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A framework for attribute selection in marketing using rough computing and formal concept analysis

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Abstract Marketing management employs various tools and techniques, including market research, to perform accurate marketing analysis. Information and communication technology provided a new dimension in marketing research to maximise the revenues and profits of the firm by identifying the chief attributes affecting decisions. In this paper, we present a hybrid approach for attribute selection in marketing based on rough computing and formal concept analysis. Our approach is aimed at handling an information system that contains numerical attribute values that are "almost similar" instead of "exact similar". To handle such an information system we use two processes—pre-process and post-process. In pre-process, we use rough set on intuitionistic fuzzy approximation space with ordering rules to find knowledge and associations, whereas in post-process we use formal concept analysis to identify the chief attributes affecting decisions.

Introduction

Marketing management is a business discipline that focusses on marketing techniques, management of a firm's marketing resources and activities. Rapid globalisation has compelled firms to market their products beyond their home country. Therefore, it is highly challenging for managers to influence the level, timing, and composition of customer demand; the size of the business; corporate culture; and industry context. To create an effective cost efficient marketing management strategy, firms must possess a detailed, objective understanding of their own business and the market in which they operate (Clancy & Krieg, 2000; Joshi, 2005; Kotler & Keller, 2006). With the introduction of information and communication technology, the buyer today is exposed to a veritable flood of information. These sources provide information about new products and services, improved versions of existing products, new uses of existing products and the like. Therefore, attribute selection in marketing is a challenging issue today. To this end, introduction of computers

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and information technology has brought about a drastic change in the recent past. Also, the use of information at the right time helps gain better knowledge in real life, and leads to knowledge mining. Although knowledge mining from databases is, increasingly, becoming important, the knowledge discovered is not always useful to users. This is because the discovered knowledge does not necessarily fit a user's interest, and may be redundant or inconsistent with a priori knowledge. Therefore, the real challenge lies in converting voluminous marketing data into knowledge, and to use this knowledge to make informed sales output appropriately. Though present-day technologies help in obtaining decisions by creating huge databases, most of the information may not be relevant. So attribute reduction becomes an important factor in handling large databases by eliminating superfluous data to enable decision making in an effective manner. Researchers have proposed many methods to mine knowledge from the voluminous data but most of the tools to mine knowledge are traditional and are crisp, deterministic and precise in character. Real life situations are quite the opposite of this. For a complete description of a real system, often one would require far more detailed data than a human being could ever recognise and simultaneously, process and understand. This leads to the extension of the concept of crisp sets, so as to model imprecise data that can enhance their modelling power. At the other end, researchers generally make statistical inferences on the existing data. This tendency gets accentuated by increased interest in making efficient use of organisational data through data mining and data warehousing (Beynon, Curry, & Morgan, 2001). Therefore, there are enough grounds for consideration of some of the newer techniques which have developed in the recent past.

The earliest of the new approaches is the notion of fuzzy sets by Zadeh (1965) that captures impreciseness in information. Since the initiation of fuzzy set theory, there have been suggestions for non classical and higher order fuzzy sets for different purposes. Keeping in view these suggestions, the different theories developed are "twofold fuzzy sets" by Dubosis and Prade (1987); "L-fuzzy set" by Goguen (1967); "toll sets" by Dubois and Prade (1993) and "intuitionistic fuzzy sets" by Atanasov (1986). However, the major difficulties lie in the determination of membership values. In general, these are situation dependent, and not so significant in dealing with dissimilar types of problems. Even for a particular situation, due to lack of information, as also possibly due to the vagueness of the information, the membership values cannot always be determined.

On the other hand, rough sets of Pawlak (1982) capture indiscernibility among objects to model imperfect knowledge. Pawlak and Skowron (2007a, 2007b, 2007c) is an alternative technique for extracting rules from the data sets. The basic definition of rough sets depends upon the notion of equivalence relations defined over a universe. A rough set, with respect to an equivalence relation, is defined through a pair of crisp sets, called the lower and upper approximations of the set. However, equivalence relations are relatively rare in practice. So, efforts have been made to make the relations less significant by removing one or more of the three requirements of an equivalence relation. The first such attempt is the study of rough sets on fuzzy approximation spaces (De, 1999) that depend upon the concept of fuzzy proximity relation. The above concept is extended to the setting of rough sets on intuitionistic fuzzy approximation space by Tripathy (2006). Its properties, applications and comparison with rough sets on fuzzy approximation space are further researched by Acharjya (2013).

As a data mining tool, rough set theory helps in obtaining decision rules about the problem. Mahapatra, Sreekumar, and Mahapatra (2010) presented an application of rough sets as a methodology for rule derivation. But, the objective of the research work, attribute selection in marketing, is missing and it has some limitations. In order to overcome the limitation, we propose an integrated model that combines rough sets on intuitionistic fuzzy approximation space with ordering relations and formal concept analysis. The motivation behind this study is that the two theories aim at different goals and summarise different types of knowledge. Rough computing is used for prediction whereas formal concept analysis is used for description. Therefore, the combination of both leads to a better model. In the integrated model, we use two processes—pre-process and post-process—to mine suitable rules and to explore the relationship between the attributes. In pre-process we use rough set on intuitionistic fuzzy approximation space and ordering rules to mine suitable rules, whereas in post-process we use formal concept analysis to better explore knowledge and the most important characteristics affecting decision making.

For completeness, the remainder of the paper is organised as follows: the second section presents the foundations of rough computing. In the third section we discuss order information table, followed by the basic idea of formal concept analysis in the fourth section. The proposed model for attribute selection is presented in the fifth section. In the sixth section, an empirical study on attribute selection in marketing is presented. The paper is concluded in the seventh section.

Foundations of rough computing

In this age of the internet, a huge repository of data is available across various domains. Therefore, it is very hard to extract useful information from the voluminous data available in the universe. So, information retrieval and knowledge representation have emerged as one of the more popular areas of recent research. Information retrieval and acquisition of knowledge are important components of an information system. But the real challenge lies in converting voluminous data into knowledge and to use this knowledge to make proper decisions. In order to transform the processed data into useful information and knowledge, there is a need for new techniques and tools. Rough set theory developed by Pawlak (1982), used to process uncertain and incomplete information, is a tool to address the above mentioned problem. One of its strengths is the attribute dependencies, and their significance among inconsistent data. At the same time, it does not need any preliminary or additional information about the data. Therefore, it classifies imprecise, uncertain or incomplete information expressed, in terms of data acquired from experience.

Rough sets

In this section we recall the definitions of basic rough set theory developed by Pawlak (1991). Let U be a finite nonempty set called the universe. Suppose $R \subseteq (U \times U)$ is an equivalence relation on U. The equivalence relation R partitions the set U into disjoint subsets. Elements of same equivalence class are said to be indistinguishable. Equivalence classes induced by R are called elementary concepts. Every union of elementary concepts is called a definable set. The empty set is considered to be a definable set, thus all the definable sets form part of Boolean algebra, and (U, R) is called an approximation space. Given a target set X, we can characterise X by a pair of lower and upper approximations. We associate two subsets $\underline{R}X$ and $\bar{R}X$ called the R-lower and R-upper approximations of X respectively and are given by:

$$\underline{R}X = \cup\{Y \in U/R : Y \subseteq X\} \tag{1}$$

$$\bar{R}X = \cup\{Y \in U/R : Y \cap X \neq \phi\} \tag{2}$$

The R-boundary of X, $BN_R(X)$ is given by $BN_R(X) = \bar{R}X - \underline{R}X$. We say X is rough with respect to R if and only if $\bar{R}X \neq \underline{R}X$, equivalently $BN_R(X) \neq \phi$. X is said to be R-definable if and only if $\bar{R}X = \underline{R}X$ or $BN_R(X) = \phi$. So, a set is rough with respect to R if and only if it is not R-definable.

Rough set on intuitionistic fuzzy approximation space

In real time environment, we face many problems while studying information retrieval from large databases. This is because of inconsistent and ambiguous datasets that are precise in character. The basic rough set philosophy for information retrieval developed by Pawlak (1991) depends upon equivalence relations. However, such types of relations are rare in practice, while studying information systems containing numerical values. Therefore, the equivalence relation is generalised to fuzzy proximity relation and it leads to rough sets on fuzzy approximation spaces (De, 1999). The fuzzy proximity relation is further generalised to intuitionistic fuzzy proximity relation (Atanasov, 1986). Thus rough set on fuzzy approximation spaces is generalised to rough set on intuitionistic fuzzy approximation spaces (Tripathy, 2006). Further it is observed that, rough set on intuitionistic fuzzy approximation spaces is a better model over rough set and rough set on fuzzy approximation space (Acharjya, 2009). However, for completeness of the paper we provide the basic notions of rough sets on intuitionistic fuzzy approximation spaces. We use standard notation μ for membership and ν for non-membership functions associated with an intuitionistic fuzzy set.

An intuitionistic fuzzy relation R on a universal set U is an intuitionistic fuzzy set defined on $(U \times U)$. An intuitionistic fuzzy relation R on U is said to be an intuitionistic fuzzy proximity relation if the following properties hold.

$$\mu_R(x, x) = 1 \quad \text{and} \quad \nu_R(x, x) = 0 \quad \text{for all } x \in U$$

$$\mu_R(x, y) = \mu_R(y, x) \quad \text{and}$$
$$\nu_R(x, y) = \nu_R(y, x) \quad \text{for all } x, y \in U$$

Let R be an intuitionistic fuzzy (IF) proximity relation on U. Then for any $(\alpha, \beta) \in J$, where $J = \{(\alpha, \beta) : \alpha, \beta \in [0, 1] \text{ and } 0 \leq (\alpha + \beta) \leq 1\}$ the (α, β)-cut, $R_{\alpha,\beta}$, of R is given by:

$$R_{\alpha,\beta} = \{(x, y) : \mu_R(x, y) \geq \alpha \text{ and } \nu_R(x, y) \leq \beta\}$$

We say that two elements x and y are (α, β)-similar with respect to R if $(x, y) \in R_{\alpha,\beta}$ and we write $xR_{\alpha,\beta}y$. In addition, we say that two elements x and y are (α, β)-identical with respect to R for $(\alpha, \beta) \in J$, written as $x\, R(\alpha, \beta)\, y$ if and only if $xR_{\alpha,\beta}y$ or there exists a sequence of elements $u_1, u_2, u_3, \cdots, u_n$ in U such that $xR_{\alpha,\beta}u_1$, $u_1R_{\alpha,\beta}u_2$, $u_2R_{\alpha,\beta}u_3$, …, $u_nR_{\alpha,\beta}y$. In the last case, we say that x is transitively (α, β)-similar to y with respect to R.

It is also easy to see that for any $(\alpha, \beta) \in J$, $R(\alpha, \beta)$ is an equivalence relation on U. We denote $R^*_{\alpha,\beta}$ the set of equivalence classes generated by the equivalence relation $R(\alpha, \beta)$ for each fixed $(\alpha, \beta) \in J$. The pair (U, R) generated in this way is an intuitionistic fuzzy approximation space (IF-approximation space). An IF-approximation space (U, R) generates usual approximation space $(U, R(\alpha, \beta))$ of Pawlak for every $(\alpha, \beta) \in J$. The rough set on X in the generalised approximation space $(U, R(\alpha, \beta))$ is denoted by $(X_{\alpha,\beta}, X_{\alpha,\beta})$ where:

$$\underline{X}_{\alpha,\beta} = \cup\{Y : Y \in R^*_{\alpha,\beta} \text{ and } Y \subseteq X\} \quad \text{and} \tag{3}$$

$$\bar{X}_{\alpha,\beta} = \cup\{Y : Y \in R^*_{\alpha,\beta} \text{ and } Y \cap X \neq \phi\} \tag{4}$$

Let X be a rough set in the generalised approximation space $(U, R(\alpha, \beta))$. Then we define the (α, β)-boundary of X with respect to R denoted by $BNR_{\alpha,\beta}(X)$ as $BNR_{\alpha,\beta}(X) = \bar{X}_{\alpha,\beta} - \underline{X}_{\alpha,\beta}$. The target set X is (α, β)-discernible with respect to R if and only if $\bar{X}_{\alpha,\beta} = \underline{X}_{\alpha,\beta}$ and X is (α, β)-rough with respect to R if and only if $\bar{X}_{\alpha,\beta} \neq \underline{X}_{\alpha,\beta}$.

Almost indiscernibility relation

An information system provides all available information and knowledge about the objects under certain consideration. Objects are only perceived or measured by using a finite number of properties without considering any semantic relationship between attribute values of a particular attribute. Therefore, in general, one uses the trivial equality relation on values of an attribute as discussed in Pawlak's rough set theory. However, in many real life applications it is observed that the attribute values are not exactly identical but almost identical (Tripathy & Acharjya, 2010). At this point we generalise Pawlak's approach of indiscernibility. Keeping this in view, the almost indiscernibility relation is generated, and is the basis of rough set on intuitionistic fuzzy approximation space as discussed in the previous section.

Let U be the universe and A be a set of attributes. With each attribute $a \in A$, we associate a set of its values V_a, called the domain of a. The pair $S = (U,A)$ will be called an information system. Let $B \subseteq A$. For a chosen $(\alpha, \beta) \in [0, 1]$ we denote a binary relation $R_B(\alpha, \beta)$ on U defined by $xR_B(\alpha, \beta)y$ if and only if $x(a)R_B(\alpha, \beta)\, y(a)$ for all $a \in B$, where $x(a) \in V_a$ denotes the value of attribute x in a. Obviously, it can be proved that the relation $R_B(\alpha, \beta)$ is an equivalence relation on U. Also, we notice that $R_B(\alpha, \beta)$ is not exactly the indiscernibility relation defined by Pawlak (1991); rather it can be viewed as an

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almost indiscernibility relation on U. For $\alpha=1$, $\beta=0$ the almost indiscernibility relation, $R_B(\alpha, \beta)$ diminishes to the indiscernibility relation. Thus, it generalises Pawlak's indiscernibility relation. The family of all equivalence classes of $R_B(\alpha, \beta)$ i.e., the partition generated by B for $(\alpha, \beta) \in [0, 1]$, will be denoted by $U/R_B(\alpha, \beta)$. If $(x, y) \in R_B(\alpha, \beta)$, then we will say that x and y are (α, β)-indiscernible. Blocks of the partition $U/R_B(\alpha, \beta)$ are referred as $B_{\alpha,\beta}$-elementary concepts. These are the basic concepts of our knowledge in rough set on intuitionistic fuzzy approximation space.

Order information table

As mentioned in the previous section, rough set philosophy is based on the assumption that in addition to crisp set theory, we have some additional information about the elements of a universe of discourse. Elements that exhibit the same information are indiscernible and form the basic building blocks that can be considered as elementary concept of knowledge about the universe.

Let $I = (U, A, \{V_a, a \in A\}, \{f_a, a \in A\})$ be an information system, where U is a finite non-empty set of objects called the universe and A is a non-empty finite set of attributes. For every $a \in A$, V_a is the set of values that attribute a may take and $f_a: U \to V_a$ is an information function. Attributes can be interpreted as features, variables, and characteristics. A special case of information systems is called information table or attribute value table where the columns are labelled by attributes and rows are labelled by objects. For example: The information table assigns a value $x(a)$ from V_a to each attribute a and object x in the universe U. With any $B \subseteq A$ there is an associated equivalence relation $IND(B)$ such that:

$$IND(B) = \{(x, y) \in U^2 : \forall\, a \in B,\, x(a) = y(a)\} \tag{5}$$

The relation $IND(B)$ is called a B-indiscernibility relation. The partition of U is a family of all equivalence classes of $IND(B)$ and is denoted by $U/IND(B)$ or U/B. If $(x, y) \in IND(B)$, then x and y are indiscernible by attributes from B.

Generalised information table may be viewed as information tables with added semantics. For the problems of knowledge acquisition, we introduce order relations on attribute values (Yao & Ying, 2001). However, it is not appropriate in case of attribute values that are almost indiscernible. An ordered information table (OIT) is defined as $OIT = \{I, \{\prec_a : a \in A\}\}$ where, I is a standard information table and $\prec_a$ is an order relation on attribute a. An ordering of values of a particular attribute a naturally induces an ordering of objects:

$$x \prec_{\{a\}} y \Leftrightarrow f_a(x) \prec_a f_a(y) \tag{6}$$

where, $\prec_{\{a\}}$ denotes an order relation on U induced by the attribute a. An object x is ranked ahead of object y if and only if the value of x on the attribute a is ranked ahead of the value of y on the attribute a. For example, a sample ordered information table of seven objects with four attributes $\{a_1, a_2, a_3, a_4\}$ is shown in Table 1, where the attribute a_1 represents qualification; a_2 represents total years of experience; a_3 represents designation, and a_4 represents salary in Indian rupees. For a subset of attributes $B \subseteq A$, we define:

Table 1 Sample information system.

Objects	Qualification	Total years of experience	Designation	Salary in Indian rupees
x_1	M. Tech.	11	Associate Professor	26,000
x_2	Ph. D.	5	Professor	38,000
x_3	Ph. D.	6	Professor	36,000
x_4	M. Tech.	10	Assistant Professor	22,000
x_5	B. Tech.	5	Assistant Professor	20,000
x_6	Ph. D.	12	Associate Professor	30,000
x_7	B. Tech.	8	Assistant Professor	18,000

$\prec_{a_1}$: Ph. D. $\prec$ M. Tech. $\prec$ B. Tech.
$\prec_{a_2}$: $12 \prec 11 \prec 10 \prec 8 \prec 6 \prec 5$
$\prec_{a_3}$: Professor $\prec$ Associate Professor $\prec$ Assistant Professor
$\prec_{a_4}$: $38000 \prec 36000 \prec 30000 \prec 26000 \prec 22000 \prec 20000 \prec 18000$

$$\begin{aligned} x \prec_B y &\Leftrightarrow x(a) \prec_a y(a)\ \forall\, a \in B \\ &\Leftrightarrow \wedge_{a\in B}\, x(a) \prec_a y(a) \\ &\Leftrightarrow \textstyle\bigcap_{a\in B} x \prec_{\{a\}} y \end{aligned}$$

It indicates that x is ranked ahead of y if and only if x is ranked ahead of y according to all attributes in B. The above definition is a straightforward generalisation of the standard definition of equivalence relations in rough set theory, where the equality relation is used.

The basic objective of order information table is to provide an order relation among all attribute values of an attribute in a qualitative information system. If the attribute values are almost identical then it is necessary to order the attribute values after getting almost equivalence classes. It helps the intuitive notion of ordering the attribute values of an attribute in an information system.

Fundamentals of formal concept analysis

In this section we recall the fundamental notions of formal concept analysis (FCA) introduced by Wille (2005) which is a conceptual tool for the analysis of data. The basic objective is to visualise the data in the form of concept lattices and thereby to make them more transparent and more easily discussible. The primary aim is to support the user in analysing and structuring a domain of interest based on mathematisation of the concept and conceptual hierarchy. It activates mathematical thinking for conceptual data analysis and knowledge processing based on a formal understanding of a concept as a unit of thought. Thus, a concept is composed of extension and intension. The extension of a formal concept is formed by all objects to which the concept applies whereas the intension consists of all attributes that exist in those objects. The set of objects, attributes and the relations between an object and an attribute in a dataset forms the basic conceptual structure of FCA known as formal context. Concepts can only live on relationships with many other concepts where the subconcept superconcept relation plays a vital role. A subconcept of a superconcept means that the extension of the subconcept is contained in the extension of the super concept. This is equivalent to the relationship that the intension of the subconcept contains the intension of the superconcept (Wormuth & Becker, 2004).

Formal context and formal concept

A formal context defined as a set structure $K = (U, A, R)$ consists of two sets U and A while R is a binary relation from U to A, *i.e.*, $R \subseteq (U \times A)$. The elements of U are called the objects whereas the elements of A are called the attributes of the context. The formal concept of the formal context (U, A, R) is defined with the help of derivation operators. The derivation operators are defined for arbitrary $X \subseteq U$ and $B \subseteq A$ as follows:

$$X' = \{a \in A : uRa \ \forall \ u \in X\} \tag{7}$$

$$B' = \{u \in U : uRa \ \forall \ a \in A\} \tag{8}$$

A formal concept of a formal context $K = (U, A, R)$ is defined as a pair (X,B) with $X \subseteq U, B \subseteq A, X = B'$ and $B = X'$. The first member X, is called the extent whereas the second member B is called the intent of the formal concept. It indicates that the formal concepts of a formal context are forming the mathematical structure of a lattice with respect to the subconcept superconcept relation (Wille, 2005). The subconcept superconcept relation can be depicted best by a lattice diagram, and hence we can derive concepts, implications sets, and association rules based on the cross table. In such a diagram, each object is attached with its object concept whereas each attribute is attached with its attribute concept.

Proposed model for attribute selection

This section proposes a model for attribute selection that consists of pre-process and post-process as shown in Fig. 1. In pre-process, we process the data after data cleaning by using rough set on intuitionistic fuzzy approximation space and ordering relation. In post-process formal concept analysis is used to identify the chief attributes affecting decisions. The main advantage of this model is that it works for both qualitative and quantitative data.

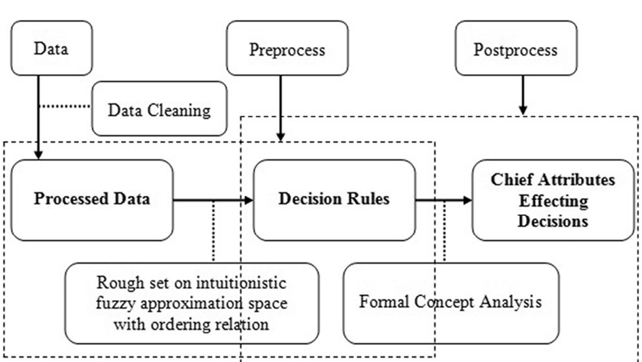

Figure 1 An abstract view of proposed model.

The basic step of any model is the identification of the right problem. Incorporation of prior knowledge is always associated with the problem definition. However the potential validity of an individual data element or pattern of data element may change from organisation to organisation because of inclusion of vagueness and incompleteness. It is very difficult for human beings to identify the chief attributes affecting decisions. Hence, it is essential to deal with incomplete and vague information in classification, data analysis, and concept formulation.

Pre-processing architecture design

This section presents pre-process architecture design of the proposed model that consists of problem undergone, target data, data cleaning, intuitionistic fuzzy proximity relation, data classification, ordering relation, and rough set rule extraction as shown in Fig. 2. For any model, the fundamental steps are problem definition and incorporation of prior knowledge.

After structuring the objectives and based on the associated attributes, a target dataset is created on which data mining is to be performed. Before further analysis, a sequence of data cleaning tasks such as consistency check, removal of noise, and data completeness is carried out to ensure that the data are as accurate as possible. Finally for each attribute, we compute the (α, β)-equivalence classes based on the almost indiscernibility relation as discussed earlier in the paper. The intuitionistic fuzzy proximity relation identifies the almost indiscernibility among the objects. This result induces the (α, β)-equivalence classes. Finally, we obtain categorical classes for obtaining decision rules on imposing order relation on this classification. We define the membership and non-membership relations of the intuitionistic fuzzy proximity relation R to identify the almost indiscernibility among the objects x_i and x_j as below:

$$\mu_R(x_i, x_j) = 1 - \frac{|V_{x_i} - V_{x_j}|}{\text{Max Range}} \tag{9}$$

$$\nu_R(x_i, x_j) = \frac{|V_{x_i} - V_{x_j}|}{2 \times \text{Max Range}} \tag{10}$$

The membership and non-membership relations are formulated in such a way that the sum of membership and non-membership values lies between [0, 1]. In addition, the relation must be symmetric. After getting the (α, β)-equivalence classes, the ordered information system is obtained by imposing the ordering relation. Further we impose rough set decision analysis to obtain decisions.

Rule generation algorithm

In this section we propose a rule generation algorithm that generates all the possible reducts by eliminating all

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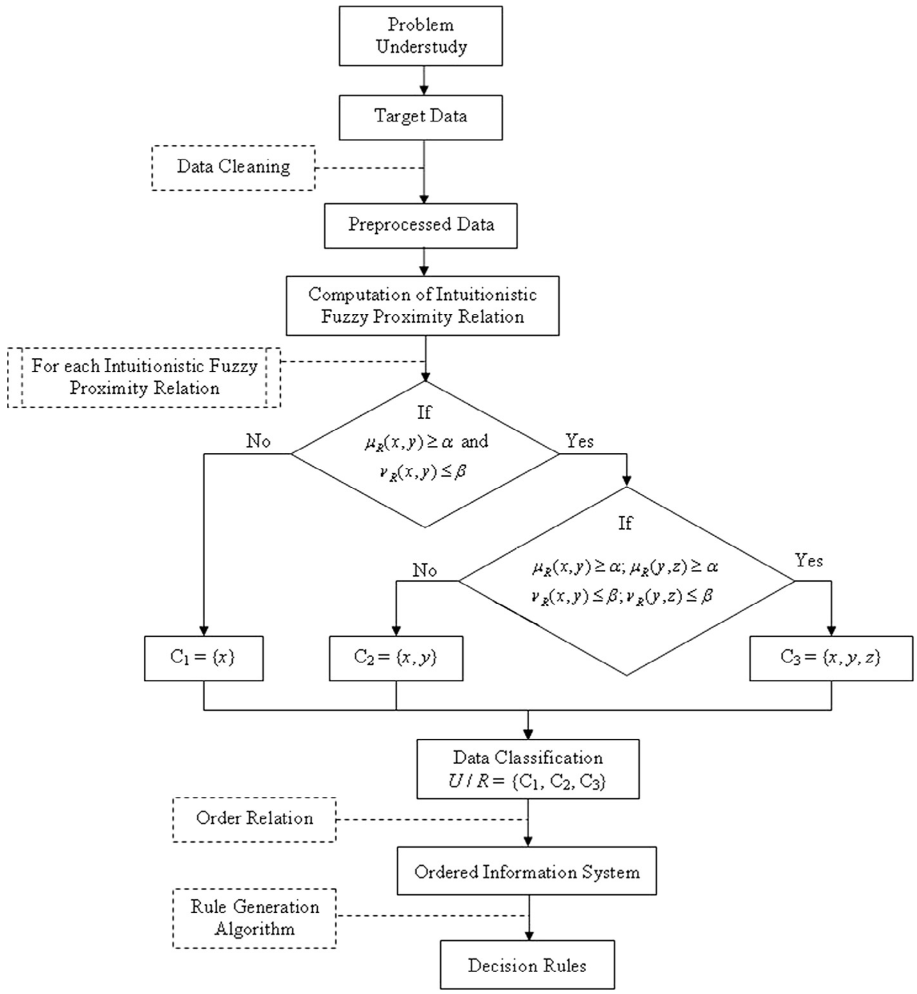

Figure 2 Pre-process architecture design.

dispensable attributes and deriving the candidate decision rules from the dataset. We apply the following steps in order to generate the decision rules.

Algorithm

Input: Target information system
Output: Candidacy decision rules

1. Set object number $i = 1$
2. Choose object i from the training data set and compute a set of reducts for all the condition attributes
3. Replace $i = i + 1$
4. If all objects have been chosen, then go to step 5; otherwise go to step 2
5. Compute the number of supporting objects for each reduct after combining the identical reducts
6. Obtain the decision rules from the selected reducts
7. Process terminates and the decisions rules are written

A numerical illustration

In this section we explain how rule generation works in pre-processing phase of our model with a numerical illustration. Consider a target dataset with 12 objects as shown in Table 2, where the attributes $a_1, a_2, a_3, \cdots, a_6$ are defined based on the problem objective. Before we compute the rules, data cleaning and data classification are carried out by using intuitionistic fuzzy proximity relation. In particular, we remove the objects x_7, x_{10}, and x_{11} that contain missing attribute values. Thus the dataset contains nine objects

Table 2 Numerical illustration dataset.

Objects	a_1	a_2	a_3	a_4	a_5	a_6	Decision
x_1	1	7	1	1	1	2	2
x_2	2	1	5	1	1	5	1
x_3	1	3	4	1	1	4	2
x_4	2	2	3	3	1	4	1
x_5	1	3	1	1	1	2	2
x_6	2	1	4	1	2	4	1
x_7	–	–	2	2	2	1	2
x_8	1	3	2	3	1	2	2
x_9	1	8	4	1	2	5	1
x_{10}	–	–	–	1	2	–	–
x_{11}	–	3	–	–	–	2	1
x_{12}	1	2	4	3	2	4	1

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$x_1, x_2, x_3, \cdots, x_6, x_8, x_9$, and x_{12}. The dataset is processed to obtain candidate decision rules.

Following steps 2–4 of the algorithm, a set of decision rules based on the attributes $a_1, a_2, a_3, \cdots, a_6$ is generated with their supporting objects. The candidate rules generated are given in Table 3. For example, rule 7 is denoted as ×3××××2. This leads to the following decision rule: If $a_2 = 3$, then the value of the decision attribute is 2. Similarly, we can also obtain the other decision rules on considering various combinations of attributes.

Post-process of proposed model

The inputs to the post-process are the decisions obtained in the pre-process. Now we present the cross table of the decision rules for decision class 1 obtained from Table 2 in Table 4. In Table 4, the rows are represented as objects and columns are represented as attributes. The relation between them is represented by a cross mark. The subconcept superconcept relation can be depicted best by a lattice diagram and we can derive concepts, implication sets, and association rules based on the cross table. In such a diagram the name of each object is attached to its represented object concept and the name of each attribute is attached to its represented attribute concept. The subconcept superconcept relation is transitive. It indicates that a concept is the subconcept of any concept that can be reached by travelling upwards from it. The corresponding lattice diagram is depicted in Fig. 3, where the nodes represent formal concepts. In particular, one concept consists of object D_{15} and attributes A_{22}, and A_{43} and is expressed as $(\{D_{15}\}, \{A_{22}, A_{43}\})$. Similarly, another concept consists of object D_{12} and attribute A_{33} and is expressed as $(\{D_{12}\}, \{A_{33}\})$. It is also observed that lines up give more general concepts whereas lines down give more specific concepts. A pair of set of objects and a set of attributes that is closed in this manner is called a formal concept.

An empirical study on attribute selection

This section demonstrates how the proposed model can be applied to get chief attributes affecting decisions. To demonstrate our model, we consider an information system of a group of companies in a country. In Table 5, we consider a few parameters for business strategies to get maximum sales, notation and their possible range of values. A company with appropriately high expenditure in marketing, advertisement, distribution, miscellaneous items, and research and development is an ideal case for getting maximum sales. But such a case is rare in practice. So, it is essential to identify the chief attributes of a company to get the maximum sales.

The companies can be judged by the sales outputs that are produced. The amount of sales can be judged by the different parameters of the companies. These parameters form the attribute set for our analysis. Here the marketing expenditure means, all expenditure incurred for corporate promotion, which includes event marketing, sales promotion, direct marketing etc. which comes to around 6%. The advertising expenditure includes promotional activities using various media such as television, newspaper, and the internet which comes to around 36%. The miscellaneous expenditure is mainly incurred through activities like corporate social responsibility and which amounts to a maximum of 28%. The distribution cost includes expenses on logistics, supply chain and the

Table 3 Candidacy rule computation.

Rule	a_1	a_2	a_3	a_4	a_5	a_6	Decision	Supporting objects
[1]	2	×	×	×	×	×	1	x_2, x_4, x_6
[2]	×	1	×	×	×	×	1	x_2, x_6
[3]	×	2	×	×	×	×	1	x_4, x_{12}
[4]	×	×	×	×	2	×	1	x_6, x_9, x_{12}
[5]	×	×	×	×	×	5	1	x_2, x_9
[6]	×	×	×	3	×	4	1	x_4, x_{12}
[7]	×	3	×	×	×	×	2	x_3, x_5, x_8
[8]	×	×	1	×	×	×	2	x_1, x_5
[9]	×	×	×	×	×	2	2	x_1, x_5, x_8
[10]	1	×	×	×	1	×	2	x_1, x_3, x_5, x_8

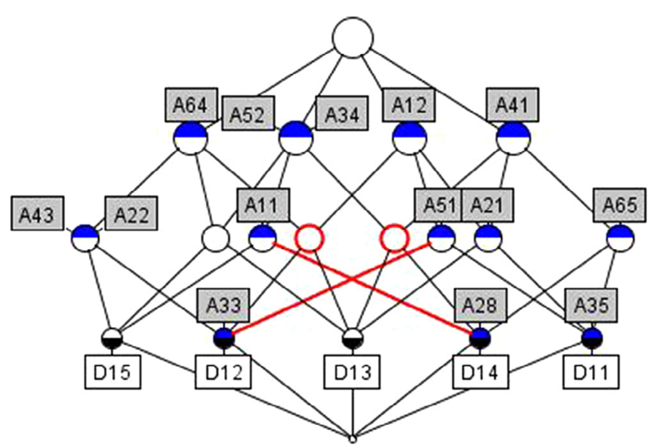

Figure 3 Lattice diagram of decision rules for decision class 1.

Table 4 Cross table of decision rules for decision class 1.

Rule	a_{11}	a_{12}	a_{21}	a_{22}	a_{28}	a_{33}	a_{34}	a_{35}	a_{41}	a_{43}	a_{51}	a_{52}	a_{64}	a_{65}
D1	–	×	×	–	–	–	–	×	×	–	×	–	–	×
D2	–	×	–	×	–	×	–	–	–	×	×	–	×	–
D3	–	×	×	–	–	–	×	–	×	–	–	×	×	–
D4	×	–	–	–	×	–	×	–	×	–	–	×	–	×
D5	×	–	–	×	–	–	×	–	–	×	–	×	×	–

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like which comes to around 24%. The investment made on new product development and other research activities is considered as part of research and development activities and amounts to around 6%. However we have not considered many other parameters that do not influence the sales of a company; this has also been done to make our analysis simple. The average of the data collected is considered to be the representative figure as tabulated below. The notations and abbreviations used in the following analysis are presented in Table 5.

In Table 6 we present the data obtained from 10 different companies. (Their identities are kept confidential.) Here, we use the notation $x_i, i = 1, 2, 3, \cdots, 10$ for different companies, for the purpose of the study, to demonstrate the model and not to probe the performance of individual companies. It is to be noted that all non-ratio figures shown in the Table 6 are 10 million INR.

Pre-process of empirical study

In this section we discuss in detail the steps of the pre-process architecture design for the empirical study taken into consideration. To demonstrate pre-process of our proposed model, data from 10 different companies are considered as in Table 6. We have designed intuitionistic fuzzy proximity relations based on the attributes and computed the "almost similarity" between them. The intuitionistic fuzzy proximity relation identifies the almost indiscernibility among the objects. This result induces the (α, β)-equivalence classes. We obtain categorical classes on imposing order relation on this classification. The membership and non-membership values corresponding to intuitionistic fuzzy proximity relations R^{a_i}; $a_i \in A$ corresponding to the attributes EM, ED, EA, EMi, ERD, and TS are computed by using equations (9) and (10). We present the intuitionistic fuzzy proximity relation for the parameters EM in Table 7. (Keeping in view the length of the paper, the intuitionistic fuzzy proximity relations for the parameters ED, EA, EMi, ERD and TS are omitted.)

On considering the almost similarity of 98% and dissimilarity of 1% *i.e.*, $\alpha \geq 0.98$ and $\beta \leq 0.01$, it is observed from Table 7 that $\mu_R(x_1, x_1) = 1$; $\nu_R(x_1, x_1) = 0$; $\mu_R(x_2, x_2) = 1$; $\nu_R(x_2, x_2) = 0$; $\mu_R(x_2, x_3) = 0.981$; $\nu_R(x_2, x_3) = 0.009$; $\mu_R(x_2, x_5) = 0.995$; $\nu_R(x_2, x_5) = 0.002$; $\mu_R(x_4, x_4) = 1$; $\nu_R(x_4, x_4) = 0$; $\mu_R(x_4, x_8) = 0.985$; $\nu_R(x_4, x_8) = 0.007$; $\mu_R(x_8, x_7) = 0.982$; $\nu_R(x_8, x_7) = 0.009$; $\mu_R(x_7, x_9) = 0.983$; $\nu_R(x_7, x_9) = 0.008$; $\mu_R(x_6, x_6) = 1$; $\nu_R(x_6, x_6) = 0$; $\mu_R(x_6, x_{10}) = 0.996$; $\nu_R(x_6, x_{10}) = 0.002$. Therefore, the companies x_2, x_3, and x_5 are (α, β)-identical. Similarly the companies x_4, x_7, x_8, and x_9 are (α, β)-identical. In addition x_6 and x_{10} are (α, β)-identical. Therefore, we get the following classification:

$$U/R^{EM} = \{\{x_1\}, \{x_2, x_3, x_5\}, \{x_4, x_7, x_8, x_9\}, \{x_6, x_{10}\}\}$$

Hence, the values of the attribute expenditure on marketing are classified into four categories namely low, average, high, and very high (v high) and thus can be ordered. Similarly, the different equivalence classes corresponding to the attributes ED, EA, EMi, ERD and TS are given below.

$$U/R^{ED} = \{\{x_1, x_3, x_4, x_6, x_8, x_{10}\}, \{x_2, x_9\}, \{x_5\}, \{x_7\}\}$$

$$U/R^{EA} = \{\{x_1, x_2, x_{10}\}, \{x_3, x_5, x_6\}, \{x_4, x_8\}, \{x_7, x_9\}\}$$

$$U/R^{EMi} = \{\{x_1, x_2, x_3\}, \{x_4, x_8, x_{10}\}, \{x_5, x_6\}, \{x_7, x_9\}\}$$

$$U/R^{ERD} = \{\{x_1, x_3\}, \{x_2, x_4, x_6, x_8, x_9\}, \{x_5, x_7, x_{10}\}\}$$

$$U/R^{TS} = \{\{x_1, x_5, x_9\}, \{x_2, x_6, x_{10}\}, \{x_3, x_7\}, \{x_4, x_8\}\}$$

From the above classification, it is clear that the values of the attributes expenditure on advertisement (EA) and expenditure on distribution (ED) are classified into four categories namely low, average, high, and very high. Similarly,

Table 5 Attribute notation description.

Attribute	Notation	Possible range
Expenditure on marketing	EM (a_1)	[1, 150]
Expenditure on distribution	ED (a_2)	[1, 600]
Expenditure on advertisement	EA (a_3)	[1, 900]
Expenditure on miscellaneous	EMi (a_4)	[1, 700]
Expenditure on research and development	ERD (a_5)	[1, 150]
Total sales	TS (a_6)	[1, 12,000]

Table 6 Small universe of information system.

Company	EM (a_1)	ED (a_2)	EA (a_3)	EMi (a_4)	ERD (a_5)	TS (a_6)
x_1	18.276	30.236	162.236	72.146	9.156	1,220.586
x_2	37.321	82.568	163.72	68.257	1.513	623.538
x_3	34.531	25.237	37.773	79.237	8.769	11,232.76
x_4	2.076	6.793	5.393	8.290	0.383	42.767
x_5	36.621	492.534	36.772	25.343	35.967	1,352.264
x_6	27.333	16.496	38.660	24.343	1.523	561.697
x_7	7.033	508.676	866.916	637.530	38.963	11,449.56
x_8	4.323	1.753	4.173	3.176	0.003	60.89
x_9	9.563	80.768	872.57	632.535	1.453	1,197.725
x_{10}	26.678	26.593	161.268	9.269	33.928	558.395

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Table 7 Intuitionistic fuzzy proximity relation for the attribute expenditure on marketing (EM).

R^{EM}	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
x_1	1,	0.873	0.892	0.892	0.878	0.94	0.925	0.907	0.942	0.944
	0	0.063	0.054	0.054	0.061	0.03	0.037	0.047	0.029	0.028
x_2	0.873	1,	0.981	0.765	0.995	0.933	0.798	0.78	0.815	0.929
	0.063	0	0.009	0.117	0.002	0.033	0.101	0.11	0.093	0.035
x_3	0.892	0.981	1,	0.784	0.986	0.952	0.817	0.799	0.834	0.948
	0.054	0.009	0	0.108	0.007	0.024	0.092	0.101	0.083	0.026
x_4	0.892	0.765	0.784	1,	0.77	0.832	0.967	0.985	0.95	0.836
	0.054	0.117	0.108	0	0.115	0.084	0.017	0.007	0.025	0.082
x_5	0.878	0.995	0.986	0.77	1,	0.938	0.803	0.785	0.82	0.934
	0.061	0.002	0.007	0.115	0	0.031	0.099	0.108	0.09	0.033
x_6	0.94	0.933	0.952	0.832	0.938	1,	0.865	0.847	0.882	0.996
	0.03	0.033	0.024	0.084	0.031	0	0.068	0.077	0.059	0.002
x_7	0.925	0.798	0.817	0.967	0.803	0.865	1,	0.982	0.983	0.869
	0.037	0.101	0.092	0.017	0.099	0.068	0	0.009	0.008	0.065
x_8	0.907	0.78	0.799	0.985	0.785	0.847	0.982	1,	0.965	0.851
	0.047	0.11	0.101	0.007	0.108	0.077	0.009	0	0.017	0.075
x_9	0.942	0.815	0.834	0.95	0.82	0.882	0.983	0.965	1,	0.886
	0.029	0.093	0.083	0.025	0.09	0.059	0.008	0.017	0	0.057
x_{10}	0.944	0.929	0.948	0.836	0.934	0.996	0.869	0.851	0.886	1,
	0.028	0.035	0.026	0.082	0.033	0.002	0.065	0.075	0.057	0

expenditure on miscellaneous (EMi) and total sales (TS) are classified into four categories namely medium, average, high, and very high. The values of the attribute expenditure on research and development (ERD) are classified into three categories namely low, average, and very high. Therefore, the ordered information system of the business strategies of different companies of Table 6 is given in Table 8. To make our analysis simpler we have assigned some value to each category. However, these values are optional and do not affect the analysis.

We employed the dataset to derive the reducts and obtain the rules. We also removed the unusual outcomes. In addition to this, the identical objects in the dataset were reduced to only one case in order to avoid unnecessary analysis. Based on the rule generation algorithm, the rules were determined. The total number of candidate rules generated for further post-process analysis is 40. The candidacy rules are further classified into four categories. The different candidate rules for medium total sales are presented in Table 9, whereas those for average total sales are presented in Table 10. Similarly, the candidate rules obtained for high total sales are presented in Table 11 and very high total sales are presented in Table 12.

Table 8 Order information system of small universe.

Company	EM (a_1)	ED (a_2)	EA (a_3)	EMi (a_4)	ERD (a_5)	TS (a_6)
x_1	Avg (3)	Low (1)	High (4)	High (4)	Avg (3)	High (4)
x_2	V high (5)	Avg (3)	High (4)	High (4)	Low (1)	Avg (3)
x_3	V high (5)	Low (1)	Avg (3)	High (4)	Avg (3)	V high (5)
x_4	Low (1)	Low (1)	Low (1)	Medium (2)	Low (1)	Medium (2)
x_5	V high (5)	High (4)	Avg (3)	Avg (3)	V high (5)	High (4)
x_6	High (4)	Low (1)	Avg (3)	Avg (3)	Low (1)	Avg (3)
x_7	Low (1)	V high (5)	V high (5)	V high (5)	V high (5)	V high (5)
x_8	Low (1)	Low (1)	Low (1)	Medium (2)	Low (1)	Medium (2)
x_9	Low (1)	Avg (3)	V high (5)	V high (5)	Low (1)	High (4)
x_{10}	High (4)	Low (1)	High (4)	Medium (2)	V high (5)	Avg (3)

$\prec_{EM}$: Very high $\prec$ High $\prec$ Average $\prec$ Low
$\prec_{ED}$: Very high $\prec$ High $\prec$ Average $\prec$ Low
$\prec_{EA}$: Very high $\prec$ High $\prec$ Average $\prec$ Low
$\prec_{EMi}$: Very high $\prec$ High $\prec$ Average $\prec$ Medium
$\prec_{ERD}$: Very high $\prec$ Average $\prec$ Low
$\prec_{TS}$: Very high $\prec$ High $\prec$ Average $\prec$ Medium

Post-process of empirical study

In this section we discuss how formal concept analysis can help in obtaining chief factors affecting decisions. The objective of this process is to use formal concept analysis to aggregate the rules that are generated in pre-process and hence to obtain the chief factors affecting the decisions. This helps the decision maker to identify the factors for total sales.

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Table 9 Candidacy rules of total sales: medium.

Rule	EM (a_1)	ED (a_2)	EA (a_3)	Emi (a_4)	ERD (a_5)	TS (a_6)
[1]	×	×	1	×	×	2
[2]	1	1	×	×	×	2
[3]	1	×	×	2	×	2
[4]	×	×	×	2	1	2

Table 10 Candidacy rules of total sales: average.

Rule	EM (a_1)	ED (a_2)	EA (a_3)	Emi (a_4)	ERD (a_5)	TS (a_6)
[1]	4	×	×	×	×	3
[2]	5	3	×	×	×	3
[3]	5	×	4	×	×	3
[4]	5	×	×	×	1	3
[5]	×	3	4	×	×	3
[6]	×	3	×	4	×	3
[7]	×	1	×	3	×	3
[8]	×	1	×	×	5	3
[9]	×	×	4	2	×	3
[10]	×	×	4	×	1	3
[11]	×	×	4	×	5	3
[12]	×	×	3	×	1	3
[13]	×	×	×	4	1	3
[14]	×	×	×	3	1	3
[15]	×	×	×	2	5	3

Table 11 Candidacy rules of total sales: high.

Rule	EM (a_1)	ED (a_2)	EA (a_3)	Emi (a_4)	ERD (a_5)	TS (a_6)
[1]	3	×	×	×	×	4
[2]	×	4	×	×	×	4
[3]	5	×	×	3	×	4
[4]	5	×	×	×	5	4
[5]	1	3	×	×	×	4
[6]	×	3	5	×	×	4
[7]	×	3	×	5	×	4
[8]	×	×	4	×	3	4
[9]	×	×	3	×	5	4
[10]	×	×	5	×	1	4
[11]	×	×	×	3	5	4
[12]	×	×	×	5	1	4
[13]	×	1	4	4	×	4

Results and discussions are categorised into four sections. In the section immediately following, we discuss the results related to very high total sales whereas results pertaining to high total sales are discussed in the section after that. In addition, we discuss the results pertaining to average total sales and medium total sales in subsequent sections.

Decision class of total sales: very high

Now we present the context table of very high total sales in Table 13, which converted eight rules of very high total sales representing the attributes obtained from pre-process. In Fig. 4, we present the lattice diagram of the context Table 13 for the decision class—very high total sales.

Table 12 Candidacy rules of total sales: very high.

Rule	EM (a_1)	ED (a_2)	EA (a_3)	Emi (a_4)	ERD (a_5)	TS (a_6)
[1]	×	5	×	×	×	5
[2]	5	1	×	×	×	5
[3]	5	×	×	×	3	5
[4]	1	×	×	×	5	5
[5]	×	×	3	4	×	5
[6]	×	×	3	×	3	5
[7]	×	×	5	×	5	5
[8]	×	×	×	5	5	5

Table 13 Context table of total sales: very high.

Rule	a_{11}	a_{15}	a_{21}	a_{25}	a_{33}	a_{35}	a_{44}	a_{45}	a_{53}	a_{55}
[1]	–	–	–	×	–	–	–	–	–	–
[2]	–	×	×	–	–	–	–	–	–	–
[3]	–	×	–	–	–	–	–	–	×	–
[4]	×	–	–	–	–	–	–	–	–	×
[5]	–	–	–	–	×	–	×	–	–	–
[6]	–	–	–	–	×	–	–	–	×	–
[7]	–	–	–	–	–	×	–	–	–	×
[8]	–	–	–	–	–	–	–	×	–	×

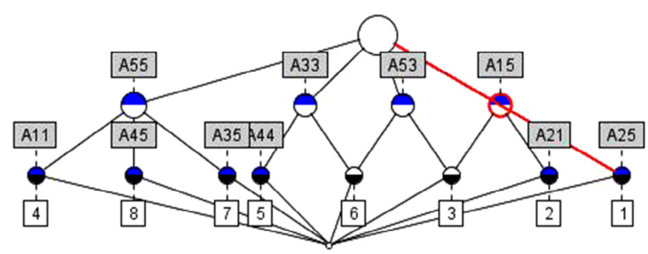

Figure 4 Lattice diagram of total sales: very high.

The implication set table for the very high total sales is presented in Table 14. Further we compute the implication relation table from the implication set Table 14 and it is given in Table 15. From the higher frequency of the implication Table 15, we can find the chief characteristics influencing the decision class very high total sales can be computed. The most important characteristics of this class are ED-low and EMi-High. The next most important characteristics that lead to this class are EM-low, EA-very high and Emi-very high. Therefore, we conclude that the total sales is very high if EA-very high, EM-low, ED-low and EMi-very high or high.

Decisions class of total sales: high

Now we present the context table of high total sales in Table 16, which converted 13 rules of high total sales representing the attributes obtained from pre-process. In Fig. 5,

Table 14 Implication set table of total sales: very high.

Implication Sets
1 <1> A21 ⇒A15
2 <1> A44 ⇒ A33
3 <1> A11 ⇒ A55
4 <1> A35 ⇒ A55
5 <1> A45 ⇒ A55
6 <0> A15 A55 ⇒ A11 A21 A25 A33 A35 A44 A45 A53
7 <0> A25 A55 ⇒ A11 A15 A21 A33 A35 A44 A45 A53
8 <0> A33 A55 ⇒ A11 A15 A21 A25 A35 A44 A45 A53
9 <0> A53 A55 ⇒ A11 A15 A21 A25 A33 A35 A44 A45
10 <0> A15 A25 ⇒ A11 A21 A33 A35 A44 A45 A53 A55
11 <0> A15 A33 ⇒ A11 A21 A25 A35 A44 A45 A53 A55
12 <0> A25 A33 ⇒ A11 A15 A21 A35 A44 A45 A53 A55
13 <0> A15 A21 A53 ⇒ A11 A25 A33 A35 A44 A45 A55
14 <0> A25 A53 ⇒ A11 A15 A21 A33 A35 A44 A45 A55
15 <0> A33 A44 A53 ⇒ A11 A15 A21 A25 A35 A45 A55
16 <0> A11 A35 A55 ⇒ A15 A21 A25 A33 A44 A45 A53
17 <0> A11 A45 A55 ⇒ A15 A21 A25 A33 A35 A44 A53
18 <0> A35 A45 A55 ⇒ A11 A15 A21 A25 A33 A44 A53

we present the lattice diagram of the context Table 16 for the decision class high total sales.

The implication set table for the high total sales is computed like the previous case. However, we have omitted this keeping in view the length of the paper. Further we compute the implication relation table from the implication set Table and this is given in Table 17. From the higher frequency of the implication Table 17, we can find the chief characteristics influencing the decision class high total sales. The most important characteristics of this class are ED-low and Emi-High; the next most important is ERD-average. Therefore, we conclude that the total sales is high if Emi-high, ED-low, and ERD-average.

Decision class of total sales: average

Now we present the context table of average total sales in Table 18, which converted 15 rules of average total sales representing the attributes obtained from pre-process. In Fig. 6, we present the lattice diagram of the context Table 18 for the decision class average total sales.

Table 15 Implication relation table of total sales: very high.

	A11	A15	A21	A25	A33	A35	A44	A45	A53	A55
A11	–	2	2	2	2	1	2	1	2	3
A15	4	–	3	3	3	4	4	4	3	–
A21	1	1	–	1	1	1	1	1	–	1
A25	4	3	4	–	3	4	4	4	3	3
A33	4	3	4	3	–	4	3	4	3	3
A35	1	2	2	2	2	–	2	1	2	–
A44	1	1	1	1	1	1	–	1	–	1
A45	1	2	2	2	2	1	2	–	2	–
A53	4	3	3	3	3	4	3	4	–	3
A55	6	6	7	6	6	4	7	6	6	–
Frequency	26	23	28	23	23	26	28	26	21	14

Table 16 Context table of total sales: high.

Rule	a_{11}	a_{13}	a_{15}	a_{21}	a_{23}	a_{24}	a_{33}	a_{34}	a_{35}	a_{43}	a_{44}	a_{45}	a_{51}	a_{53}	a_{55}
[1]	–	×	–	–	–	–	–	–	–	–	–	–	–	–	–
[2]	–	–	–	–	–	×	–	–	–	–	–	–	–	–	–
[3]	–	–	×	–	–	–	–	–	–	×	–	–	–	–	–
[4]	–	–	×	–	–	–	–	–	–	–	–	–	–	–	×
[5]	×	–	–	–	×	–	–	–	–	–	–	–	–	–	–
[6]	–	–	–	–	×	–	–	–	×	–	–	–	–	–	–
[7]	–	–	–	–	×	–	–	–	–	–	–	×	–	–	–
[8]	–	–	–	–	–	–	–	×	–	–	–	–	–	×	–
[9]	–	–	–	–	–	–	×	–	–	–	–	–	–	–	×
[10]	–	–	–	–	–	–	–	–	×	–	–	–	×	–	–
[11]	–	–	–	–	–	–	–	–	–	×	–	–	–	–	×
[12]	–	–	–	–	–	–	–	–	–	–	–	×	×	–	–
[13]	–	–	–	×	–	–	–	×	–	–	×	–	–	–	–

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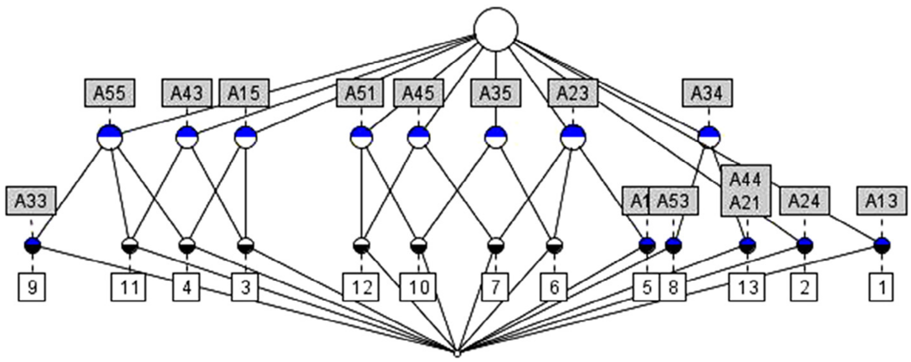

Figure 5 Lattice diagram of total sales: high.

Table 17 Implication relation table of total sales: high.

	A11	A13	A15	A21	A23	A24	A33	A34	A35	A43	A44	A45	A51	A53	A55
A11	–	2	2	2	1	2	2	2	1	2	2	1	2	2	2
A13	9	–	8	9	8	8	9	8	8	8	9	9	8	9	8
A15	9	8	–	9	8	8	8	8	8	8	9	8	8	9	7
A21	1	1	1	–	1	1	1	1	1	1	1	1	1	–	1
A23	7	8	8	9	–	8	9	9	8	8	9	8	8	9	8
A24	9	9	8	9	8	–	9	8	8	8	9	8	8	9	8
A33	2	2	1	2	2	2	–	2	2	1	2	2	2	2	1
A34	10	9	9	9	9	9	10	–	9	9	9	9	9	9	9
A35	7	7	7	8	7	7	8	7	–	7	8	7	8	8	7
A43	8	9	8	9	8	8	8	8	8	–	9	8	8	9	7
A44	1	1	1	1	1	1	1	1	1	1	–	1	1	–	1
A45	7	7	7	8	8	8	8	7	7	7	8	–	8	8	7
A51	7	6	6	7	6	6	7	6	7	6	7	7	–	7	6
A53	1	1	1	–	1	1	1	1	1	1	–	1	1	–	1
A55	10	10	9	10	9	9	8	9	9	8	10	9	9	10	–
Frequency	89	79	76	92	77	78	89	78	78	75	92	79	81	91	73

Table 18 Context table of total sales: average.

Rule	a_{14}	a_{15}	a_{21}	a_{23}	a_{33}	a_{34}	a_{42}	a_{43}	a_{44}	a_{51}	a_{55}
[1]	×	–	–	–	–	–	–	–	–	–	–
[2]	–	×	–	×	–	–	–	–	–	–	–
[3]	–	×	–	–	–	×	–	–	–	–	–
[4]	–	×	–	–	–	–	–	–	–	×	–
[5]	–	–	–	×	–	×	–	–	–	–	–
[6]	–	–	–	×	–	–	–	–	×	–	–
[7]	–	–	×	–	–	–	–	×	–	–	–
[8]	–	–	×	–	–	–	–	–	–	–	×
[9]	–	–	–	–	–	×	×	–	–	–	–
[10]	–	–	–	–	–	×	–	–	–	×	–
[11]	–	–	–	–	–	×	–	–	–	–	×
[12]	–	–	–	–	×	–	–	–	–	×	–
[13]	–	–	–	–	–	–	–	–	×	×	–
[14]	–	–	–	–	–	–	–	×	–	×	–
[15]	–	–	–	–	–	–	×	–	–	–	×

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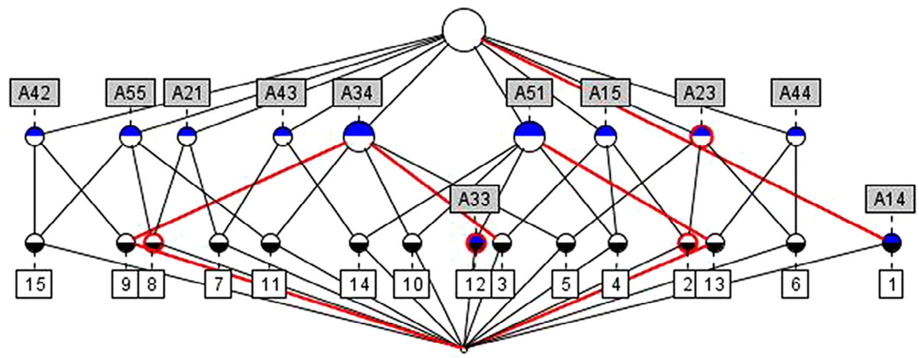

Figure 6 Lattice diagram of total sales: average.

Table 19 Implication relation table of total sales: average.

	A14	A15	A21	A23	A33	A34	A42	A43	A44	A51	A55
A14	–	8	8	8	9	8	8	8	8	8	8
A15	8	–	8	8	8	7	8	8	8	7	8
A21	6	6	–	6	7	6	6	7	6	6	7
A23	6	6	6	–	7	6	6	6	7	6	6
A33	4	3	4	4	–	3	4	3	3	1	4
A34	7	6	7	7	7	–	7	7	7	6	7
A42	7	7	7	7	8	7	–	7	7	7	7
A43	7	7	8	7	7	7	7	–	7	7	7
A44	7	7	7	8	7	7	7	7	–	7	7
A51	9	9	9	9	6	8	9	9	9	–	9
A55	6	6	7	6	7	6	6	6	6	6	–
Frequency	67	64	71	70	73	65	68	68	68	61	70

Table 20 Context table of total sales: medium.

Rule	a_{11}	a_{21}	a_{31}	a_{42}	a_{51}
[1]	–	–	×	–	–
[2]	×	×	–	–	–
[3]	×	–	–	×	–
[4]	–	–	–	×	×

The implication set table for the average total sales has been computed but is omitted due to the length restriction of this article. Further we compute the implication relation table from the implication set table and this is given in Table 19. From the higher frequency of the implication Table 19, we can find the chief characteristics influencing the decision class average total sales. The most important of the characteristics of this class is EA-average; the next most important is ED-low. Therefore, we conclude that the total sales is average because of EA-average and ED-low.

Decision class of total sales: medium

Now we present the context table of medium total sales in Table 20, which converted four rules of medium total sales representing the attributes obtained from pre-process. In

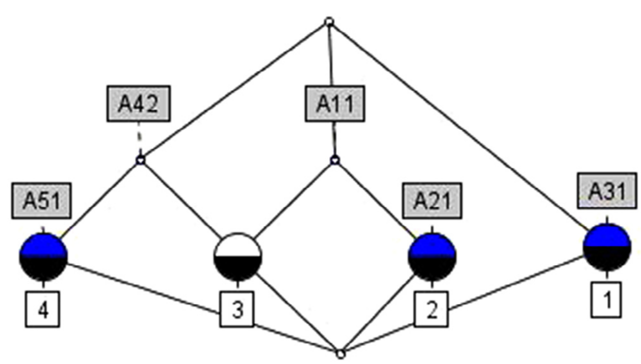

Figure 7 Lattice diagram of total sales: medium.

Fig. 7, we present the lattice diagram of the context table, Table 20, for the decision class medium total sales.

The implication set table for the medium total sales has been computed but omitted due to the length restriction of this article. Further we compute the implication relation table from the implication set table and this is given in Table 21. From the higher frequency of the implication Table 21, we can find the chief characteristics influencing the decision class medium total sales. The most important characteristics of this class are ED-low and ERD-low; the next most important is EA-low. Therefore, we conclude that the total sales is medium because of ED-low, ERD-low and EA-low.

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Table 21 Implication relation table of total sales: medium.

	A11	A21	A31	A42	A51
A11	–	2	2	1	2
A21	1	–	1	–	1
A31	1	2	–	1	2
A42	1	2	2	–	2
A51	–	1	1	1	–
Frequency	3	7	6	3	7

Future research directions

This article hybridises the concept of rough set on intuitionistic fuzzy approximation spaces and formal concept analysis. In pre-process we have used rough set on intuitionistic fuzzy approximation spaces to get the almost equivalence classes. Further research can be carried out on replacing rough set on intuitionistic fuzzy approximation space with fuzzy rough set or intuitionistic fuzzy rough set. Indeed, future selection can also be carried out by using swarm intelligence algorithms. The results obtained through all these techniques can also be compared for more insights. Additionally, future work can also be carried out in the direction of multigranulation and its applications.

Conclusion

Rough set on intuitionistic fuzzy approximation spaces extends the notion of traditional rough sets. Ordering of objects is a fundamental issue in decision making and plays a vital role in uncertain information analysis. In this study, a layout for performing the ordering of objects using rough sets on intuitionistic fuzzy approximation spaces is carried out before employing the rule generation algorithm. Further these rules are explored to identify the chief characteristics affecting the decisions total sales by using formal concept analysis. This helps the decision maker with a priori detection of the total sales. To this end, we have taken a real life example of 10 companies according to different attributes and shown how analysis can be carried out for identifying chief attributes affecting decisions. The chief characteristics of total sales-medium are expenditure on distribution-low, expenditure on R&D-low and expenditure on advertisement-low. The chief characteristics of total sales-average are expenditure on advertisement-average and expenditure on distribution-low. The chief characteristics of total sales-high are expenditure on miscellaneous-high, expenditure on distribution-low, and expenditure on R&D-average. Finally, the chief characteristics of total sales-very high are expenditure on advertisement-very high, expenditure on marketing-low, expenditure on distribution-low and expenditure on miscellaneous-high. The results obtained in pre-process can further be processed with the help of domain intelligence experts to obtain more specific characteristics of attributes affecting decisions. We believe that, formal concept analysis can be used to find more information regardless of the type of rule based soft computing. In addition, we believe that the proposed model is useful for decision makers to take decisions.

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