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Engineering Science and Technology, an International Journal ■■ (2015) ■■–■■

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Engineering Science and Technology, an International Journal

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Full Length Article

Unsteady MHD radiative flow and heat transfer of a dusty nanofluid over an exponentially stretching surface

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ARTICLE INFO

Article history:
Received 11 April 2015
Received in revised form
12 May 2015
Accepted 6 June 2015
Available online

Keywords:
MHD
Dusty fluid
Nanofluid
Stretching/shrinking sheet
Volume fraction
Non-uniform heat source/sink

ABSTRACT

We analyzed the unsteady magnetohydrodynamic radiative flow and heat transfer characteristics of a dusty nanofluid over an exponentially permeable stretching surface in presence of volume fraction of dust and nano particles. We considered two types of nanofluids namely Cu-water and CuO-water embedded with conducting dust particles. The governing equations are transformed into nonlinear ordinary differential equations by using similarity transformation and solved numerically using Runge–Kutta based shooting technique. The effects of non-dimensional governing parameters namely magneticfield parameter, mass concentration of dust particles, fluid particle interaction parameter, volume fraction of dust particles, volume fraction of nano particles, unsteadiness parameter, exponential parameter, radiation parameter and suction/injection parameter on velocity profiles for fluid phase, dust phase and temperature profiles are discussed and presented through graphs. Also, friction factor and Nusselt numbers are discussed and presented for two dusty nanofluids separately. Comparisons of the present study were made with existing studies under some special assumptions. The present results have an excellent agreement with existing studies. Results indicated that the enhancement in fluid particle interaction increases the heat transfer rate and depreciates the wall friction. Also, radiation parameter has the tendency to increase the temperature profiles of the dusty nanofluid.

1. Introduction

Momentum and heat transfer of dusty fluids and nanofluids have tremendous applications in engineering and sciences. In the past few decades, researchers have been focusing on analyzing the heat and mass transfer characteristics of either dusty or nanofluids through different channels. In the present study we are taking initiation to discuss the momentum and heat transfer characteristics of a dusty nanofluid over a stretching surface by considering volume fraction of nano particles (in nanometers) embedding with conducting dust particles (in micrometers). Through this study our aim is to find the metals or metallic oxides which give good thermal enhancement by embedding into different nanofluids. The convective flow of dusty viscous fluids has a variety of applications like wastewater treatment, combustion and petroleum transport, power plant piping etc. Heat transport in nanofluids plays a major role in heat transfer enhancement in the renewable energy systems, material processing and industrial thermal management.

Choi [1] was the first person who introduced the term nanofluid. He found the enhanced thermal conductivity of the base fluid embedded with nanometer sized particles. Marble [2] discussed about the impact of dusty gases on fluid mechanics. Chakrabarti and Gupta [3] analyzed the MHD flow and heat transfer characteristics of a flow over a stretching surface. Makinde and Aziz [4] studied boundary layer flow of a nanofluid over a stretching surface. The effects of radiation on unsteady convective flow of an ethylene glycol based nanofluid over a vertical plate were presented by Sandeep et al. [5]. Akbar et al. [6] discussed the radiation effects on MHD stagnation point flow of nano fluid toward a stretching surface by considering convective boundary conditions. Optimized analytical solution for oblique flow of a Casson-nano fluid was analyzed by Nadeem et al. [7]. Gireesha et al. [8] discussed magnetohydrodynamic flow and heat transfer characteristics of dusty fluid past a stretching surface. Boundary layer flow and heat transfer of a nanofluid over a non linear stretching/shrinking sheet was discussed by Zaimi et al. [9]. Mohankrishna et al. [10] studied radiation effects on unsteady MHD natural convection flow of a nanofluid past an infinite vertical plate with heat source. Akbar et al. [11,12] studied the stagnation point

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Peer review under responsibility of Karabuk University.

http://dx.doi.org/10.1016/j.jestch.2015.06.004

flow of a CNT suspended nanofluid and Prandtl fluid over stretching and shrinking sheets. Partial slip effect on non-aligned stagnation point flow of a nanofluid over a stretching convective surface was discussed by Nadeem et al. [13]. Stagnation point flow of a nanofluid over a stretching sheet by considering slip effects was illustrated by Malvandi et al. [14]. Prasad et al. [15] discussed the effect of variable viscosity on MHD viscoelastic flow over a stretching surface.

Radiation and chemical reaction effects of a flow through porous media over a vertical channel were discussed by Sandeep et al. [16]. Swati Mukhopadhyay [17] studied MHD boundary layer flow past an exponentially stretching surface embedded with thermally stratified medium. MHD flow of a non-Newtonian nanofluid over an impermeable surface with heat source/sink was analyzed by Ramesh et al. [18]. Anwar et al. [19] discussed the conjugate effects on heat and mass transfer in nanofluids over a nonlinear stretching sheet. An exact solution for unsteady magnetohydrodynamic oscillatory flow of a Maxwell fluid in a porous medium was illustrated by Khan et al. [20]. Conjugate effects of radiation flux on double diffusive magnetohydrodynamic free convection flow of a nanofluid past a power law stretching sheet was studied by Anwar et al. [21]. Debnath and Ghosh [22] studied unsteady MHD flow of a dusty fluid between two oscillating plates. Pulsatile flow and heat transfer characteristics of dusty fluid flow over infinite annular pipe were illustrated by Datta and Dalal [23]. MHD boundary layer flow over a stretching surface by considering slip effects was analyzed by Rushi Kumar [24]. Wang and Mujumdar [25] gave a detailed description about heat transfer characteristics of a nanofluid. The effects of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over an exponentially stretching surface in the presence of internal heat generation or absorption was discussed by Elbashbeshy et al. [26]. Ferdows et al. [27] considered a permeable unsteady stretching sheet and analyzed the boundary layer flow and heat transfer of a nanofluid with viscous dissipation effect. Philip et al. [28] discussed thermo physical properties of different nanofluids. Stagnation-point flow of a nanofluid over a nonlinear stretching sheet was analyzed by Anwar et al. [29]. Khalid et al. [30] presented an exact solution for free convection flow of nanofluids in presence of ramped wall temperature. Anwar et al. [31] illustrated Dufour and Soret effects on free convection flow of a nanofluid past a power law stretching sheet.

Buongiomo [32] explained about convective heat transport phenomenon in nanofluids. Heat transfer characteristics of a flow over a nonlinear stretching sheet with non-uniform heat were presented by Mahentesh et al. [33]. Shejkholeslami et al. [34] discussed heat transfer characteristics of a MHD nanofluid over a rotating system. Unsteady MHD flow and heat transfer behavior of dusty fluid at different physical properties was analyzed by Attia [35]. MHD effects on convective flows by considering volume fraction of dust particles were analyzed by Ibrahim Saidu et al. [36]. Very recently Ramana Reddy et al. [37] studied radiation and chemical reaction effects on unsteady MHD dusty viscous flow with heat source/sink. Unsteady MHD free convection flow of casson fluid past a vertical plate embedded in a porous medium was studied by Khalid et al. [38]. Stagnation-point flow and heat transfer behavior of Cu-water nanofluid toward horizontal and exponentially stretching/shrinking cylinders was illustrated by Sulochana and Sandeep [39]. Anwar et al. [40] discussed the influence of MHD and radiation on stagnation point flow of nanofluid toward a nonlinear stretching sheet. The effective thermal conductivity influenced by particle shape was discussed by Hamilton and Crosser [41]. The importance of thermal radiation and its applications was discussed by Cortell [42].

To the authors' knowledge no studies have been reported yet on unsteady MHD radiative flow and heat transfer characteristics of a dusty nanofluid over an exponentially permeable stretching surface in presence of volume fraction of dust and nano particles. In this study we considered two types of nanofluids namely Cu-water and CuO-water embedded with dust particles. The governing equations are transformed into nonlinear ordinary differential equations by using similarity transformation and then solved numerically using Runge–Kutta based shooting technique. The effects of non-dimensional governing parameters on velocity profiles for fluid and dust phases and temperature profiles are discussed and presented through graphs. Also, friction factor and Nusselt numbers are discussed and presented for two dusty nanofluids separately.

2. Mathematical analysis

Consider an unsteady two dimensional MHD boundary layer flow of a dusty nanofluid over a permeable stretching/shrinking surface. It is assumed that the velocity of the stretching surface is $U_w(x,t)=\dfrac{U_0}{(1-ct)}e^{Ex/L}$, E is the exponential parameter. The flow starts at $t=0$ and it is steady at $t<0$. The surface is along x-axis and y-axis is normal to it. A uniform magnetic field B is applied along the y-direction. Non-uniform heat source/sink is considered. The dust particles are assumed to be uniform in size. Spherical shaped nano and dust particles are considered. Number density of dust particles along with volume fraction of dust and nano particles are taken into account. The temperature maintained near the surface is $T_w(x,t)=T_\infty+\dfrac{T_0}{1-ct}e^{Ex/2L}$. Under above assumptions the boundary layer equations of dusty nanofluid can be written in the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho_{nf}(1-\phi_d)\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=(1-\phi_d)\mu_{nf}\frac{\partial^2 u}{\partial y^2}+KN(u_p-u)-\sigma B^2 u, \tag{2}$$

$$\frac{\partial u_p}{\partial x}+\frac{\partial v_p}{\partial y}=0, \tag{3}$$

$$Nm\left(\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=KN(u-u_p), \tag{4}$$

$$(\rho c_p)_{nf}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{N}{\tau_\upsilon}(u_p-u)^2-\frac{\partial q_r}{\partial y}, \tag{5}$$

with the boundary conditions

$$u=U_w(x,t),\quad v=V_w(x,t),\quad T=T_w(x,t)\quad at\quad y=0,$$

$$u\to 0,\quad u_p\to 0,\quad v_p\to v,\quad T\to T_\infty\quad as\quad y\to\infty, \tag{6}$$

where (u,v) and (u_p,v_p) are the velocity components of the fluid and particle phase respectively in x,y directions, ϕ_d is the volume fraction of the dust particles, ρ_{nf} and μ_{nf} are the nanofluid density and dynamic viscosity respectively, K is the stokes resistance, N is the number density of the dust particles, m is the mass concentration of dust particles, σ is the electrical conductivity, $B=B_0e^{Ex/2L}/\sqrt{1-ct}$ is the magnetic field imposed along the y-axis, $(\rho c_p)_{nf}$ is the specific heat capacitance of nanofluid and k_{nf} is the effective thermal conductivity of nano-fluid, q_r is the radiative heat flux and t refers to the time.

Where $U_w(x,t)=\dfrac{U_0}{1-ct}e^{Ex/L}$ is the sheet velocity, c is the positive constant which measures the unsteadiness, $V_w(x,t)=-S\sqrt{\dfrac{U_0\nu_f}{2L(1-ct)}}e^{Ex/2L}$ is the suction velocity, $S>0$ for suction, $S<0$ for injection, U_0 is the reference velocity, L is the characteristic length, $T_w(x,t)=T_\infty+\dfrac{T_0}{1-ct}e^{Ex/2L}$ is the temperature distribution near the surface, T_0 is the reference temperature. The radiative heat flux q_r under Rosseland approximation has the form

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{7}$$

where σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient. The temperature differences within the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature. Expanding T^4 using Taylor series and neglecting higher order terms yields

$$T^4 \cong 4T_\infty{}^3 T - 3T_\infty{}^4, \tag{8}$$

The nanofluid constants are given by

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},$$
$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s + (n-1)k_f) - \phi(n-1)(k_f - k_s)}{(k_s + (n-1)k_f) + \phi(k_f - k_s)}, \tag{9}$$

To measure the thermal conductivity of the nanofluid, k_{nf} for different shapes of nano particles, we adopted the following formula, which is given by Hamilton and Crosser [41]. Here n is the nano particle shape, for spherical shaped nano particles $n = 3$ and for cylindrical shaped particles $n = 3/2$ where ϕ is the volume fraction of the nano particles. The subscripts f and s refer to fluid and solid properties respectively.

Now, we introduce the following similarity transformations to convert the governing equations into the corresponding coupled nonlinear ordinary differential equations:

$$u = \frac{U_0}{1-ct}e^{Ex/L}f'(\eta), \quad v = -E\left(\frac{\nu_f U_0}{2L(1-ct)}\right)^{1/2} e^{Ex/2L}\{f(\eta) + \eta f'(\eta)\},$$
$$u_p = \frac{U_0}{1-ct}e^{Ex/L}F'(\eta), \quad v_p = -E\left(\frac{\nu_f U_0}{2L(1-ct)}\right)^{1/2} e^{Ex/2L}\{F(\eta) + \eta F'(\eta)\},$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \left(\frac{U_0}{2L\nu_f(1-ct)}\right)^{1/2} e^{Ex/2L}y,$$
$$T - T_\infty = \frac{T_0}{(1-ct)}e^{Ex/2L}\theta(\eta), \tag{10}$$

By substituting equations (7) to (10) into equations (2), (4), (5) and (6) and equating the coefficients of $(x/L)^0$ on both sides, we get

$$\left.\begin{array}{l}\frac{(1-\phi_d)}{(1-\phi)^{2.5}}f''' - (1-\phi_d)(1-\phi+\phi(\rho_s/\rho_f))E(2f'^2 - ff'' + A[2f' + \eta f'']) \\ \qquad + 2\alpha\beta(F' - f') - Mf' = 0,\end{array}\right\} \tag{11}$$

$$FF'' - 2F'^2 - A[2F' + \eta F''] + \beta(f' - F') = 0, \tag{12}$$

$$\left.\begin{array}{l}\frac{1}{\Pr\left(1-\phi+\phi\left((\rho c_p)_s/(\rho c_p)_f\right)\right)}\left[\left(\frac{k_{nf}}{k_f} + \frac{4R}{3}\right)\theta'' + 2\Pr\alpha\beta Ec(F' - f')^2\right] \\ \qquad + E(f\theta' - \theta f' - A(\eta\theta' + 4\theta)) = 0,\end{array}\right\} \tag{13}$$

Subject to boundary conditions

$$f(\eta) = S, \quad f'(\eta) = \lambda, \quad \theta(\eta) = 1 \quad at \quad \eta = 0,$$

$$f'(\eta) = 0, \quad F'(\eta) = 0, \quad F(\eta) = f(\eta) + \eta f'(\eta) - \eta F'(\eta),$$
$$\theta(\eta) = 0 \quad as \quad \eta \to \infty, \tag{14}$$

where primes denote differentiation with respect to η, E is the exponential parameter, $A = cL/U_0$ is the unsteadiness parameter, $M = 2\sigma B_0{}^2 L/\rho_f U_0$ is the magnetic field parameter, $\alpha = Nm/\rho_f$ is the mass concentration of the dust particles, $\beta = (1-ct)L/U_0\tau_v$ is the fluid particle interaction parameter, where $\tau_v = m/K$ is the relaxation time of the dust particles, $R = 4\sigma^* T_\infty^3/k_f k^*$ is the radiation parameter, Pr is the Prandtl number, $Ec = U_0^2/(c_p)_f T_0$ is the Eckert number.

For engineering interest the friction factor C_f and Nusselt number Nu_x are given by

$$C_f \operatorname{Re}_x^{1/2} = (1-\phi)^{-1/2} f''(0), \tag{15}$$

$$Nu_x \operatorname{Re}_x^{-1/2} = -(k_{nf}/k_f)\theta'(0), \tag{16}$$

where $\operatorname{Re}_x = U_0 L/\nu_f$ is the local Reynolds number.

3. Results and discussion

The coupled ordinary differential equations (11) to (13) subject to the boundary conditions (14) are solved numerically using Runge–Kutta based shooting technique. Comprehensive numerical computations have been carried out for various values of non dimensional governing parameters, namely volume fraction of dust particles ϕ_d, volume fraction of nano particles ϕ, unsteadiness parameter A, mass concentration of the dust particles α, fluid particle interaction parameter β, magneticfield parameter M, exponential parameter E, radiation parameter R and suction/injection parameter S on velocity and temperature profiles for Cu-water and CuO-water dusty nanofluids and then the results are presented in terms of graphs. Also, friction factor and Nusselt numbers are discussed and presented through tables. For numerical results we considered $Ec = 3, \Pr = 6.2, \phi = \phi_d = 0.1, M = R = 1, A = 0.4, E = 0.1, \alpha = 1.2, \beta = 0.5, \eta = 11$. These values are kept as common in the entire study except the varied values as shown in respective figures and tables. The thermophysical properties of water, Copper and Copper Oxide are displayed in Table 1.

Figs. 1 and 2 depict the effect of volume fraction of dust particles (ϕ_d) on velocity and temperature profiles of Cu-water and CuO-water dusty nanofluids. It is evident from figures that increase in volume fraction of dust particles depreciates the velocity profiles of the fluid and dust phases along with the temperature profiles. This is due to the fact that the volume occupied by the dust particles per unit volume of mixture is more then it develops the mass concentration of the dust phase and reduces the velocity and thermal boundary layer thickness. Figs. 3 and 4 illustrate the effect of volume fraction of nano particles (ϕ) on velocity and temperature profiles of Cu-water and CuO-water dusty nanofluids. It is clear from the figures that enhancement in volume fraction of nano particles increases the velocity profiles of the fluid and dust phases along with temperature profiles. This is due to the reason that increase in volume fraction of nano particles enhances the thermal and velocity boundary layer thicknesses. It was noticed that the development in the temperature profiles of CuO-water dusty nanofluid is more compared with temperature profiles of Cu-water dusty nanofluid due to hike in volume fraction of nano particles.

Table 1
Thermophysical properties of base fluid and different nanoparticles.

	$\rho(Kg\,m^{-3})$	$c_p(J\,Kg^{-1}\,K^{-1})$	$k(Wm^{-1}\,K^{-1})$
H_2O	997.1	4179	0.613
CuO	6320	531.8	76.5
Cu	8933	385	401

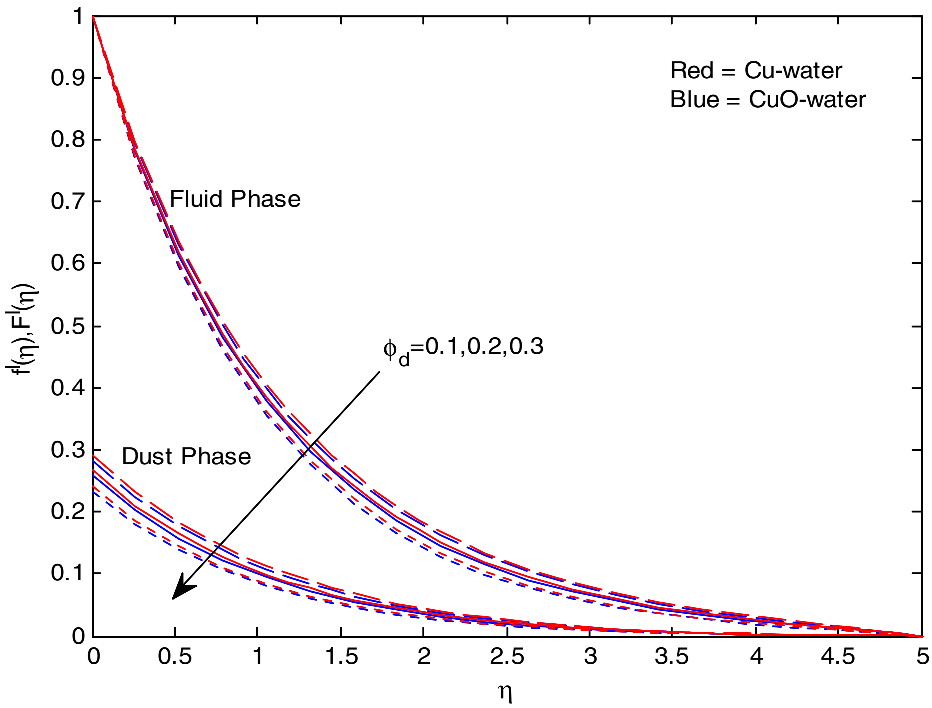

Fig. 1. Variation in velocity profiles of fluid and dust phases for different values of ϕ_d.

The influence of unsteadiness parameter (A) on velocity and temperature profiles of the flow are displayed in Figs. 5 and 6. It is observed from the figures that an increase in unsteadiness parameter declines the velocity profiles of the dust phase and temperature profiles. But we noticed enhancement in the velocity profiles of the fluid phase. This agrees with the general fact that the acceleration of the flow is high then the velocity boundary layer of dust phase becomes thin. The hike in unsteadiness parameter improves the

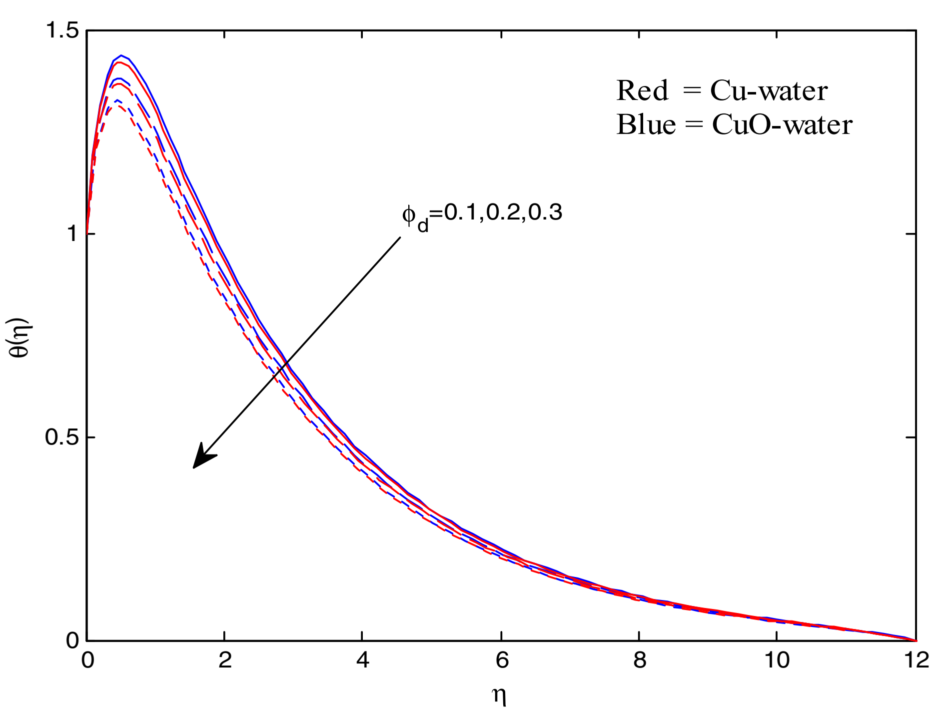

Fig. 2. Variation in temperature profiles for different values of ϕ_d.

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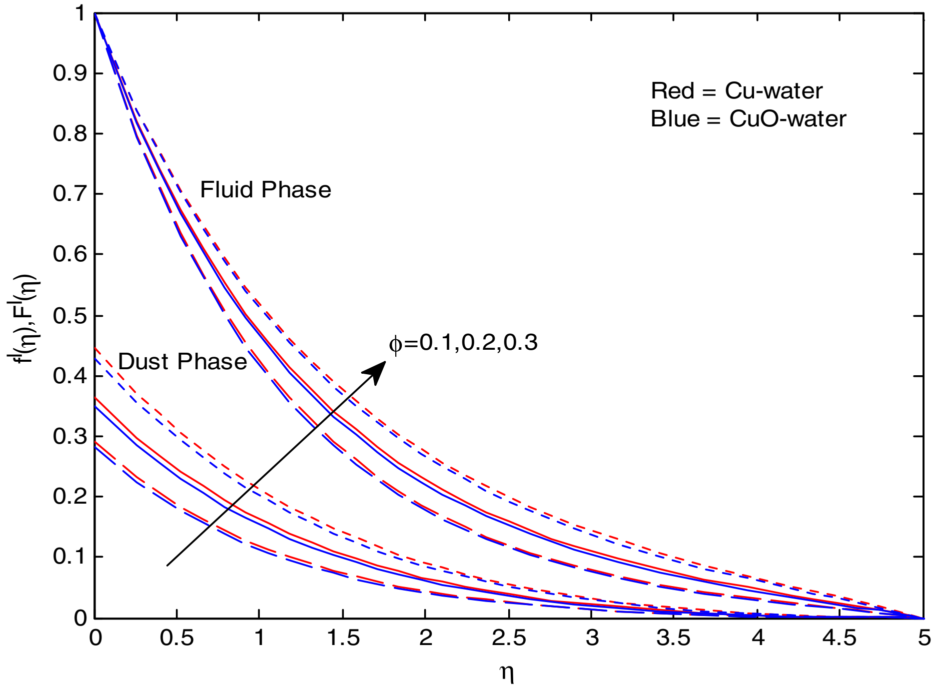

Fig. 3. Variation in velocity profiles of fluid and dust phases for different values of ϕ.

thermal conductivity of the flow after certain peak time. Figs. 7 and 8 show the influence of fluid particle interaction parameter on velocity and temperature profiles of Cu-water and CuO-water dusty nanofluids. It is noticed from the figures that a rise in the value of β declines the velocity profiles of the fluid phase and improves the velocity profiles of the dust phase along with the temperature profiles. We may explain this phenomenon as: the increase in fluid particle interaction parameter causes the development of the internal opposite forces to the flow till dust phase reaches the velocity of the fluid phase.

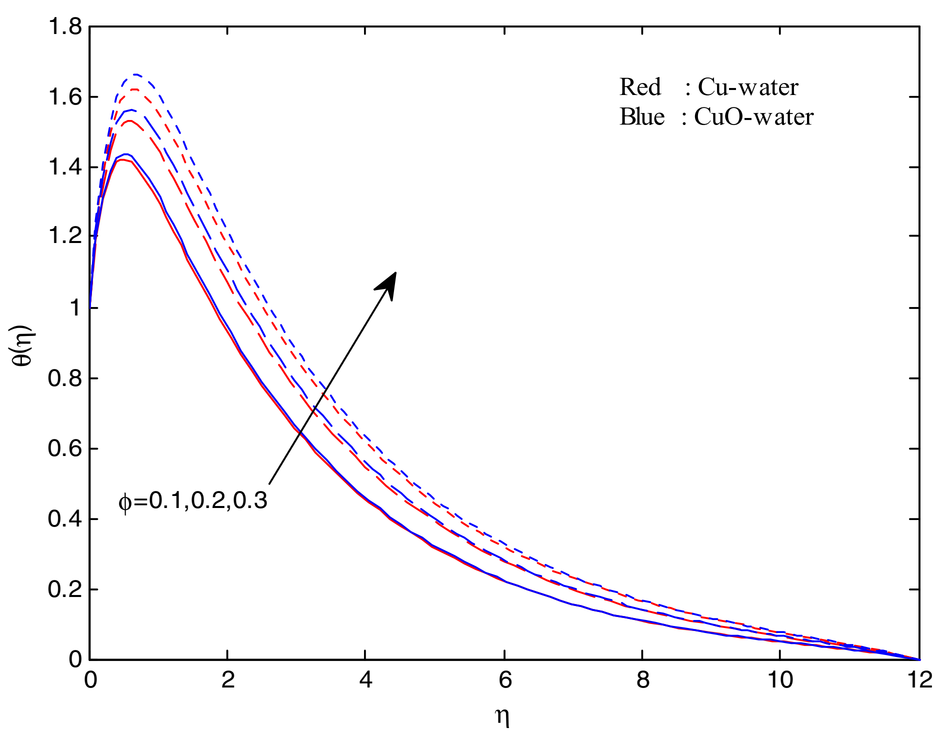

Fig. 4. Variation in temperature profiles for different values of ϕ.

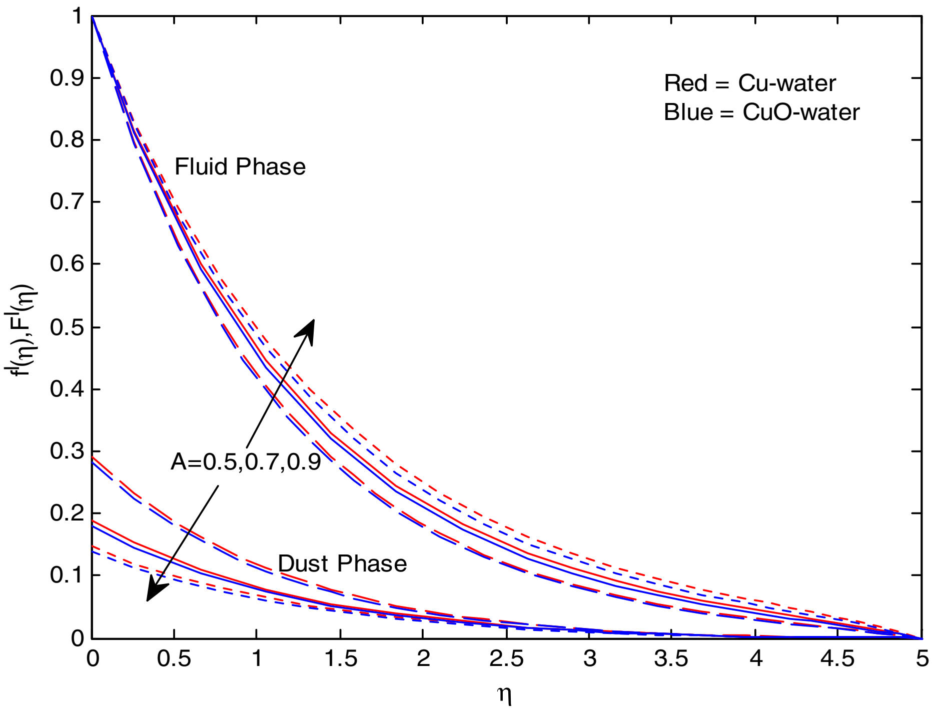

Fig. 5. Variation in velocity profiles of fluid and dust phases for different values of *A*.

The effect of mass concentration of dust particles (α) on velocity and temperature profiles of the flow is depicted in Figs. 9 and 10. It is clear that an increase in mass concentration of dust particles depreciates the velocity profiles of the fluid and dust phases and enhances the temperature profiles of the flow. Physically this means that enhancement in the mass concentration of the dustparticles increases the weight of the particle phase, which slows down the velocity profiles of the flow. But these hikes in particle mass help to develop the thermal conductivity. Figs. 11 and 12 display the variation in velocity and temperature profiles for different values of magneticfield parameter (M). It is evident from the figures that an increase in the magneticfield parameter decreases the

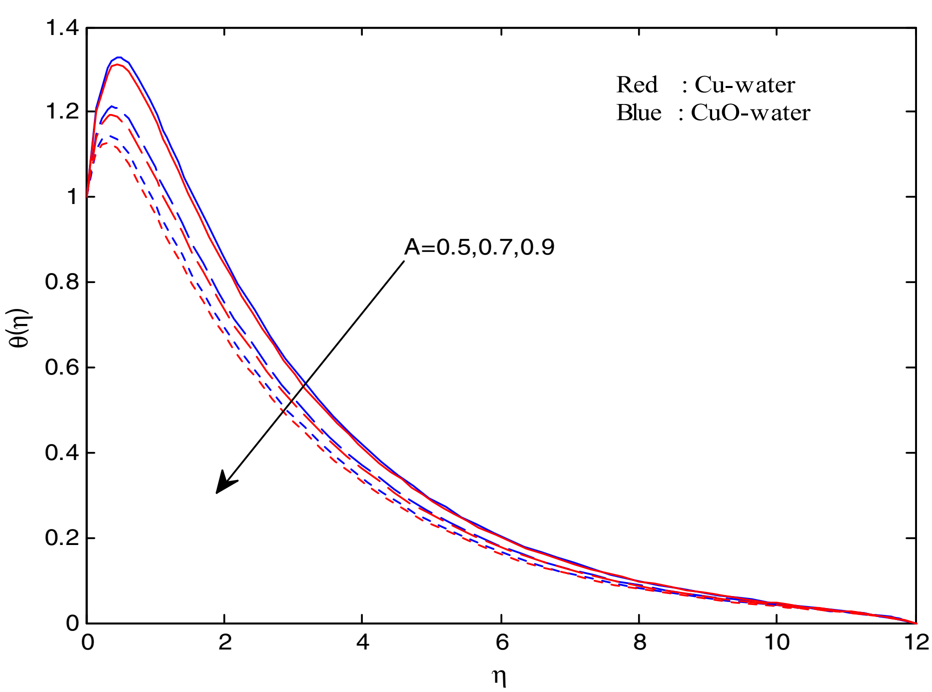

Fig. 6. Variation in temperature profiles for different values of *A*.

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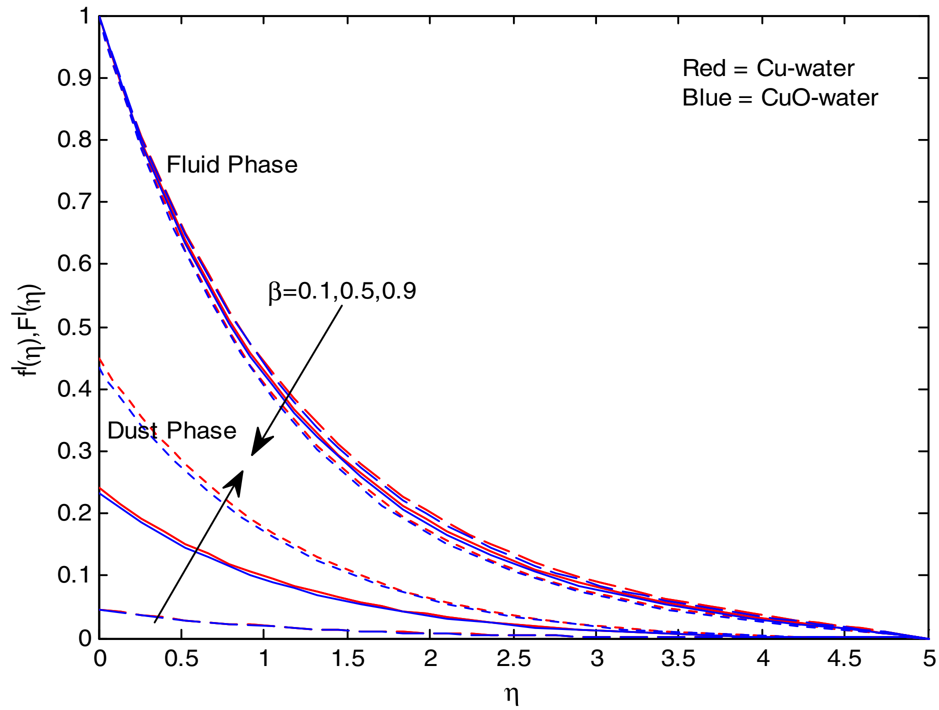

Fig. 7. Variation in velocity profiles of fluid and dust phases for different values of β.

velocity profiles and increases the temperature profiles of the flow. This is due to the fact that a raise in the magneticfield parameter develops the opposite force to the flow, which is called Lorentz force. This force declines the velocity boundary layer and improves the thermal boundary layer. We noticed an interesting result that increases in magneticfield parameter hike the temperature profiles of CuO-water dusty nanofluid significantly compared with Cu-water dusty nanofluid. The effect of radiation parameter (R) on temperature profiles is illustrated in Fig. 13. It is observed that the enhancement in R hikes the temperature profiles of both Cu-water and CuO-water dusty nanofluids. This may happen due to the domination of Stefan–Boltzmann constant and less influence of mean

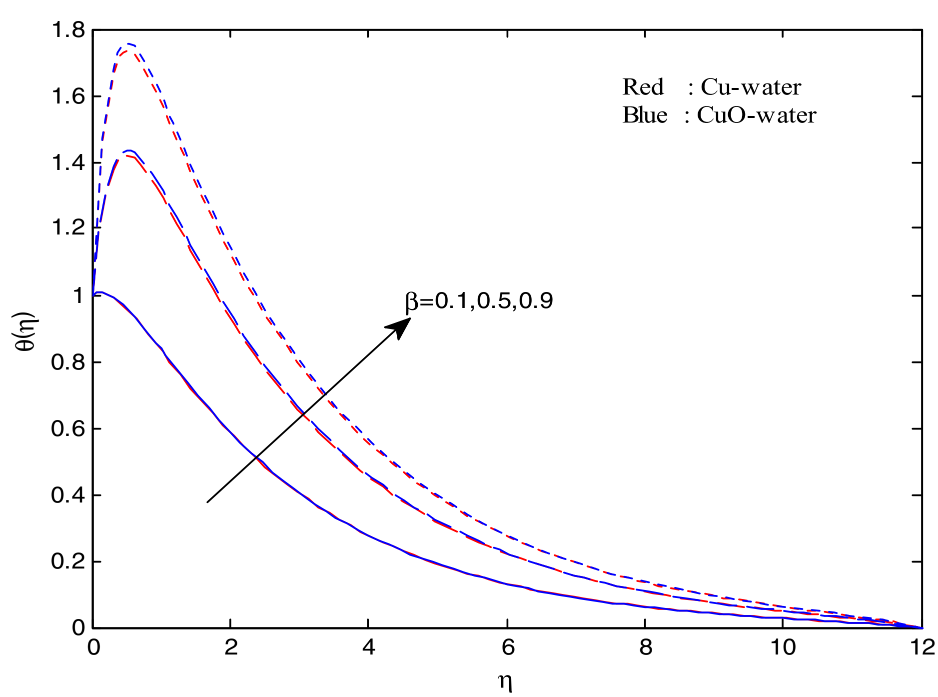

Fig. 8. Variation in temperature profiles for different values of β.

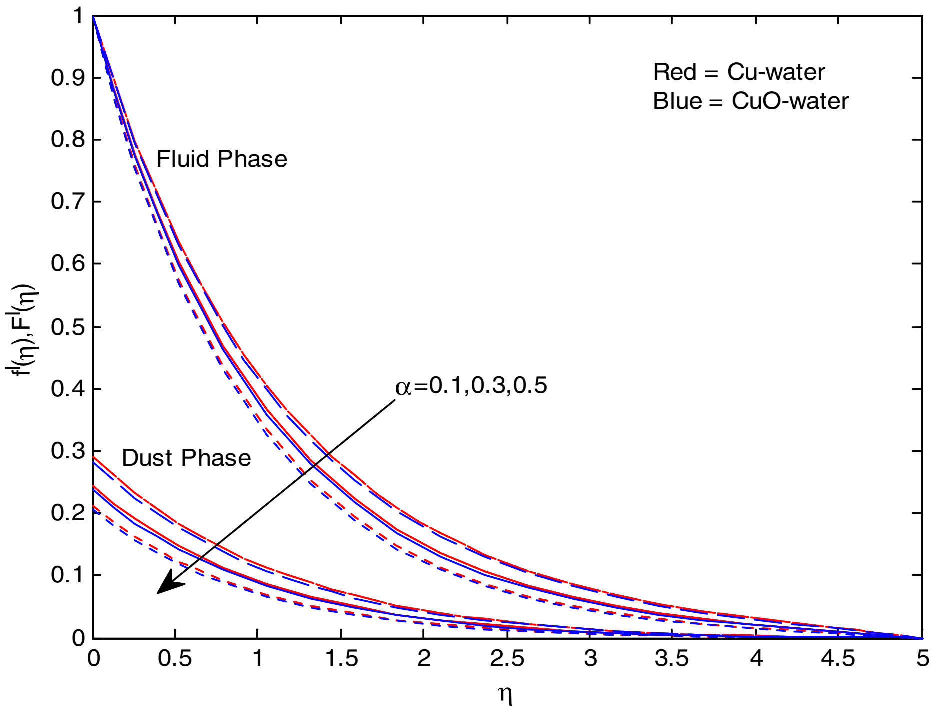

Fig. 9. Variation in velocity profiles of fluid and dust phases for different values of α.

absorption coefficient in Rosseland approximation. The influence of exponential parameter (E) on velocity and temperature profiles of Cu-water and CuO-water dusty nanofluids is illustrated by Figs. 14 and 15. It is evident that the increase in exponential parameter enhances the velocity profiles of both fluid and dust phases and depreciates the temperature profiles. It is interesting to note that the wall temperature depreciates in entire boundary for positive values of exponential parameter E. Figs. 16 and 17 illustrate the effect of suction/injection (S) parameter on velocity and temperature profiles of the flow. It is observed that the increases in the value of S depreciate the velocity and temperature profiles of the fluid phase and enhance the velocity profiles of the dust phase. This agrees with

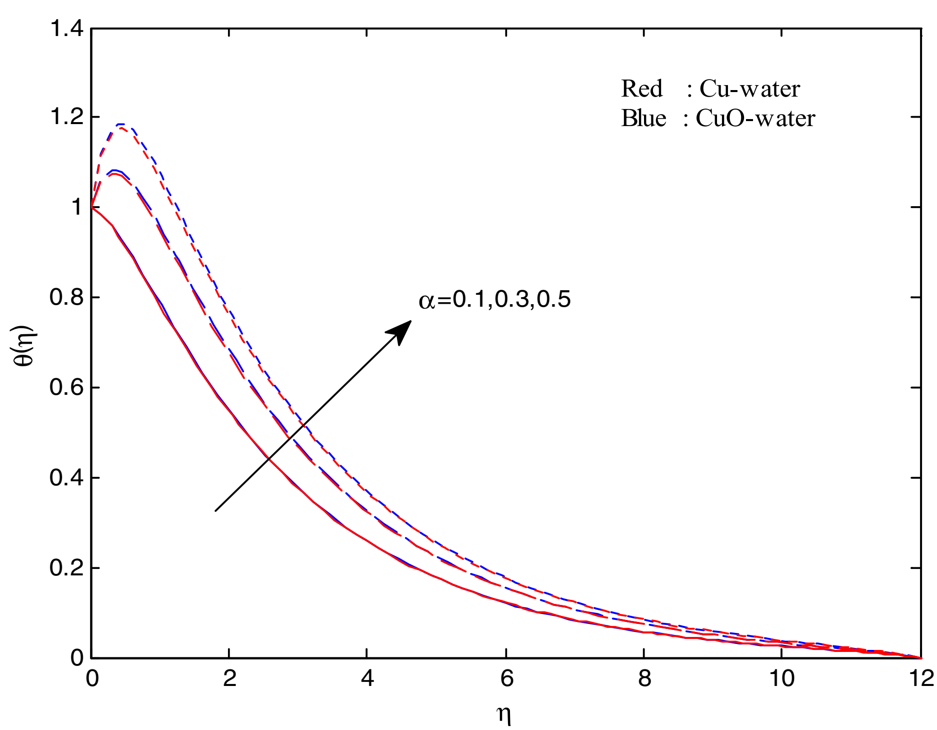

Fig. 10. Variation in temperature profiles for different values of α.

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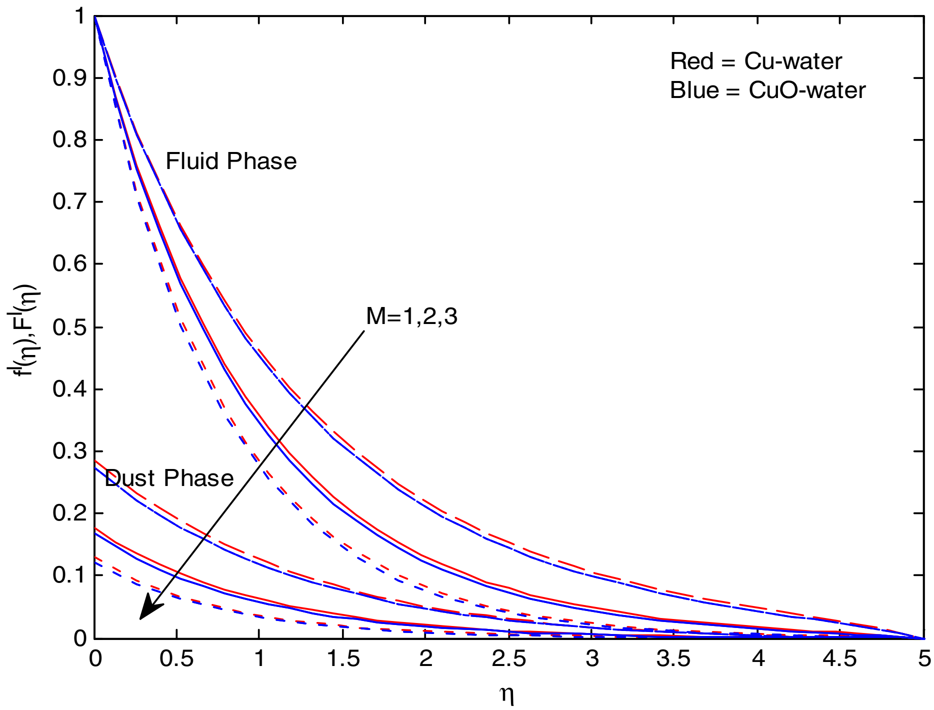

Fig. 11. Variation in velocity profiles of fluid and dust phases for different values of *M*.

the general physical behavior of suction/injection parameter. Fig. 18 depicts the influence of nano particle shape on the temperature profiles of the flow. It is clear that the enhancement in the value of n depreciates the temperature profiles of the flow. From this we can conclude that cylindrical shaped nano particles ($n = 3/2$) show better heat transfer performance compared with spherical shaped nano particles ($n = 3$).

Table 2 displays the comparison of the present results with existing results of Elbashbeshy et al. [26] for reduced Nusselt number. Present results have an excellent agreement with existing results

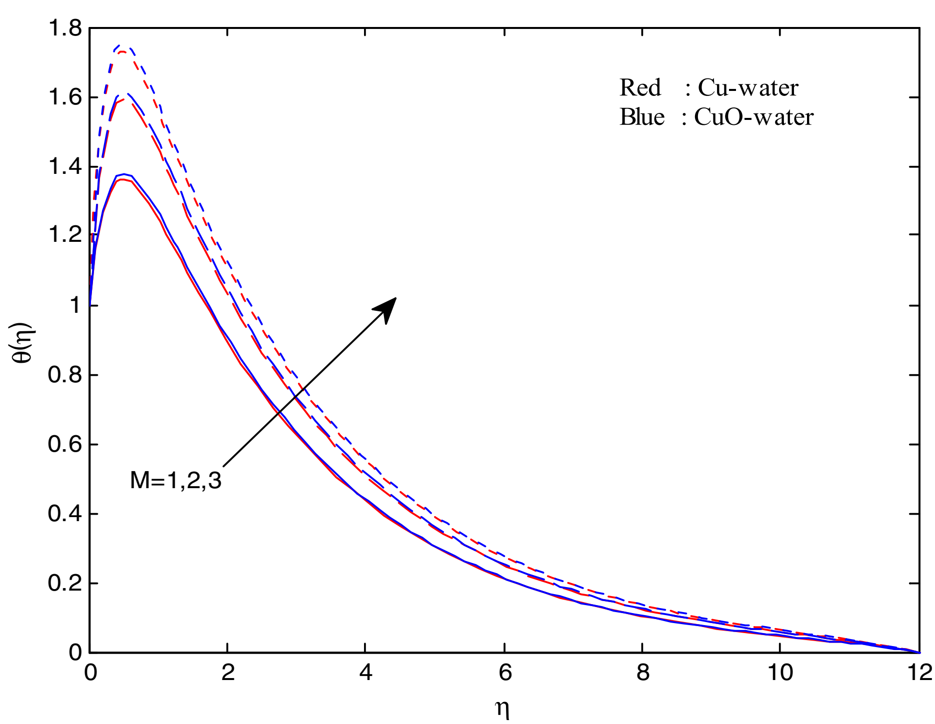

Fig. 12. Variation in temperature profiles for different values of *M*.

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Table 2
Comparison of the values of $-\theta'(0)$ when $M = A = \beta = R = E = \phi = \phi_d = N = S = 0$.

Pr	Elbashbeshy et al. [26]	Present results
0.72	0.76728	0.7672761
1	0.95478	0.9547823
2	1.47146	1.4714581
3	1.86907	1.8690721
5	2.50013	2.5001301
10	3.66037	3.6603723

under some special assumptions. This shows the validity of the present results along with the numerical technique we used in this study. Tables 3 and 4 respectively display the influence of governing parameters on skin friction coefficient and local Nusselt number for Cu-water and CuO-water dusty nanofluids. It was observed from the tables that increase in magneticfield parameter and volume fraction of dust particles declines the friction factor along with the heat transfer rate. Enhancement in unsteadiness parameter and fluid particle interaction parameter reduces the skin friction

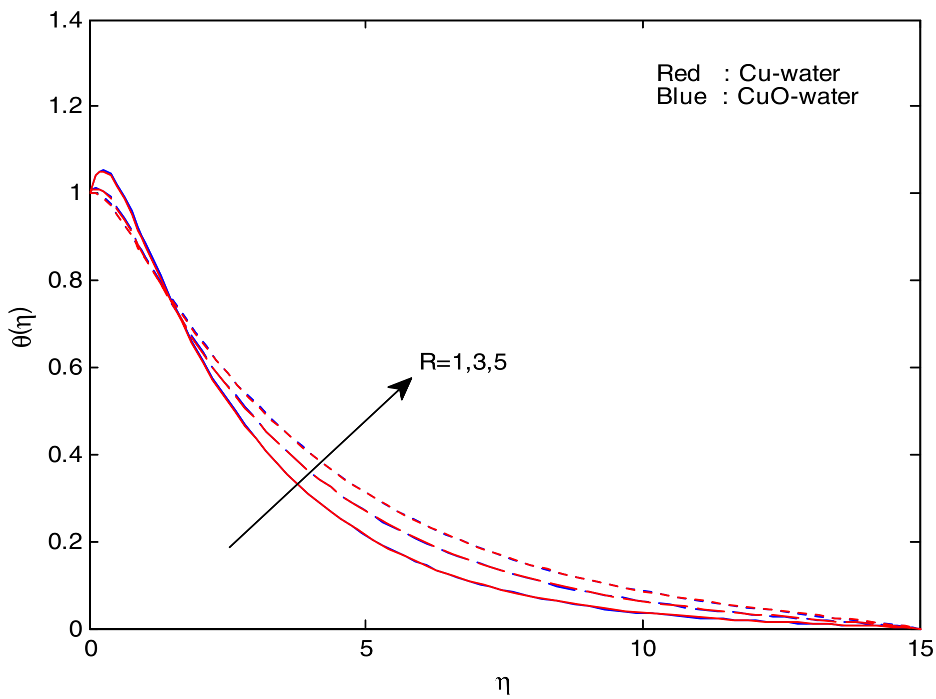

Fig. 13. Variation in temperature profiles for different values of *R*.

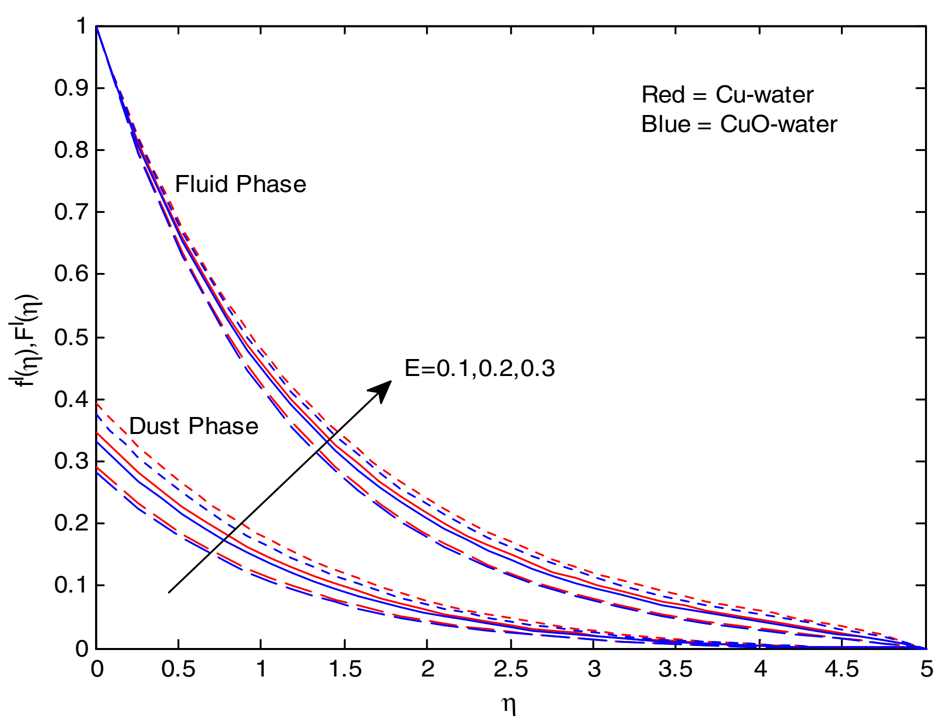

Fig. 14. Variation in velocity profiles of fluid and dust phases for different values of *E*.

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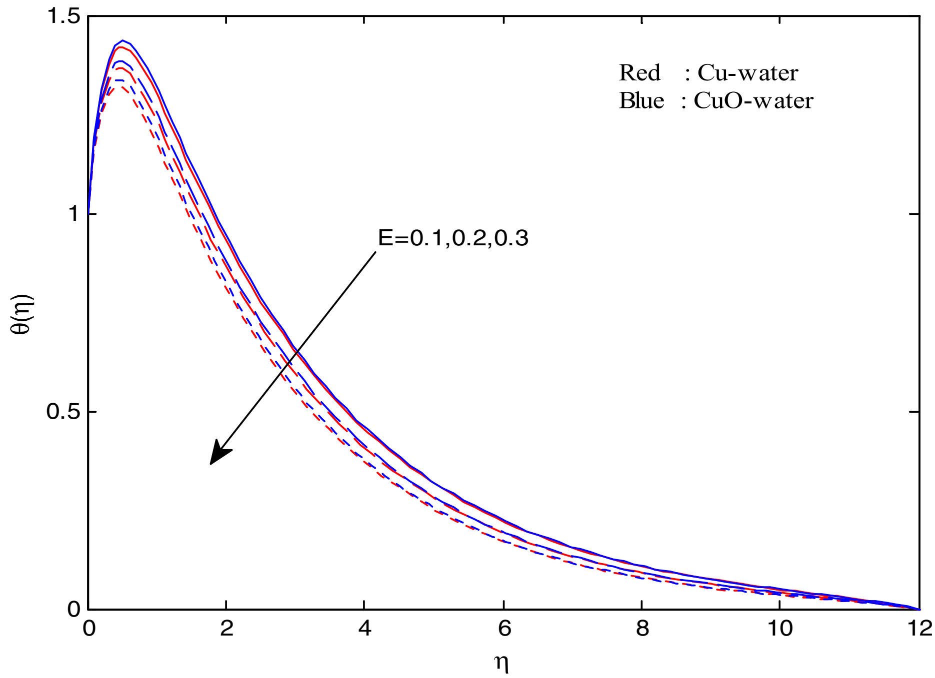

Fig. 15. Variation in temperature profiles for different values of *E*.

Table 3
Variation in $f''(0)$ and $-\theta'(0)$ for Cu-water dusty nanofluid.

M	R	ϕ	ϕ_d	β	A	S	$f''(0)$	$-\theta'(0)$
1							−1.561840	2.254144
2							−1.839995	1.808287
3							−2.081123	1.539637
	1						−1.561840	2.254144
	2						−1.561840	1.542794
	3						−1.561840	1.157965
		0.1					−1.561720	2.352369
		0.2					−1.365966	2.567185
		0.3					−1.265964	2.766619
			0.1				−1.561840	2.254144
			0.2				−1.649159	2.092565
			0.3				−1.754082	1.926384
				0.1			−1.123516	0.120926
				0.5			−1.561840	2.254144
				0.9			−1.918901	4.204056
					0.5		−1.561840	2.254144
					0.7		−1.747527	3.247056
					0.9		−1.918901	4.204056
						−1	−1.698309	1.675188
						0	−1.622546	1.893080
						1	−1.561840	2.254144

Table 4
Variation in $f''(0)$ and $-\theta'(0)$ for CuO-water dusty nanofluid.

M	R	ϕ	ϕd	β	A	S	$f''(0)$	$-\theta'(0)$
1							−1.541720	2.302369
2							−1.817069	1.843278
3							−2.056565	1.567336
	1						−1.541720	2.302369
	2						−1.541720	1.577610
	3						−1.541720	1.185285
		0.1					−1.541720	2.302369
		0.2					−1.345966	2.547185
		0.3					−1.245964	2.736619
			0.1				−1.541720	2.302369
			0.2				−1.628161	2.135593
			0.3				−1.732163	1.964310
				0.1			−1.107813	0.127414
				0.5			−1.541720	2.302369
				0.9			−1.897238	4.288405
					0.5		−1.541720	2.302369
					0.7		−1.726478	3.314185
					0.9		−1.897238	4.288405
						−1	−1.648902	1.729464
						0	−1.588282	1.944349
						1	−1.541720	2.302369

coefficient and enhances the Nusselt number. Radiation parameter does not show any influence on friction factor but it decreases the heat transfer rate. A rise in the volume fraction of nano particles and suction/injection parameter enhances the friction factor and heat transfer rate.

4. Conclusions

This study presents the MHD radiative flow and heat transfer characteristics of a dusty nanofluid over an exponentially permeable stretching surface in presence of volume fraction of dust and nano particles. In this study we considered two types of nanofluids namely Cu-water and CuO-water embedded with dust particles. The governing equations are transformed into nonlinear ordinary differential equations by using similarity transformation and then solved numerically using Runge–Kutta based shooting technique. The effects of non-dimensional governing parameters on velocity profiles for fluid and dust phases and temperature profiles are discussed and presented through graphs. Also, friction factor and Nusselt numbers are discussed and presented for two dusty nanofluids separately.

From this study the conclusions are as follows:

- An increase in the volume fraction of the nano particles enhances the friction factor and heat transfer rate.

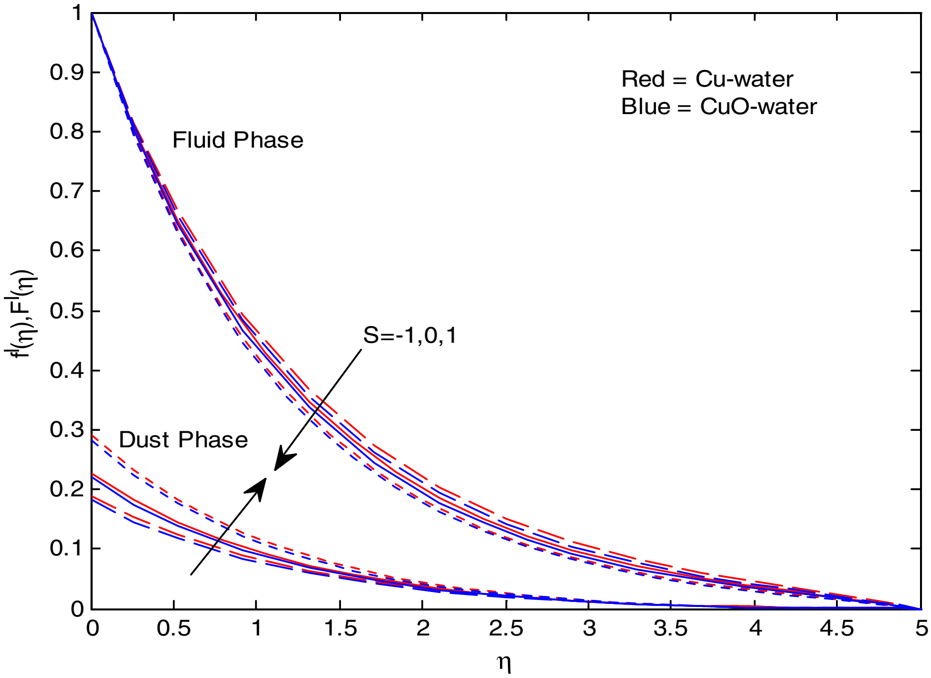

Fig. 16. Variation in velocity profiles of fluid and dust phases for different values of *S*.

- Magneticfield parameter has the tendency to reduce the skin friction coefficient and Nusselt number.
- A rise in the value of fluid particle interaction parameter enhances the heat transfer rate and reduces the friction factor.
- Heat transfer performance of CuO-water dusty nanofluid is good compared with Cu-water dusty nanofluid.
- Suction/injection parameter has the tendency to reduce the temperature profiles of the flow.

Acknowledgements

The authors wish to express their thanks to the very competent anonymous referees for their valuable comments and

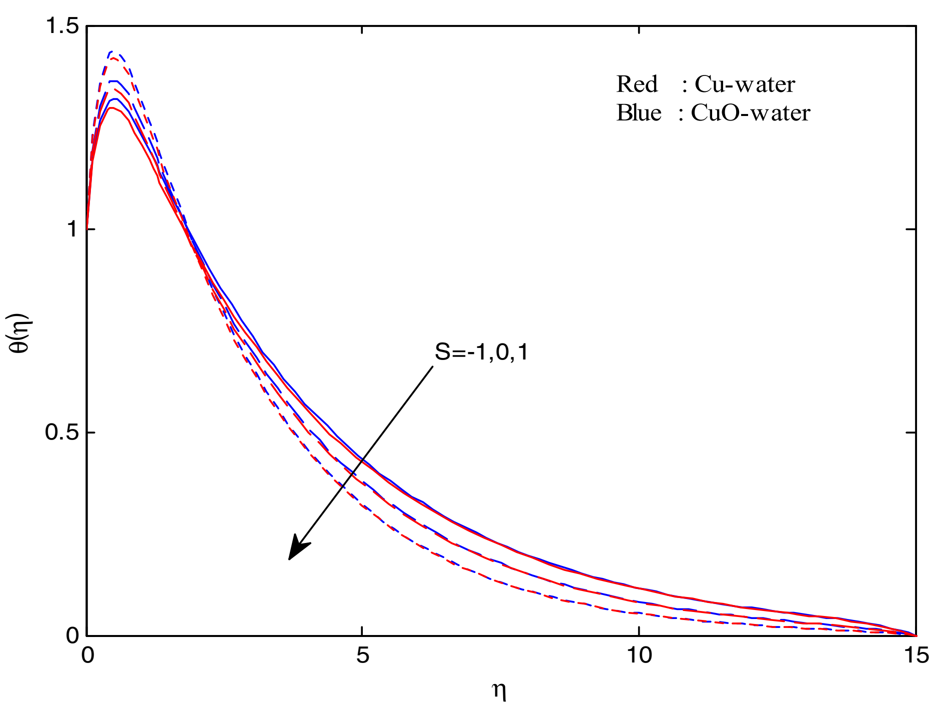

Fig. 17. Variation in temperature profiles for different values of *S*.

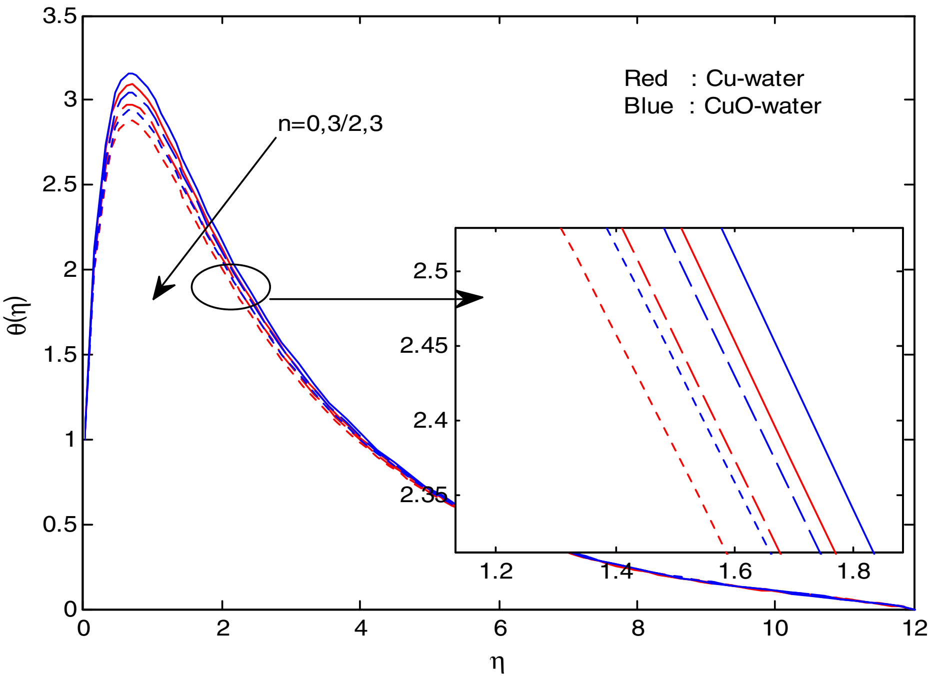

Fig. 18. Variation in temperature profiles for different values of *n*.

suggestions. Authors from Gulbarga University acknowledge the UGC for financial support under the UGC Dr.D.S.Kothari Post-Doctoral Fellowship Scheme (No.F.4-2/2006 (BSR)/MA/13-14/0026).

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