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A DEA approach for optimization of multiple responses in Electrical Discharge Machining of AISI D2 steel

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Abstract

Present research proposes an optimization methodology for the selection of best process parameters in multi-response situation. Experiments have been conducted on a die-sinking electric discharge machine under different conditions of process parameters. A response surface methodology (RSM) is adopted to establish effect of various process parameters such as discharge current (Ip), pulse on time (Ton), duty factor (τ) and flushing pressure (Fp) on four important responses like material removal rate (MRR), tool wear rate (TWR), surface roughness (Ra) and circularity (r_1/r_2) of machined component. Since the natures of responses are contradicting in nature, it is difficult to find a single combination of machining parameters that provides the best performance satisfying all responses simultaneously. In order to achieve best machining condition, an equivalent single response capable of representing all individual responses is needed. The work includes data envelopment analysis (DEA) to obtain relative efficiency for each experimental run treating as decision making unit (DMU). Each DMU is evaluated using LINGO software to obtain relative efficiency. The relative efficiency is ranked in ascending order and average ranked value (ARV) is calculated to find the optimal solution. Finally, the optimal setting capable of improving all the responses simultaneously is found to be Ip=7 amp, Ton= 200 μs, τ = 90%, and Fp = 0.4 kg/cm^2. With this best combination of factorial level, the experimental values of responses are obtained as MRR=13.9600 mm^3/min, TWR=0.0201 mm^3/min, Ra=4.9300 μm and circularity=0.8401.

Keywords: RSM; DEA; MRR; TWR; DMU; Relative efficiency.

Nomenclature

Ip	discharge current (amp)
Ton	pulse on time (μs)
τ	duty factor (%)
Fp	flushing pressure (kg/cm^2)
T	time of machining
Greek symbols	
ρ_w	density of work piece
ρ_t	density of tool material

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1. Introduction

Electrical Discharge Machining (EDM) is a non-traditional machining process which is more efficient than traditional machining process due to ease of machining of difficult-to-machine materials with complex shapes. EDM is used for machining of toughened and high strength conductive materials which is hard enough to cut by traditional processes. It has many applications in manufacturing sectors especially industries like aerospace, ordinance, automobile and general engineering [1]. In EDM, the material removal mechanism is owing to spark erosion process. Due to spark, huge amount of heat is generated which is sufficient to melt or vaporize the material along with tool and the molten mass is removed by flushing of dielectric. So the tool profile is transferred to work piece. From exhaustive literature review, it is found that there are few controllable parameters such as discharge current (Ip), pulse on time (Ton), duty factor (τ) and flushing pressure (fp) largely influence machining performance [2, 3]. In this work, response surface methodology (RSM) has been used to study effect of various parameters on machining performance and develop empirical relations between controllable factors and responses. However, conventional techniques are suitable for optimization of single response problems. When number of responses is more than one, the conventional techniques breaks down. It is difficult to obtain best parametric combination that optimizes all the responses simultaneously. In this study, four responses such as material removal rate, tool wear rate, surface roughness, and circularity are considered.

Generally, in a multi-response optimization problem, the responses may be of three types, some responses may be "larger-the-best", some may be "nominal-the-best" and some may be "smaller-the-best" type. In such cases, the multiple responses are converted into an equivalent single response using technique for order preference by similarity to ideal solution (TOPSIS), principal component analysis (PCA), and fuzzy logic. The sum of the weighted responses has employed assigning a weight for each response to optimize a multi-response problem [4]. Regression technique based approach has been used by Reddy et al. [5] to optimize the multi-response problem. But unfortunately, regression approaches increase the complexity of computational process and thus, require statistical skills. PCA is employed to transform the multi-responses into some uncorrelated ones [6]. The principal components are then utilized to find the optimal factor levels for multiple-responses. But, PCA has limited application because of multivariate normally distributed random variable error terms. To alleviate these problems, a mathematical tool called data envelopment analysis (DEA) is used in this work to convert multi-responses into a single response. In DEA method, each experimental run is treated as a decision making unit (DMU). DEA is basically a fractional mathematical programming technique to evaluate the relative efficiency from homogeneous DMUs having multiple inputs and multiple outputs [7]. The relative efficiency so obtained is treated as representative single response for the multiple responses.

2. Material and method

The experiments are conducted on an Electronica Electra plus PS 50ZNC die sinking electric discharge machine (figure 1). The manufacturer-supplied EDM oil is taken as dielectric medium. These experiments have been conducted to investigate the effect of discharge current (Ip), pulse-on-time (Ton), duty factor (τ) and flushing pressure (fp) on responses. Here, duty factor (τ), is defined as τ =Ton/(Ton+Toff) in percentage where Toff is the pulse-off -time. In recent years, high strength to weight and high strength to volume material is preferred for various engineering applications. AISI D2 is one such material which is an air-hardening, high carbon, high chromium tool steel. Due to presence of large volume of carbides in the microstructure, it has an excellent abrasion resistance. The composition of D2 tool steel is C (1.55%), Mn (0.6%), Si (0.6%), Cr (11.8%), Mo (0.8%), V (0.8%) and rest is iron. Its other properties at room temperature (25 °C) are Density=7.7×103 kg/m^3, Poisson's Ratio=0.27-0.3, Elastic Modulus=190-210GPa, Tensile strength=1736MPa, Hardness=57HRC, Thermal conductivity=20w/mk, Thermal Expansion=10.4× 10-6/ºC at temperature 20-100°C or more. Since a large amount of heat is dealt in EDM owing to spark, the tool should be of a good conductive material with high melting point. Therefore, pure copper in cylindrical shape of 25mm diameter is taken as the tool material. The EDM operation is performed on D2 steel having 6mm thick and 85mm diameter cylindrical work piece. Initially the weight of tool and work piece is measured in a in a high precision electronic weight measuring machine manufactured by Sansui Electronics (P) Ltd. (least count 10^{-3}). Then, the tool is connected in tool holder, work piece is placed in position and dielectric is allowed to fill the tank. The experiment is conducted as per Box-Behnken RSM design and final weight of tool and work piece is noted down. There are twenty-seven experimental runs to be performed in Box-Behnken RSM design with three levels of four factors and three center points. The layout of experimental runs is shown in table 1. Each experiment is run for one hour and four responses are obtained as:

(i) $\mathrm{MRR} = \frac{1000 \times \Delta W_w}{\rho_w \times T}$ (ii) $\mathrm{TWR} = \frac{1000 \times \Delta W_t}{\rho_t \times T}$

(where ΔW_w and ΔW_t are the weight of material removed from work piece and tool respectively, ρ_w and ρ_t are the density of work piece and tool respectively, T is the time of machining).
(iii) Roughness is measured by portable stylus type profilometer talysurf (Taylor Hobson, Surtronic 3+)
(iv) Circularity is calculated as the ratio of minimum to maximum ferret's diameter (figure 2) [8]. The diameters are measured using magnified photographs obtained through microscope (RADIAL INSTRUMENT with Samsung camera attachment, 45-X magnification).

Figure 1. Die Sinker EDM Model: PS 50ZNC

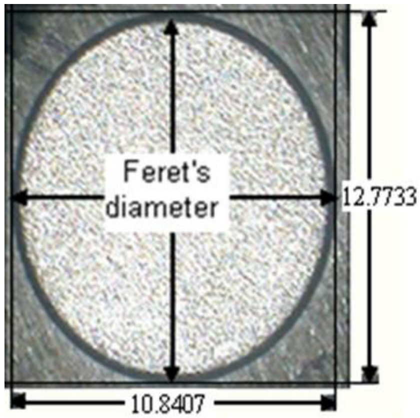

Figure 2. Ferret's diameter.

2.1. Data Envelopment Analysis (DEA)

DEA is a mathematical tool applied when multiple inputs and multiple outputs make the comparison difficult [9]. This technique is based on linear programming (LP) used to calculate relative efficiency for a set of experiment where each experiment is known as decision making unit (DMU).The relative efficiency is the ratio of weighted sum of the DMU's of outputs and weighted sum of the DMU's of inputs [10, 11].

Relative efficiency = $E_k = \dfrac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}}$ where k is DMU number.

In this paper, the CCR model of DEA is used to calculate the relative efficiency. This technique includes 'n' DMU's and each DMU with 'm' inputs and 's' outputs to be evaluated. Suppose individually evaluated DMU on any trial be designated as DMUo, where 'o' is DMU number ranging from 1 to n. The relative efficiency, Eo, of DMUo with inputs of x_{io} (i=1,……,m) and outputs of y_{ro} (r=1,……,s) is evaluated by CCR model as follows:

$$E_0 = \mathrm{Max}\theta = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

$$\text{Subjected to: } \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1$$

$$j = 1, \ldots, n;\quad u_1, u_2, \ldots, u_s \ge 0;\quad v_1, v_2, \ldots, v_m \ge 0$$

where u_r is the virtual weight for r^{th} output and v_i are the virtual weights for i^{th} input and θ is the scalar. The objective function for relative efficiency is the ratio of the sum of the weighted outputs with respect to the sum of the weighted inputs. The first constraint ensures that relative efficiency E_j lies between zero and one for all the n DMUs. The CCR model is a nonlinear function which is difficult to program, therefore it can be transformed into a linear model by setting the sum of the weighted inputs equal to one. This model is called the "input oriented" CCR model, which can be expressed as follows:-

$$E_o = \text{Max}\ \theta = \sum_{r=1}^{s} u_r y_{ro} \quad \text{subjected to:} \quad \sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} \le \sum_{i=1}^{m} v_i x_{ij}$$

$$j = 1, \ldots, n; \quad u_1, u_2, \ldots, u_s \ge 0; \quad v_1, v_2, \ldots, v_m \ge 0$$

3. The proposed approach

Product/process have quality characteristic that describes their performance relative to customer requirements or expectations. Accordingly responses can be divided into three main types: the smaller-the-best (STB), the nominal-the-best (NTB), and the larger-the-better (LTB) responses. In practice, all the responses may not be same category. In this regard, the proposed approach for solving the multi-response problem using DEA is outlined in the following steps:

Step 1 Characterize the responses

Assume n experiments are conducted utilizing RSM and treat each experiment as a DMU. As mentioned earlier, the relative efficiency is defined as the sum of weighted outputs divided by the sum of the weighted inputs. Typically, higher efficiency indicates better performance which can be achieved if the sum of the weighted outputs increases and/or the sum of the weighted inputs decreases. To enhance the relative efficiency of each DMU and achieve the desired target of each quality response, set the input and output of each DMU as follows: (i) If responses are STB type, set responses as inputs for all DMUs (ii) If responses are LTB type set response as output for all DMUs [12].

Step 2 Normalize all the responses X_{ij} so that $0 \le Z_{ij} \ge 1$, where Z_{ij} is the normalized value. Normalization is carried out to avoid the scaling effect of responses measured in different scales. For responses of larger-the-better and smaller-the-better type, normalization is carried out using equation shown below.

$$Z_{ij} = \frac{x_{ij} - \min\{x_{ij}, j = 1,2,\ldots,n\}}{\max\{x_{ij}, j = 1,2,\ldots,n\} - \min\{x_{ij}, j = 1,2,\ldots,n\}}$$

Step 3 Solve each DMU by the input oriented CCR model to evaluate the relative efficiency. LINGO 10 software package is used to find the relative efficiency of DMU’s.

Step 4 Rank the DMU according to relative efficiency from low to high in ascending order so that least relative efficiency assigned rank 1. The average of ranked values (AVR_{ij}) is calculated for factor i and level j to find out the most favorable factor and its level combination which have better quality and productivity. The factor level at which AVR is higher provides better performance. To analyze the effect of factors, L_j is calculated as $L_j = \max_j\{AVR_{ij}\} - \min_j\{AVR_{ij}\}$. Larger L_j value indicates more significant effect of factor i. The suitable level (j^*) for controllable factor i is obtained by $j^* = \{j \mid \max_j \{AVR_{ij}\}\}$ [9].

4. Results and discussions

The responses are conflicting in unit, therefore responses are to be normalized to make them unit less and minimize the variation. The desired responses are, MRR is larger-the-best type, TWR is smaller-the-best type, Ra is smaller-the-best type, and circularity is larger-the-best type. The normalized values are lies in between 0 and 1.The normalized response data are shown in table 1 using the equation described in step 2. The twenty seven experiments are taken as twenty seven DMUs and LINGO linear program is proposed to solve it by CCR model DEA. Larger-the-best is taken as output and smaller-the-best is taken as input to find the relative efficiency (table 2). It is observed that relative efficiency is one for three DMUs. So, the DEA cannot distinguish here to find out the best DMU, so average ranked value (ARV) method is adopted to find out the best solution. The relative efficiency and its ranking are shown in table 2. The average ranked value is shown in table 3. From table 3, it is observed that the variation of AVR is highest in case of duty factor and therefore duty factor has highest effect on relative efficiency. The AVR values of each factor are plotted against respective levels to find the optimal solution as shown figure 3. Figure 3 shows that the optimal setting is Ip=7 amp, Ton= 200 μs, τ = 90%, and Fp = 0.4 kg/cm^2.

Table 1. RSM design and normalized response values

Expt. No.	Ip (A)	Ton (μs)	τ (%)	Fp (bar)	MRR (mm^3/min)	TWR (mm^3/min)	Ra (μm)	Circularity
1	3	100	85	0.3	0.123935	0.299107	0.267196	0.635328
2	7	100	85	0.3	0.87929	1	1	0.732194
3	3	300	85	0.3	0	0.123825	0	0
4	7	300	85	0.3	0.624895	0.56203	0.449735	0.669516
5	5	200	80	0.2	0.284237	0.119596	0.457672	0.732194
6	5	200	90	0.2	0.436221	0.211466	0.529101	0.732194
7	5	200	80	0.4	0.294347	0.214756	0.661376	0.48433
8	5	200	90	0.4	0.456442	0.211466	0.507937	0.660969
9	3	200	80	0.3	0.141271	0.117246	0.216931	0.088319
10	7	200	80	0.3	0.678523	0.605733	0.928571	0.746439
11	3	200	90	0.3	0.113169	0.167763	0.060847	0.632479
12	7	200	90	0.3	1	0.368421	0.73545	0.823362
13	5	100	85	0.2	0.381102	0.62688	0.57672	0.980057
14	5	300	85	0.2	0.175468	0.292763	0.349206	0.498575
15	5	100	85	0.4	0.395612	0.43891	0.777778	0.487179
16	5	300	85	0.4	0.195199	0.156955	0.164021	0.777778
17	3	200	85	0.2	0.10681	0.167763	0.134921	0.618234
18	7	200	85	0.2	0.779162	0.433741	0.753968	0.757835
19	3	200	85	0.4	0.087892	0	0.161376	0.626781
20	7	200	85	0.4	0.803625	0.774671	0.986772	0.632479
21	5	100	80	0.3	0.334624	0.56203	0.746032	0.603989
22	5	300	80	0.3	0.152474	0.211466	0.441799	0.77208
23	5	100	90	0.3	0.494926	0.430451	0.547619	0.749288
24	5	300	90	0.3	0.263925	0.255404	0.275132	0.467236
25	5	200	85	0.3	0.335438	0.299107	0.31746	0.626781
26	5	200	85	0.3	0.337886	0.167763	0.433862	1
27	5	200	85	0.3	0.327286	0.255404	0.507937	0.817664

Table 2. Relative Efficiency and its ranking.

Expt. No.	Ip (A)	Ton (μs)	τ (%)	Fp (bar)	Relative Efficiency	Ranking
1	3	100	85	0.3	0.39918	06
2	7	100	85	0.3	0.5979	13
3	3	300	85	0.3	0	01
4	7	300	85	0.3	0.9106	23
5	5	200	80	0.2	0.64435	15
6	5	200	90	0.2	0.71285	18
7	5	200	80	0.4	0.41478	07
8	5	200	90	0.4	0.74887	20
9	3	200	80	0.3	0.47587	09
10	7	200	80	0.3	0.52445	10
11	3	200	90	0.3	1	25
12	7	200	90	0.3	1	25
13	5	100	85	0.2	0.45336	08
14	5	300	85	0.2	0.39484	04
15	5	100	85	0.4	0.38095	03
16	5	300	85	0.4	0.99177	24
17	3	200	85	0.2	0.68462	17
18	7	200	85	0.2	0.75092	21
19	3	200	85	0.4	1	25
20	7	200	85	0.4	0.57237	11
21	5	100	80	0.3	0.32747	02
22	5	300	80	0.3	0.39884	05
23	5	100	90	0.3	0.63515	14
24	5	300	90	0.3	0.65953	16
25	5	200	85	0.3	0.72494	19
26	5	200	85	0.3	0.78027	22
27	5	200	85	0.3	0.57671	12

Table 3. ARV and variation of factors.

Factor	level 1	level 2	level 3	(Max-Min)ARV
Ip (A)	13.8333	12.6000	17.1667	3.3333
Ton (μs)	7.6667	17.0667	12.1667	9.4000
τ (%)	8.0000	13.9333	19.6667	11.6667
Fp (bar)	13.8333	13.4667	15.0000	1.5333

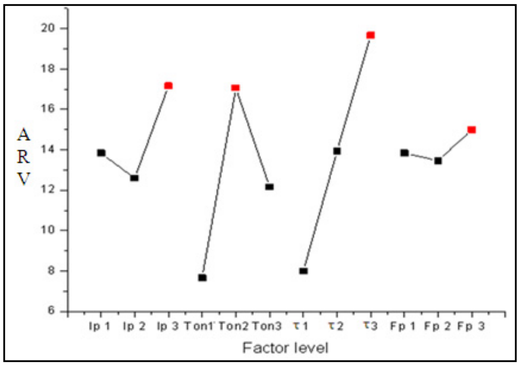

Figure 3. Optimal factor levels

Regression models have been developed for responses to predict MRR, TWR, Ra, circularity. The models are shown below.

MRR= 150.70858-11.20975×Ip+0.04986×Ton-3.14208×τ-8.30117×Fp-2.1645E-003×Ip×Ton+0.116×Ip×τ +0.71975×Ip×Fp -3.24200E-004×Ton×τ+1.73250E-003×Ton×Fp+0.067100×τ×Fp+0.38853×Ip^2-6.31158E-005×Ton^2 +0.016478×τ^2 -0.95333×Fp^2

TWR= -0.44570+0.014174×Ip-5.84041E-004×Ton+0.011068×τ+0.054865×Fp-1.39813E-005×Ip×Ton-3.06125E-004×Ip×τ +0.027054×Ip×Fp+3.72900E-006×Ton×τ+5.55250E-005×Ton×Fp-2.02250E-003×τ×Fp+1.16135E-003×Ip^2+6.45204E-007×Ton^2-5.76983E-005×τ^2-0.052158 × Fp^2

Ra= 66.15469+0.77854×Ip+5.37917E-03×Ton-1.586×τ+26.61667×Fp-1.3375E-03×Ip×Ton-3.5E-03×Ip×τ+0.97500×Ip×Fp +6.00000E-005×Ton×τ-0.036500×Ton×Fp-0.42500×τ×Fp+0.012604×Ip^2+2.91667E-007×Ton^2 +9.81667E-003×τ^2+22.29167×Fp^2

Circularity= -0.35143+0.048048×Ip+4.63458E-004×Ton+0.025032×τ-0.32537×Fp+2.51250E-005×Ip×Ton-4.1E-004×Ip×τ -5.87500E-003×Ip×Fp-7.90000E-006×Ton×τ+6.77500E-004×Ton×Fp+3.10000E-003×τ×Fp-1.38854E-003×Ip^2-3.75417E-007×Ton^2-1.29167E-004×τ^2-0.10542×Fp^2

5. Confirmative test

Confirmative test is carried out with optimal setting to check the validation of model. The predicted and experimental values are shown in table 4. It is observed that the predicted and experimental values are nearly equal. Therefore the model can be validated inside and outside the boundary.

Table 4. Comparison of predicted and experimental value

	Predicted value	Experimental value
MRR (mm^3/min)	14.7209	13.9600
TWR (mm^3/min)	0.0239	0.0201
Ra (μm)	5.8104	4.9300
Circularity	0.8382	0.8401

6. Conclusions

In this research, it is observed that the data envelopment analysis (DEA) methodology along with ARV approach works satisfactorily and yields acceptable results as well as finding suitable condition among a large number of alternative processes for generation of a desired quality and productivity in EDM process. It is concluded that the best quality and productivity achieved at Ip=7 amp, Ton= 200 μs, τ = 90%, and Fp = 0.4 kg/cm^2. With this best combination of factorial level, the experimental values of responses are obtained as MRR=13.9600 mm3/min, TWR=0.0201 mm3/min, Ra=4.9300 μm and circularity=0.8401 which are nearly equal to the predicted result obtained from regression model. Thus, DEA method has the ability to hold the multiplicity of inputs and outputs and an easy optimization technique to find the best alternatives. In this work, DEA is coupled with design of experiment approach for multi-response optimization in a non-traditional machining process.

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