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A study on technical efficiency of a DMU (review of literature)

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Abstract. In this research paper the concept of technical efficiency (due to Farell) [1] of a decision making unit (DMU) has been introduced and the measure of technical and cost efficiencies are derived. Timmer's [2] deterministic approach to estimate the Cobb-Douglas production frontier has been proposed. The idea of extension of Timmer's [2] method to any production frontier which is linear in parameters has been presented here. The estimation of parameters of Cobb-Douglas production frontier by linear programming approach has been discussed in this paper. Mark et al. [3] proposed a non-parametric method to assess efficiency. Nuti et al. [4] investigated the relationships among technical efficiency scores, weighted per capita cost and overall performance Gahe Zing Samuel Yank et al. [5] used Data envelopment analysis to assess technical assessment in banking sectors.

1. Introduction

Farrell [1] introduced the concept of technical efficiency, of a decision making unit, measured in relation to a linear homogeneous production frontier. If two inputs are combined to produce one output, the linear homogeneous frontier may be expressed as,

$$\begin{aligned} u &= f(x_1, x_2) \\ l &= f\left(\frac{x_1}{u}, \frac{x_2}{u}\right) \\ &= f\left(\hat{x}_1, \hat{x}_2\right) \end{aligned} \tag{1.0.1}$$

A linear homogeneous production frontier admits constant returns to scale.

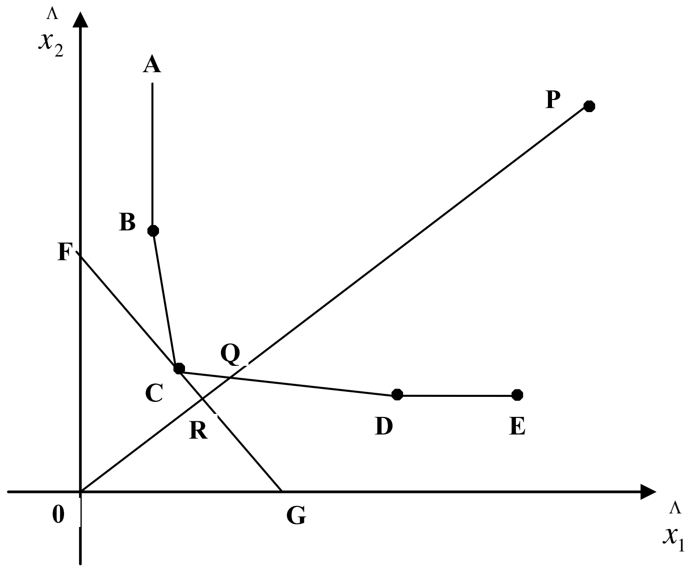

Figure (1.1)

The piecewise linear curve constituted by the line segments AB, BC, CD and DE is the unit output isoquant, which is nothing but the locus of all input combinations capable of producing one unit of output. The region bound below by the piecewise linear isoquant is the production possibility set. Let us denote this by L (1).

$$L(1) = \left\{ \hat{x} : \hat{x} \text{ produces unit output} \right\}$$

The production unit P is inefficient. Reducing its inputs radially in the direction of origin, it can attain technical efficiency.

$$\begin{aligned} F(u_0, x_0) &= Min \ \{\lambda : \lambda x_0 \in L(u_0)\} \\ &= Min \left\{ \lambda : \lambda \frac{x_0}{u_0} \in L(1) \right\} \\ &= Min \left\{ \lambda : \lambda \hat{x}_0 \in L(1) \right\} \\ &= F\left(1, \hat{x}_0\right) \end{aligned}$$

$0 \le F(u_0, x_0) \le 1$. Where $F(u_0, x_0)$ is the Farrell's input technical efficiency measure. In terms of figure (1.1), $F(u_0, x_0) = \dfrac{OQ}{0P}$.

Farrell's technical efficiency measure is conditioned on constant returns to scale. The line segment FG is the cost line, $C = p_1 \hat{x}_1 + p_2 \hat{x}_2$ .The technically efficient input vector need not be cost efficient. Cost efficiency is attained at a point where the cost line is tangent to the isoquant. Production cost is minimized at C.

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$$Q(u_0, p) = \underset{x}{Min} \quad \{px : x \in L(u_0)\}$$

$$= \underset{x}{Min}\left\{px : \frac{x}{u_0} \in L(1)\right\}$$

$$= u_0 \quad \underset{\hat{x}}{Min}\left\{p\,\hat{x} : \hat{x} \in L(1)\right\}$$

$$\frac{Q(u_0, p)}{u_0} = \underset{\hat{x}}{Min}\left\{p\,\hat{x} : \hat{x} \in L(1)\right\}$$

Actual cost incurred to produce unit output is, $p\,\hat{x}_0$ .Farrell's cost efficiency is measured by the ratio:

$$CE = \frac{Q(u_0, p)/u_0}{p\,x_0/u_0} = \frac{Q(u_0, p)}{p\,x_0}$$

In terms of figure (1.1), the measure of cost efficiency may be expressed as, $CE = \frac{OQ}{OP}$

Failure to operate at cost efficient point on the isoquant leads to allocative inefficiency.

$$CE = \frac{Q(u_0, p)}{p\,x_Q} \;x\; \frac{px_Q}{p\,x_0} = AE \text{ x } TE$$

Thus, allocative efficiency is defined as, $AE \quad = \quad \frac{Q(u_0, p)}{p\,x_Q} = \frac{OR}{OQ}$

2. Productive efficiency of DMU

The Farrell's [1] input efficiency measures can be easily extended to any homogeneous production frontier is

$$f(\lambda x) = \lambda^{\theta} f(x) \tag{2.0.1}$$

f (x) is homogeneous production frontier.
Its degree of homogeneity is θ. If θ =1 ⇒ Returns to scale are constant,θ<1 ⇒ RTS are decreasing and θ>1 ⇒ RTS are increasing.

To measure productive efficiency, parametric production frontiers may be used. One frontier production function widely used in empirical production analysis is the Cobb- Douglas production frontier.

$$u = A \prod_{i=1}^{m} x_i^{\alpha_i} \;,0 \le \alpha_i \le 1,\; \forall I \text{ and } A > 0 \tag{2.0.2}$$

(2.0.2) admits variable returns to scale depending on the value taken by the sum, $\sum_{i=1}^{m} \alpha_i = \theta$. The production frontier can be transformed into a form that admits constant returns to scale.

$$u_1 = A \prod_{i=1}^{m} x_i^{\beta_i} \text{ , where} \beta_i = \frac{\alpha_i}{\theta} \text{ and } \sum_{i=1}^{m} \beta_i = 1 \tag{2.0.3}$$

The input quantity space and the price space are dualistically related. The dual factor minimal cost implied by (2.0.2) is,

$$Q(u,p) = r\ u^{\frac{1}{\theta}} \prod_{i=1}^{m} p_i^{\alpha_i/\theta}\ \text{, where } \theta = \sum_{i=1}^{m} \alpha_i \tag{2.0.4}$$

If a producer whose efficiency is under evaluation is inefficient technically, then we have,

$$u_1 \le A \prod_{i=1}^{m} x_i^{\alpha_i}$$

Let us contract inputs radially to the extent that brings equality.

$$u = A \prod_{i=1}^{m} (\lambda_1 x_i)^{\alpha_i} = = \lambda^{\sum \alpha_i} A \prod x_i^{\alpha_i} = \lambda_0 = \lambda^{\sum \alpha_i} = \frac{u}{A \prod_{i=1}^{m} x_i^{\alpha_i}}$$

$0 \le \lambda_0 \le 1$ and λ_0 measures pure technical efficiency

$$\lambda = \left(\frac{u}{A \prod_{i=1}^{m} x_i^{\alpha_i}} \right)^{1/\theta}$$ $, 0 \le \lambda_1 \le 1$ and λ measure overall technical efficiency

$$\lambda_c = \frac{Q(u_0, p)}{p\, x_0} = \frac{r\ u^{\frac{1}{\theta}} \prod x_{i0}^{\alpha_i/\theta}}{\sum_{i=1}^{m} p_i\, x_{i0}}$$ and $0 \le \lambda_c \le 1$.Where λ_c measures overall productive efficiency of the decision making unit.

In parametric approach of efficiency measurement it is mandatory to specify a production frontier a priori and estimate its parameters. If the specification fails to represent reality, the specification error emanates to play its role. Consequently, the efficiency estimates will be inept to describe reality. A subsection

Some text.

2.1 Estimation of Cobb-Douglas Production Frontier

Timmer's [2] proposed a deterministic approach to estimate the Cobb-Douglas production frontier. The method minimizes the average value of the slacks. For j[th] producer we have, Mean value of all the slacks is

$$\bar{\in} = \sum_{j=1}^{n} \in_j / n$$

The parameters are estimated by the linear programming approach.

$$Min\ \bar{\in}$$

$$subject\ to$$

$$\ln A + \sum_{i=1}^{n} \alpha_i \ln x_{ij} - \in_j = \ln u_j \tag{2.1.1}$$

j=1, 2…… n and $0 \le \lambda_i \le 1$.Where ln A is unrestricted for sign.

The above problem may be rewritten as,

$$\underset{(\alpha_0,\alpha)}{Min} \quad \alpha_0 + \sum_{i=1}^{n} \alpha_i \ \ln \ \overline{x_i}$$

subject to

$$\alpha_0 \ + \sum_{i=1}^{n} \alpha_i \ \ln \ x_{ij} \ \geq \ \ln \ u_j \tag{2.1.2}$$

j=1,2,.....,n and $0 \le \lambda_i \le 1$, *i=1,2,....,m.* where α_0 is unrestricted for sign.

The linear programming problem (2.1.1) and (2.1.2) are one and the same. They imply the same estimates of the parameters. A number of deficiencies follow:

The LP estimates cannot be tested for statistical significance. Specification bias of unknown magnitude is a possibility. As the optimization problem is a linear programming problem with n constraints and (m+1) parameters, the optimal solution emerges assigning zeroes to some of the output elasticity, implying that changes in these division variables of any magnitude will not bring any change in the output.

2.2 Estimation of timmer's method

Timmer's [2] method can be extended to any production frontier which is linear in parameters. The translog production frontier provides a smooth production frontier, continuous everywhere on the surface, is a generalized production frontier, linear in parameters. It is Taylor's series second order approximation of an arbitrary production function.

$$\ln \ u = \alpha_0 + \sum_{i=1}^{m} \alpha_i \ln x_i \ + \frac{1}{2} \sum_{i=1}^{m} \sum_{k=1}^{m} \alpha_{ij} \left(\ln \ x_i\right)\left(\ln \ x_k\right) \tag{2.2.1}$$

Since α_{ij} are partial derivatives, they are symmetric that is $\alpha_{ij} = \alpha_{ji}$ (2.2.2)

If the translog frontier is linear homogeneous in inputs, we impose the following restrictions on the parameters:

$$\sum_{j=1}^{n} \alpha_{ik} = 0, \sum_{i=1}^{m} \alpha_{ik} = 0, \text{and} \sum_{i=1}^{m} \alpha_0 = 1 \tag{2.2.3}$$

Timmer's [2] slack based efficiency can be measured solving the following linear programming problem:

$$\underset{(\alpha_0,\alpha)}{Min} \quad \alpha_0 + \sum_{i=1}^{m} \alpha_i \ \overline{\ln x_i} \ + \ \frac{1}{2} \sum_{i=1}^{m} \sum_{k=1}^{m} \alpha_{ik} \overline{\left(\ln x_i\right)\left(\ln \ x_k\right)}$$

subject to

$$\alpha_0 + \sum_{i=1} \alpha_i \ln x_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{k=1}^{m} \alpha_{ik} \left(\ln x_{ij}\right)\left(\ln \ x_{kj}\right) \geq \ln \ u_j, \ j = 1, 2, ...n$$

α_0 and α_{ik} are unrestricted for sign. $\alpha_i \geq 0$, *i=1,2,.....,m*

$$\sum_{i=1}^{m} \alpha_i = 1, \sum_{i=1}^{m} \alpha_{ik} = 0 \quad , \qquad k = 1, 2, ..., m$$

$$\sum_{k=1}^{m} \alpha_{ik} = 0 \quad , \qquad i = 1,2,.............., m$$

$\alpha_{ik} = \alpha_{ki}$, i=1,2,…..,m and k=1,2,…..,m

The slacks of the optimal solution reveal technical efficiencies. Let $(\hat{\alpha}_0, \hat{\alpha})$ be an optimal solution of (2.2.3)

$$\hat{\alpha}_0 + \sum_{i=1} \hat{\alpha}_i \ln x_{ij} + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_{ik} (\ln x_{ij})(\ln x_{kj}) - \in_j = \ln u_j$$

$$\ln \hat{u}_j - \hat{\in}_j = \ln u_j \, ,$$

$$- \hat{\in}_j = \ln u_j - \ln \hat{u}_j$$

$$- \hat{\in}_j = \ln \frac{u_j}{\hat{u}_j}$$

$$\frac{u_j}{\hat{u}_j} = e^{-\hat{\in}_j} , \quad j = 1,2,......., n$$

$$0 \leq \frac{u_j}{\hat{u}_j} \leq 1 \; , j = 1,2,......., n$$

2.3 Maximum likelihood estimation

Stochastic frontiers [6, 7] provide technical efficiency estimates amenable for statistical tests of significance. They also yield non-zero estimates of output elasticities with respect to the inputs. The inefficient output may be expressed as

$y = f(x, \alpha) + u - v$, where y $\in R_+$ and $x \in R_+^m$ *u* and v are scalar valued random variables. While *u* is a two sided random variable that assumes both positive and negative values, v is a non-negative random variable. α is the vector of frontier parameters. *u* is assumed to follow normal distribution with mean value *0* and variance σ_u^2, u follows N (0, σ_u^2). v is assumed to follow either exponential distribution or the one sided normal distribution. The method of estimation often is the 'maximum likelihood'. The simplest of the stochastic estimation is the method of corrected least squares. The method attempts to estimate the production function model by the method of ordinary least squares. The most negative residual can be reduced zero. This process transforms all the residuals non-negative adjusting the intercept term suitably. The fundamental hypothesis is that technical efficiency differences affect the intercept leaving the slope parameter values intact.

3. Conclusions

In the above research article input technical efficiency measure and the measure of cost efficiency are proposed. The overall productive efficiency of the decision making unit has been obtained in the above discussion. In this paper the extension of Timmer's method to any production frontier which is linear in parameters is presented.

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