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A Pragmatic Approach to Understand Hebbian Cell Assembly

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ABSTRACT

Formed at the cerebral cortex, neuron cell assemblies are regarded as basic units in cortical representation. Proposed by Hebb, these cell assemblies are regarded as the distributed neural representation of relevant objects, concepts or constellations. Each cell assembly contains a group of neurons having strong mutual excitatory connections. During a stimulus, these cells get activated. This activation either performs a given action or represent a given percept or concept in brain. This theory is in the strongest connection of the problem of concept forming in the brain. The challenge is to model coordinated activity among neurons in brain mathematically. The need of modelling it mathematically enables this paper to give clear view of functionality of Hebbian cell assembly. Therefore this paper proposes a pragmatic approach to Hebbian cell assemblies using mathematical model grounded in lattice based formalism that utilizes Galois connections. During this proposal, the authors also show the connections of the proposal to cognitive model of memory in particularly long-term memory (LTM).

KEYWORDS

Associative Memory, Cell Assemblies, Clique, Cognitive Models, Formal Concept Analysis, Galois Lattice, Hebbian Cell

1. INTRODUCTION

Understanding brain in terms of its mechanism and processing of information is one of the fundamental research areas (Wang et al, 2006). It is observed that the brain processes the information through a combined activity of large number of neurons (Braitenberg, 1978). For modelling this neural information processing, Canadian Neuropsychologist D. O. Hebb in 1949 proposed a hypothesis named 'Cell Assembly' (Hebb, 2005). The notion of cell assembly describes the clusters of neurons which are being activated during a certain mental process. Therefore, the excitatory synapses are formed among cluster of neurons. In general, when two neurons or cells those are repeatedly active at the same time will become associated so the action at one encourages the movement in other. Hence, when a subset of neurons in the assembly fires due to a stimulus, the entire assembly will be activated through mutual excitation. Idea of the cell assembly is utilized freely to depict clusters of neurons that execute the given action or represent the given percept or concept in brain (Huyck & Passmore, 2013). Hebb's theory is often quoted with the phrase "*cells that fire together, wires*

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together" (Huyck & Passmore, 2013). This phrase explains that thousands of neurons are triggered and form a neural network by every thought, feeling, experience and physical sensation. When the experience is repeated over and over, brain learns to trigger the same neurons each time. It can be beneficial to have neurons wired together as it can help us to learn, store and recall information in an effective way. It is established that sensory perceptions and abstract thoughts of human corresponds to the activity of cell assemblies (Wang et al., 2017). The question arises "How the cell assemblies represent, memorize and compute the information". To address this question, Fay in 1975 (Fay & Takacs, 1975), has proposed lattice based formalism in brain nerve net. Further, this model proved that the brain tissue is isomorphic to the Galois lattice. Their work has just introduced the idea of Galois connections for the brain research without elaboration and systematic discussion. Lattice based models based approach without having employed formal approach of Formal Concept Analysis (FCA) (Wille, 2005).

Extending on the work proposed by Fay (Fay & Takacs, 1975) under Willey's FCA approach, our paper is aimed to provide a clear view of understanding and representing the information in cell assemblies. The proposed work provides the pragmatic approach for formation of topological structure and functioning of Hebbian cell assemblies under FCA. Further, we have interpreted dual isomorphism with regard to inclusion relation among the lattices of effectors and receptors set.

The paper is organized as follows. Section 2 describes the background information of cell assembly and FCA. Section 3 describes the proposed work and its explanation; to assess the proposed work, experiments are conducted in Section 4; anatomical relevance related to the proposed work is given in Section 5 and finally the conclusion is given in Section 6.

2. BACKGROUND

The brief introduction and related work of Hebbian cell assembly and FCA are provided in this section. In addition, we also define the main objective that we are aiming to achieve in this work.

2.1. Hebbian Cell Assembly

The term 'cell assembly' was introduced by Donald O. Hebb in order to model the brain and the complex process of thought. Two components that Hebb tries to explain are learning and memory. Hilgard and Marquis (Hilgard & Marquis, 1940), proposed a postulates which states that every action causes prompt reverberated activity in brain. These are transient memory of the stimulus. Therefore, memory is totally dependent on neural activity instead of basic changes in the cerebrum. Hebb builds on their idea and proposes "cell assembly". The concept of cell assembly has been used to answer few queries as shown in Table 1.

Table 1. Concept of cell assembly

S.No.	Question	Solution
1	How the objects and concepts are represented in brain?	The objects and concepts are represented as cell assembly instead of single neuron
2	How learning is performed in our brain?	By creating associations between concepts through excitatory connection between cell assemblies.
3	How brain stores information and perform recall operation?	Information is stored as cell assembly and is recalled by simulation in an assembly would activate the entire group.

The pictorial representation of neuronal assembly (Dubuc, 2017) is shown in Figure 1. In the diagram given below brain's capability emerged from large population of neurons that are highly interconnected and processing information in parallel. These neurons connect to each other and form a neuronal assembly via synaptic connection. In particular, neuronal assembly is just a group of neurons that are simultaneously active in response to a certain input. Mapping of these neurons represent the objects and attributes in the brain. Further, the neuronal assembly depicts the lattice formalism in FCA.

Hebb's theory states that "When an axon of cell A is near enough to excite cell B, and repeatedly and permanently takes place in firing it, some growth process or metabolic changes takes place in one or both cells such that **A**'s efficiency as one of the cell firing **B** is increased". The theory is summarized as when two cells fires at the same time repeatedly (or when one fires causing the other to fire) chemical changes occur in both so that the two tend to connect more strongly. For example, a cell say 'hat', will fire the other cells associated with it such as the size, color, and shape etc. through many subtle triggers. The association becomes stronger when the cell is firing repeatedly. Let us represent 'hat' as cell **A** and its features such as 'color' as cell **B.** When we think of hat i.e. **A** the other cell associated with it gets fire i.e. cell **B.** The association between cells **A** and **B** becomes stronger when the cell **A** gets fired again and again. This association in Hebb's law is shown in Figure 2 and is mathematically represented as

$$\mathbf{R}_{\mathbf{AB}} = \mathbf{A} \times \mathbf{B} \tag{1}$$

Where $\mathbf{R}_{\mathbf{AB}}$ is the association or connection from cell **A** to **B**.

Figure 1. Neural cell assemblies in brain

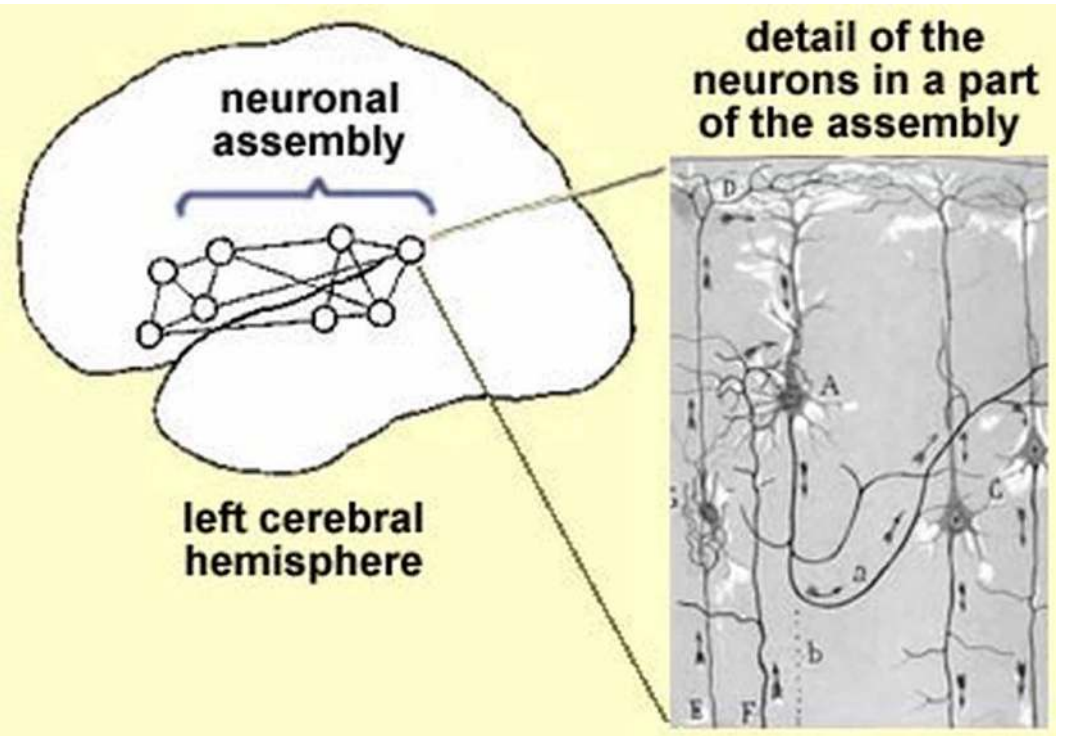
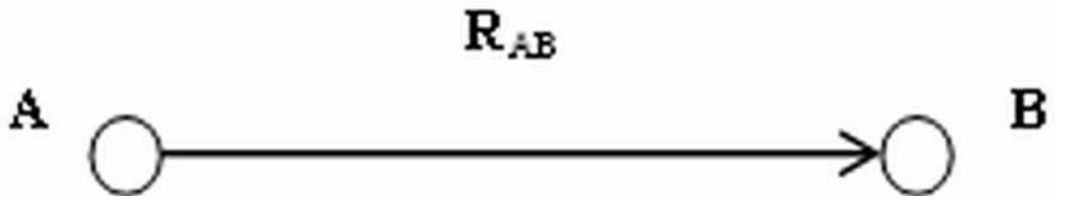


Figure 2. Connection between cells



Researchers from various disciplines perceive that the cell assembly is considered as essential part of cortical representation (Wennekers, 2006; Braitenberg, 1978). According to this theory, synaptic plasticity is responsible for forming assemblies. Thus, the coactivation of cells increases the connectivity within an assembly. Experimental support has found in this regard (Bi & Poo, 2001). The Hebbian model has additionally observed numerous applications over the years (Wennekers & Palm, 2000; Wennekers & Palm, 2007; Huyck & Orengo, 2005).

As mentioned, the qualitative nature of Hebb's theory paved a way for different mathematical models (Lansner & Fransén, 1992; Lin, Osan & Tsein, 2006; Lopes-dos-Santos, Conde-Ocazionez, Nicolelis, Ribeiro & Tort, 2011).The theory was experimented with different models starting with the spinal motor neurons, pulse generating motor neurons, spiking motor neuron model (Ranhel, 2012) and so on. Further, computational advantages of Hebb's theory was established in different recurrent artificial neural network models (Fransén, Lansner & Liljenström, 1993; Lansner, 2009) such as Hopfield networks, Long short term memory network, where the pattern completion and competition between cell assemblies by using motor neurons as excitatory neurons is demonstrated. The result obtained shows the believability of Hebbian cell assembly. Further, Sommer and Palm (1998) demonstrates the biological believable form of Cross Bidirectional (CB) retrieval which also contributes to the cell assembly theory. Wenneker and Palm (2006) have shown how the Hebbian cells can be extended by decision rules and synaptic patterns.

2.2. Formal Concept Analysis

FCA was introduced by a German mathematician, Rudolf Wille in 1982 (Wille, 2005). It may be understood as "Applied Abstract Algebra" or more specifically as "Applied Lattice Theory". FCA investigates the data which depicts relationship between the set of objects and the set of attributes. Various models based on FCA are introduced (Singh, Cherukuri & Li, 2017; Annapurna & Cherukuri, 2013; Cherukuri, Dias &Vieira, 2015; Cherukuri & Srinivas, 2010; Cherukuri, 2012; Shivhare & Cherukuri, 2016; Shivhare et al., 2017) in many areas according to the need. Cognitive functionalities Bidirectional Associative Memory has been modelled using FCA (Cherukuri, Ishwarya & Loo, 2015). Their work concentrated on the functionalities such as learning, memorizing, recalling the memorized. Several researchers revealed that FCA can be used in cell assembly as well (Wenneker, 2009; Endres, Földiák & Priss, 2009; Lin, Osan & Tsien, 2006). FCA has been applied in various frameworks. In order to model the class of human ways of problem solving and information processing is a challenging task. Yao (2016) introduced a three- way decision for human problem solving. A novel cognitive systems based on FCA have been established that exactly describes the human cognitive process (Yao, 2009). Extending upon this work, researchers have combined granular approaches to concept learning from cognitive computing viewpoint (Xu, Pang & Luo, 2014; Li, Mei, Xu & Qian, 2015; Li, Hang, Qi, Qian & Liu, 2017). The basic definitions of classical FCA are listed below:

Definition 1: (Formal Context) (Poelmans, Ignatov, Kuznetsov & Dedene, 2013) A formal context is a triple **K** = (**U**, **V**, **R**), where **U** is a non-empty set of objects, **V** is a non-empty set of attributes and **R** is the binary relation between the objects (**U**) and attributes (**V**). For a pair $x \in \mathbf{U}$ and

$y \in \mathbf{V}$, if there exists $x\mathbf{R}y$, we write $(x,y) \in \mathbf{R}$. Further, we say that the object x has the attribute y. The formal context is considered as a binary information table. In this table, objects are tabulated along rows and the characteristics or attributes are tabulated along columns. A cell (x, y) is marked by '$\times$' if the attribute y belongs to the object x represents a formal context.

Definition 2: (Concept forming operators) (Poelmans, Ignatov, Kuznetsov & Dedene, 2013) In any formal context **K**, a pair of operators can be defined as follows: $^{\uparrow}$: $2^{\mathbf{U}} \rightarrow 2^{\mathbf{V}}$ and $^{\downarrow}$: $2^{\mathbf{V}} \rightarrow 2^{\mathbf{U}}$ for every $\mathbf{X} \subseteq \mathbf{U}$ and $\mathbf{Y} \subseteq \mathbf{V}$:

$$\mathbf{X}^{\uparrow} = \{y \in \mathbf{V} / \text{ for each } x \in \mathbf{X} : (x,y) \in \mathbf{R}\}$$

$$\mathbf{Y}^{\downarrow} = \{x \in \mathbf{U} / \text{for each } y \in \mathbf{Y} : (x,y) \in \mathbf{R}\}$$

Definition 3: (Formal Concept) (Poelmans, Ignatov, Kuznetsov & Dedene, 2013) Given a formal context **K**= (**U**, **V**, **R**), a formal concept is defined as a pair $(\mathbf{X},\mathbf{Y})$ of an object subset $\mathbf{X} \subseteq \mathbf{U}$ and an attribute subset $\mathbf{Y} \subseteq \mathbf{V}$ such that X= $\mathbf{Y}^{\downarrow}$ and Y = $\mathbf{X}^{\uparrow}$. Further the subsets **X** and **Y** are respectively known as extent and intent of the concept $(\mathbf{X},\mathbf{A})$.

Definition 4: (Concept Lattice) (Poelmans, Ignatov, Kuznetsov & Dedene, 2013) The concepts of a formal context **K** = (**U**, **V**, **R**) are ordered by the relation $\leq$ as $(\mathbf{X}_1, \mathbf{Y}_1) \leq (\mathbf{X}_2, \mathbf{Y}_2) \Leftrightarrow \mathbf{X}_1 \subseteq \mathbf{X}_2$ & $\mathbf{Y}_2 \subseteq \mathbf{Y}_1$ where $(\mathbf{X}_1, \mathbf{Y}_1)$ and $(\mathbf{X}_2, \mathbf{Y}_2)$ are concepts. $(\mathbf{X}_1, \mathbf{Y}_1)$ is called sub-concept of $(\mathbf{X}_2, \mathbf{Y}_2)$ and $(\mathbf{X}_2, \mathbf{Y}_2)$ is called as super-concept of $(\mathbf{X}_1, \mathbf{Y}_1)$. The relation $\leq$ is found to be a partial order relation. The partially ordered sub-concepts and super-concepts together form a lattice called concept lattice and is represented as CL (**U**, **V**, **R**).

Literature has witnessed the connection among the biological models and mathematical frameworks (Wang, 2007; Wang et al., 2007). Fay & Takacs (1975) have proposed a lattice based approach towards cell assembly. By regarding neurons as lattice vertices and connection among neurons as the lattice edges, authors have analysed the neuron structures in group. Further, Wenneker (2009) has proposed the hierarchal nature of cell assemblies where the model can learn the lattices incrementally. They have demonstrated an interpretation of neuron firing pattern as a concept lattice about stimulus response mapping in the sense of FCA. Moreover, concept lattices are used to model the effects of sparse neural coding (Endres et al., 2009). Rumbell (2013) has introduced an implementation in networks of spiking neurons for cortical functions. Baudot, (2019) has provided a detailed review on how mathematical structures such as lattice theory and algebraic principles help modeling the cognition in an information topology perspective. Dragoi (2020) have brought the similarity among the functionality of neurons in the human brain and the social networks. Very recently, Ishwarya & Kumar (2020) have discussed how three-way FCA can be used to model the human cognition. In particular they have discussed the quantum aspects that this model exhibit which are similar to the cognitive aspects. Therefore, aforementioned literature proves that FCA has been utilized in Hebbian cell assembly as well.

2.3. Problem Description

To understand the functionality of brain beyond its neurons and synapses, Hebb's has proposed cell assemblies. According to Hebb, a group of strongly interconnected active neurons represents cell assembly. As a result of their high interconnectivity, the activation in one part of assembly can activate the entire assembly (Legendy, 1967; Palm, 2012). Understanding this activation is one of

the main challenges towards modeling the way knowledge is represented, stored and retrieved in brain. Therefore, binding of such assembly would give the premise by which complex cognitive processes such as memory recall, thinking, planning and decision making could be done (Sejnowski, 1994). Some of the interesting investigations have been done in formalizing the Hebb's postulate mathematically. One of the approaches is formal associative memory (Lansner, 2009; Shivhare & Cherukuri, 2017a; Wennekers, 2009; Cherukuri et al., 2015; Shivhare & Cherukuri, 2017b) which were more or less in Hebb's proposal. In another interesting work, Fay & Takacs (1975) proposed a lattice based formalism. The model can understand the activity pattern in large number of neurons during cognitive activities. Further, this model has anatomical and psychological relevance and significance. However the limitation of their work is that they have provided the idea of Galois connection for brain research without elaboration and systematic discussion. From this understanding of the literature, in this paper, we provide a pragmatic approach towards understanding cell assembly formation using FCA framework.

Based on these observations, following research questions are addressed in this paper:

1. How to model the collective behavior of neurons?
2. How to model cell formalism using mathematical framework?

In order to address above mentioned problem, we aim at the following proposal in the paper:

1. Introduce the mathematical lattice based approach, FCA for cell formalism, cell assembly and clique assembly formation.
2. Provide the anatomical relevance of the proposed work.

By introducing these proposals, we are able to provide a clear view of understanding and representing the information in cell assemblies.

3. PROPOSED WORK

It is observed that cell assembly contains neurons that are mutually connected. It should be noted that no other neuron has full connectivity to this assembly. With the advancement in computation, there is a need to develop formal approaches which helps in modeling the activity of the cell assemblies. The proposed work models the functionalities of cell assembly through FCA. This work is an extension of lattice based formalism in brain nerve net by Fay & Takacs (1975) in the direction of literature dealing with cell assemblies (Wille, 2005; Wennekers, 2009). In this section, we will describe how a mathematical lattice approach contributes towards the formation of topological structure and functioning of Hebbian cell assemblies.

It is observed that the brain is viewed as an algebraic structure (Fay & Takacs, 1975). The modelling of cell assembly can be performed by the theoretical means of algebraic structure. Cell assembly is used to describe a group of neurons that perform a given action or represent a given percept or concept. For modeling cell assembly mathematically, two sets are considered for interpretation which is the set of receptors and the set of effectors. These sets are further regarded as objects or attributes. Further, binary relation is assumed between receptors and effectors. Hence, each formal concept is regarded as a cell assembly containing set of receptors (extent) and set of effectors (intent). Since, we have considered binary excitation; the relation among these cells is binary. Consider the two finite sets of receptors and effectors **U** and **V** respectively, such that

$$\mathbf{U}= \{u_1, u_2, \ldots\ldots u_i\} \tag{2}$$

Table 2. Mapping between Hebbian Cell Assembly and FCA

Biological Model	Cell Assembly	FCA
Neurons	Cell	Objects/Attributes
Group of Neurons	Cell Assembly/Clique	Formal Concepts
Collection of Neurons	Receptor cell assembly	Extent
Collection of Neurons	Effector cell assembly	Intent
Axon	Synapse	Relation
Biological Neural Network	Clique Assembly	Concept Lattice

$$\mathbf{V}= \left\{v_1, v_2, \ldots\ldots v_3\right\} \tag{3}$$

The relation **R** between the elements of **U** and **V** is written as:

$$u_i \mathbf{R} v_j$$

The above relation represents that the axon of neuron u_i excite the other neuron v_j. Further the relation is considered as connectivity relation. Therefore, we can find a many to many relationships among the neurons, where several neurons are connected or related to several other neurons. From such connection, we find receptors and effectors subsets that are maximal by using clique formation process. Each maximal subset is considered as concept or a cell clique. Every clique contains the assembly of receptor cells and effector cells. For a better understanding we have mapped the notations of Hebbian cell assembly with the notation of FCA as shown in Table 2. This table provides all the notions that are basis for the current work.

As per this proposal, we regard each neuron as either receptor or effector. Hence a group of neurons form a receptor or effector cell assembly.

3.1. Formation of Connectivity Matrix

Consider two finite sets **U** and **V** represented by receptors and effectors respectively. The connection/relation between them is represented by **R**.

Definition 5: For a given receptor cell $x \in \mathbf{X}$ where $\mathbf{X} \subseteq \mathbf{U}$, the corresponding set of effectors that possess x is denoted $\mathbf{R}(x)$, defined as:

$$\mathbf{R}\left(x\right) = \left\{y \in \mathbf{V} : x\mathbf{R}y\right\} \subseteq \mathrm{V} \tag{4}$$

Definition 6: For a given effector cell $y \in \mathbf{Y}$, where $\mathbf{Y} \subseteq \mathbf{V}$, the corresponding set of receptors that possess y is denoted by $\mathbf{R}(y)$, defined as:

$$\mathbf{R}\left(y\right) = \left\{x \in \mathbf{U} : x\mathbf{R}y\right\} \subseteq \mathbf{U} \tag{5}$$

The notation $\mathbf{R}(x)$ and $\mathbf{R}(y)$ represents an association between the effector cells and receptor cells. By using the definitions defined above one can form the connectivity among the receptors and effectors. This connectivity is represented as connectivity matrix which further known as formal context in FCA. By extending the notations of $\mathbf{R}(x)$ and $\mathbf{R}(y)$, for each receptor x and effector y one can easily determine the relationships between the set of receptors and set of effectors. Table 3 shows the connectivity matrix which contains receptors along the row and effectors along the column.

Table 3. Connectivity matrix

	a	b	c	D
1	×	-	×	-
2	-	×	×	-
3	-	×	×	-
4	×	-	×	×

3.2. Formation of Many to Many Relations

For finding many to many relations among the two sets i.e. the receptors and effectors, clique generation process is employed (Cherukuri & Singh, 2014) and shown in Table 4. In this algorithm, for a pair of receptors and effectors (**U**, **V**) at current clique, the corresponding lower cliques are found. Among these found lower cliques, the algorithm will return the maximal general clique. The found lower clique is assigned as current clique for whom lower cliques are found further. This iterative process is repeated till all the possible many to many relations among receptors and effectors are discovered. As a result, all possible cliques which are a pair of set of receptors and effectors will be obtained. These cliques exhibit all meaningful associations between receptors and effectors.

Consider the connectivity matrix shown in Table 3. According to algorithm, first we find the maximally general cliques.

- Take all the receptors {1, 2, 3, 4} given in the connectivity matrix. Check whether it shares any effector by using operators $\uparrow\downarrow$ as shown in the algorithm. Thus, $\{1, 2, 3, 4\}^{\uparrow} = \{c\}$ & $\{c\}^{\downarrow} = \{1, 2, 3, 4\}$. Therefore, {{1, 2, 3, 4} {c}} is the concept placed in top of the cell assembly.
- The current clique is {{1, 2, 3, 4} {c}}. Now, find the next clique by exploring the effector with effector assembly {c}. The candidates for this clique are: $\{\{a,c\}^{\downarrow}, \{a,c\}^{\downarrow\uparrow}\} = \{\{1, 4\} \{a, c\}\}$

$\{\{b,c\}^{\downarrow}, \{b, c\}^{\downarrow\uparrow}\} = \{\{2, 3\} \{b, c\}\}$
$\{\{c,d\}^{\downarrow}, \{c, d\}^{\downarrow\uparrow} = \{\{4\} \{a, c, d\}\}$

From the above candidates, {{1, 4} {a, c}}, {{2, 3} {b, c}}, {{4} {a, c, d}} are the lower cliques for the {{1, 2, 3, 4} {c}}.

- Next, consider the current clique as {{1, 4} {a, c}}. Again, find the next clique. The candidates for this clique is {{4} {a, c, d}} which is the only lower clique of the current clique.
- The next current clique is now {{2, 3}, {b, c}}. Again find the next clique. There is no candidate for this clique.
- Now, the current clique is {{4} {a, c, d}}.There is no further candidates for this clique.

Table 4. Clique generation algorithm

Input: Connectivity matrix (**U**, **V**, **R**)
Output: maximal Clique (receptors, effectors)
C = (**U**, **U**$^{\uparrow}$)
while CurrentClique ≠ empty
NextClique = empty
for each (**X**, **Y**) ∈ CurentClique
LowerCliques = FindLowerCliques (**X**, **Y**)
for each $\left(\mathbf{X}_1, \mathbf{Y}_1\right) \in$ LowerCliques
if ($\left(\mathbf{X}_1, \mathbf{Y}_1\right)$) $\notin$ **C**)
C = **C** $\cup$ ($\mathbf{X}_1, \mathbf{Y}_1$); // adding this clique with the list of all cliques
NextClique = NextClique $\cup$ ($\mathbf{X}_1, \mathbf{Y}_1$ **);** // set this clique as the next clique
End **if**
End **for**
CurrentClique = NextClique; // Assign next clique as current clique
End **while**
FindLowerCliques (**X, Y**)
Candidates = empty;
For each $m \in \mathbf{M} \setminus \mathbf{Y}$
$\mathbf{X}_1 = (\mathbf{Y} \cup \{m\})^{\downarrow}$
$\mathbf{Y}_1 = (\mathbf{X}_1)^{\uparrow}$
If ($\mathbf{X}_1$, $\mathbf{Y}_1^{\uparrow}$) ∉ Candidates **then**
Candidates = Candidates $\cup$ ($\mathbf{X}_1, \mathbf{Y}_1$);
End **for**
return maximal Cliques

- There is no clique at current clique. So from the last step $\mathbf{X}_1 = (\mathbf{Y} \cup \{m\})^{\downarrow}$ and $\mathbf{Y}_1 = (\mathbf{X}_1)^{\uparrow}$ of the algorithm, $\mathbf{X}_1 = \{a, b, c, d\}^{\downarrow} = \{\ \}$. Therefore, $\mathbf{Y}_1 = \{\ \mathbf{X}_1\ \}^{\uparrow} = \{a, b, c, d\}$. Since, ($\mathbf{X}_1$, $\mathbf{Y}_1$) is not a candidate. Thus, {{ } {a, b, c, d}} is the last next clique. The final cell assembly is shown in Figure 2. This cell assembly is a lattice structure as in FCA.

List of cliques obtained from the connectivity matrix is given in Table 5 using algorithm shown in Table 4 is given below:

The complexity of an algorithm is **O(|C||U||V²|)**, where **U** is the set of receptors, **V** is the set of effectors and **C** is the set of obtained list of cliques.

Table 5. List of cliques

S.No.	Cliques (Formal Concepts)	
	Receptors	Effectors
1.	{1, 2, 3, 4}	{c}
2.	{2, 3}	{b, c}
3.	{1, 4}	{a, c}
4.	{4}	{a, c, d}
5.	{ }	{a, b c, d}

Definition 7: (Set of Maximal Effectors): For any given subset of receptor cells **X**, the maximal subset of effectors $\mathbf{X}^{\uparrow}$ can be defined as:

$$\mathbf{X}^{\uparrow} = \{y \in \mathbf{V} | y \in R(x) \text{ for every } x \in U\}$$

$$= \bigcap_{x \in \mathbf{R}} \mathbf{R}(x) \tag{6}$$

By the maximal subset of effectors ($\mathbf{X}^{\uparrow}$), we mean that the maximal subset of effector cells that are having relation in common with the cells of receptor subset. For example, {a, c} is a set of maximal effectors for the receptors subset {1,4}.

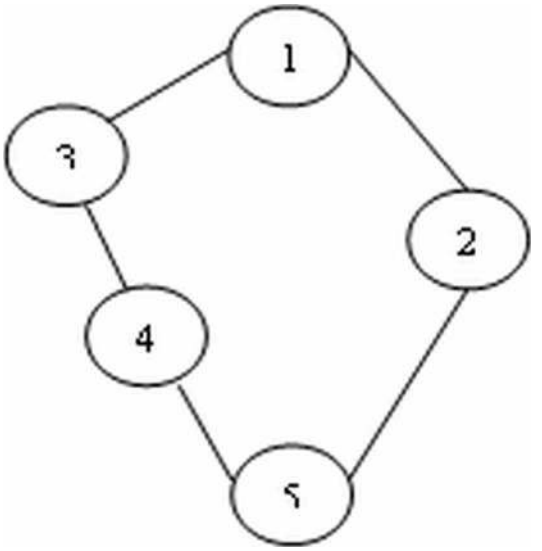
Definition 8: (Set of Maximal Receptors) For any given subset of effector cells **Y**, the maximal subset of receptors $\mathbf{Y}^{\downarrow}$ can be defined as:

$$\mathbf{Y}^{\downarrow} = \{x \in \mathbf{U} | x \in R(y), \text{for every } y \in \mathbf{V}\}$$

$$= \bigcup_{y \in R} R(y)$$

By the maximal subset of receptors ($\mathbf{Y}^{\downarrow}$), we mean that the maximal subset of the receptor cells that are having relation in common with the cells of effector subset. For example the subset {4} is a maximal set of receptors for the effectors subset {a, c, d}.

Figure 3. Clique assemblies



From the Table 3 {{4} {a, c}} is not a clique as it is not maximal. There are other receptors such as {1} which are having relation with a set of effectors {a, c}. Similarly, {{1, 4}, {a}} is not a clique as there is other effectors such as {c} which are having relation with the set of receptors {1, 4}.

Here the operators ($\uparrow\downarrow$) are sufficient to reflect the unique association between receptors and effectors cells.

Definition 9: (Maximal clique) A clique (**X, Y**) which consists of a pair of receptors (**X**) and effectors (**Y**) is said to be maximal iff it follows the below conditions:

$$\mathbf{X} = \left(\mathbf{R}\left(\mathbf{R}\left(x\right)\right)\right) \tag{8}$$

The subset **X** is said to be closed under binary relation **R**

$$\mathbf{Y} = \left(\mathbf{R}\left(\mathbf{R}\left(y\right)\right)\right) \tag{9}$$

The subset **Y** is said to be closed under binary relation **R**.

It should be noted that by maximal or closed subsets we mean that the subset contains all the effectors cells that are having relation in common with the elements or cells of receptor subsets and vice versa. For example, {{4} {a, c, d}} is a maximal clique because there is no other maximal receptors or maximal effectors which are having relation with each other.

3.3. Proposed Approach

According to the above theory proposed, in this section we will describe on how to form the cell assemblies through excitatory connection.

Step 1: Input the body of knowledge i.e. the receptors (**U**) and effectors (**V**).
Step 2: Build the connectivity matrix from knowledge (**U**, **V**, **R**)
Step 2.1: Compute corresponding set of effectors (**Y**) that possess the given receptor cells (**X**) using definition 1.
Step 2.2: Compute corresponding set of receptors (**X**) that possess the given effector cell (**Y**) using definition 2.
Step 3: Form the clique from connectivity matrix as follows:
Step 3.1: Perform the clique generation process of FCA as shown in Table 4
Step 4: Check the activation of cell assembly.
Step 4.1: Perform the maximal conditions on sets of receptors and effectors.
Step 4.1.1: Find the set of maximal effectors using definition 7.
Step 4.1.2: Find the set of maximal receptors using definition 8.
Step 4.2: Check whether a pair of maximal receptors and effectors activate cell assembly using definition 5.
Step 4.3: Continue the steps 4.1. to 4.2 for finding all possible activation of cell assembly.
Step 5: Output Cell Assembly/ Clique.

It should be noted that the most generalized cliques are at top of the cell assembly and most specialized cliques are at the bottom of the cell assembly. When we move up in the cell the number of receptors increases and the effectors decreases while the number of effectors increases and receptors decreases when we down the cell assembly. This structure is similar to the pyramid structure of neural clique assemblies (Lin, Osan & Tsien, 2006).

In this article, the proposed approach is inspired by the following aspects:

- **Computational Models** (Fay & Takacs, 1975; Cherukuri, Ishwarya & Loo, 2015; Wennekers, 2009; Wennekers & Palm, 2007): With regard to the computational model, Fay (Fay & Takacs, 1975) have described the lattice based formalism in brain nerve net and proved that the brain tissue is isomorphic to the Galois lattice. In 2009 Wenneker (Wennekers, 2009), have suggested the interpretation of neuron firing pattern in the sense of FCA. Further, they have extended the work of Belohlavek's BAM implementation of concept lattice by proposing a model that can learn the lattices incrementally. FCA based approach is proposed in order to model the functionality of BAM (Cherukuri, Ishwarya & Loo, 2015). The computational model of cell assembly is not only limited to these model. Wenneker and Palm (Wennekers & Palm, 2007) has shown how Hebbian cells can be extended by decision rules and synaptic pattern.
- **Biological Relevance** (Hebb, 2005; Wang, 2007; Smith, 2006): The biological relevance also provides a strong basis for the proposed approach. The analogy to the natural property of cell assembly as a group of that wire and fire together provides the strong connection for the proposed work (Hebb, 2005). Smith (Smith, 2006) gives the justification by saying that different neurons population appeared to detect different forms of information about the stimulus response. Parts of neurons get activated only when information meaningful to the object is present. When the neurons are active they categorize the region as containing the meaningful feature of an object. This gives the assumption of forming cell assembly. Further, object-attribute relation model (OAR) (Wang, 2007) is developed which gives the justification of receptors and effectors cells and the binary relation among them.

4. ANATOMICAL SIGNIFICANCE

In this section, anatomical relevance related to our proposed approach is discussed briefly. Each neuron is a cell. We consider each object (receptors)/attributes (effectors) as a neuron. Hence, extent

which is a maximal set of objects is a receptor cell assembly and intent which is a maximal set of attributes is effector cell assembly. These two assemblies when grouped together constitute a clique which is a concept. As it is pointed out, concept is the basic unit of thought in human cognition (Wang, 2007). Therefore, clique assembly is a concept. Hence, formal concept is a clique that can be regarded as ganglion which is connecting/combining closed set of receptors cells i.e. receptor assembly and closed set of effector cells i.e. effector assembly. Cell assembly forms a basis for formation of ganglion. Anatomically ganglion is a structure that contains a number of nerve cell bodies that are linked by synapses. The connection established between the effector and receptors cells is regarded as the synapse.

According to Lin et al., (2006), the anatomical layout of hierarchical neural clique assembly contributes towards long term memory (LTM) formation. Further references and analysis given in Lin et al., (2006) has shown that formation of neural clique assemblies and hierarchical organization of such cell structures are witnessed in hippocampal regions of humans, animals such as rats. This observation provides evidence for existence of such neural clique assemblies in living organisms and provides the justification for the proposed mathematical approach. Further in modelling long term memory (LTM), Wang (Wang, 2007) has proposed to use an object-attribute relation (OAR) model. The model describes the LTM by stating that LTM consists of dynamic and partially interconnected neurons. By saying partial neurons, means that not each neurons are connected to each and every neurons. A connection between the pair of neurons via synapse represents the association between the concepts. This provides the justification for the assumption of effector and receptor cells in the brain and the binary relation among them. Moreover, according to Smith (Smith, 2006), with respect to stimulus response different population of neurons appeared to detect different types of information. The information which is detected by these neurons is meaningful to particular objects. This gives the justification for the assumption of clique formation. Further, parts of neurons get activated only when information relevant to the object is present. When these neurons are active they categorize a region. This is the assumption of forming cell assemblies.

In order to process the stimuli, brain identifies the objective importance of stimuli. Further numerous neurons process in parallel to understand the properties of the objects that are identified through sensory inputs Seyedielmabad (2020). For example when we look at an apple brain process several properties of the apple such as its color, size, shape, taste, etc. Brain is able to associate all these properties that are decided at different locations and provides a unified representation of single object i.e. apple in this case. Similarly when multiple objects are notices using different properties of those objects, brain is able to associate them with the help of common properties among them. The problem becomes more complicated when we need to associate two distinct objects. Hence, these anatomical properties of brain have paved a way in modeling the Hebbian cell assembly mathematically.

5. EXPERIMENTAL ANALYSIS

This section demonstrates the implementation of the proposed method on the sample context in order to understand the formation of cell assembly more clearly.

5.1 Illustration of Proposed Approach

Consider the body of knowledge shown in Table 6 consists of 8 receptors cells (objects) and 15 effectors cells (attributes). This table represents the characteristics of animals. The receptors cells represent the animals as object along the row {Cat, Mouse, Horse, Brontosaurus, Mammoth, Ant, Spider, Snake} and the effectors cells represent the characteristics of animal as attributes along the column {Has fur/Hair, Herbivore, Exoskeleton, Large, Small, 4 legs, 8 legs, 0 legs, Vertebrate, Extinct, Mammal, Sociable, Domestic, Insect, Reptile} of the table. The animal having the particular characteristics is denoted by 'Y' and others are denoted by 'N'. The list of all possible cliques is shown in Table 7.

Table 6. Characteristics of animals

	Has Fur/ Hair	Herbivore	Exoskeleton	Large	Small	4 legs	8 legs	0 legs	Vertebrate	Extinct	Mammal	Sociable	Domestic	Insect	Reptile
Cat	Y	N	N	N	Y	Y	N	N	Y	N	Y	Y	Y	N	N
Mouse	Y	Y	N	N	Y	Y	N	N	V	N	Y	Y	Y	N	N
Horse	Y	Y	N	Y	N	Y	N	N	Y	N	Y	Y	Y	N	N
Brontosaurus	N	Y	N	Y	N	Y	N	N	Y	Y	N	N	N	N	Y
Mammoth	Y	Y	N	Y	N	Y	N	N	Y	Y	N	N	N	N	N
Ant	N	Y	Y	N	Y	N	N	N	N	N	N	Y	N	N	N
Spider	N	N	Y	N	Y	N	N	Y	Y	N	N	N	Y	Y	N
Snake	N	N	N	N	Y	N	N	Y	Y	N	N	N	Y	N	Y

The Hasse diagram shown in Figure 4 denotes the representation of connectivity of matrix in memory. Now, we will demonstrate on how cell assembly is formed through FCA. Following steps need to be taken care:

5.1.1. Step 1: Input the Body of Knowledge i.e. Receptors and Effectors

The receptors and effectors in the body of knowledge is{Cat, Mouse, Horse, Brontosaurus, Mammoth, Ant, Spider, Snake} and {Has fur/Hair, Herbivore, Exoskeleton, Large, Small, 4 Legs, 8 Legs, 0 Legs, Vertebrate, Extinct, Mammal, Sociable, Domestic, Insect, Reptile}.

5.1.2. Step 2: Build the Connectivity Matrix from Knowledge

For forming the connectivity matrix, compute the corresponding set of effectors for a given receptors. Let us take any receptors from body of knowledge i.e. {mammoth} and compute its corresponding set of effectors which is related to given receptors by using

$$R\left(x\right) = \left\{y \in V, xRy\right\} \subseteq V \, .$$

Therefore, R(mammoth) = {Has fur/Hair, Herbivore, Large, 4 Legs, Vertebrate, Extinct, Mammal}.

The Hasse diagram shown in Figure 4 denotes the representation of connectivity of matrix in memory.

Continue this process for all the receptors of the body of knowledge. Now, compute the corresponding set of receptors for a given effectors. Let us take {mammal} and compute its corresponding set of receptors which is related to a given effectors by using

$$R\left(y\right) = \left\{x \in U, xRy\right\} \subseteq U$$

Therefore, R(mammal) ={Cat, Mouse, Horse, Mammoth}. Continue this process for all effectors provided in the body of knowledge. This connectivity among receptors and effectors is represented as connectivity matrix which is further known as formal context in FCA. Table 6 represents the connectivity matrix and its representation in memory is shown in Figure 4.

5.1.3. Step 3: Find Many to Many Relations from the Connectivity in Order to Form Cliques

In order to form many to many relations for creating all possible cliques (formal concepts) i.e. a pair of set of receptors and set of effectors perform the clique generation process given in Table 4.

5.1.4. Step 4: Perform the Activation of Cell Assembly

In order to perform the activation of cell assembly, we need to find the maximal set receptors and effectors by using definitions 7 and 8 mentioned above.

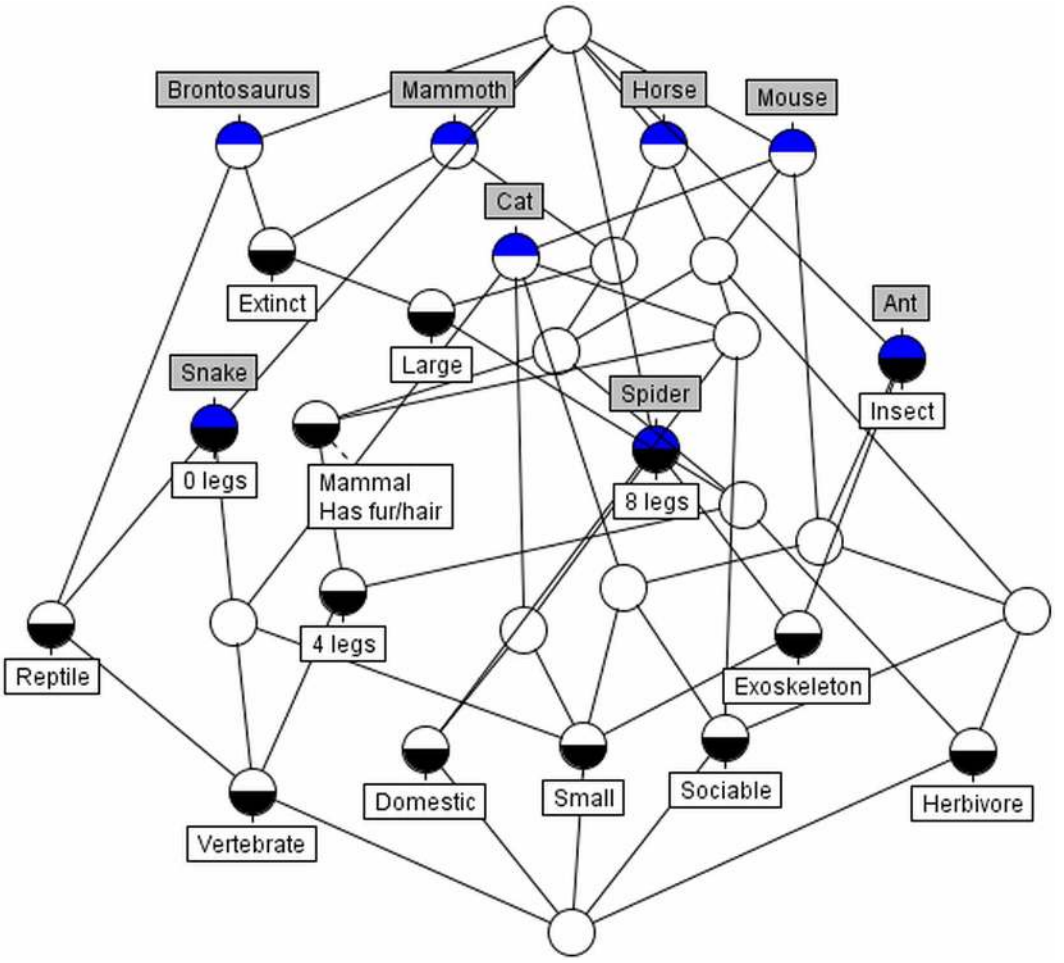
Consider the receptor cell {Mammoth} and check the activation of this particular receptor. For performing activation of cell assembly related to {Mammoth}, we need to find the maximal set of effectors. Therefore, (Mammoth)* = {Has fur/Hair, Herbivore, Large, 4 Legs, Vertebrate, Extinct, Mammal}. Next step is to check whether the receptors and effectors set formed are maximal or not. {Has fur/Hair, Herbivore, Large, 4 Legs, Vertebrate, Extinct, Mammal}* = {Mammoth}. Figure 5 shows the activation of cell assembly related to receptor cell {Mammoth}.

Similarly, the activation of cell assembly related to effector cell {Mammal} is shown in Figure 6.

Table 7. List of cliques (Formal Concepts)

S.No.	Cliques (Formal Concepts)	
	Receptors	Effectors
1.	{Cat, Mouse, Ant, Spider, Snake}	{Small}
2.	{Cat, Mouse, Horse, Ant}	{Sociable}
3.	{Mouse, Horse, Brontosaurus, Mammoth, Ant}	{Herbivore}
4.	{Cat, Mouse, Horse, Spider, Snake}	{Domestic}
5.	{Cat, Mouse, Horse, Brontosaurus, Mammoth, Snake}	{Vertebrate}
6.	{Ant, Spider}	{Exoskeleton, Small}
7.	{Cat, Mouse, Ant}	{Small, Sociable}
8.	{Cat, Mouse, Spider, Snake}	{Small, Domestic}
9.	{Mouse, Horse, Ant}	{Herbivore, Sociable}
10.	{Cat, Mouse, Horse, Snake}	{Vertebrate, Domestic}
11.	{Cat, Mouse, Horse, Brontosaurus, Mammoth}	{4 legs, Vertebrate}
12.	{Brontosaurus, Mammoth}	{Vertebrate, Reptile}
13.	{Spider}	{Exoskeleton, Small, 8 legs, Domestic}
14.	{Mouse, Ant}	{Herbivore, Small, Sociable}
15.	{Cat, Mouse, Snake}	{Small, Vertebrate, Domestic}
16.	{Cat, Mouse, Horse, Mammoth}	{Has fur/Hair, 4 legs, Vertebrate, Mammal}
17.	{Mouse, Horse, Brontosaurus, Mammoth}	{Herbivore, 4 legs, Vertebrate}
18.	{Ant}	{Herbivore, Exoskeleton, Small, Sociable, Domestic}
19.	{Snake}	{Small, 0 legs, Vertebrate, Domestic, Retile}
20.	{Cat, Mouse, Horse}	{Has fur/Hair, 4 legs, Vertebrate, Mammal, Sociable, Domestic}
21.	{Mouse, Horse, Mammoth}	{Has fur/Hair, Herbivore, 4 legs, Vertebrate, Mammal}
22.	{Horse, Brontosaurus, Mammoth}	{Herbivore, Large, 4 legs, Vertebrate}
23.	{Cat, Mouse}	{Has fur/Hair, Small, 4 legs, Vertebrate, Mammal, Sociable, Domestic}
24.	{Mouse, Horse}	{Has fur/Hair, Herbivore, 4 legs, Vertebrate, Mammal, Sociable, Domestic}
25.	{Horse, Mammoth}	{Has fur/Hair, Herbivore, large, 4 legs, Vertebrate, Mammal}
26.	{Brontosaurus, Mammoth}	{Herbivore, Large, 4 legs, Vertebrate, Extinct}
27.	{Mouse}	{Has fur/Hair, Herbivore, Small, 4 legs, vertebrate, Mammal, Sociable, Domestic}
28.	{Horse}	{Has fur/Hair, Herbivore, Large, 4 legs, Vertebrate, Mammal, Sociable, Domestic}
29.	{Mammoth}	{Has fur/Hair, Herbivore, Large, 4 legs, Vertebrate, Extinct, Mammal}
30.	{Brontosaurus}	{Herbivore, Large, 4 legs, Vertebrate, Extinct, Reptile}
31.	{Cat, Mouse, Horse, Brontosaurus, Mammoth, Ant, Spider, Snake}	{empty}
32.	{empty}	{Has fur/Hair, Herbivore, Exoskeleton, Large, Small, 4 legs, 8 legs, 0 legs, Vertebrate, Extinct, Mammal, Sociable, Domestic, Insect, Reptile}

Figure 4. Hasse diagram representation of memory

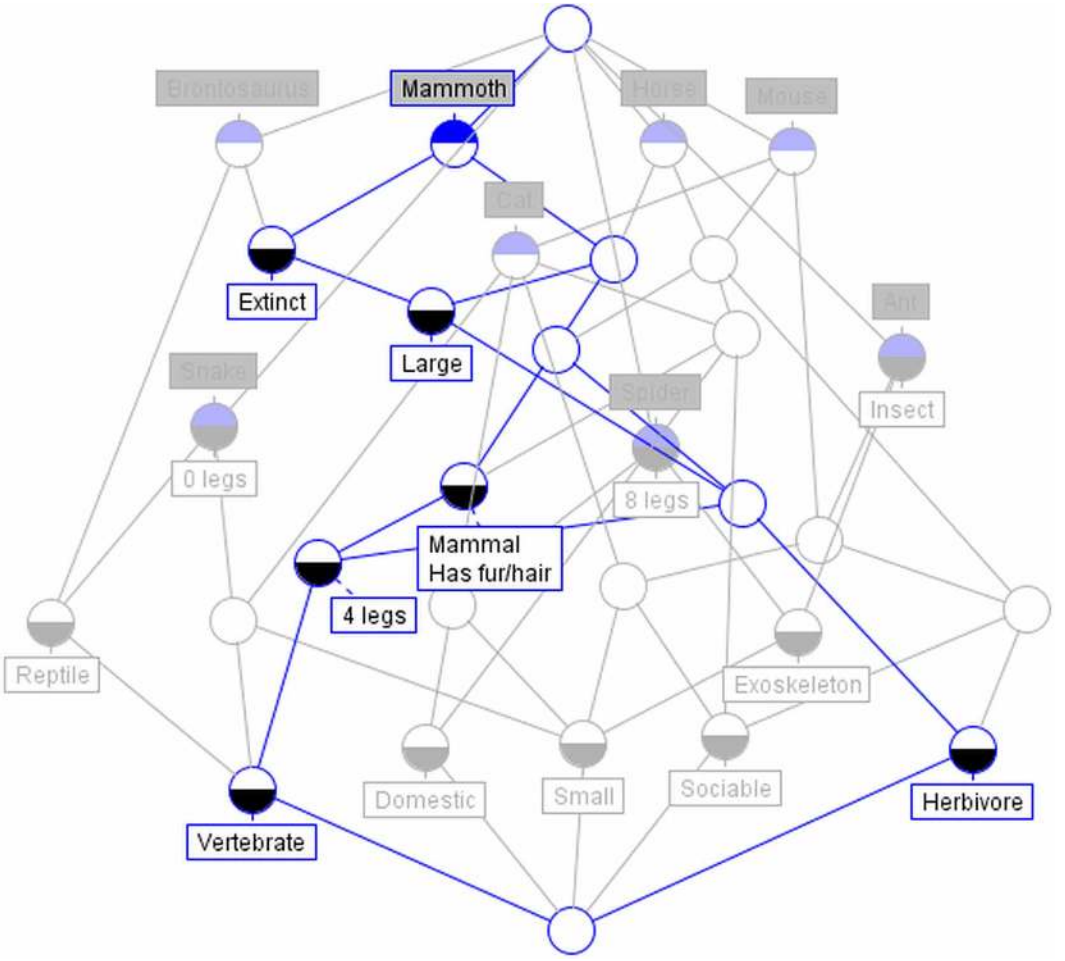


The proposed model would serve as more steps towards understanding and representing the information in cell assemblies. Each cell assembly can be modeled as a concept since by being in assembly implies maximal connectivity between the receptors (object) and effectors (attributes). Thus, there are no other receptors that are influenced or related with all the cells in the effectors set and vice-versa. It should be noted that the cell assembly represents the lattice structure in FCA. Thus, we introduced a mathematical approach based on FCA to model the Hebbian cell assembly.

5.2. Discussion

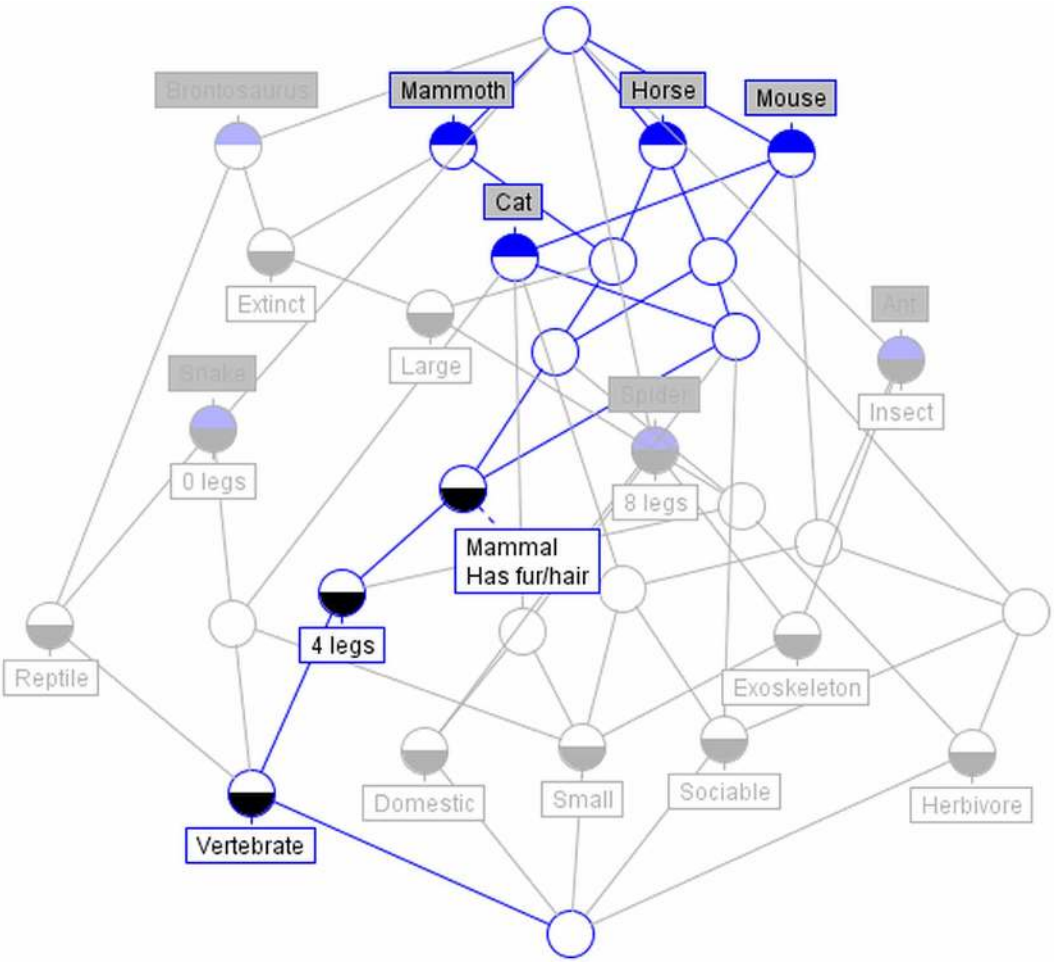
In this paper we have provided a mathematical lattice based approach for cell formalism cell assembly and clique assembly formation. There are biological and anatomical justifications in the literature about formation of such cell concentrations and assemblies in living organisms. Hence the proposal in this paper provides a pragmatic approach towards such formalisms. If we can view the entire brain as an algebraic structure such as lattice (Fay & Takacs, 1975), graph (Palm, 1981), and one can study the functionalities and formations, operations by theoretical means of such algebraic structures. Therefore, the fundamental aspect of proposal is to view brain as an algebraic structure.

Figure 5. Clique assemblies of mammoth



In this article, neurons in the brain can be categorized as receptor and effector cells. Real world things or objects are represented by receptor cells. Similarly ideas that surround these objects or attributes that makeup the objects are represented by effector cells. As per the proposed algebraic approach, the closed sets of receptor cells and closed sets of effector cells forms the same structure i.e. algebraic structure of objects or things is same as that of algebraic structure among attributes in brain. This modelling helps to address one of the major challenges in understanding the activity pattern in large neuronal population during cognitive activities (Lin, Osan & Tsien, 2006). The structure of cell assembly is similar to the pyramid structure of neural clique assemblies as suggested by Lin (Lin, Osan & Tsien, 2006). While Fay (Fay & Takacs, 1975) has discussed the lattice based formalism in brain nerve nets, we extend this work by the Willey's FCA (Wille, 2005) approach that has formalized the lattice properties for knowledge representation and processing tasks. Based on this approach we have interpreted dual isomorphism with regard to inclusion relation among the lattices of effectors and receptors sets.

Figure 6. Clique assemblies of mammal



6. CONCLUSION

Main objective of this work is to propose a mathematical model, grounded in FCA, for representing Hebbian assembly of neurons also known as cell assemblies. This work concentrated on the various fundamental aspects of Hebb's postulates such as formation and representation of cell assembly as well as storing and recalling information in cell assemblies. While proposing, we have considered each neuron as a cell represented as receptors and effectors. The set of receptors and set of effectors are closed under binary relation (Fay & Takacs, 1975). Hence, each formal concept is regarded as a cell assembly/clique containing set of receptors and effectors. Algorithm for generating clique assemblies is proposed. The model is illustrated on animal dataset which describes the characteristics of an animal. It is expected that modeling Hebb's postulates through the proposed framework will help in understanding and implementing the neural activity during cognitive phenomena such as learning, recalling and thinking.

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