* (2014) presented Joule heating effects on MHD natural convection flows in presence of pressure stress work and viscous dissipation from a horizontal circular Cylinder.

All of the previously mentioned studies assumed no slip boundary conditions. The non-adherence of the fluid to a solid boundary is known as velocity slip. It is a phenomenon that has been observed under certain circumstances (1998). The slip flow problem of laminar boundary layer is of considerable practical interest. Microchannels which are at the forefront of today's turbomachinery technologies are widely being considered for cooling of electronic devices, micro heat exchanger systems, etc. If the characteristic size of the flow system is small or the flow pressure is very low, slip flow happens. If the characteristic size of the flow system tends to the molecular mean free path, continuum physics is no longer suitable. In no-slip-flow, as a requirement of continuum physics, the flow velocity is zero at a solid–fluid interface and the fluid temperature instantly closest to the solid walls is equal to that of the solid walls. The fluids exhibiting boundary slip find applications in technology such as in the polishing of artificial heart valves and internal cavities. Recently, taking slip flow condition at the boundary, many researchers (2006; 2002; 2002; 2008; 2009) investigated the different flow problems over a stretching sheet. Off late, Chauhan and Olkha (2011) investigated the slip flow of second grade fluid past a stretching sheet in a porous medium by considering the power-law surface temperature/heat flux. Swati Mukhopadhyay and Gorla (2009) studied the effects of partial slip on boundary layer flow past a permeable exponential stretching sheet in presence of thermal radiation. Mukhopadhyay *et al*. (2012) analyzed the Lie group analysis of MHD boundary layer slip flow past a heated stretching sheet in presence of heat source/sink. Mukhopadhyay and Andersson (2009) studied the Effects of slip and heat transfer analysis of flow over an unsteady stretching surface. Saghafian *et al.* (2015) presented a numerical study on slip flow heat transfer in micro-poiseuille flow using perturbation method. Malvandi *et al.* (2015). Studied the boundary layer slip flow and heat transfer of nanofluid induced by a permeable stretching sheet with convective boundary condition.

In this paper an attempt is made to investigate the thermal radiation effects on MHD boundary layer slip flow past a permeable exponential stretching surface in the presence of Joule heating and viscous dissipation. The governing boundary layer equations are solved using Runge-Kutta fourth order technique along with shooting method.

2. MATHEMATICAL ANALYSIS

A two dimensional boundary layer flow of a viscous incompressible electrically conducting and radiating fluid over a porous stretching surface is considered. The *x*-axis is taken along the stretching surface in the direction of the motion and *y*-axis perpendicular to it. The flow is confined to y > 0. Two equal and opposite forces are applied along the *x*-axis so that the wall is stretched keeping the origin fixed. The fluid is assumed to be gray, absorbing emitting but non scattering. A uniform magnetic fluid is applied in the direction perpendicular to be stretching surface. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible. Then under the above assumptions, in the absence of an input electric field, the governing boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\sigma B^2}{\rho C_p}u^2 +\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

where, *u* and *v* are the velocity components in the *x* and *y* directions, respectively, ν is the kinematic viscosity, σ is the electric conductivity, *B* is the magnetic induction, ρ is the fluid density, *T* is the temperature of the fluid, *k* is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure, μ is the coefficient of fluid viscosity, q_r is the radiative heat flux. The second, third and fourth terms on the right-hand side of the energy Eq. (3) signifies the Joule heating, viscous dissipation and radiation.

The boundary conditions for the velocity and temperature fields are

$$\left.\begin{aligned} u &= U + N\nu\frac{\partial u}{\partial y}, \mathrm{v} = -V(x), \\ T &= T_w + D\frac{\partial T}{\partial y} \end{aligned}\right\} \quad at \quad y = 0,$$
$$u \to 0, \quad T \to 0 \quad as \quad y \to \infty. \tag{4}$$

where $U = U_0 e^{X/L}$ is the stretching velocity, $T_w = T_0 e^{X/L}$ is the temperature at the sheet, U_0, T_0 are the reference velocity and temperature respectively, $N = N_1 e^{-X/2L}$ is the velocity slip factor which changes with *x*, N_1 is the initial value of velocity slip factor and $D = D_1 e^{-X/2L}$ is the thermal slip factor which changes with *x*, D_1 is the initial value of thermal slip factor. The no slip case is recovered for $N = D = 0$. $V(x) > 0$ is the velocity of suction and $V(x) < 0$ is the velocity of blowing, $V(x) = V_0 e^{X/2L}$, a special type of velocity at the wall is considered. V_0 is the initial strength of suction.

By employing Rosseland approximation (Sajid and Hayat (2008)), the radiative heat flux q_r is given by

$$q_r = -\frac{4}{3}\frac{\sigma^*}{k^*}\frac{\partial T^4}{\partial \bar{y}} \tag{5}$$

where σ* is the Stefan-Boltzmann constant and *k** is the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature differences within the flow field are sufficiently small, then Eq. (5) can be linearized by expanding T^4 into the Taylor series about T_∞, which after neglecting higher-order terms takes the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

In view of Eqs. (5) and (6), Eq. (3) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\left(1 + \frac{16\sigma^* T_\infty^3}{3K^* k}\right)\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B^2}{\rho C_p}u^2 + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{7}$$

To obtain similarity solutions, it is assumed that the magnetic field $B(x)$ is of the form

$$B = B_0 e^{x/2L} \tag{8}$$

where B_0 is the constant magnetic field.
In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced

$$u = U_0 e^{x/L} f'(\eta), v = -\sqrt{\frac{\nu U_0}{2L}} e^{x/2L}\{f(\eta) + \eta f'(\eta)\},$$
$$T = T_0 e^{x/2L}\theta(\eta), \eta = \sqrt{\frac{U_0}{2\nu L}} e^{x/2L} y, \quad M = \frac{2\sigma B_0^2 L}{\rho U_0},$$
$$\Pr = \frac{k}{\nu\rho Cp}, \quad Nr = \frac{4\sigma T_\infty^3}{kk^*}, \quad Ec = \frac{U_0^2}{T_0 Cp}. \tag{9}$$

In view of Eqs. (8) and (9), the governing Eqs. (2) and (7) reduce to the dimensionless form

$$f''' + ff'' - 2(f')^2 - Mf' = 0 \tag{10}$$

$$\left(1 + \frac{4}{3}Nr\right)\theta'' + \Pr(f\theta' - f'\theta) + \Pr Ec\, e^{3X/2}\left\{M(f')^2 + (f'')^2\right\} = 0 \tag{11}$$

The corresponding boundary conditions are

$$f(0) = S, f'(0) = 1 + \lambda f''(0), \theta(0) = 1 + \delta\theta'(0),$$
$$f'(\eta) \to 0 \quad \theta(\eta) \to 0 \quad as \quad \eta \to \infty \tag{12}$$

where primes denotes the ordinary differentiation with respect to *η*, *M* is the magnetic parameter, *L* is the characteristic length of the plate, *X* is the *X*-location, *Nr* is the radiation parameter, *Pr* is the Prandtl number and *Ec* is the Eckert number.

A close look at the Eq. (11) reveals that, in mixed convection due to viscous fluid, the temperature profiles are not similar because the *x*-coordinate cannot be eliminated from the equation. Although local non-similarity solutions have been found for some boundary layer flows dealing with viscous fluid, the technique is hard to extend to in this case. Thus, for ease of analysis, it was decided to proceed with finding local similarity solutions for the governing Eq. (11). That is, taking $X = x/L$ and then one can still study the effects of various parameters on different profiles at any given *X*- location.

For the type of flow under consideration, the main physical quantities of interest are the skin friction coefficient $f''(0)$ and the local Nusselt number– $\theta'(0)$, which represent the wall shear stress and the heat transfer rate at the surface, respectively. Our task is to investigate how the values of $f''(0)$ and $-\theta'(0)$ vary with the radiation parameter *Nr*, magnetic parameter *M*, Prandtl number Pr and Eckert number *Ec*.

3. SOLUTION OF THE PROBLEM

The governing boundary layer equations (10) and (11) subject to the boundary conditions (12) is solved numerically by using Runge-Kutta fourth order method along with shooting technique. First of all higher order non- linear differential equations (10) and (11) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique. The resultant initial value problem is solved by employing Runge - Kutta fourth order technique. The step size $\Delta\eta = 0.01$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient and the Nusselt number which are respectively proportional to $f''(0)$ and $-\theta'(0)$ are also sorted out and their numerical values are presented in a tabular form.

Table 1 Values of the heat transfer coefficient, $-\theta'(0)$ for different values of *Pr* when *M* = *Nr* = *Ec* = 0

Pr	Magyari and Keller (2000)	Bidin and Nazar (2009)	El-Aziz (2009)	Ishak (2011)	Mukhopadhyay and Gorla (2009)	Present Results
1	0.954782	0.9547	0.954785	0.9548	0.9547	0.95495
2		1.4714		1.4715	1.4714	1.47142
3	1.869075	1.8691	1.869074	1.8691	1.8691	1.86904
5	2.500135		2.500132	2.5001	2.5001	2.50011
10	3.660379		3.660372	3.6604	3.6603	3.66035

Table 2 Values of the heat transfer coefficient, $-\theta'(0)$ for different values of *Pr* and *Nr* when *M* = *Ec* = 0

	Bidin and Nazar (2009)		Nadeem *et al.* (2011)		Mukhopadhyay and Gorla (2009)		Present Results	
Pr\ *Nr*	0.5	1.0	0.5	1.0	0.5	1.0	0.5	1.0
1	0.6765	0.5315	0.680	0.534	0.6765	0.5315	0.67961	0.53142
2	1.0735	0.8627	1.073	0.863	1.0734	0.8626	1.07352	0.86331
3	1.3807	1.1214	1.381	1.121	1.3807	1.1213	1.38071	1.12141

4. RESULTS AND DISCUSSION

In order to get a physical insight into the problem, a representative set of numerical results is shown graphically in Figs.1-20, to illustrate the influence of physical parameters viz., the magnetic parameter *M*, the radiation parameter *Nr*, the Eckert number *Ec*, the *X*-location, the velocity slip parameter λ, the thermal slip parameter δ and the suction/injection parameter *S* on the velocity $f'(\eta)$, temperature $\theta(\eta)$, shear stress $f''(\eta)$ and temperature gradient $\theta'(\eta)$.

In order to test the accuracy of the present method, our results are compared with those of Magyari and Keller (2000), Bidin and Nazar (2009), El-Aziz (2009), Ishak (2011), Nadeem *et al.* (2011) and Mukhopadhyay and Gorla (2009) (for reduced cases) and found that there is an excellent agreement, as shown in Tables 1 and 2. Throughout the calculations, the parametric values are fixed to be *Pr* = 0.7, *Nr* = 0.1, *S* = 0.1, *Ec* = 0.1, *M* = 1.0, *X* = 0.5, λ = 0.1, δ = 0.1, unless otherwise indicated.

Figures 1 and 2 illustrate the velocity and shear stress for different values of the magnetic parameter *M*. It is observed that the velocity decreases as the magnetic parameter increases (Fig.1). This is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the magnetic parameter. From Fig. 2, it is noticed that the magnitude of shear stress decreases initially with an increase in the magnetic parameter but it increases significantly after a certain distance η normal to the sheet.

Figures 3 and 4 depict the temperature and temperature gradient for different values of the magnetic parameter *M*. It is observed that the temperature increases as the magnetic parameter increases (Fig.3). From Fig. 4, it is clear that the temperature gradient increases initially with an increase in the magnetic parameter but it decreases significantly after a certain distance η normal to the sheet.

The influence of thermal radiation *Nr* on the temperature and temperature gradient are shown in Figs. 5 and 6 respectively, in the presence of suction, velocity and thermal slips at the boundary.

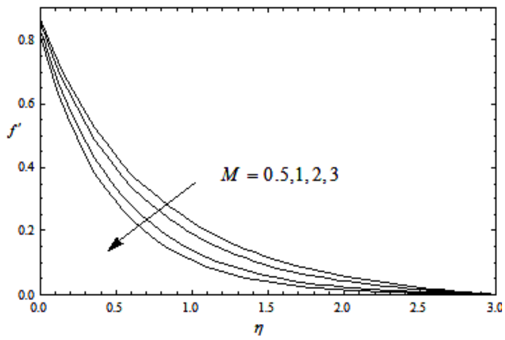

Fig. 1. Velocity profiles for different values of M.

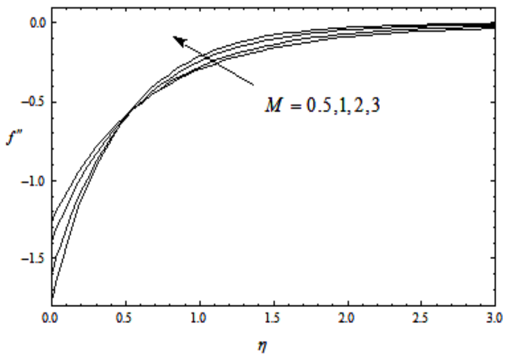

Fig. 2. Effect of M on the surface skin friction.

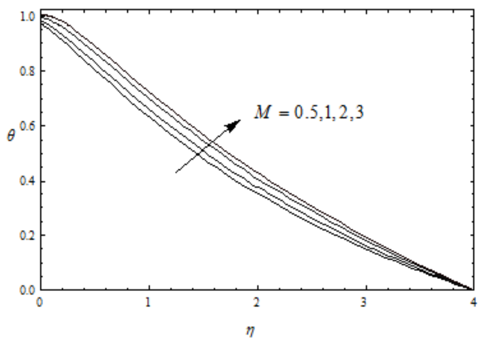

Fig. 3. Temperature profile doffrent values of M.

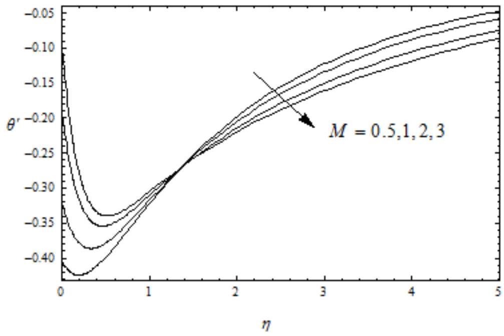

Fig. 4. Effect of M on the surface heat transfer.

It is found that the temperature increases as the radiation parameter *Nr* increases (Fig. 5). Larger values of *Nr* sound dominance of thermal radiation over conduction. Consequently larger values of *Nr* are indicative of larger amount of radiative heat energy being poured into the system, causing rise in θ(η) and θ′(η)(Fig. 6).

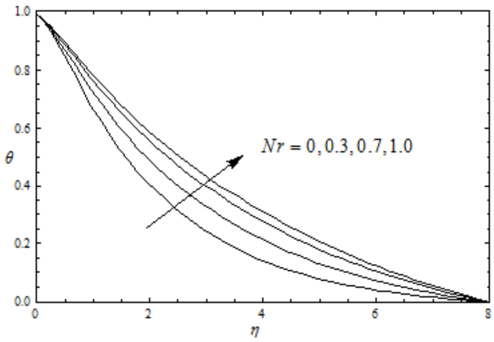

Fig. 5. Temperaure profiles for diffrenet values of *Nr*.

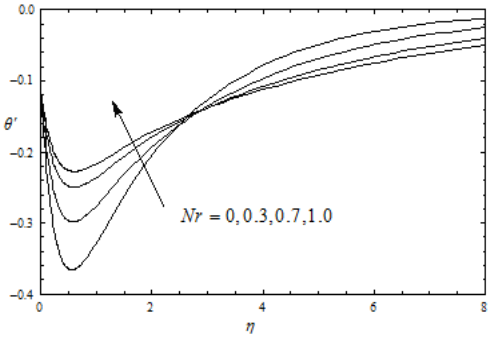

Fig. 6. Effect of *Nr on the suface heat transfer*.

For different values of the viscous dissipation parameter i.e., the Eckert number *Ec* on the temperature and temperature gradient are shown in Figs. 7 and 8 respectively, in the presence of suction, velocity and thermal slips at the boundary. The Eckert number *Ec* express the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conservation of kinetic energy into internal energy by work done against the viscous fluid stress. The positive Eckert number implies cooling of the sheet i.e., loss of heat from the sheet to the fluid. It is found that the temperature increases as the Eckert number *Ec* increases (Fig. 7). From Fig. 8, it is seen that the temperature gradient increases initially with an increase in the Eckert number but it decreases significantly after a certain distance η normal to the sheet.

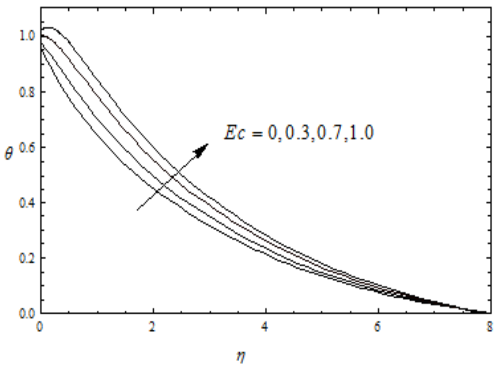

Fig. 7. Temperature profiles for different values of *Nr*.

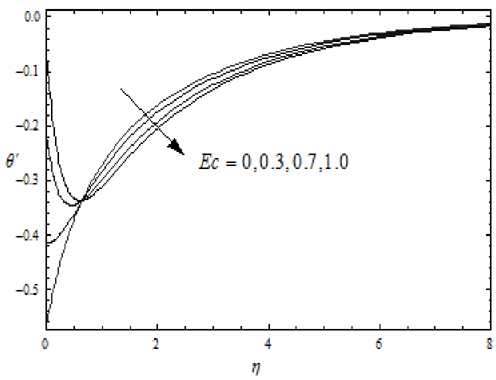

Fig. 8. Effect of *Ec* on the surface heat transfer.

Figures 9 and 10 show the effects of the *X*-location on the temperature and temperature gradient. It is clear from Fig. 9 that the thermal boundary layer thickness increases with the increase of *X* but with significant effect near the stretching sheet. It can be seen from Fig.10 that the temperature gradient increases initially with an increase in the Eckert number but it decreases significantly after a certain distance η normal to the sheet.

The effect of velocity slip parameter λ on velocity and shear stress in the presence of suction at the wall are shown in Fig. 11 and 12 respectively. From Fig. 11, it is observed that the rate of transport decreases with the increasing distance (η) normal to the sheet. In all cases the velocity vanishes at some large distance from the sheet. With the increasing λ, the stream wise component of the velocity is found to decrease. When slip occurs, the flow velocity near the sheet is no longer equal to the stretching velocity of the sheet. With the increase in λ, such slip velocity increases and consequently fluid velocity decreases because under the slip condition, the pulling of the stretching sheet can be only partly transmitted to the fluid. It is noted that λ has a substantial effect on the solutions. Fig. 12 shows that initially, the magnitude of shear stress increases with the increasing values of velocity slip parameter λ. This feature prevails up to certain heights and then the process is slowed down.

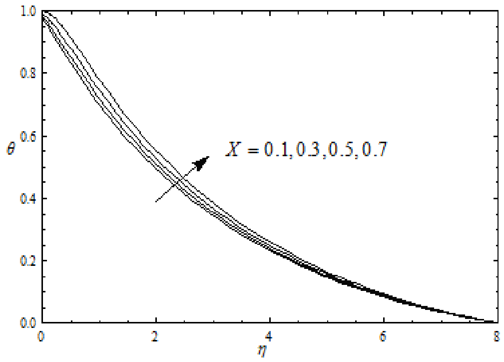

Fig. 9. Temperature profile for different values of *X*.

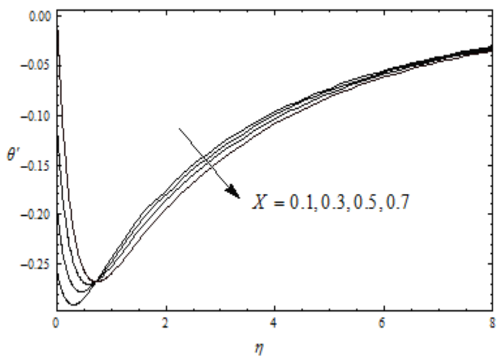

Fig. 10. Effect of *X* on the surface heat transfer.

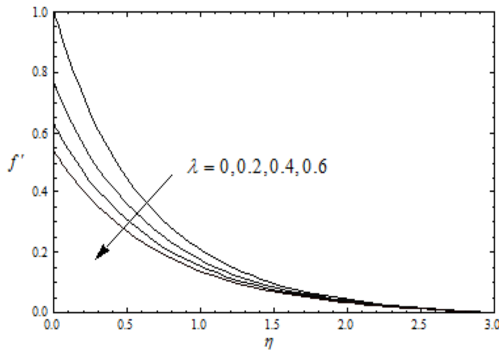

Fig. 11. Velocity profiles for different values of λ.

Figures 13 and 14 present the temperature and temperature gradient for different values of velocity slip parameter in presence of suction. It is observed that the temperature decreases initially but after a certain distance from the sheet it increases with an increase in λ (Fig. 13). Temperature gradient exhibits a fluctuating nature (Fig. 14). From this figure, it is very clear that temperature gradient decreases initially with increasing velocity slip λ but after a certain distance from the sheet, it increases and it finally decreases with λ.

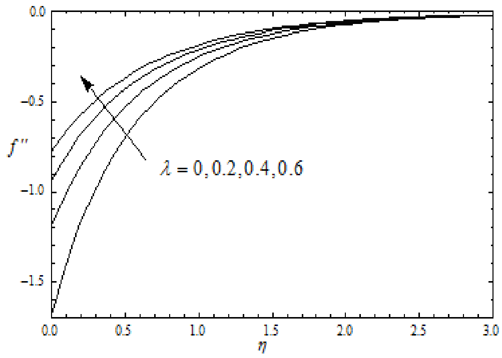

Fig. 12. Effect of λ on the surface skin friction.

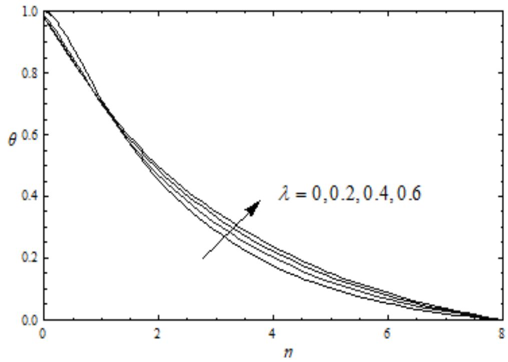

Fig. 13. Temperature profiles for different values of λ.

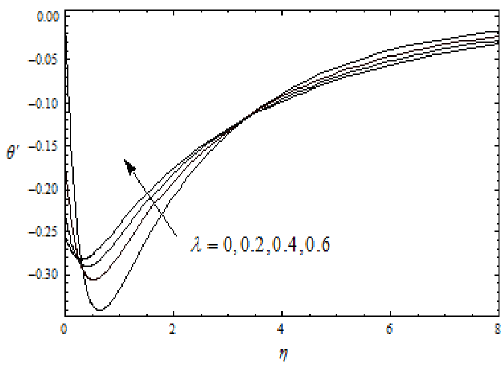

Fig. 14. Effect of λ on the surface heat transfer.

Figures 15 and 16 depict the temperature and temperature gradient for different values of the thermal slip parameter δ respectively. From Fig. 15, it is found that, initially the temperature decreases with thermal slip parameter δ but after a certain distance η normal to the sheet, such feature is smeared out. With the increase of thermal slip parameter δ, less heat is transferred to the fluid from the sheet and so temperature is found to decrease. On the other hand, temperature gradient increases with increasing thermal slip parameter δ (Fig. 16).

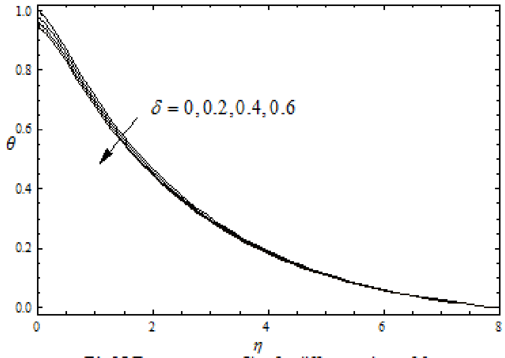

Fig. 15. Temperature profiles for different values δ.

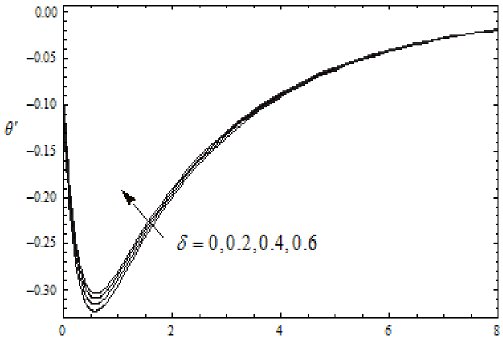

Fig. 16. Effect of λ on the surface heat transfer.

The effect of suction parameter S on velocity and shear stress are shown in Figs. 17 and 18 respectively in the presence of slip at the boundary for exponentially stretching sheet. It is observed that velocity decreases significantly with increasing suction parameter whereas fluid velocity is found to increase with blowing (Fig. 17). From Fig. 18, it is very clear that, the magnitude of shear stress decreases initially with the suction parameter S but it increases significantly after certain distance η normal to the sheet. It is observed that, when the wall suction (S > 0) is considered, this causes a decrease in the boundary layer thickness and the velocity field is reduced. $S = 0$ represents the case of non porous stretching sheet. Opposite behavior is noted for blowing ($S < 0$).

Figures 19 and 20 represent the temperature and temperature gradient for different values of variable suction parameter S in presence of slip. It is seen that temperature decreases with increasing suction parameter whereas it increases due to blowing (Fig. 19). Temperature overshoot is noted for blowing (S < 0). This feature prevails up to certain heights and then the process is slowed down and at a far distance from the wall temperature vanishes. The temperature gradient decreases initially with the suction parameter S but it increases after a certain distance g normal to the sheet. Far away from the wall, such feature is smeared out. However, opposite behavior is noted for wall injection (S < 0) (Fig. 4b).

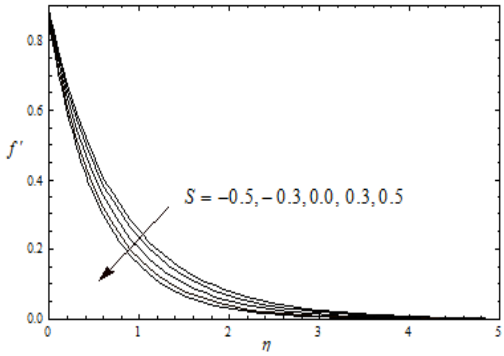

Fig. 17. Velocity profile for different values of *S*.

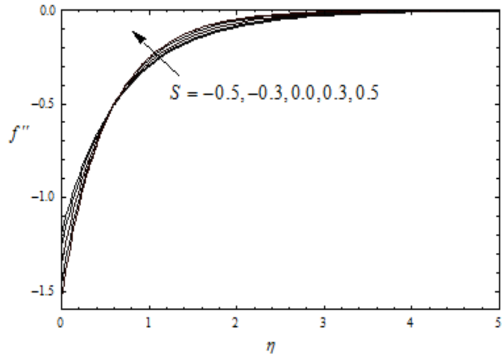

Fig. 18. Effect of *S* on the surface skin friction.

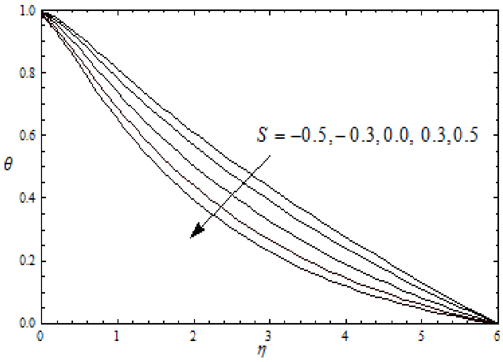

Fig. 19. Temperature profiles for different values of *S*.

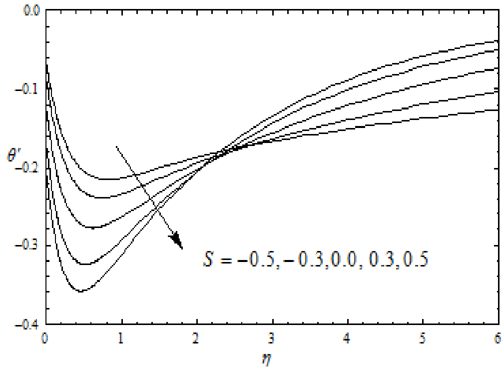

Fig. 20. Effect of *S* on the surface heat transfer.

The effects of various governing parameters on the skin friction coefficient $f''(0)$ and the heat transfer coefficient $-\theta'(0)$ are shown in Tables 3 and 4. From Table 3, it is found that the local skin friction coefficient $f''(0)$ increases as λ increases, while it decreases with increase in the magnetic parameter M or the variable suction parameter S. Here, the values of shear stress near the plate are negative because of the plate is stretching exponentially. From Table 4, it is observed that the heat transfer coefficient $-\theta'(0)$ increases as Pr or S increases, whereas it decreases as M or Nr or Ec or X or λ or δ increases.

Table 3 Effect of the skin-friction for different values of *M, λ, S*

M	Λ	S	$f''(0)$
0.5	0.1	0.1	-1.26307
1.0	0.1	0.1	-1.38502
2.0	0.1	0.1	-1.59107
1.0	0.1	0.1	-1.38502
1.0	0.2	0.1	-1.18601
1.0	0.4	0.1	-0.92947
1.0	0.1	-0.3	-1.24966
1.0	0.1	0.0	-1.34906
1.0	0.1	0.3	-1.45675

Table 4 Effect of the heat transfer coefficient $-\theta'(0)$, for different values of *M, Nr, Pr, Ec, X, λ, δ, S*

M	Nr	Pr	Ec	X	S	Λ	Δ	$-\theta'(0)$
0.5	0.1	0.7	0.1	0.5	0.1	0.1	0.1	0.545439
1.0	0.1	0.7	0.1	0.5	0.1	0.1	0.1	0.497706
2.0	0.1	0.7	0.1	0.5	0.1	0.1	0.1	0.425623
1.0	0.1	0.7	0.1	0.5	0.1	0.1	0.1	0.471862
1.0	0.7	0.7	0.1	0.5	0.1	0.1	0.1	0.335352
1.0	1.0	0.7	0.1	0.5	0.1	0.1	0.1	0.300091
1.0	0.1	0.5	0.1	0.5	0.1	0.1	0.1	0.379196
1.0	0.1	0.7	0.1	0.5	0.1	0.1	0.1	0.471862
1.0	0.1	2.0	0.1	0.5	0.1	0.1	0.1	0.912733
1.0	0.1	0.7	0.1	0.5	0.1	0.1	0.1	0.471862
1.0	0.1	0.7	0.3	0.5	0.1	0.1	0.1	0.292948
1.0	0.1	0.7	0.5	0.5	0.1	0.1	0.1	0.114035
1.0	0.1	0.7	0.1	0.1	0.1	0.1	0.1	0.512224
1.0	0.1	0.7	0.1	0.3	0.1	0.1	0.1	0.495047
1.0	0.1	0.7	0.1	0.5	0.1	0.1	0.1	0.471862
1.0	0.1	0.7	0.1	0.5	0.3	0.1	0.1	0.386837
1.0	0.1	0.7	0.1	0.5	0.0	0.1	0.1	0.443613
1.0	0.1	0.7	0.1	0.5	0.3	0.1	0.1	0.533032
1.0	0.1	0.7	0.1	0.5	0.1	0.1	0.1	0.471862
1.0	0.1	0.7	0.1	0.5	0.1	0.4	0.1	0.430189
1.0	0.1	0.7	0.1	0.5	0.1	0.6	0.1	0.406990
1.0	0.1	0.7	0.1	0.5	0.1	0.1	0.1	0.471862
1.0	0.1	0.7	0.1	0.5	0.1	0.1	0.4	0.403854
1.0	0.1	0.7	0.1	0.5	0.1	0.1	0.6	0.368452

CONCLUSIONS

The present study gives numerical solutions for steady MHD boundary layer slip flow and radiative heat transfer over an exponentially stretching surface in the presence of joule heating and viscous dissipation. The effect of suction parameter on a viscous incompressible fluid is to suppress the velocity field which in turn causes the enhancement of the skin friction coefficient. The results pertaining to the present study indicate that due to slip, velocity decreases. The temperature increases with increasing values of the radiation parameter or magnetic parameter or Eckert number.

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