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A Method for Decomposition of Fuzzy Formal Context

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Abstract

Recently, fuzzy FCA becomes popular in researchers for representing the knowledge precisely. In this study we present a method for decomposition of given fuzzy formal context into several crisp formal contexts with help of distinct granules of fuzzy relation. The presented method reduces the lattice of given fuzzy formal context into several lattices for refining the interested patterns. From the obtained crisp formal contexts, we can generate the formal concepts in order to visualize them in lattice structure for describing the knowledge of given research field. Proposed method can be helpful for the researchers for reducing number of concepts, visualization of concepts for distinct fuzzy relations, and reduction of fuzzy lattice into several lattices in fuzzy context related fields.

Keywords: Context decomposition; Formal concept analysi;, Fuzzy concepts; Fuzzy formal context; Fuzzy relation.

Introduction

Formal Concept Analysis (FCA) presented by Wille at early 80's in mathematical order and lattice theory [1]. Now a day, FCA has become an effective mathematical tool for knowledge discovery and representation. From the formal context, FCA discovers concepts and visualizes them in the form of generalization and specialization in a heirachical order called as concept lattice [2]. Fuzzy logic in FCA represents vague and imprecise information by assigning membership value; hence it is a generalization of classical FCA [3-4]. Several algorithms are available in fuzzy FCA for generating fuzzy concepts discussed by Belohlavek [4]. How to handle large fuzzy formal context for knowledge discovery is open problem for research communities at present time. Several methods are proposed earlier on context

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decomposition, matrix decomposition, fuzzy lattice decomposition, attribute reductions and fuzzy K-means clustering in FCA [5-19]. Recently, Konecny et al presented a method for reducing fuzzy lattice [15]. Fuzzy graph used in fuzzy FCA for building the fuzzy lattice by Ghosh et al [16]. Gely discussed modular decomposition [17], Dubois et al provided possibility theory [18], and Cross et al presented fuzzy factor analysis of fuzzy formal context [19].

Main purpose of knowledge reduction from a given fuzzy formal context is (a) to reduce the number of concepts, (b) visualize them through lattice structure, (c) for describing the knowledge of given research field [20-22]. In this paper a method is proposed, which decompose the fuzzy formal context into several crisp formal contexts for knowledge discovery. Rest of the paper is organized as follows. Section 2 provides brief background about fuzzy FCA. We propose the method in section 3. Section 4 provides illustration of proposed method followed by conclusions, acknowledgements and references.

2. Fuzzy Formal Concept Analysis

A fuzzy formal context is a triplet **F**= (**O**, **P**, $\underset{\sim}{R}$) where **O** is set of objects, **P** is set of attributes and $\underset{\sim}{R}$ is a fuzzy relation on $\mathbf{O}\times\mathbf{P}$. Relation $(o,a)\in\underset{\sim}{R}$ represent that fuzzy object $o\in\mathbf{O}$ has membership value $\mu(o,a)$ in [0,1] with fuzzy attributes $a\in\mathbf{P}$ [19, 23]. For objects, attributes and fuzzy relation of any fuzzy formal context several possibilities are discussed in Table 1. From these possibilities we can conclude that, most of the conditions fuzzy relation completely given for generating the concepts. Hence a generalized method for decomposition of the fuzzy formal context can depends on fuzzy relation of fuzzy formal context.

Table 1. Possibilities for fuzzy formal context

Conditions	Objects	Attributes	Fuzzy relation
(a)	Complete	Complete	Incomplete
(b)	Incomplete	Complete	Complete
(c)	Complete	Incomplete	Complete
(d)	Incomplete	Complete	Complete
(e)	Crisp	Fuzzy	Complete
(f)	Fuzzy	Crisp	Complete
(g)	Fuzzy	Fuzzy	Complete
(h)	Crisp	Crisp	Complete

For a fuzzy formal context **K**, let $L^{\mathbf{O}}$ $([0,1]^{O},\subseteq)$ and $L^{\mathbf{P}}$ $([0,1]^{P},\subseteq)$ be two posets then the operator (↑) $f(o)$: $L^{\mathbf{O}}([0,1]^{O})\rightarrow L^{\mathbf{P}}([0,1]^{P})$ and (↓) g (a): $L^{\mathbf{P}}([0,1]^{P})\rightarrow L^{\mathbf{O}}([0,1]^{O})$ are defined $\forall$ $o\subseteq\mathbf{O}$ and $a\subseteq\mathbf{P}$ where L is residuated lattice [24],

- $f(o) = O^{\uparrow} =\{\ \mu_a(\alpha)\,|\,\alpha = \min \mu_{\underset{\sim}{R}}(o,a),\ \forall\ o\in\mathbf{O}\}$,
- $g(a) = P^{\downarrow} =\{\ \mu_o(\alpha)\,|\ \alpha = \min \mu_{\underset{\sim}{R}}(o,a),\ \forall\ a\in\mathbf{P}\}$

The operator $O^{\uparrow}$ is called as extent- intent fuzzy operator and $P^{\downarrow}$ is called as intent-extent fuzzy operator either fuzzy concept forming operators.

A fuzzy lattice is a partial ordered set of ($\underset{\sim}{R}$; ≤), in which for every pairs of (o,a) $\exists$ a sup= $o\vee a$ and an inf $= o\wedge a$. A residuated lattice $\mathbf{L} = (\mathrm{L},\wedge,\vee,\otimes,\rightarrow,0,1)$ is the finite structure of truth values of object and its properties. **L** is complete residuated lattice iff [4],

(1). $(\mathrm{L},\wedge,\vee,0,1)$ is a complete lattice.

(2). (L,⊗,1) is commutative monoid. (i. e. ⊗ is commutative and associative means $a \otimes 1 = 1 \otimes a = a$, $\forall\ a \in$ **L).**

(3). ⊗and → are binary operation called multiplication and residuum, respectively.

The ⊗ and → operators are defined distinctly by Lukaswiech, Gödel, and Goguen t-norms and their residua [16]. FC_K is set of fuzzy formal concepts generated from a given fuzzy formal context **F.** If set FC_K follows the ordering principle of set using the super and sub hierarchy properties of lattice $(o_2, a_2) \leq (o_1, a_1) :\Leftrightarrow\ o_2 \subseteq o_1\ (\Leftrightarrow\ a_2 \supseteq a_1)$ for every fuzzy concepts, then it forms a complete fuzzy Galois lattice $L_{FC_K} = (FC_K, \leq)$. The fuzzy concept lattice have a inf (0,0,…,0) and sup (1,1,…,1) for every fuzzy concepts [25].

3. Proposed Method

Step 1. For a discrimination level α which is a real number in [0, 1], an α -cut can be defined for a fuzzy relation. Hence, we find the distinct fuzzy relations $\mu(o,a) \in \underset{\sim}{R}$ between objects and attributes available in given fuzzy formal context and denote as α .

Step 2. Then given fuzzy formal context can be decomposed into several formal contexts for distinct value of α defined in step 1. The decomposed formal context for α is represented as K_α. Each of the formal context K_α represents partition for corresponding value of the α . The fuzzy relation belongs to the same decomposed formal context iff $K_\alpha = \{(o,a) \mid \mu(o,a) \geq \alpha,\ o \in \mathbf{O},\ a \in \mathbf{P}\}$, So K_α has become crisp formal context for α .

Step 3. Hence, a given fuzzy formal context can be represented by these decomposed contexts αK_α with the help of standard fuzzy union operator defined as below:

- **F**$=\bigcup_{\alpha} \alpha K_\alpha$, $\forall \alpha \in \underset{\sim}{R}$ where, αK_α is a fuzzy formal context with fuzzy relation α .

Step 4. The decomposed formal context satisfies the following properties:

- $\mu_{\alpha K_\alpha}(o,a) = \alpha . \mu_{K_\alpha}(o,a)\ \forall\ K_\alpha, \alpha \in \underset{\sim}{R}$.
- If $\alpha_1 \geq \alpha_2$ then $K_{\alpha_1} \subseteq K_{\alpha_2}$ for every α defined in step 1.
- The number of concepts generated from each crisp context (K_α) depends on given fuzzy formal context.

4. Illustrations

For illustration of proposed method we have considered a fuzzy context shown in Table 2, [16] in which distinct discrimination levels are {0.0, 0.5, 1.0} denoted as α [22]. For α =0.0 we get same fuzzy context shown in Table 2, since it is a trivial case. The decomposed formal contexts, for α =0.5 is $K_{0.5}$ and for α =1.0 is $K_{1.0}$, are shown in Table 3 and Table 4 respectively. Next, the given fuzzy formal context **F** can be represented as **F**=0.5 $K_{0.5} \cup 1.0\, K_{1.0}$. Here we can also observed that, $1.0 \geq 0.5$, then $K_{1.0} \subseteq K_{0.5}$.

Table 2. Fuzzy formal context (**O**, **P**, $\underset{\sim}{R}$)

	P_1	P_2	P_3	P_4	P_5	P_6
O_1	0.0	1.0	0.5	0.5	1.0	0.0
O_2	1.0	1.0	1.0	0.0	0.0	0.0
O_3	0.5	0.5	0.0	0.0	0.0	1.0
O_4	0.0	0.0	0.0	1.0	0.5	0.0
O_5	0.0	0.0	1.0	0.5	0.0	0.0
O_6	0.5	0.0	0.0	0.0	0.0	0.0

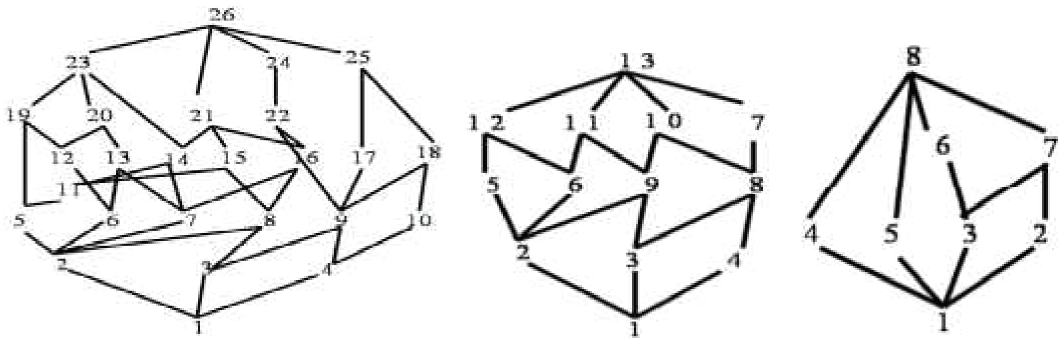

Fig. 1. (a) Fuzzy concept lattice for Table 2; (b) Lattice for Table 3; (c) Lattice for Table 4.

Table 3. Context for $K_{0.5}$

	P_1	P_2	P_3	P_4	P_5	P_6
O_1	0	1	1	1	1	0
O_2	1	1	1	0	0	0
O_3	1	1	0	0	0	1
O_4	0	0	0	1	1	0
O_5	0	0	1	1	0	0
O_6	1	0	0	0	0	0

Table 4. Context for $K_{1.0}$

	P_1	P_2	P_3	P_4	P_5	P_6
O_1	0	1	0	0	1	0
O_2	1	1	1	0	0	0
O_3	0	0	0	0	0	1
O_4	0	0	0	1	0	0
O_5	0	0	1	0	0	0
O_6	0	0	0	0	0	0

All fuzzy formal concepts generated from Table 2 are:

1. $\{\{\phi\}, \{P_1,P_2,P_3,P_4,P_5,P_6\}\}$
2. $\{\{0.5/O_1\}, \{P_2,P_3,P_4,P_5\}\}$
3. $\{\{O_2\}, \{P_1,P_2,P_3\}\}$
4. $\{\{0.5/O_3\}, \{P_1,P_2,P_6\}\}$
5. $\{\{0.5/O_1, 0.5/O_5\}, \{P_3,P_4\}\}$
6. $\{\{0.5/O_1, 0.5/O_4\}, \{P_4,P_5\}\}$
7. $\{\{O_1\}, \{P_2,0.5/P_3,0.5/P_4,P_5\}\}$
8. $\{\{0.5/O_1,O_2\}, \{P_2,P_3\}\}$
9. $\{\{O_2,0.5/O_3\}, \{P_1,P_2\}\}$
10. $\{\{O_3\}, \{0.5/P_1,0.5/P_2,P_6\}\}$
11. $\{\{0.5/O_1,O_5\}, \{P_3,0.5/P_4\}\}$
12. $\{\{0.5/O_1,O_4\}, \{P_4,0.5/P_5\}\}$
13. $\{\{O_1,0.5/O_4\}, \{0.5/P_4,P_5\}\}$
14. $\{\{O_1,O_5\}, \{0.5/P_3,0.5/P_4\}\}$
15. $\{\{0.5/O_1,O_2,O_5\}, \{P_3\}\}$
16. $\{\{O_1,O_2\}, \{P_2,0.5/P_3\}\}$
17. $\{\{O_2,0.5/O_3,0.5/O_6\}, \{P_1\}\}$
18. $\{\{O_2,O_3\}, \{0.5/P_1,0.5/P_2\}\}$

19. $\{\{0.5/O_1, O_4, 0.5/O_5\}, \{P_4\}\}$
20. $\{\{O_1, O_4\}, \{0.5/P_4, 0.5/P_5\}\}$
21. $\{\{O_1, O_2, O_5\}, \{0.5/P_3\}\}$
22. $\{\{O_1, O_2, 0.5/O_3\}, \{P_2\}\}$
23. $\{\{O_1, O_4, O_5\}, \{0.5/P_4\}\}$
24. $\{\{O_1, O_2, O_3\}, \{0.5/P_2\}\}$
25. $\{\{O_2, O_3, O_6\}, \{0.5/P_1\}\}$
26. $\{\{O_1, O_2, O_3, O_4, O_5, O_6\}, \phi\}$

The lattice for fuzzy concepts generated from Table 2 is shown in Figure 1(a) [16].

All formal concepts generated from Table 3 are:

1. $\{\{\phi\}, \{P_1, P_2, P_3, P_4, P_5, P_6\}\}$
2. $\{\{O_1\}, \{P_2, P_3, P_4, P_5\}\}$
3. $\{\{O_2\}, \{P_1, P_2, P_3\}\}$
4. $\{\{O_3\}, \{P_1, P_2, P_6\}\}$
5. $\{\{O_1, O_4\}, \{P_4, P_5\}\}$
6. $\{\{O_1, O_5\}, \{P_3, P_4\}\}$
7. $\{\{O_2, O_3, O_6\}, \{P_1\}\}$
8. $\{\{O_2, O_3\}, \{P_1, P_2\}\}$
9. $\{\{O_1, O_2\}, \{P_2, P_3\}\}$
10. $\{\{O_1, O_2, O_3\}, \{P_2\}\}$
11. $\{\{O_1, O_2, O_5\}, \{P_3\}\}$
12. $\{\{O_1, O_4, O_5\}, \{P_4\}\}$
13. $\{\{O_1, O_2, O_3, O_4, O_5, O_6\}, \phi\}$.

The lattice for concepts generated from Table 3 is shown in Figure 1(b). All formal concepts generated from Table 4 are:

1. $\{\{\phi\}, \{P_1, P_2, P_3, P_4, P_5, P_6\}\}$
2. $\{\{O_1\}, \{P_2, P_5\}\}$
3. $\{\{O_2\}, \{P_1, P_2, P_3\}\}$
4. $\{\{O_3\}, \{P_6\}\}$
5. $\{\{O_4\}, \{P_4\}\}$
6. $\{\{O_2, O_5\}, \{P_3\}\}$
7. $\{\{O_1, O_2\}, \{P_2\}\}$
8. $\{\{O_6\}, \phi\}$.

The lattice for concepts generated from Table 4 is shown in Figure 1(c). We can observe that,

- Fuzzy context shown in Table 2 is decomposed into crisp contexts as shown in Table 3 and 4.
- The number of concepts generated from Table 3 and 4 are lesser than generated from Table 2.
- The fuzzy lattice shown in Figure1 (a) is reduced into lattices as shown in Figure 1(b) and 1(c). The lattice shown in Figure 1(c) is similar to the lattice build by projection method with respect to attributes for same fuzzy context [13].
- The specialized and generalized objects and attributes for Figure 1(a) are approximately same in reduced lattice shown in Figure 1(b) and Figure 1(c) respectively.

We believe that, presented method can be helpful for the researchers in the field of (a) Information retrieval, (b) Data mining, (c) Frequent item set mining, (d) Gene expression data, (e) Decompositions of fuzzy formal context for knowledge discovery (f) Reducing the number of concepts (g) Reduction of fuzzy lattice into several lattices for discovering interested patterns in data and other fuzzy context related fields.

5. Conclusions

In this paper a method is proposed for decomposition of a given fuzzy formal context into several crisp contexts for knowledge discovery and representation. Proposed method decomposes the given fuzzy formal context with help of distinct granules of fuzzy relation which is denoted as α between [0, 1]. From these decomposed formal contexts, we can generate the concepts and visualize them in the lattice structure. Hence proposed method reduces the number of concepts and also the lattice structure. However, the number of concepts generated from each decomposed context depends on given fuzzy context. Also, concept lattices of these crisp formal contexts show approximately same specialized and generalized concepts which can be observed in Figure 1(a), 1(b) and 1(c). In future this method can be extended for other t- co norm operators.

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