ARTICLE IN PRESS
Ain Shams Engineering Journal (2014) xxx, xxx–xxx

Ain Shams University
Ain Shams Engineering Journal
www.elsevier.com/locate/asej
www.sciencedirect.com

CIVIL ENGINEERING

Vector machine techniques for modeling of seismic liquefaction data

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Received 10 September 2013; revised 3 December 2013; accepted 24 December 2013

KEYWORDS
Support Vector Machine;
Least Square Support Vector Machine;
Relevance Vector Machine;
Liquefaction;
Probability

Abstract This article employs three soft computing techniques, Support Vector Machine (SVM); Least Square Support Vector Machine (LSSVM) and Relevance Vector Machine (RVM), for prediction of liquefaction susceptibility of soil. SVM and LSSVM are based on the structural risk minimization (SRM) principle which seeks to minimize an upper bound of the generalization error consisting of the sum of the training error and a confidence interval. RVM is a sparse Bayesian kernel machine. SVM, LSSVM and RVM have been used as classification tools. The developed SVM, LSSVM and RVM give equations for prediction of liquefaction susceptibility of soil. A comparative study has been carried out between the developed SVM, LSSVM and RVM models. The results from this article indicate that the developed SVM gives the best performance for prediction of liquefaction susceptibility of soil.

1. Introduction

There is a lot of engineering problems that require the analysis of uncertain and imprecise information. Generally, the development of proper model to explain past behaviors or predict future ones is a difficult task due to incomplete understanding of the problem. Soft computing technique is generally used to solve this type of problem. This technique is developed by Zadeh Iizuka [1]. The most commonly used soft computing technique is Artificial Neural Network (ANN). ANN has been used to solve different problems in engineering [2–6]. However, ANN has the following limitations.

- Unlike other statistical models, ANN does not provide information about the relative importance of the various parameters [7].
- The knowledge acquired during the training of the model is stored in an implicit manner and hence it is very difficult to come up with reasonable interpretation of the overall structure of the network [8].
- In addition, ANN has some inherent drawbacks such as slow convergence speed, less generalizing performance, arriving at local minimum and over-fitting problems.

This article adopts three soft computing techniques {Support Vector Machine (SVM), Least Square Support Vector Machine (LSSVM) and Relevance Vector Machine (RVM)} for prediction of liquefactions susceptibility of soil. Geotechnical

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Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

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http://dx.doi.org/10.1016/j.asej.2013.12.004

engineers use the different soft computing techniques for prediction of seismic liquefaction potential of soil [9–13]. The database has taken from the work of Hanna et al. [14]. The dataset contains information about depth of the soil layer (z), corrected standard penetration blow numbers ($N_{1,60}$), percent finest content less than 75 μm ($F \leqslant 75$ μm, %), depth of ground water table (d_w), total and effective overburden stresses ($\sigma_{vo}, \sigma'_{vo}$), threshold acceleration (a_t), cyclic stress ratio (τ_{av}/σ'_{v0}), shear wave velocity (V_s), internal friction angle of soil (ϕ'), earthquake magnitude (M_w), maximum horizontal acceleration at ground surface (a_{max}) and status of soil (status of soil means the condition of soil after earthquake). SVM is a new soft computing technique introduced by Vapnik [15]. There are lots of applications of SVM in engineering [16–20,11–15]. LSSVM is a modified version of SVM [21]. Researchers have successfully used LSSVM for solving different problems [22–26]. RVM was introduced by Tipping [27]. The application of RVM is demonstrated in various literatures [16,28–31]. This article gives equations for prediction of liquefaction susceptibility of soil based on the developed SVM, LSSVM and RVM models. A comparative study has been presented between the developed SVM, LSSVM and RVM models.

2. Details of SVM

SVM was developed based on Structural Risk Minimization Principle [15]. Let us consider the following training dataset (D)

$$D = (x_1, y_1), (x_2, y_2), \ldots, (x_l, y_l), x_i \in R^N \text{ and } y_i \in \{+1, -1\} \quad (1)$$

where x is input, R^N is N-dimensional vector space, and y is output.

In this article, a value of -1 is assigned to the liquefied sites while a value of $+1$ is assigned to the non-liquefied sites so as to make this a two-class classification problem. This study uses z, $N_{1,60}$, $F \leqslant 75$ μm, $d_w, \sigma_{vo}, \sigma'_{vo}, a_t, \tau_{av}/\sigma'_{v0}$, V_s, ϕ', M_w, and a_{max} as input variables. So, $x = [z, N_{1,60}, F \leqslant 75\mu m, d_w, \sigma_{v0}, \sigma'_{v0}, a_t, \tau_{av}/\sigma'_{vo}, V_s, \phi', M_w, a_{\max}]$.

SVM uses the following form for prediction of y.

$$y = \text{sign}(w.\phi(x) + b) \quad (2)$$

$\phi(x)$ represents a high-dimensional feature space which is nonlinearly mapped from the input space x, w is weight and b is bias. The following optimization problem has been used to determine the value of w and b [15].

$$\text{Minimize}: \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}\xi_i$$

$$\text{Subjected to}: y_i(w.x_i + b) \geqslant 1 - \xi_i \quad (3)$$

The constant $0 < C < \infty$, a parameter defines the trade-off between the number of misclassification in the training data and the maximization of margin and ξ_i is called slack variable. This optimization problem (4) is solved by Lagrangian Multipliers [15] and its solution is given by,

$$y = sign\left(\sum_{i=1}^{l}\alpha_i y_i K(x_i, x) + b\right) \quad (4)$$

where α_i is Lagrange multipliers and $K(x_i,x)$ is kernel function.

This article uses the above SVM for prediction of liquefaction susceptibility of soil. To develop SVM, the data have been divided into the following two groups:

Training Dataset: This is required to construct the SVM model. This article uses 434 datasets out of 620 as training dataset.

Testing Dataset: This is used to verify the developed SVM. The remaining 185 datasets have used a testing dataset.

Polynomial function ($K(x_i, x) = \{(x_i.x) + 1\}^d$, d = degree of polynomial) has been used as a kernel function. Input variables have been normalized between 0 and 1. The program of SVM has been constructed by MATLAB.

3. Details of LSSVM

This section will describe a brief introduction of LSSVM. The details of LSSVM have been given by Suykens and Vandewalle [21]. The main difference between SVM and LSSVM is that LSSVM uses a set of linear equations for training while SVM uses a quadratic optimization problem [31].

Let us consider the following training dataset (D)

$$D = (x_1, y_1), (x_2, y_2), \ldots, (x_l, y_l), x_i \in R^N \text{ and } y_i \in \{+1, -1\} \quad (5)$$

where x is input, R^N is N-dimensional vector space, and y is output.

In LSSVM, a value of -1 is assigned to the liquefied sites while a value of $+1$ is assigned to the non-liquefied sites so as to make this a two-class classification problem. This study uses z, $N_{1,60}$, $F \leqslant 75$ μm, $d_w, \sigma_{vo}, \sigma'_{vo}, a_t, \tau_{av}/\sigma'_{v0}$, V_s, ϕ', M_w, and a_{max} as input variables. So, $x = [z, N_{1,60}, F \leqslant 75\mu\text{m}, d_w, \sigma_{v0}, \sigma'_{v0}, a_t, \tau_{av}/\sigma'_{vo}, V_s, \phi', M_w, a_{\max}]$.

LSSVM uses the following equation for prediction of y.

$$y = sign[w^T\phi(x) + b] \quad (6)$$

$\phi(x)$ represents a high-dimensional feature space which is nonlinearly mapped from the input space x, w is weight and b is bias.

LSSVM adopts the following optimization problem for determination of w and b.

$$\text{Min}: \frac{1}{2}w^T w + \frac{\gamma}{2}\sum_{i=1}^{l}e_i^2$$

$$\text{Subject to}: e_i = y_i - (w^T\varphi(x_i) + b), i = 1, \ldots, l \quad (7)$$

This optimization problem (4) is solved by Lagrangian Multipliers [21], and its solution is given by

$$y = sign\left(\sum_{i=1}^{l}\alpha_i y_i K(x_i, x) + b\right) \quad (8)$$

where α_i is Lagrange multipliers and $K(x_i.x)$ is kernel function. This study adopts radial basis function ($K(x_i, x) = \exp\left\{-\frac{(x_i - x)^T(x_i - x)}{2\sigma^2}\right\}$ where σ is width of radial basis function) as kernel function.

It should be noted that for model calibration and verification using LSSVM, the same training data sets, testing data sets and normalization technique previously used for the SVM modeling are utilized and LSSVM is implemented using the MATLAB software.

4. Details of RVM

RVM was developed to introduce the Bayesian principle to the SVM model [32]. Let us consider a set of example of input vectors $\{x_i\}_{i=1}^{N}$ is given along with a corresponding set of targets $y = \{y_i\}_{i=1}^{N}$. In this study, y_i should be 0 for "Liquefaction" and +1 for "No Liquefaction". RVM uses the following equation for prediction of y_i.

$$y_i = w^T\psi(x_i) + \varepsilon_i \text{ and } \varepsilon_i \sim N(0, \sigma^2) \quad (9)$$

where $\psi(x_i)$ is basis function.

Assuming a Bernoulii distribution for $P(y/x)$, the likelihood is written as [28]:

$$P(y/w) = \prod_{i=1}^{N}\sigma\{y(x_i; w)\}^{y_i}[1 - \sigma\{y(x_i; w)\}]^{1-y_i} \quad (10)$$

We cannot integrate the weights analytically. The RVM adopts the following separable Gaussian prior, with a distinct hyper-parameter, α_i, for each weight,

$$p(w/\alpha) = \prod_{i=1}^{N}N(w_i/0, \alpha_i^{-1}) \quad (11)$$

The optimal parameters of the model are then derived by minimizing the penalized negative log-likelihood,

$$log\{P(y/w)p(w/\alpha)\} = \sum_{i=1}^{N}[y_i log y_i + (1 - y_i) log(1 - y_i)] - \frac{1}{2}w^T A w \quad (12)$$

If we differentiate twice Eq. (10), the expression is given below [33]:

$$\nabla w \nabla w log p(w/y, \alpha) = -(\Phi^T B \Phi + A) \quad (13)$$

where $B = diag(\beta_1, \ldots, \beta_N)$ is a diagonal matrix with $\beta_n = \sigma\{y(x_n)\}[1 - \sigma\{y(x_n)\}]$

The following equation has been used for updating hyper-parameter

$$\alpha_i^{new} = \frac{1 - \alpha_i \sum_{ii}}{\mu_i^2} \quad (14)$$

where μ_i is the ith posterior mean weight, $\sum_{ii}$ is the ith diagonal element of the posterior weight covariance. The process is repeated until the ultimate goal is met. The property of this optimization problem is that the value of many w will be zero. The nonzero weights are called relevance vectors.

RVM uses the same training dataset, testing dataset and normalization technique as used by SVM and LSSVM. Radial basis function has been used as basis function. RVM has been developed by MATLAB.

5. Results and discussion

The performance of developed SVM, LSSVM, and RVM has been assessed by using the following equation.

$$\text{Testing/Training performance (\%)} = \left(\frac{\text{No of data predicted accurately by SVM, LSSVM and RVM}}{\text{Total data}}\right) \times 100 \quad (15)$$

For SVM, the design value of C and d has been determined by trail and error approach. Fig. 1 shows the fluctuation of testing performance and number for support vector with C. From Fig. 1, it is clear that the number of support vectors is decreasing with an increase in C. For the best SVM model, the less number of support vector as well as high testing performance (%) is desirable. SVM produces best testing performance (%) and lowest number of support vector for $C = 130$ and $d = 4$. The design value of $C(C = 130)$ and $d(d = 4)$ produces 96.32% training performance, 86.49% testing performance and 112 support vectors. The developed SVM gives the following equation (by putting $K(x_i, x) = \{(x_i.x) + 1\}^d$, $d = 4$, $b = 0$ and $l = 435$ in Eq. (4)) for prediction of liquefaction susceptibility of soil.

$$y = sign\left(\sum_{i=1}^{435}\alpha_i y_i\{(x_i x) + 1\}^4\right) \quad (16)$$

Fig. 2 shows the value of α_i.

The design value of γ and σ has been determined by trail and error approach in the LSSVM model. Fig. 3 shows the variation in testing performance (%) with γ. It is observed from Fig. 3 that the developed LSSVM gives best performance at $\gamma = 180$ and $\sigma = 10$. The developed LSSVM produces 97.24% training performance and 85.41% testing performance. The following equation (by putting $K(x_i, x) = \exp\left\{-\frac{(x_i - x)^T(x_i - x)}{2\sigma^2}\right\}$, $\sigma = 10$, $l = 435$, and $b = 1.320$ in Eq. (8)) has been presented from the developed LSSVM.

$$y = \text{sign}\left(\sum_{i=1}^{435}\alpha_i y_i \exp\left\{-\frac{(x_i - x)^T(x_i - x)}{200}\right\} + 1.320\right) \quad (17)$$

The values of α have been depicted in Fig. 4.

For RVM, the design value of σ has been determined by trail and error approach. Fig. 5 illustrates the variation in testing performance (%) and number of relevance vector with σ. It can be seen from Fig. 5 that the testing performance (%) and number of relevance vector increase with an increase σ. Fig. 5 also shows that the developed RVM gives best performance at $\sigma = 0.06$ and number of relevance vector = 265. The developed RVM produces 86.44% training performance and 74.59% testing performance. The developed RVM gives the following equation for prediction of liquefaction susceptibility of soil.

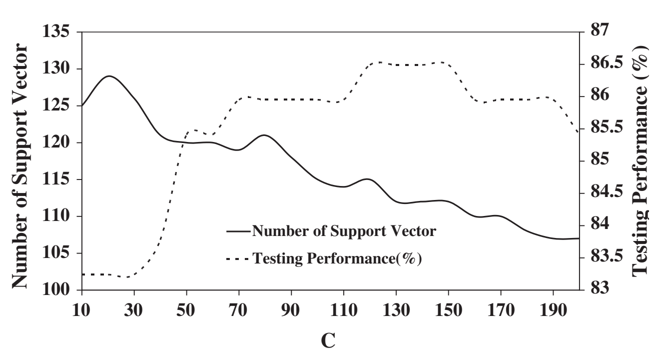

Figure 1 Variation in testing performance (%) and number of support vector with C.

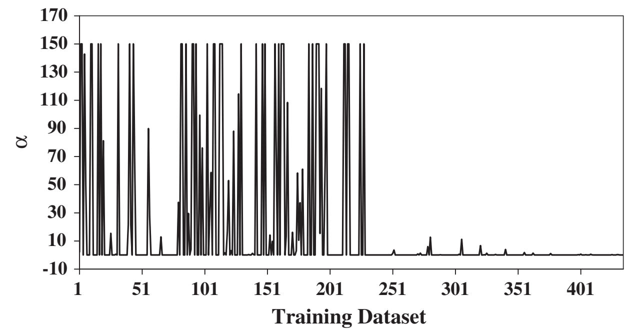

Figure 2 Values of α for the SVM model.

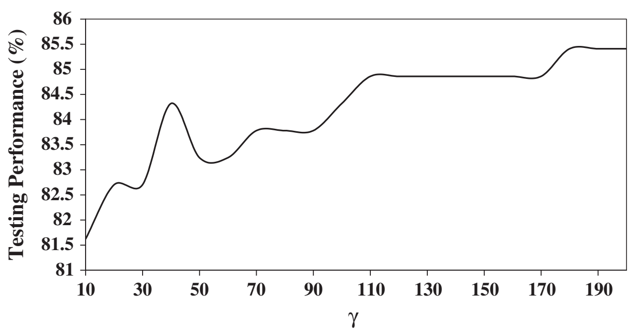

Figure 3 Variation in testing performance (%) with γ.

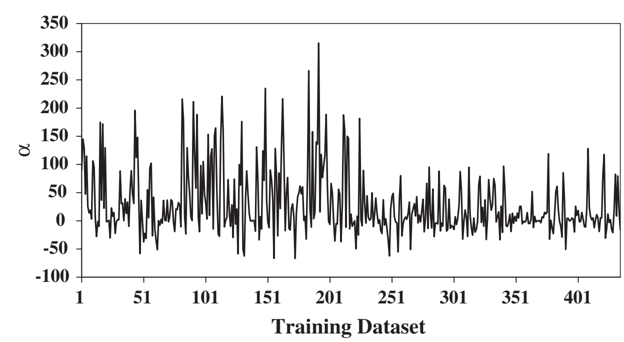

Figure 4 Values of α for the LSSVM.

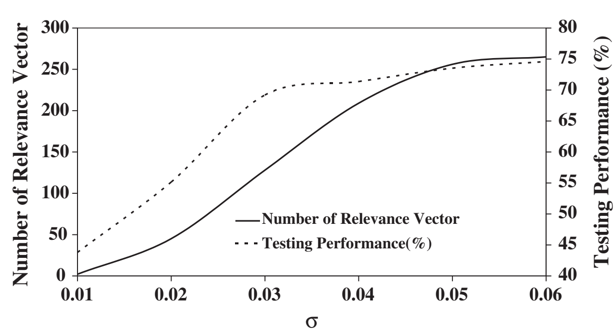

Figure 5 Variation in testing performance (%) and number of relevance vector with σ.

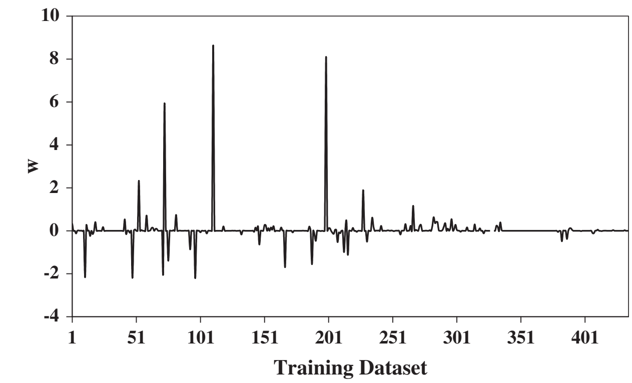

Figure 6 Values of w for the RVM.

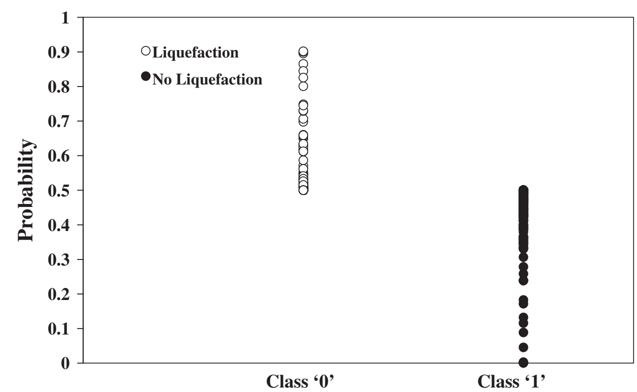

Figure 7 Probability of the training and testing dataset from the RVM.

$$y = \sum_{i=1}^{435} w_i \exp\left\{-\frac{(x_i - x)^T (x_i - x)}{0.0072}\right\} \tag{18}$$

Fig. 6 shows the value of w.

The developed RVM gives the probabilistic output. Fig. 7 depicts the probability of training and testing dataset. It is observed from Fig. 7 that the liquefiable soil fell within the 0.5–1 probability range and most non-liquefiable soil fell within the 0–0.5 range. Thus, the RVM probabilistic output can be used to determine the liquefaction susceptibility of soil. If the output is less than 0.5, the probability of liquefaction is decreased. If the output is more than 0.5, the probability of liquefaction is increased.

A comparative study has been carried out between the developed SVM, LSSVM and RVM models. Table 1 shows the comparison. The developed SVM and LSSVM outperform the RVM. The performance of the SVM and LSSVM is comparable. Receiver Operating Characteristics (ROC) has been developed for the SVM, LSSVM and RVM models. Fig. 8 shows the ROC curves. The area under ROC curve is maximum for the SVM. The developed RVM gives the minimum area under ROC curve. Therefore, the performance of SVM is best. The developed SVM and RVM use 112 and 265 training dataset for final model respectively. So, the SVM and RVM produce sparse solution. Whereas, the developed LSSVM model uses all training dataset for final prediction. Therefore, it does not produce any sparse solution. The

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Table 1 Comparison between SVM, LSSVM and RVM models.

Model	Training performance (%)	Testing performance (%)
SVM	96.32	86.49
LSSVM	97.24	85.41
RVM	86.44	74.59

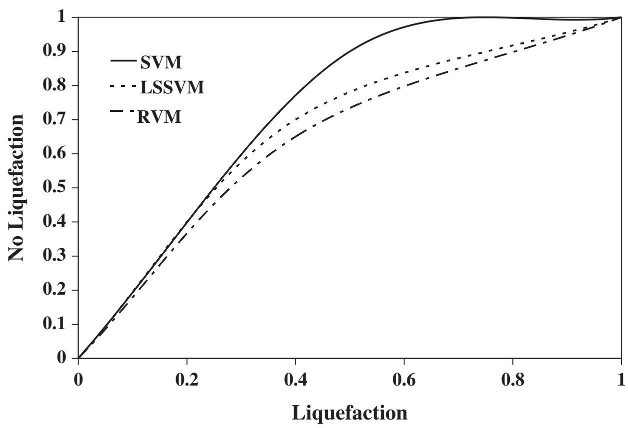

Figure 8 ROC curves of the developed SVM, LSSVM and RVM models.

developed SVM and LSSVM uses two tuning parameters (For SVM: C and d; For LSSVM: γ and σ). Whereas, there is only one tuning parameter for the RVM model.

6. Conclusion

This article has described SVM, LSSVM and RVM for prediction of liquefaction susceptibility of soil. 620 data have been utilized to develop the SVM, LSSVM and RVM models. The performance of the SVM and LSSVM is better than that of the RVM. The developed equations can be used by the users for determination of liquefaction susceptibility of soil. The developed SVM and LSSVM produce almost same performance. The obtained probability from the RVM can be used to determine uncertainty. In summary, it can be concluded that SVM, LSSVM and RVM can be used for solving different problems in engineering.

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