A method for solving equally spread symmetric fuzzy assignment problems using branch and bound technique

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A method for solving equally spread symmetric fuzzy assignment problems using branch and bound technique

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Abstract. The fuzzy assignment model depicts a special case of fuzzy linear programming problem (F.L.P.P) in which the objective is to assign the number of origins to the same number of destinations at minimum total cost. The fuzzy assignment should be framed on one-to-one basis. In this research article it is developed a method using the Branch and Bound Technique has been developed for symmetric fuzzy assignment (equally spread) of jobs. Also in this method fuzzy optimal solution has been obtained without converting the symmetric fuzzy assignment problem (equally spread) as a crisp.

1. Introduction

The fuzzy assignment model describes a special case of fuzzy linear programming problem. Tanaka et al [1,2] and Zimmermann[3]were done a revolutionary work on such fuzzy linear programming problems. So far different methods have been proposed to solve different kinds of FLPP.In 1965, Lotfi Zadeh [4] has introduced fuzzy sets which provide as a new mathematical tool to deal with uncertainty of information.

Das et.al [6] presented Ranking of trapezoidal intuitionistic fuzzy numbers. Kuhn[7], The Hungarian method for the assignment and transportation problems. Kalaiarasi et.al [8] Optimization of fuzzy assignment model with triangular fuzzy numbers. Different kinds of fuzzy assignment problems are solved in the papers [9,10,11,12].

Here the main objective is to assign the number of origins to the same number of destinations at minimum total cost. The assignment should be made on one-to-one basis. In most number of cases the departments will have the specialists for operating certain critical or sophisticated machinery/equipment while in some other cases anybody can operate any machine. Whatever the type of step may it be, it is necessary for a manager to see that maximum work is to be taken out from his man power resources which are precious and scarce. This can be done only when right job is allotted to right man.

Generally, all the men cannot have same efficiency and knowledge on all the jobs even if anybody can do job. A person can do a job fast and another person may be slow in doing the same job. A person can do a job fast and another job slow. If the jobs are assigned accordingly to their efficiencies, the maximum output can be derived in minimum time.

The proven scientific Hungarian assignment method is the most suitable method for job shop production developed by Hungarian mathematician D. Konig. It is also called “Floods Technique or Fast Food Technique”. It works on the principle of reducing the cost matrix to opportunity cost which shows the relative penalties associated with assigning resources to an activity as opposed to making

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the best or least cost assignment. If we can reduce the cost matrix to the extent of possessing at least one zero in each row and column, then it will be possible to make optimal assignments.

It develops a new method using branch and bound technique. It is a step-by-step approach towards the optimal solution, out of the space of all feasible solutions of a problem which is finite in number. The set of all feasible solutions are continuously branched into smaller subsets and a lower bound (minimum cost in the case of cost matrix) is found within each subset. The next branching is made from the latest (higher order) active bound. The accuracy of the optimal solution depends only on the number of solutions verified before the end of the search.

In most of the methods fuzzy assignment problem has been transformed into crisp assignment problem in the linear programming problem form and solved by using Branch and bound. Robust's ranking method [5] for the fuzzy numbers is one of them.

2. Preliminaries

2.1. Triangular fuzzy number

$\tilde{A} = (a_1, a_2, a_3)$is a real fuzzy number and it is a fuzzy subset of the real number R with Membership function $\mu(\tilde{A})$ satisfies the below conditions,

- $\mu(\tilde{A})$ is continuous from R to $[0,1]$
- $\mu(\tilde{A})$ is strictly increasing and continuous on $[a_1, a_2]$
- $\mu(\tilde{A})$ is strictly decreasing and continuous on $[a_2, a_3]$

2.2. Spread of triangular fuzzy number

Left Spread of the triangular fuzzy number is $Left(\tilde{A}) = (a_2 - a_1)$ and Right Spread of the triangular fuzzy number is $Right(\tilde{A}) = (a_3 - a_2)$. Based on this we can classify the fuzzy number into three types, as follows:

Left Spread Fuzzy Number

If $Left(A) > Right(\tilde{A})$ we can say the triangular fuzzy number Ã is a left spread fuzzy number.

Right Spread Fuzzy Number

If $Left(\tilde{A}) < Right(\tilde{A})$ we can say the triangular fuzzy number Ã is a right spread fuzzy number.

Equally Spread Fuzzy Number

If $Left(\tilde{A}) = Right(\tilde{A})$ we can say the triangular fuzzy number Ã is a equally spread fuzzy number.

2.3. Relation between two triangular fuzzynumbers (Symmetric)

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be the two non-negative symmetric triangular fuzzy numbers with equal spread. Then the relation between those two can be defined as follows:

1. If$a_1 < b_1$ then $\tilde{A} < \tilde{B}$
2. If $a_1 > b_1$ then $\tilde{A} > \tilde{B}$
3. If $a_1 = b_1$ then $\tilde{A} = \tilde{B}$

3. Proposed method for solving symmetric fuzzy assignment problem (Equally Spread type)

Step-1

Find the least time of each job and compute the total time taken to complete all the jobs. This number is a lower bound on all solutions. Therefore no assignment is possible with a time lesser than the lower bound.

Step-2

Consider the feasible solution into n subsets. Assign Job-1 to candidate C1 and the remaining jobs are assigned to remaining candidates with least after ignoring "one job one candidate" assumption. Calculate the total time.

Step-3

Assign Job-1 to C_2 and the remaining jobs to remaining candidates with least time. Repeat this process until Job-1 is assigned to all the n candidates. Calculate the total time at each step.

Step-4

Construct the tree denoting the total time below the corresponding node and mention the assignment over the arrow (branches). The next branching is made from the terminal node which is the least total time. If there is a tie, then the terminal node at the higher level is branched for unique solution.

Step-5

Repeat steps (2) to (4) by taking Job-2, Job-3,…., Job-n in the place of Job-1.

Step-6

If the minimum total time happens to be at any of the terminal node at the $(n-1)^{th}$ level, the optimality is attained. Now the optimal assignment is given by the path starting from the initial node to the terminal node together with the missing job and candidate assignment.

In the following problem it can be observed by finding fuzzy optimal solution that new proposed method is effective than the existing.

4. Numerical example

Here we consider a numerical example with the data of symmetric triangular fuzzy numbers (all are equally spread type). By proposed method we can solve this kind of problems without converting them to a crisp valued assignment problem.

Machines

Jobs	**M_1**	**M_2**	**M_3**	**M_4**
J_1	(15,17,19)	(12,14,16)	(14,16,18)	(2,4,6)
J_2	(13,15,17)	(2,4,6)	(12,14,16)	(10,12,14)
J_3	(7,9,11)	(14,16,18)	(12,14,16)	(11,13,15)
J_4	(6,8,10)	(5,7,9)	(9,11,13)	(13,15,17)

Table 1. Fuzzy valued assignment problem

Solution using proposed method

First find the minimum time of each job and calculate the total time taken for completion of all the jobs.

Jobs	**Minimum Time**	**Corresponding Machine**
J_1	(2,4,6)	M_4
J_2	(2,4,6)	M_2
J_3	(7,9,11)	M_1
J_4	(5,7,9)	M_2
TOTAL	**(16,24,32)**	

Table 2. The minimum total time

This denotes the minimum total time for the solution. But this is not feasible since M_2 cannot do more than one Job. Now let us check the total time of all the Jobs.

Job	**Time**	**Machine**
J_1	(15,17,19)	M_1
J_2	(2,4,6)	M_2
J_3	(11,13,15)	M_4
J_4	(5,7,9)	M_2
TOTAL	**(33,41,49)**	

Table 3. If J_1 is assigned to M_1

Job	Time	Machine
J_1	(12,14,16)	M_2
J_2	(10,12,14)	M_4
J_3	(7,9,11)	M_1
J_4	(6,8,10)	M_1
TOTAL	**(35,43,51)**	

Table 4. If J_1 is assigned to M_2

Job	Time	Machine
J_1	(14,16,18)	M_3
J_2	(2,4,6)	M_2
J_3	(7,9,11)	M_1
J_4	(5,7,9)	M_2
TOTAL	**(28,36,44)**	

Table 5. If J_1 is assigned to M_3

Job	Time	Machine
J_1	(2,4,6)	M_4
J_2	(2,4,6)	M_2
J_3	(7,9,11)	M_1
J_4	(5,7,9)	M_2
TOTAL	**(16,24,32)**	

Table 6. If J_1 is assigned to M_4

Therefore the first branch of the solution tree is as follows

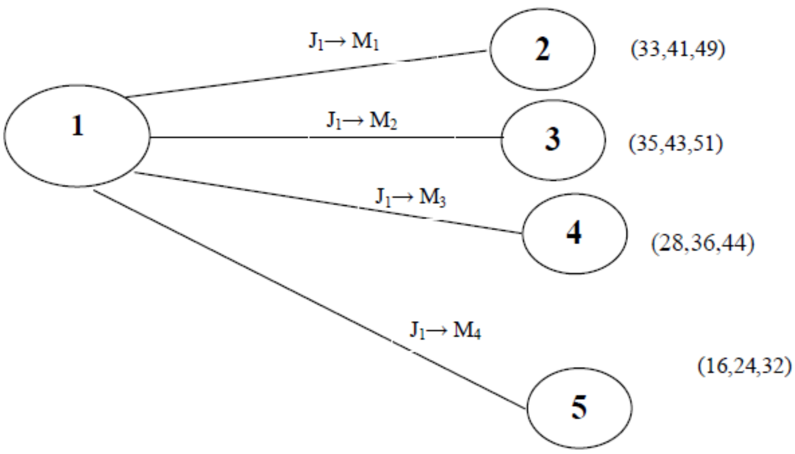

Figure 1. First branch of the solution tree

$J_1 \rightarrow M_4$ is minimum. So for$J_1 \rightarrow M_4$.

Now the reduced problem becomes

Machines

Jobs	**M_1**	**M_2**	**M_3**
J_2	(13,15,17)	(2,4,6)	(12,14,16)
J_3	(7,9,11)	(14,16,18)	(12,14,16)
J_4	(6,8,10)	(5,7,9)	(9,11,13)

Table 7. Reduced problem

Again let us change the machines for J_2, J_3, and J4 and calculate the time.

Job	**Time**	**Machine**
J_1	(2,4,6)	M_4
J_2	(13,15,17)	M_1
J_3	(12,14,16)	M_3
J_4	(9,11,13)	M_3
Total	**(36,44,52)**	

Table 8. J_2 assigned to M_1

Job	**Time**	**Machine**
J_1	(2,4,6)	M_4
J_2	(2,4,6)	M_2
J_3	(7,9,11)	M_1
J_4	(6,8,10)	M_1
Total	**(17,25,33)**	

Table 9. J_2 assigned to M_2

Job	**Time**	**Machine**
J_1	(2,4,6)	M_4
J_2	(12,14,16)	M_3
J_3	(7,9,11)	M_1
J_4	(6,8,10)	M_1
Total	**(27,35,43)**	

Table 10. J_2 assigned to M_3

Second branch of the solution tree is as follows

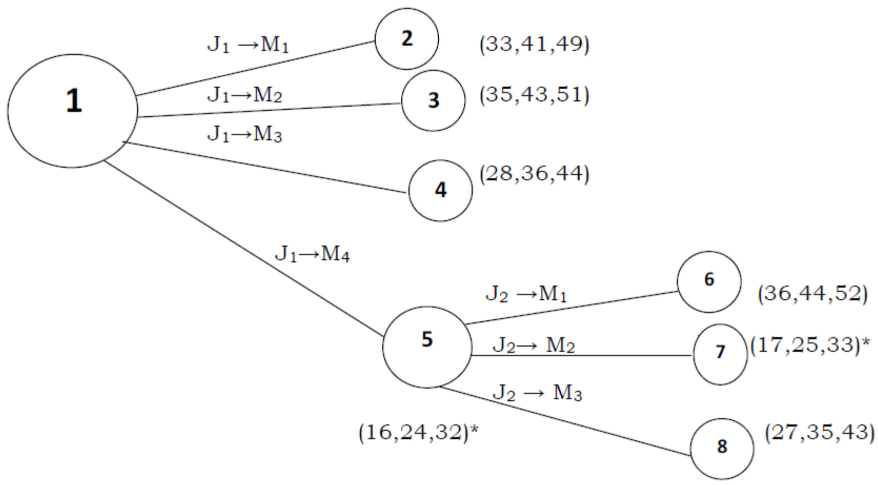

Figure 2. Second branch of the solution tree

Here (17, 25, 33) is minimum time the nodes 2, 3, 4, 6, 8 are having total time greater than or equal to (17, 25, 33), they are not considered for further branching.

Job	Time	Machine
J_1	(2,4,6)	M_4
J_2	(2,4,6)	M_2
J_3	(7,9,11)	M_1
J_4	(9,11,13)	M_3
Total	**(20,28,36)**	

Table 11. If J_3 is assigned to M_1

Job	Time	Machine
J_1	(2,4,6)	M_4
J_2	(2,4,6)	M_2
J_3	(12,14,16)	M_3
J_4	(6,8,10)	M_1
Total	**(22,30,38)**	

Table 12. If J_3 is assigned to M_3

The optimal assignment is

$J_1 \rightarrow M_4$ = (2, 4, 6)
$J_2 \rightarrow M_2$ = (2, 4, 6)
$J_3 \rightarrow M_1$ = (7, 9, 11)
$J_4 \rightarrow M_3$ = (9, 11, 13)
Total time = (20, 28, 36) Hours

5. Conclusions

The proposed method having simple steps to compute the fuzzy optimal solution for the symmetric fuzzy assignment problems (Equally spread). These kind of problems can be solved by proposed method easily without converting them as a crisp. Minimum time for each job is considered for assigning it machine. Fuzzy assignment problems require computing reduced cost matrix. Drawing minimum number of lines to cover all zeros needs is critical and a complex computation. Opportunity matrix is revised repeatedly to get the fuzzy optimal solution. Height of solution tree in Branch and Bound is n, the number of jobs. Width of tree reduces from n nodes at level 1 to one node at level n. For any assignment problem Branch and Bound technique requires computing n (n+1)/2 nodes.

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