A New Approach to Interval-Valued Fuzzy Soft Sets and Its Application in Decision-Making

B.K. Tripathy, T.R. Sooraj and R.K. Mohanty

Abstract Soft set (SS) theory was introduced by Molodtsov to handle uncertainty. It uses a family of subsets associated with each parameter. Hybrid models have been found to be more useful than the individual components. Earlier interval-valued fuzzy set (IVFS) was introduced as an extension of fuzzy set (FS) by Zadeh. Yang introduced the concept of IVFSS by combining and soft set models. Here, we define IVFSS through the membership function approach to define soft set by Tripathy et al. very recently. Several concepts, such as complement of an IVFSS, null IVFSS, absolute IVFSS, intersection, and union of two IVFSSs, are redefined. To illustrate the application of IVFSSs, a decision-making (DM) algorithm using this notion is proposed and illustrated through an example.

Keywords SS · FS · FSS · IVFS · IVFSS · DM

1 Introduction

Fuzzy set introduced by Zadeh [1] in 1965 has been found to be a better model of uncertainty and has been extensively used in real-life applications. To bring topological flavor into the models of uncertainty and associate family of subsets of a universe to parameters, SS model was proposed in 1999 [2]. The study on SS was carried out by Maji et al. [3, 4]. As mentioned in the abstract, hybrid models obtained by suitably combining individual models of uncertainty have been found to be more efficient than their components. Following this trend Maji et al. [5] put forward the concept of FSS as a hybrid model from FS and SS. Tripathy et al. [6] defined soft sets through their

B.K. Tripathy (✉) · T.R. Sooraj · R.K. Mohanty
SCOPE, VIT University, Vellore, Tamilnadu, India
e-mail: tripathybk@vit.ac.in

T.R. Sooraj
e-mail: soorajtr19@gmail.com

R.K. Mohanty
e-mail: rknmohanty@gmail.com

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characteristic functions. This approach has been highly authentic and helpful in defining the basic operations, such as the union, intersection, and complement of soft sets. Similarly, it is expected that defining membership function for fuzzy soft sets will systematize many operations defined upon them as done in [7, 8]. Extending this approach further, we introduce the membership functions for IVFSS in this paper. In [2], some applications of SS were discussed. In [3], an application to decision-making is proposed. This study was further extended to the context of FSSs by Tripathy et al. [8], where they identified some drawbacks in [3] and took care of these drawbacks while introducing an algorithm for decision-making. In this paper, we have carried this study further using IVFSS in handling the problem of multi-criteria decision-making. This notion further extended in [9–14].

It is well known that IVFSS introduced by Yang [15] is a more realistic model of uncertainty than the fuzzy set. In [16], an application of IVFSS is given. This concept is extended in [17] by taking parameters as fuzzy entities. Here, we follow the definition of soft set proposed in [6] in defining IVFSS and redefine the basic operations on them. The highlight of this work is the introduction of a decision-making algorithm that uses IVFSS, and we illustrate the suitability of this algorithm in real-life situations. In addition, it generalizes the algorithm introduced in [8] while keeping the authenticity intact.

2 Definitions and Notions

By $P(U)$ and $I(U)$, we denote the power set and the fuzzy power set of U, respectively.

Definition 2.1 (*Soft Set*) A pair (F, E) is called as a soft set over the universal set U, where

$$F: E \to P(U) \tag{2.1}$$

The pair (U, E), which is a combination of a universal set U and a parameter set E, is called a soft universe.

Definition 2.2 We denote a FSS over (U, E) by (F, E), where

$$F: E \to I(U) \tag{2.2}$$

Let $I([0, 1])$ denote the set of all closed subintervals of [0, 1].

Definition 2.3 (IVFS) An IVFS X on a universe U is a mapping, such that

$$\mu_X: U \to Int([0, 1]) \tag{2.3}$$

Moreover, $\forall x \in U, \mu_X(x) = [\mu_X^-(x), \mu_X^+(x)] \subseteq [0, 1]$. Here, $\mu_X^-(x)$ and $\mu_X^+(x)$ represent as the lower and upper degrees of membership of x to X.

3 Interval-Valued FSS

In this section, we follow the membership function approach introduced in [7] to define IVFSS. The basic operations on IVFSS are also redefined. Let (*F, E*) be an IVFSS. We associate with (*F, E*) a family of parameterized membership functions $\mu_{(F,E)} = \left\{\mu^a_{(F,A)} | a \in E\right\}$ as in (3.1).

Definition 3.1 Given $a \in E$ and $x \in X$, we define

$$\mu^a_{(F,E)}\ (\mathrm{x}) = [\alpha, \beta] \in \mathrm{I}([0, 1]) \tag{3.1}$$

For any two IVFSS (*F*, *E*) and (*G*, *E*), we define the following operations.

Definition 3.2 The union of (*F, E*) and (*G, E*) is the IVFSS (*H, E*), and $\forall a \in E$ and $\forall x \in U$, we have

$$\begin{aligned}(F,E) \cup (G,E)(x) &= \max[\mu^a_{(F,E)}(x), \mu^a_{(G,E)}(x)] \\ &= [\max(\mu^{a-}_{(F,E)}(x), \mu^{a-}_{(G,E)}(x)), \max(\mu^{a+}_{(F,E)}(x), \mu^{a+}_{(G,E)}(x))]\end{aligned} \tag{3.2}$$

where $\mu^{a-}_{(F,E)}$ and $\mu^{a+}_{(F,E)}$ denotes the lower and upper membership value of the IVFSS.

Definition 3.3 The intersection of (*F, E*) and (*G, E*) is the IVFSS (*H, E*), and $\forall a \in E$ and $\forall x \in U$, we have

$$\begin{aligned}(F,E) \cap (G,E)(x) &= \min[\mu^a_{(F,E)}(x), \mu^a_{(G,E)}(x)] \\ &= [\min(\mu^{a-}_{(F,E)}(x), \mu^{a-}_{(G,E)}(x)), \min(\mu^{a+}_{(F,E)}(x), \mu^{a+}_{(G,E)}(x))]\end{aligned} \tag{3.3}$$

Definition 3.4 (*F, E*) is said to be interval valued fuzzy soft subset of (*G, E*), $(F,E) \subseteq (G,E)$. Then, $\forall a \in E$, $\forall x \in U$,

$$\mu^{a+}_{(F,E)}(x) \le \mu^{a+}_{(G,E)}(x) \text{ and } \mu^{a-}_{(F,E)}(x) \le \mu^{a-}_{(G,E)}(x) \tag{3.4}$$

Definition 3.5 We say that (*F, E*) is equal to (*G, E*) written as (*F, E*) = (*G, E*) if $\forall x \in U$,

$$\mu^{a+}_{(F,E)}(x) = \mu^{a+}_{(G,E)}(x) \text{ and } \mu^{a-}_{(F,E)}(x) = \mu^{a-}_{(G,E)}(x) \tag{3.5}$$

Definition 3.6 For any two IVFSSs (*F, E*) and (*G, E*) over a common soft universe (*U, E*), we define the complement (*H, E*) of (*G, E*) in (*F, E*) as $\forall a \in E$ and $\forall x \in U$.

$$\mu_{(H,E)}^{a+}(x) = \max\left\{0, \mu_{(F,E)}^{a+}(x) - \mu_{(G,E)}^{a+}(x)\right\} \text{ and } \mu_{(H,E)}^{a-}(x) = \max\left\{0, \mu_{(F,E)}^{a-}(x) - \mu_{(G,E)}^{a-}(x)\right\} \tag{3.6}$$

Definition 3.7 The complement of an IVFSS over a soft universe (*U, E*) can be derived from the above Definition 3.6 by taking (*F, E*) as *U* and (*G, E*) as (*F, E*). We denote it by $(F, E)^c$ and clearly $\forall x \in U$ and $\forall e \in E$,

$$\begin{aligned} \mu_{(F,E)^c}^{e+}(x) &= \max(0, \mu_U^{e+}(x) - \mu_{(F,E)}^{e+}(x)) \text{ and} \\ \mu_{(F,E)^c}^{e-}(x) &= \max(0, \mu_U^{e-}(x) - \mu_{(F,E)}^{e-}(x)) \end{aligned} \tag{3.7}$$

It can be seen easily that

$$\mu_{(F,E)^c}^{e+}(x) = 1 - \mu_{(F,E)}^{e+}(x)) \text{ and } \mu_{(F,E)^c}^{e-}(x) = 1 - \mu_{(F,E)}^{e-}(x)) \tag{3.8}$$

4 Application of IVFSS in Decision-Making

Tripathy et al. [8] rectified some of the issues in [3] and provided suitable solutions for the problems in that paper. In addition, there is the concept of negative and positive parameters that was introduced.

Consider the case of an interview conducted by an organization, where interview performance of each candidate is analyzed by a panel. Here, we assign some parameters to evaluate the performance of each candidate. Some parameters are communication skills, personality, reactivity, etc.

We denote a set of candidates as U = {c_1, c_2, c_3, c_4, c_5, c_6} and *E* be the parameter set given by E = {knowledge, communication, presentation, reaction, other activities}. We denote the parameters as e_1, e_2, e_3, e_4, and e_5 for further calculations, where e_1 denotes knowledge, e_2 denotes communication, e_3 denotes presentation, e_4 denotes reaction, and e_5 denotes other activities. Consider an IVFSS (*U, E*) which describes the 'performance of a candidate'.

Table 1 shows the IVFSS of performance of candidates in a selection process. In the case of IVFSSs, we need to consider three cases.

Table 1 Tabular representation of IVFSS

U	e_1	e_2	e_3	e_4	e_5
c_1	0.2–0.4	0.3–0.5	0.8–0.9	0.4–0.7	0.6–0.9
c_2	0.4–0.8	0.6–0.9	0.2–0.5	0.7–1	0.5–0.6
c_3	0.5–0.8	0.7–0.9	0.7–0.8	0.8–1	0.5–0.7
c_4	0.6–0.8	0.5–0.9	0.8–1	0.5–0.9	0.7–0.8
c_5	0.1–0.4	0.9–1	0.3–0.6	0.1–0.5	0.8–1
c_6	0.9–1	0.5–0.7	0.1–0.3	0.2–0.4	0.3–0.7

(i) Pessimistic
(ii) Optimistic
(iii) Neutral

Neutral values are obtained by taking the average of pessimistic values and optimistic values.

$$neutral\,value = \frac{pesimistic + optimistic}{2} \tag{4.1}$$

4.1 Algorithm

1. Input the IVFSS.
2. Get the priority of the parameters from the user which lies in [−1, 1]. The default priority value for a parameter is 0 (Zero), which means that the parameter has no impact on decision-making and can be opted out from further computation.
3. Extract the pessimistic, optimistic, and neutral values from the interval valued fuzzy sets.
4. Do the following steps for pessimistic, optimistic, and neutral values of IVFSSs.
   a. Multiply the priority values with the corresponding parameter values to get the priority table.
   b. Compute the row sum of each row in the priority table (PT).
   c. Construct the comparison table (CT) by finding the entries as differences of each row sum with those of all other rows.
   d. Compute the row sum for each row in the CT to get the score.
   e. Assign rank to each candidate based on the CT values.
5. Construct the decision table based on the results we got in the above step, and final decision can be taken by the sum of all the ranks.
6. The object having highest value in the final decision column is to be selected. If more than one candidate is having the same rank-sum, then the candidate having higher value under the highest absolute priority column is selected and will continue like this.

In the case of pessimistic decision-making, we consider the lower membership value of each parameter, in the case of optimistic decision-making, we need to take the highest membership value of each parameter, and in the case of neutral decision–making, we need to take average of both pessimistic and optimistic values. First, we are considering the pessimistic case. The values of the pessimistic case are shown in Table 2.

Table 2 Pessimistic values

U	e_1	e_2	e_3	e_4	e_5
c_1	0.2	0.3	0.8	0.4	0.6
c_2	0.4	0.6	0.2	0.7	0.5
c_3	0.5	0.7	0.7	0.8	0.5
c_4	0.6	0.5	0.8	0.5	0.7
c_5	0.1	0.9	0.3	0.1	0.8
c_6	0.9	0.5	0.1	0.2	0.3

Table 3 Priority table for pessimistic case

U	e_1	e_2	e_3	e_4	e_5
c_1	0.14	0.09	0.16	−0.2	0.24
c_2	0.28	0.18	0.04	−0.35	0.2
c_3	0.35	0.21	0.14	−0.4	0.2
c_4	0.42	0.15	0.16	−0.25	0.28
c_5	0.07	0.27	0.06	−0.05	0.32
c_6	0.63	0.15	0.02	−0.1	0.12

Table 4 CT for pessimistic case

c_i	c_j							
	c_1	c_2	c_3	c_4	c_5	c_6	Row sum	Rank
c_1	0	0.08	−0.07	−0.33	-0.24	−0.39	−0.95	5
c_2	−0.08	0	−0.15	−0.41	-0.32	−0.47	−1.43	6
c_3	0.07	0.15	0	−0.26	-0.17	−0.32	−0.53	4
c_4	0.33	0.41	0.26	0	0.09	−0.06	1.03	2
c_5	0.24	0.32	0.17	−0.09	0	−0.15	0.49	3
c_6	0.39	0.47	0.32	0.06	0.15	0	1.39	1

Here, the panel is assigning some priority to the parameters. The priorities for the parameters e_1, e_2, e_3, e_4, and e_5 are 0.7, 0.3, 0.2, −0.5, and 0.4, respectively. With the help of this priority values, we create a priority table, as shown in Table 3.

The CT is formed as in step 4c of the algorithm, which is shown in Tables 4, 5, and 6.

Similarly, we need to find the comparison table for optimistic and neutral cases. The comparison table for optimistic decision-making is given in Table 5.

The CT for neutral decision-making is shown in Table 6.

The final decision can be taken as the average of pessimistic, optimistic, and neutral decision-making. It shown in Table 7.

From this table, we can see that candidate c_6 is the best choice. The next choices are in the order of c_4, c_5, c_3, c_2, and c_1.

Table 5 CT optimistic decision-making

	c_1	c_2	c_3	c_4	c_5	c_6	Row sum	Rank
c_1	0	−0.05	−0.15	−0.28	−0.23	−0.36	−1.07	6
c_2	0.05	0	−0.1	−0.23	−0.18	−0.31	−0.77	5
c_3	0.15	0.1	0	−0.13	−0.08	−0.21	−0.17	4
c_4	0.28	0.23	0.13	0	0.05	−0.08	0.61	2
c_5	0.23	0.18	0.08	−0.05	0	−0.13	0.31	3
c_6	0.36	0.31	0.21	0.08	0.13	0	1.09	1

Table 6 CT for neutral decision-making

	c_1	c_2	c_3	c_4	c_5	c_6	Row sum	Rank
c_1	0	0.015	−0.11	−0.305	−0.235	−0.375	−1.01	6
c_2	−0.015	0	−0.125	−0.32	−0.25	−0.39	−1.1	5
c_3	0.11	0.125	0	−0.195	−0.125	−0.265	−0.35	4
c_4	0.305	0.32	0.195	0	0.07	−0.07	0.82	2
c_5	0.235	0.25	0.125	−0.07	0	−0.14	0.4	3
c_6	0.375	0.39	0.265	0.07	0.14	0	1.24	1

Table 7 Decision table

	Pessimistic	Optimistic	Neutral	Row sum	Final decision
c_1	5	6	6	17	6
c_2	6	5	5	16	5
c_3	4	4	4	12	4
c_4	2	2	2	6	2
c_5	3	3	3	9	3
c_6	1	1	1	3	1

5 Conclusions

In [6], the notion of soft set was defined using the characteristic function approach, which was further extended in [8] to take care of FSS. Here, we further extended the approach to define IVFSS and redefined all basic operations on them. These are elegant and authentic. In addition, an algorithm to handle decision-making where the input data is in the form of IVFSS is proposed. A suitable example illustrates the application of the algorithm in real-life situations.

References

1. Zadeh, L.A.: Fuzzy sets. Inf. Control **8**, 338–353 (1965)
2. Molodtsov, D.: Soft set theory—first results. Comput. Math Appl. **37**, 19–31 (1999)
3. Maji, P.K., Biswas, R., Roy, A.R: An application of soft sets in a decision making problem. Comput. Math. Appl. **44**, 1007−1083 (2002)
4. Maji, P.K., Biswas, R., Roy, A.R.: Soft set theory. Comput. Math Appl. **45**, 555–562 (2003)
5. Maji, P.K., Biswas, R., Roy, A.R.: Fuzzy Soft Sets. J. Fuzzy Math. **9**(3), 589–602 (2001)
6. Tripathy, B.K., Arun, K.R.: A new approach to soft sets soft multisets and their properties. Int. J. Reason.-based Intell. Syst. **7**(3), 244–253 (2015)
7. Sooraj, T.R., Mohanty, R.K., Tripathy, B.K.: Fuzzy soft set theory and its application in group decision making. In: Advanced Computing and Communication Technologies. Advances in Intelligent Systems and Computing, vol. 452, pp.171−178 (2015)
8. Tripathy, B.K., Sooraj, T.R., Mohanty, R.K.: A new approach to fuzzy soft set theory and its application in decision making. In: Computational Intelligence in Data Mining—Volume 2, pp. 305–313 (2016)
9. Mohanty, R.K., Sooraj, T.R., Tripathy, B.K.: An Application of IVIFSS in medical diagnosis decision making. Int. J. Appl. Eng. Res. (IJAER) **10**(92), 85–93 (2015)
10. Mohanty, R.K., Sooraj, T.R., Tripathy, B.K.: IVIFSS and decision making. In: Proceedings of ICDECT-2016. Springer (2016)
11. Tripathy, B.K., Mohanty, R.K., Sooraj, T.R., Arun, K.R.: A new approach to intuitionistic fuzzy soft sets and their application in decision making. Adv. Intell. Syst. Comput. **439**, 93–100 Springer (2016)
12. Tripathy, B.K., Mohanty, R.K., Sooraj, T.R., Tripathy, A.: A modified representation of IFSS and its usage in GDM. In: Smart Innovation, Systems and Technologies, vol. 50, pp. 365−375 (2016)
13. Tripathy, B.K., Mohanty, R.K., Sooraj, T.R.: On intuitionistic fuzzy soft sets and their application in decision making. In: Proceedings of ICSNCS-2016, NewDelhi (2016)
14. Tripathy, B.K., Mohanty, R.K., Sooraj, T.R.: On intuitionistic fuzzy soft set and its application in group decision making. In: Proceedings of ICETETS-2016, Thanjavur (2016)
15. Yang, X.B., Lin, T.Y., Yang, J.Y., Li, Y., Yu, D.: Combination of interval-valued fuzzy set and soft set. Comput. Math Appl. **58**(3), 521–527 (2009)
16. Tripathy, B.K., Sooraj, T.R., Mohanty, R.K.: A new approach to interval-valued fuzzy soft sets and its application in group decision making. In: proceedings of CDCS-2015, Kochi (2015)
17. Tripathy, B.K., Panigrahi, A.: Interval-valued intuitionistic fuzzy parameterized soft set theory and its application in decision making. In: Proceedings of 10th international conference on intelligent systems and control (ISCO 2016), vol. 2, pp. 385–390 (2016)