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A non-linear study of fluctuating fluid flow on MHD mixed convection through a vertical permeable plate

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Abstract. In this paper, an analytical solution for an unsteady (independent of time), MHD mixed convection, two-dimensional (x and y), laminar, viscous flow of an incompressible fluid through a vertical permeable plate in a porous medium was developed with these assumptions: (i) the suction velocity (which is normal to the plate) and the free stream velocity both fluctuate with respect to time with a fixed mean; (ii) the wall temperature is constant; (iii) difference between the temperature of the plate and the free stream is moderately large due to the free convection currents. Based on the physical configuration of the model, the governing equations are derived and are non-dimensionalized using dimensionless parameters. The resultant nonlinear partial differential equations are solved using double regular perturbation technique analytically. The results are computed numerically to understand the behaviour of the fluid (i.e., effects of MHD, viscosity, body force etc.) for various non-dimensional parameters involving like Grashof number Gr , Prandtl number Pr , Hartmann number M , Eckert number E , the Viscous ratio λ and so on for velocity and temperature. These results are found to be in good agreement with known results available in the literature in the absence of few physical parameters. The numerical values of the above said flow is discussed through graphs on velocity and temperature.

1. Introduction

The study of fluid flow using heat and mass transfer has a variety of applications in medicine, science and technology like blood flow in a tube, tidal waves, wind power, geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors and underground energy transport" and so on. A comprehensive literature regarding the above subject has been mentioned in recent books [1], [2], [3], [4] and [5]. Many authors gave a large amount of preference to the study of boundary layer phenomena due to their wide range of applications in several engineering and industrial fields. The analysis of free and forced convection about a vertical plate embedded in a porous medium for various fluids under different boundary conditions was examined by Vafai and Thiyagaraja [6], Kim and Vafai [7,8].

Typical practical problems which arise in the aircraft design like "response to atmospheric gusts, aerofoil lift hysteresis at the stall, flutter phenomena involving wing, panel, and stalling flutter as well as the prediction of flow over helicopter rotor blades and through turbomachinery blade cascades" etc. Because of this, studies on laminar flow due to free-stream fluctuating are of first importance in aerodynamic flow problems. Lighthill [9] studied such flows to the external unsteady fluctuations about a mean value. Rapits [10] extended his ideas on when the free stream velocity oscillates with respect



to time about a constant mean through the infinite porous plate. Also, Stuart studied the unsteady temperature field along with the velocity field (i) by taking the assumption that there is no heat exchange between the plate and the fluid and (ii) there is a difference between the plate and the free stream temperature. Based on the conclusions of Rapits [10] an attempt was made by Soundalgekar [11], assuming that (i) the plate temperature oscillates with respect to time about a constant mean; (ii) the free convective currents are present in the boundary layer and (iii) the flow is very slow and hence viscous dissipative effects are negligible. In this problem, the coupled linear differential equations were solved by Soundalgekar and observed that the temperature field was not at all affected by the free convective currents.

In many practical problems, porous media have been used to provide effective cooling devices. So it is interesting to study the free convection current effects on the oscillatory flow over the boundary layer theory. This has been demonstrated by many authors in the literature (Cheng and Minkowycz [12], Rudraiah and Nataraj [13], Vafai and Tien [14], Kim and Vafai [15]) by considering Darcy/non-Darcy equation with boundary and inertia effects past a vertical plate embedded in a porous medium significantly. The works mentioned above are concerned with the study of steady convection when an impermeable vertical plate is embedded in a porous medium. It has been observed that a suction or injection at the plate control heating by controlling the boundary layer. Unsteady convection in the absence of porous media discussed by Rudraiah et al. [16] and Kaviany [17] with constant suction, unsteady free stream velocity and found "reversal of flow at small Prandtl numbers in the boundary layer close to a plate and predicted change in the nature of flow due to more cooling/heating of the plate". Later on, this problem has been extended by Rudraiah to an infinite vertical porous plate with uniform free stream velocity away from the plate. Goma and Taweel [18] illustrated the effects of oscillatory flow on heat transfer for both transient and time average heat-transfer rates. Effects of "unsteady mixed convection boundary-layer flow along a symmetric wedge with variable surface temperature was investigated by Hossain et al. [19] and also the effects of free convection currents on "the oscillatory flow of a polar fluid through a porous medium in the presence of variable wall heat flux" by Patil [20]. However, for an effective convective cooling/heating, it is important to study "the unsteady mixed convection on a vertical heated permeable plate embedded in a high porosity porous medium with fluctuating free stream and suction velocities without disturbing the uniform temperature maintained at the vertical plate.

The objective of the present work is, therefore, to study this problem analytically with oscillatory suction at the vertical porous plate and oscillatory free stream velocity away from the plate under the influence of viscous, body force, porous media and external constraint of the magnetic field. To achieve the objective of the present work, the plan of this work is as follows. In the next section, we consider a vertical porous plate embedded in a fluid saturated porous medium with oscillatory suction velocity at the vertical plate. We also give the conservation equations for momentum and energy with suitable boundary and inertia effect by considering Darcy-Lapwood-Brinkman equation. The relevant physical parameters are also discussed. These basic equations involve variable coefficients which are solved analytically using regular perturbation technique. These solutions are numerically computed and the results are discussed in the last section.

2. Mathematical analysis

We consider a two-dimensional (x and y), unsteady (independent of time), Boussinesq, viscous fluid through a porous medium bounded by a vertical infinite porous plate. We assume an oscillatory suction velocity and the free stream velocity away from the porous plate, about a fixed mean value in a direction parallel to the x -axis. Here, the porous plate is taken along the x -axis with the direction opposite to the direction of gravity and the y -axis is the direction normal to the porous plate. The vertical porous plate is maintained at a constant temperature. Since the flow extends to infinity in the x -direction, so all flow variables are functions of y and t only (see Figure.1) except the pressure p . Under these approximations, the basic equations of motion are:

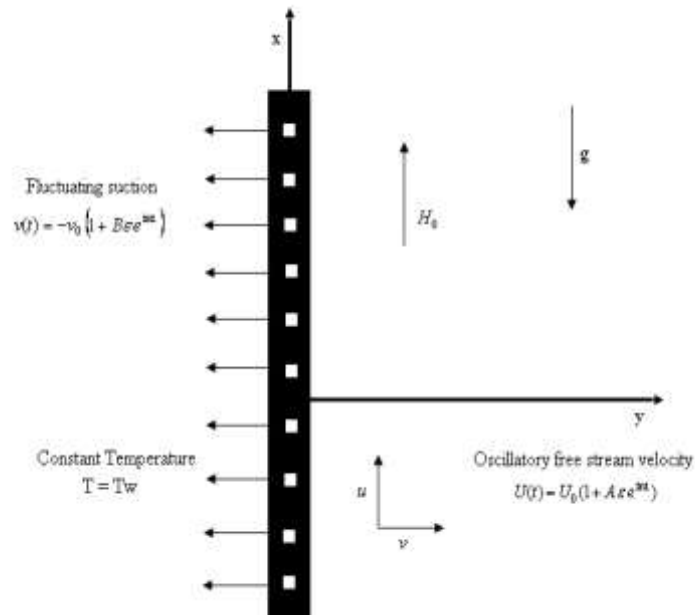


Figure 1. Physical Configuration

Equation of continuity

$$\frac{\partial v}{\partial y} = 0, \quad (1)$$

Equation of momentum

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\bar{v}}{k} u - \frac{\sigma B_0^2 u}{\rho_0} + \beta \bar{g} (T - T_\infty), \quad (2)$$

$$\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} - \frac{\bar{v}}{k} v, \quad (3)$$

Conservation of energy

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{k}{\rho_0 C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho_0 C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\mu_p}{\rho_0 C_p k} u^2 - \frac{\sigma B_0^2}{\rho_0 C_p} u^2. \quad (4)$$

We assume the following assumptions to find an approximate analytical solution for the above governing equations. (i) x varies from $-\infty$ to $+\infty$, all physical parameters are independent of x except pressure; (ii) density is constant throughout the momentum equation except for body force; (iii) density is constant for incompressible fluid; (iv) $B_0 = \mu H_0$; (v) from the equation of state we consider density (ρ) is a function of temperature only, i.e., $\rho = \rho_0 [1 - \beta(T - T_\infty)]$; (vi) the fluctuating free-stream and suction velocities respectively as

$$U(t) = U_0(1 + A\varepsilon e^{int}); \quad v(t) = -v_0(1 + B\varepsilon e^{int}), \quad (5)$$

here, the negative sign represents the suction towards the porous plate. Also, the BC's are defined as follows based on the physical configuration which is shown in Figure 1:

$$\begin{aligned} y = 0 : \quad & u = 0, \quad T = T_w, \\ y \rightarrow \infty : \quad & u \rightarrow U(t), \quad T \rightarrow T_\infty. \end{aligned} \quad (6)$$

Here, BC's are derived from the above-said assumptions, that the free stream velocity is fluctuating with time and maintaining uniform temperature away from the plate as well as at the plate. The governing free stream velocity equation is

$$\frac{dU}{dt} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} - \frac{\bar{v}}{k} U - \frac{\sigma B_0^2 U}{\rho_0}. \quad (7)$$

We solve the above non-linear PDE's using (2) in (7), we get

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \frac{dU}{dt} + v \frac{\partial^2 u}{\partial y^2} - \frac{\bar{v}}{k} (u - U) - \frac{\sigma B_0^2 U}{\rho_0} (u - U) + \beta \bar{g} (T - T_\infty) \quad (8)$$

These equations are made dimensionless using

$$y^* = \frac{v_o y}{\nu}, \quad u^* = \frac{u}{U_o}, \quad t^* = \frac{v_o^2 t}{\nu}, \quad n^* = \frac{\nu n}{v_o^2}, \quad v^* = \frac{v}{v_o}, \quad k^* = \frac{\nu^2 k}{v_o^2}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad (9)$$

$$\text{Pr} = \frac{\nu}{k}, \quad \text{Gr} = \frac{\beta \nu g (T_w - T_\infty)}{U_o^2 v_o^2}, \quad \text{M} = \frac{\sigma B_0^2 \nu}{\rho_o v_o^2}, \quad E = \frac{U_o^2}{C_p (T_w - T_\infty)}, \quad U^* = \frac{U}{U_o}, \quad \lambda = \frac{\bar{v}}{\nu}.$$

and for simplicity neglect the asterisks (*), we get

$$\frac{\partial u}{\partial t} - v_o (1 + B\varepsilon e^{\text{int}}) \frac{\partial u}{\partial y} = \frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + \frac{\lambda}{k} (U - u) + M (U - u) + \text{Gr} \theta, \quad (10)$$

$$\frac{\partial \theta}{\partial t} - v_o (1 + B\varepsilon e^{\text{int}}) \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + E \left[\left(\frac{\lambda}{k} - M \right) u^2 + \left(\frac{\partial u}{\partial y} \right)^2 \right]. \quad (11)$$

The resultant BC's in dimensionless form are

$$\begin{aligned} y = 0 : \quad u = 0, \quad \theta = 1, \\ y \rightarrow \infty : \quad u \rightarrow (1 + \varepsilon A e^{\text{int}}), \quad \theta \rightarrow 0. \end{aligned} \quad (12)$$

3. Method of solution

In order to solve the above PDE's (10) and (11), we make use of the double regular perturbation method (one for ε and another for E), we assume the solutions of the form

$$u(y, t) = u_0(y) + \varepsilon u_1(y) e^{\text{int}} + O(\varepsilon^2), \quad (13)$$

$$\theta(y, t) = \theta_0(y) + \varepsilon \theta_1(y) e^{\text{int}} + O(\varepsilon^2).$$

Where ε is the perturbation parameter, which is a very small quantity.

Substituting equation (13) into (10)-(11) and comparing like powers of ε on both sides, we get

The zeroth order (independent of ε) steady equations and its BC's are

$$u_0'' + u_0' - \left(M + \frac{\lambda}{k} \right) u_0 = -\text{Gr} \theta_0 - \left(M + \frac{\lambda}{k} \right), \quad (14)$$

$$\theta_0'' + \text{Pr} \theta_0' = -E \text{Pr} \left[\left(\frac{\lambda}{k} - M \right) u_0^2 + u_0' \right]. \quad (15)$$

$$y = 0 : \quad u_0 = 0, \quad \theta_0 = 1, \quad (16)$$

$$y \rightarrow \infty : \quad u_0 \rightarrow 1, \quad \theta_0 \rightarrow 0.$$

The first order (coefficient of ε) unsteady equations and its BC's are

$$u_1'' + u_1' - \left(in + M + \frac{\lambda}{k} \right) u_1 = -B u_0' - \theta_1 \text{Gr} - \left(M + \frac{\lambda}{k} \right) A, \quad (17)$$

$$\theta_1'' + \text{Pr} \theta_1' - in \text{Pr} \theta_1 = B \theta_0' - 2E \left[\left(\frac{\lambda}{k} - M \right) u_0 u_1 + u_0' u_1' \right]. \quad (18)$$

$$\begin{aligned} y = 0 : u_1 = 0, \quad \theta_1 = 1, \\ y \rightarrow \infty : u_1 \rightarrow A, \quad \theta_1 \rightarrow 0. \end{aligned} \quad (19)$$

Equations (14), (15), (17) and (18) are coupled equations in u_0, θ_0, u_1 and θ_1 . To solve these equations, we assume

$$\begin{aligned} u_0(y) &= u_{01}(y) + E u_{02}(y) + O(E^2); \\ u_1(y) &= u_{11}(y) + E u_{12}(y) + O(E^2); \\ \theta_0(y) &= \theta_{01}(y) + E \theta_{02}(y) + O(E^2); \\ \theta_1(y) &= \theta_{11}(y) + E \theta_{12}(y) + O(E^2). \end{aligned} \quad (20)$$

Substituting equation (20) in (14) and (17) and comparing like powers of E , we have
The zeroth order (independent of E) steady equations and its BC's are

$$u_{01}'' + u_{01}' - \left(M + \frac{\lambda}{k}\right) u_{01} = -Gr \theta_{01} - \left(M + \frac{\lambda}{k}\right), \quad (21)$$

$$\theta_{01}'' + Pr \theta_{01}' = 0, \quad (22)$$

$$\begin{aligned} y = 0 : u_{01} = 0, \quad \theta_{01} = 1, \\ y \rightarrow \infty : u_{01} \rightarrow 1, \quad \theta_{01} \rightarrow 0. \end{aligned} \quad (23)$$

The first order (coefficient of E) unsteady equations and its BC's are

$$u_{02}'' + u_{02}' - \left(M + \frac{\lambda}{k}\right) u_{02} = -Gr \theta_{02}, \quad (24)$$

$$\theta_{02}'' + Pr \theta_{02}' = -\left[\left(\frac{\lambda}{k} - M\right) u_{01}'^2 + u_{01}''^2\right]. \quad (25)$$

$$\begin{aligned} y = 0 : u_{02} = 0, \quad \theta_{02} = 0, \\ y \rightarrow \infty : u_{02} \rightarrow 0, \quad \theta_{02} \rightarrow 0. \end{aligned} \quad (26)$$

Similarly, substituting equations (20) in (15) and (18), we get

The zeroth order (independent of E) steady equations and its BC's are

$$u_{11}'' + u_{11}' - \left(in + M + \frac{\lambda}{k}\right) u_{11} = -Bu_{01}' - Gr \theta_{11} - \left(M + \frac{\lambda}{k}\right) A, \quad (27)$$

$$\theta_{11}'' + Pr \theta_{11}' - in Pr \theta_{11} = B \theta_{01}'. \quad (28)$$

$$\begin{aligned} y = 0 : u_{11} = 0, \quad \theta_{11} = 0, \\ y \rightarrow \infty : u_{11} \rightarrow A, \quad \theta_{11} \rightarrow 0. \end{aligned} \quad (29)$$

The first order (coefficient of E) unsteady equations and its BC's are

$$u_{12}'' + u_{12}' - \left(in + M + \frac{\lambda}{k}\right) u_{12} = -Bu_{02}' - Gr \theta_{12}, \quad (30)$$

$$\theta_{12}'' + Pr \theta_{12}' - in Pr \theta_{12} = B Pr \theta_{02}' + Pr u_{01}'(y) u_{11}' - 2 Pr \left[\left(\frac{\lambda}{k} - M\right) u_{01}' u_{11}'\right]. \quad (31)$$

$$\begin{aligned} y = 0 : u_{12} = 0, \quad \theta_{12} = 0, \\ y \rightarrow \infty : u_{12} \rightarrow 0, \quad \theta_{12} \rightarrow 0. \end{aligned} \quad (32)$$

Solving the differential equations (21) to (32) using the corresponding boundary conditions and with suitable simplification, we get

$$u(y,t) = \left[(1 + A_1 e^{-Pr y} + A_2 e^{-R_2 y}) + E (A_3 e^{-2R_2 y} + A_4 e^{-R_8 y} + A_5 e^{-2Pr y} + A_6 e^{-(R_2+Pr)y} + A_7 (y+1) e^{-Pr y} + A_8) \right] + \varepsilon \left[(A_9 e^{-R_2 y} + A_{10} e^{-R_4 y} + A_{11} e^{-R_6 y} + A_{12} e^{-Pr y} + A_{13}) + E \left(\begin{array}{l} A_{14} e^{-R_2 y} + A_{16} e^{-2R_2 y} + A_{17} e^{-R_6 y} + A_{18} e^{-R_8 y} \\ + A_{19} e^{-R_{10} y} + A_{20} (y+1) e^{-Pr y} + A_{21} e^{-2Pr y} \\ + A_{22} e^{-(R_2+Pr)y} + A_{23} e^{-(R_2+R_6)y} + A_{24} e^{-(R_2+R_4)y} \\ + A_{25} e^{-(R_4+Pr)y} + A_{26} e^{-(R_6+Pr)y} + A_{27} \end{array} \right) \right] e^{int} \quad (33)$$

$$\theta(y,t) = \left[(e^{-Pr y}) + E (B_1 e^{-R_2 y} + B_2 e^{-2R_2 y} + B_3 (y+1) e^{-R_2 y} + B_4 e^{-2Pr y} + B_5 e^{-(R_2+Pr)y}) \right] + \varepsilon \left[(B_6 e^{-R_4 y} + B_7 e^{-Pr y}) + E \left(\begin{array}{l} B_8 e^{-R_2 y} + B_9 e^{-2R_2 y} + B_{10} e^{-R_4 y} + B_{11} e^{-R_6 y} + B_{12} e^{-R_{10} y} \\ + B_{13} (y+1) e^{-Pr y} + B_{14} e^{-2Pr y} + B_{15} e^{-(R_2+Pr)y} \\ + B_{16} e^{-(R_2+R_6)y} + B_{17} e^{-(R_2+R_4)y} + B_{18} e^{-(R_6+Pr)y} \\ + B_{19} e^{-(R_4+Pr)y} + B_{20} \end{array} \right) \right] e^{int} \quad (34)$$

where,

$$R_{1,2} = R_{7,8} = \frac{-1 \pm \sqrt{1+4K_1}}{2}; \quad R_{3,4} = \frac{-Pr \pm \sqrt{Pr^2 + 4Pr in}}{2}; \quad R_{5,6} = \frac{-1 \pm \sqrt{1+4K_2}}{2}; \quad R_{9,10} = \frac{-Pr \pm \sqrt{Pr^2 - 4K_4}}{2}.$$

The constants A_i ($i=1$ to 27) and B_i ($i=1$ to 20) are the functions of non-dimensional parameters involved in the problem. For want of space, the expressions for them are omitted here but given in the appendix. However, they are numerically computed and used in computing u and θ .

4. Results and Discussion

The numerical computation is performed for velocity and temperature for various values of non-dimensionalized parameters which are involved in the physical model. The graphical representation of velocity and temperature are depicted from Figures 2-10. The variations of velocity and temperature for distinct positive and negative values of Grashof number Gr is shown in Figures 2-3. Figure 2 shows that, the mean velocity of air increases due to more cooling of the vertical permeable plate by the free convection currents. Because in the process of cooling the plate the free convection currents are carried away from the plate to the free stream as the free stream is in the upward direction" so the free convection currents induce the mean velocity to increase. Also, the mean velocity decreases for negatives of Gr because the flow of air moving in the upward direction both near and away from the vertical permeable plate, is being opposed by the free convection currents traveling towards the vertical permeable plate and hence the mean velocity decreases. Thus the mean flow of air is reversed, when the vertical permeable wall is heated by the free convection currents is observed from Figure 2. Figure 3 represents the plot of temperature for various positive and negative values of Gr and is observed that for cooling of the vertical permeable wall the temperature decreases from the plate and also far away from it.

Figures 4-5 shows that, the variations of velocity and temperature for non-uniform values of Pr for the fluids. Figure 4 represents that, the velocity variation for different fluids from air to mercury. For small Prandtl number (0.71 to 7), the variation of velocity is large for cooling of the vertical permeable wall. For higher values of Prandtl number ($Pr > 3$), the variation of velocity is very less because of viscous dissipation. Figure 5 represents the behavior of temperature for different fluids ($Pr = 0.71$ to 7) due to more cooling of the vertical permeable wall ($Gr > 0$). From Figure 5, the temperature decreases near the vertical permeable wall for higher values of Prandtl number. Figures 6-7 shows the variations of velocity and temperature for different values of Eckert number E for the fluids. The velocity increases near the plate and the temperature decreases as increasing the Eckert number E is observed from Figures 6 and 7.

Figures 8 and 9 illustrates, the variation of velocity and temperature for different values of Hartmann number M which is a measure of Lorentz force to viscous force in a finitely conducting fluid. As the Hartmann number M increases due to more cooling of the vertical permeable wall ($Gr > 0$) the velocity of the fluid decreases due to an increase of Lorentz force to viscous force. An opposite behavior is observed for the temperature of the fluid for the increase of Lorentz force to viscous force. Figure 10 shows that, the velocity and temperature for dissimilar values of a permeable parameter K of the porous media. If the permeability of the porous media increases there is an increase in variation for the velocity of the fluid but much variation is not seen for the temperature of the fluids. Similarly, an opposite behavior is being observed for the velocity and temperature due to heating of the vertical permeable wall for different values of Pr, E, M and K .

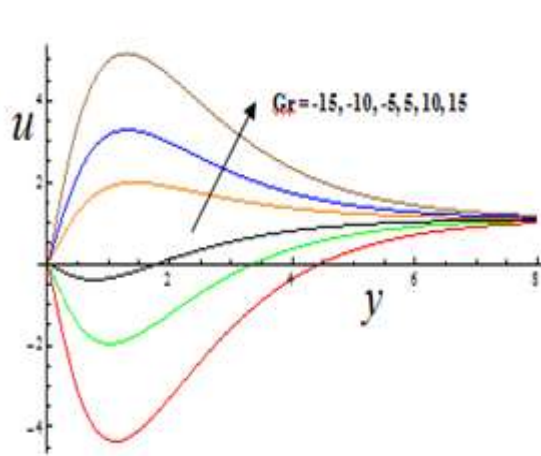


Figure 2. Velocity for non-uniform values of Gr . ($Pr = 0.71, M = 1, E = 0.01, K = 0.5, \varepsilon = 0.1$)

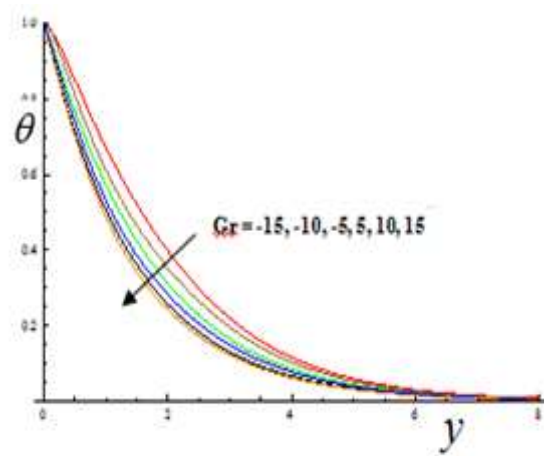


Figure 3. Temperature for non-uniform values of Gr . ($Pr = 0.71, M = 1, E = 0.01, K = 0.5, \varepsilon = 0.1$)

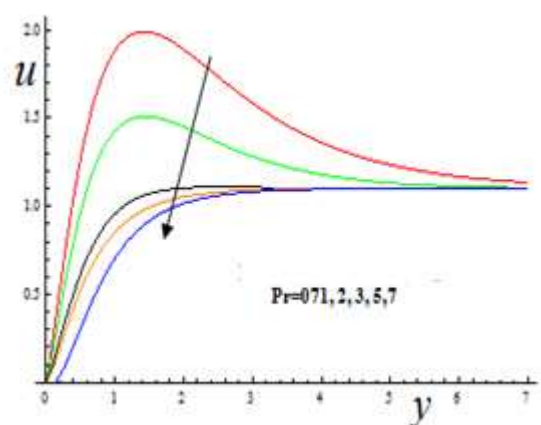


Figure 4. Velocity for non uniform values of Pr . ($Gr = 5, M = 1, E = 0.01, K = 0.5, \varepsilon = 0.1$)

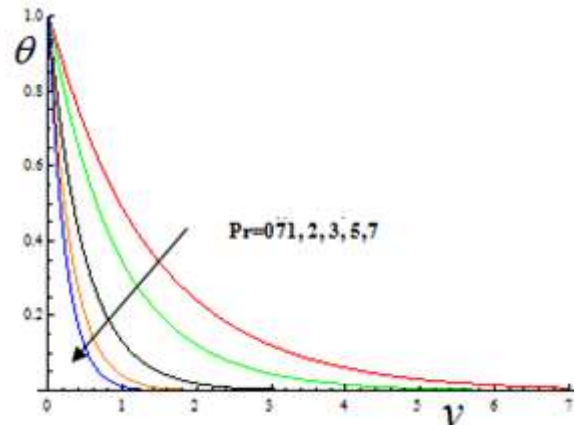


Figure 5. Temperature for non uniform values of Pr ($Gr = 5, M = 1, E = 0.01, K = 0.5, \varepsilon = 0.1$)

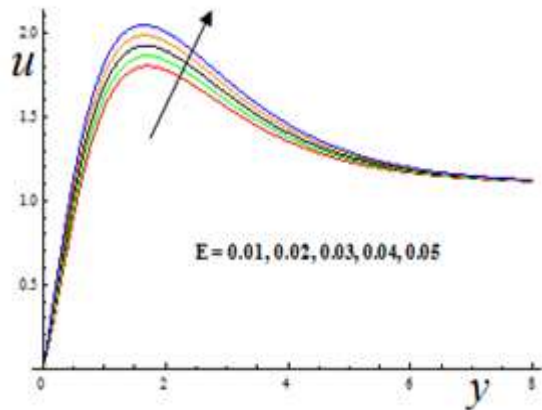


Figure 6. Velocity for non-uniform values of E . ($Gr = 5, Pr = 0.71, M = 1, K = 0.5, \varepsilon = 0.1$)

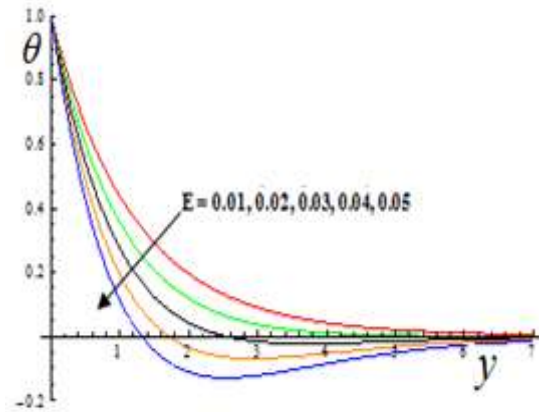


Figure 7. Temperature for non-uniform values of E . ($Gr = 5, Pr = 0.71, M = 1, K = 0.5, \varepsilon = 0.1$)

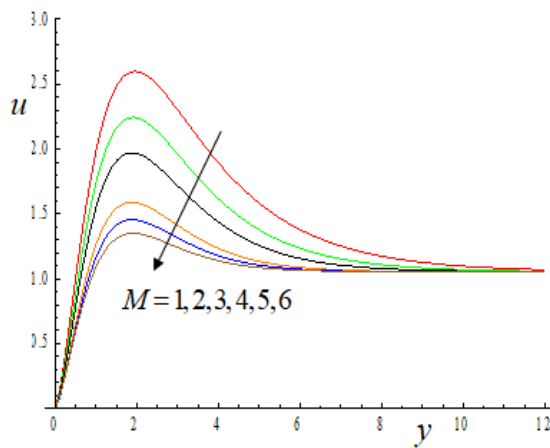


Figure 8. Velocity for non-uniform values of M . ($Gr = 5, Pr = 0.71, E = 0.01, K = 0.5, \varepsilon = 0.1$)

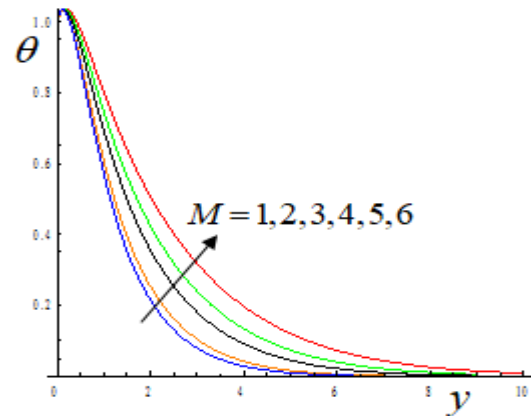


Figure 9. Temperature for non-uniform values of M . ($Gr = 5, Pr = 0.71, E = 0.01, K = 0.5, \varepsilon = 0.1$)

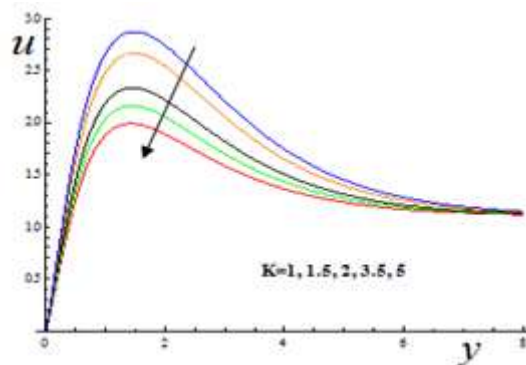


Figure 10. Velocity for non-uniform values of K . ($Gr = 5, Pr = 0.71, M = 1, E = 0.01, \varepsilon = 0.1$)

5. Conclusions

Based on the above analysis, the following conclusions are drawn:

- (i) As Grashof number increases the velocity increases, temperature decreases and in the case of negative values the velocity decreases and temperature increases.
- (ii) As Eckert number enhances the velocity increases and temperature decreases.
- (iii) As Prandtl number increases the velocity decreases and temperature increases. In the case of small Prandtl numbers the variation is large and for higher values variation is less.
- (iv) Hartmann number increases, the velocity decreases, and temperature increases.
- (v) As increases permeability of the porous medium, there is an increase in the velocity but much variation is not seen for the temperature.

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Appendix

$$A_1 = -\left(\frac{-Gr}{Pr^2 - Pr - K_1} + 1\right); A_2 = \frac{-Gr}{Pr^2 - Pr - K_1}; A_3 = \frac{g_{17}}{4R_2^2 - 2R_2 - K_1}; A_5 = \frac{g_{18}}{4Pr^2 - 2Pr - K_1};$$

$$A_4 = -\left(\frac{g_{16}}{Pr^2 - Pr - K_1} + \frac{g_{17}}{4R_2^2 - 2R_2 - K_1} + \frac{g_{18}}{4Pr^2 - 2Pr - K_1} + \frac{g_{19}}{(R_2 + Pr)^2 - (R_2 + Pr) - K_1} + \frac{2Pr - 1}{(Pr^2 - Pr - K_1)^2}\right);$$

$$\begin{aligned}
A_6 &= \frac{g_{19}}{(R_2 + \text{Pr})^2 - (R_2 + \text{Pr}) - K_1}; A_7 = \frac{g_{16}}{\text{Pr}^2 - \text{Pr} - K_1} + \frac{g_{20}}{\text{Pr}^2 - \text{Pr} - K_1} y; A_8 = \frac{2 \text{Pr} - 1}{(\text{Pr}^2 - \text{Pr} - K_1)^2}; \\
A_4 &= \frac{-B(g_1 + 1)R_2}{R_2^2 - R_2 - K_2}; A_{10} = \frac{g_2 Gr}{R_4^2 - R_4 - K_2}; A_{13} = \frac{K_1 A}{K_2}; A_{13} = \frac{K_1 A}{K_2}; A_{16} = \frac{2B g_{22} R_2 - Gr g_{60}}{4R_2^2 - 2R_2 - K_2}; \\
A_{11} &= -\left(\frac{-B(g_1 + 1)R_2}{R_2^2 - R_2 - K_2} + \frac{B \text{Pr} g_1 + g_2 Gr}{\text{Pr}^2 - \text{Pr} - K_2} + \frac{g_2 Gr}{R_4^2 - R_4 - K_2} + \frac{K_1 A}{K_2} \right); A_{12} = \frac{B \text{Pr} g_1 + g_2 Gr}{\text{Pr}^2 - \text{Pr} - K_2}; \\
A_{17} &= \frac{-Gr g_{43}}{R_6^2 - R_6 - K_2} + g_{85}; A_{18} = \frac{B R_8 g_{61}}{R_8^2 - R_8 - K_2}; A_{19} = \frac{-g_{67} Gr}{R_{10}^2 - R_{10} - K_2}; A_{21} = \frac{2B g_{23} \text{Pr} - Gr g_{59}}{4\text{Pr}^2 - 2\text{Pr} - K_2}; \\
A_{20} &= \frac{B g_{21} \text{Pr} - B g_{25} - Gr g_{58}}{\text{Pr}^2 - \text{Pr} - K_2} + \frac{g_{71}}{\text{Pr}^2 - \text{Pr} - K_2} y; A_{22} = \frac{B g_{24} (R_2 + \text{Pr}) - Gr g_{62}}{(R_2 + \text{Pr})^2 - (R_2 + \text{Pr}) - K_2}; \\
A_{23} &= \frac{-Gr g_{63}}{(R_2 + R_6)^2 - (R_2 + R_6) - K_2}; A_{24} = \frac{-Gr g_{64}}{(R_2 + R_4)^2 - (R_2 + R_4) - K_2}; A_{27} = \frac{Gr g_{47}}{K_2}; \\
A_{25} &= \frac{-Gr g_{66}}{(R_4 + \text{Pr})^2 - (R_4 + \text{Pr}) - K_2}; A_{26} = \frac{-Gr g_{65}}{(R_6 + \text{Pr})^2 - (R_6 + \text{Pr}) - K_2}; B_1 = \frac{L_{12}}{R_2^2 - \text{Pr} R_2}; \\
B_2 &= \frac{g_8}{\text{Pr}^2}; B_3 = -\left(\frac{g_{70}}{4R_2^2 - 2R_2 - \text{Pr}} + \frac{g_8}{\text{Pr}^2} + \frac{g_9}{(R_2 + \text{Pr})^2 - \text{Pr}(R_2 + \text{Pr})} + \frac{L_{12}}{R_2^2 - \text{Pr} R_2} \right) - \frac{L_1}{\text{Pr}} y; \\
B_4 &= \frac{g_8}{\text{Pr}^2}; B_5 = \frac{g_9}{(R_2 + \text{Pr})^2 - \text{Pr}(R_2 + \text{Pr})}; B_6 = -\frac{B \text{Pr}}{in}; B_{10} = \frac{2 g_5 K_3}{R_4^2 - \text{Pr} R_4 + K_4}; B_{14} = \frac{g_{31}}{K_4}; \\
B_8 &= \left(\frac{-B \text{Pr} R_2 g_{14}}{R_2^2 - \text{Pr} R_2 + K_4} - \frac{2 K_3 g_6 (g_1 + 1)}{R_2^2 - R_2 \text{Pr} + K_4} + \frac{2 g_3 K_3}{R_2^2 - \text{Pr} R_2 + K_4} \right); B_{11} = \frac{2 g_7 K_3}{R_6^2 - \text{Pr} R_6 + K_4}; \\
B_9 &= \left(\frac{-2 B \text{Pr} R_2 g_{10}}{4R_2^2 - 2\text{Pr} R_2 + K_4} - \frac{2 K_3 g_3 (g_1 + 1)}{4R_2^2 - 2R_2 \text{Pr} + K_4} + \frac{2 \text{Pr} R_2^2 g_3 (g_1 + 1)}{4R_2^2 - 2R_2 \text{Pr} + K_4} \right); B_{12} = -(g_{48} + \dots + g_{58}); \\
B_{13} &= \left(\frac{-2 B \text{Pr}^2 g_{11}}{2\text{Pr}^2 + K_4} + \frac{2 K_3 g_4 g_1}{2\text{Pr}^2 + K_4} + \frac{-2 \text{Pr}^3 g_1 g_4}{2\text{Pr}^2 + K_4} \right) + g_{56} y; B_{15} = (g_{30} + g_{35} + g_{39} + g_{50} + g_{53}); \\
B_{16} &= \frac{2(\text{Pr} R_2 R_6 - K_3) g_7 (g_1 + 1)}{(R_2 + R_6)^2 - \text{Pr}(R_2 + R_6) + K_4}; B_{17} = \frac{2(\text{Pr} R_2 R_4 - K_3) g_5 (g_1 + 1)}{(R_2 + R_4)^2 - \text{Pr}(R_2 + R_4) + K_4}; K_4 = -in + \text{Pr}; \\
K_3 &= -\text{Pr} \left(\frac{la}{k} - M \right); B_{18} = \frac{2(K_3 - \text{Pr}^2 R_6) g_1 g_7}{(R_6 + \text{Pr})^2 - \text{Pr}(R_6 + \text{Pr}) + K_4}; B_{19} = \frac{2(K_3 - \text{Pr}^2 R_4) g_1 g_5}{(R_4 + \text{Pr})^2 - \text{Pr}(R_4 + \text{Pr}) + K_4}; \\
B_{20} &= \frac{2 g_6 K_3}{K_4}; K_1 = M + \frac{la}{k}; K_2 = in + M + \frac{la}{k}; g_1 = \frac{-Gr}{\text{Pr}^2 - \text{Pr} - K_1}; g_2 = \frac{B \text{Pr}}{in}; g_6 = \frac{K_1 A}{K_2}; \\
g_3 &= \frac{-B(g_1 + 1)R_2}{R_2^2 - R_2 - K_2}; g_4 = \frac{B \text{Pr} g_1 + g_2 Gr}{\text{Pr}^2 - \text{Pr} - K_2}; g_5 = \frac{g_2 Gr}{R_4^2 - R_4 - K_2}; g_8 = g_1^2 (K_3 - \text{Pr}^3); \\
g_7 &= -\left(\frac{-B(g_1 + 1)R_2}{R_2^2 - R_2 - K_2} + \frac{B \text{Pr} g_1 + g_2 Gr}{\text{Pr}^2 - \text{Pr} - K_2} + \frac{g_2 Gr}{R_4^2 - R_4 - K_2} + \frac{K_1 A}{K_2} \right); g_9 = 2(R_2 \text{Pr}^2 - K_3) g_1 (g_1 + 1);
\end{aligned}$$

$$\begin{aligned}
g_{10} &= \frac{g_1}{4R_2^2 - 2R_2 - \text{Pr}}; g_{12} = \frac{g_9}{(R_2 + \text{Pr})^2 - \text{Pr}(R_2 + \text{Pr})}; g_{11} = \frac{g_8}{\text{Pr}^2}; g_{13} = \frac{-L_1}{\text{Pr}}; g_{14} = \frac{L_{12}}{R_2^2 - \text{Pr}R_2}; \\
g_{16} &= -Gr g_{15}; g_{17} = -Gr g_{10}; g_{18} = -Gr g_{11}; g_{19} = -Gr g_{12}; g_{21} = \frac{g_{16}}{\text{Pr}^2 - \text{Pr} - K_1}; g_{20} = -Gr g_{13}; \\
g_{15} &= -\left(\frac{g_7}{4R_2^2 - 2R_2 - \text{Pr}} + \frac{g_9}{(R_2 + \text{Pr})^2 - \text{Pr}(R_2 + \text{Pr})} + \frac{g_8}{\text{Pr}^2} + \frac{L_{12}}{R_2^2 - \text{Pr}R_2} \right); g_{22} = \frac{g_{17}}{4R_2^2 - 2R_2 - K_1}; \\
g_{23} &= \frac{g_{18}}{4\text{Pr}^2 - 2\text{Pr} - K_1}; g_{25} = \frac{g_{20}}{\text{Pr}^2 - \text{Pr} - K_1}; g_{24} = \frac{g_{19}}{(R_2 + \text{Pr})^2 - (R_2 + \text{Pr}) - K_1}; g_{27} = \frac{-B\text{Pr}^2 g_{15}}{K_4}; \\
g_{26} &= \frac{2\text{Pr} - 1}{(\text{Pr}^2 - \text{Pr} - K_1)^2}; g_{29} = \frac{-2B\text{Pr}^2 g_{11}}{2\text{Pr}^2 + K_4}; g_{28} = \frac{-2B\text{Pr}R_2 g_{20}}{4R_2^2 - 2\text{Pr}R_2 + K_4}; g_{32} = \frac{-B\text{Pr}R_2 g_{13}}{R_2^2 - \text{Pr}R_2 + K_4}; \\
g_{30} &= \frac{-B\text{Pr}(R_2 + \text{Pr})g_{12}}{(R_2 + \text{Pr})^2 - \text{Pr}(R_2 + \text{Pr}) + K_4}; g_{31} = -B\text{Pr}^2 g_{13}; g_{33} = \frac{-2K_3 g_7 (g_1 + 1)}{(R_2 + R_6)^2 - \text{Pr}(R_2 + R_6) + K_4}; \\
g_{34} &= \frac{-2K_3 g_3 (g_1 + 1)}{4R_2^2 - 2R_2\text{Pr} + K_4}; g_{35} = \frac{-2K_3 g_4 (g_1 + 1)}{(R_2 + \text{Pr})^2 - \text{Pr}(R_2 + \text{Pr}) + K_4}; g_{37} = \frac{-2K_3 g_6 (g_1 + 1)}{R_2^2 - R_2\text{Pr} + K_4}; \\
g_{36} &= \frac{-2K_3 g_5 (g_1 + 1)}{(R_2 + R_4)^2 - \text{Pr}(R_2 + R_4) + K_4}; g_{38} = \frac{2K_3 g_1 g_7}{(R_6 + \text{Pr})^2 - \text{Pr}(R_6 + \text{Pr}) + K_4}; g_{40} = \frac{2K_3 g_4 g_1}{2\text{Pr}^2 + K_4}; \\
g_{57} &= \frac{g_{31}\text{Pr}}{K_4}; g_{39} = \frac{2K_3 g_1 g_3}{(R_2 + \text{Pr})^2 - \text{Pr}(R_2 + \text{Pr}) + K_4}; g_{41} = \frac{2K_3 g_1 g_5}{(R_4 + \text{Pr})^2 - \text{Pr}(R_4 + \text{Pr}) + K_4}; \\
g_{42} &= \frac{2K_3 g_6 g_1}{K_4}; g_{43} = \frac{2g_7 K_3}{R_6^2 - \text{Pr}R_6 + K_4}; g_{44} = \frac{2g_3 K_3}{R_2^2 - \text{Pr}R_2 + K_4}; g_{45} = \frac{2g_4 K_3}{K_4}; g_{47} = \frac{2g_6 K_3}{K_4}; \\
g_{46} &= \frac{2g_5 K_3}{R_4^2 - \text{Pr}R_4 + K_4}; g_{48} = \frac{2\text{Pr}R_2 R_6 g_7 (g_1 + 1)}{(R_2 + R_6)^2 - \text{Pr}(R_2 + R_6) + K_4}; g_{49} = \frac{2\text{Pr}R_2^2 g_3 (g_1 + 1)}{4R_2^2 - 2R_2\text{Pr} + K_4}; \\
g_{50} &= \frac{2\text{Pr}^2 R_2 g_4 (g_1 + 1)}{(R_2 + \text{Pr})^2 - \text{Pr}(R_2 + \text{Pr}) + K_4}; g_{51} = \frac{2\text{Pr}R_2 R_4 g_5 (g_1 + 1)}{(R_2 + R_4)^2 - \text{Pr}(R_2 + R_4) + K_4}; g_{54} = \frac{-2\text{Pr}^3 g_1 g_4}{2\text{Pr}^2 + K_4}; \\
g_{52} &= \frac{-2\text{Pr}^2 R_6 g_7 g_1}{(R_6 + \text{Pr})^2 - \text{Pr}(R_6 + \text{Pr}) + K_4}; g_{53} = \frac{-2\text{Pr}g_1 R_2 g_3 \text{Pr}}{(R_2 + \text{Pr})^2 - \text{Pr}(R_2 + \text{Pr}) + K_4}; \\
g_{58} &= (g_{27} + g_{38} + g_{42} + g_{45} + g_{30}); g_{55} = \frac{-2\text{Pr}^2 g_5 g_1 R_4}{(R_4 + \text{Pr})^2 - \text{Pr}(R_4 + \text{Pr}) + K_4}; g_{56} = \frac{g_{31}}{K_4}; \\
g_{59} &= \frac{-2B g_{11} \text{Pr}^2 + 2(K_3 - \text{Pr}^3)g_1 g_4}{2\text{Pr}^2 + K_4}; g_{60} = \frac{-2B\text{Pr}R_2 g_{10} + 2(\text{Pr}R_2^2 - K_3)g_3 (g_1 + 1)}{4R_2^2 - 2\text{Pr}R_2 + K_4}; \\
g_{61} &= \frac{2g_3 g_6 (g_1 + 1)K_3 - 2B\text{Pr}R_2 g_{14}}{R_2^2 - \text{Pr}R_2 + K_4}; g_{62} = (g_{30} + g_{35} + g_{39} + g_{50} + g_{53}); \\
g_{63} &= \frac{2(\text{Pr}R_2 R_6 - 2K_3)g_7 (g_1 + 1)}{(R_2 + R_6)^2 - \text{Pr}(R_2 + R_6) + K_4}; g_{64} = \frac{2(\text{Pr}R_2 R_4 - K_3)g_5 (g_1 + 1)}{(R_2 + R_4)^2 - \text{Pr}(R_2 + R_4) + K_4};
\end{aligned}$$

$$g_{65} = \frac{2(K_3 - \text{Pr}^2 R_6) g_1 g_7}{(R_6 + \text{Pr})^2 - \text{Pr}(R_6 + \text{Pr}) + K_4}; g_{67} = \frac{2(K_3 - \text{Pr}^2 R_4) g_1 g_5}{(R_4 + \text{Pr})^2 - \text{Pr}(R_4 + \text{Pr}) + K_4};$$

$$g_{67} = -(g_{58} + g_{59} + \dots + g_{65} + g_{66}); g_{68} = \frac{B(g_{21} \text{Pr} - g_{25}) - Gr g_{58}}{\text{Pr}^2 - \text{Pr} - K_2}; g_{70} = \frac{2B g_{23} \text{Pr} - Gr g_{59}}{4\text{Pr}^2 - 2\text{Pr} - K_2};$$