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An EOQ model for weibull distribution deterioration with time-dependent cubic demand and backlogging

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Abstract. In this article we introduce an economic order quantity model with weibull deterioration and time dependent cubic demand rate where holding costs as a linear function of time. Shortages are allowed in the inventory system are partially and fully backlogging. The objective of this model is to minimize the total inventory cost by using the optimal order quantity and the cycle length. The proposed model is illustrated by numerical examples and the sensitivity analysis is performed to study the effect of changes in parameters on the optimum solutions.

1. Introduction

It is important to maintain and control the inventories of deteriorating items for the modern industries. Deterioration is defined as decay, dryness, spoilage, obsolescence, damage, pilferage, evaporation. Ghare and Schrader [5] initiated an EOQ model for deterioration taking fixed rate. Covert and Philip [2] derived an inventory system under conditions of constant demand and instantaneous delivery, the distribution of the time to deteriorating items is represented by weibull distribution. Jalan et al. [7] studied an inventory model for deteriorating items with linearly increasing demand rate and shortages. Wu et al. [15] derived an inventory of items that deteriorate at a weibull distribution and demand rate with a continuous function of time. Giri et al. [6] generalized this model with the ramp-type demand and weibull deterioration by taking single-period EOQ model. Dye [3] derived an economic order quantity model for deteriorating items which follows the weibull distribution. Mukhopadhyay et al. [9] formulated the replenishment a policy with demand rate is used price-dependent. Srichandan et al. [11] obtained an inventory system for perishable items under the inflationary conditions and two parameter weibull distributions for deteriorating items.

These models consider that, when there is a shortage, all the customers will be waiting for the arrival of the next replenishment (complete backlogging case) or all customers who are not served leave the system (lost sales case). Commonly, in many real-life situations, there are customers willing to wait for the next replenishment until they satisfy their demands, while others do not want to or cannot wait and leave the system. In such situations, the partial backlogging was used to formulate the model. In this line, there are many researchers contributing numerous papers. Abad [1] studied the optimal pricing problem of the perishability and shortages. Wee [14] developed an inventory model with a quantity discount, pricing and partial backordering. Wu [16] generalized an EOQ model with Weibull distribution deterioration and shortages are allowed partial backlogging. Skouri and Papachristos [10] formulated an inventory model for deteriorating items, time dependent demand, backlogging rate is an exponentially decreasing. Wu [17] derived this model by taking Weibull distribution deterioration and shortages. Dye and Ouyang [4] presented an EOQ model for perishable with goods stock-level dependent selling rate and shortages are considered partial backlogging rate. Yan [18] established an inventory model for perishable goods with freshness-dependent demand rate and partial backlogging. Valliathal and Uthayakumar [13] described an EOQ model with general ramp type demand, deteriorating of the items and backlogging of unfulfilled demand. Karthikeyan and Santhi [8] presented an inventory model for deteriorating items in which shortages are not allowed and demand rate is a cubic function of time and salvage cost.

Recently, Umakanta [12] introduced an EOQ model for weibull deterioration with demand rate as quadratic and partially backlogging. They have not considered the effect of cubic demand, complete backlogging and without shortage. In this paper, we developed an economic order quantity model for weibull deteriorating items and partial backlogging. In addition, the effect of cubic demand, complete backlogging and without shortage are considered. The two parameter weibull distribution is used to represent the time to deterioration. Here in this model, the total inventory cost is minimized. Numerical examples are provided for with and without shortage model and sensitivity analysis is performed to show the effect of changes in the parameters of the optimum solution.

2. Assumptions and notation of the model

2.1 Assumptions

(1) The replenishment rate is infinite and the lead time is zero.

(2) The demand rate is time dependent cubic function, i.e., $D(t)=a+bt+ct^2+dt^3$ a $\geq$ 0, $b\neq 0$, $c\neq 0$, $d\neq 0$.

(3) T is the length of the replenishment cycle, Q is the order quantity per cycle. t_1, t_2 and Q are decision variables.

(4) $I_1(t)$: Inventory level at time $t\left(0\leq t\leq t_1\right)$ in which the product has demand and deterioration.

(5) $I_2(t)$: Inventory level at time $t\left(t_1\leq t\leq t_1+t_2\right)$ in which the product has shortage.

(6) The rate of deterioration $\theta(t)=\alpha\beta t^{\beta-1}$, follows two parameter Weibull distribution, where $\alpha\left(0<\alpha<<1\right)$ is the scale parameter, $\beta>1$ is the shape parameter. The assumed that the deterioration increases with time $t>0$.

(7) The unsatisfied demand is backlogged at a rate $\exp\left(-\delta\left(t_1+t_2-t\right)\right)$, where $\left(t_1+t_2-t\right)$, is the time up to the next replenishment and δ is the backlogging parameter $0\leq\delta\leq 1$.

2.2 Notations

To develop this mathematical model, the following notations are used throughout this paper:

A : Order cost per unit,

C : Purchase cost per unit,

HC : Holding cost per unit time, $H(t)=g+h(t),\ g>0,\ h>0$

C_1 : Shortage cost per unit per unit time,

C_2 : Lost sales per unit,

C_d : Deterioration cost per unit of deteriorated item,

δ : Backlogging parameter is δ which lies in $0\leq\ \delta\leq\ 1$

t_1: Length of time interval with positive or zero inventory level in a replenishment cycle, where $t_1\geq 0$

t_2 : Length of time interval with negative inventory (or shortages) in a cycle, where $t_2\geq 0$

Q : Order quantity per cycle, i.e. $Q = Q_0 + BI$
T : Length of the inventory cycle,
$I(t)$: Inventory level at time t,
$I_1(t)$: Inventory level at time t, $0 \le t \le t_1$
$I_2(t)$: Inventory level at time t, $t_1 \le t \le t_1 + t_2$
Q_0 : The maximum inventory level,
BI : The maximum inventory level during the stockout period,
TC : Total relevant inventory cost,

3. Model formulation

3.1 The model with partial backlogging

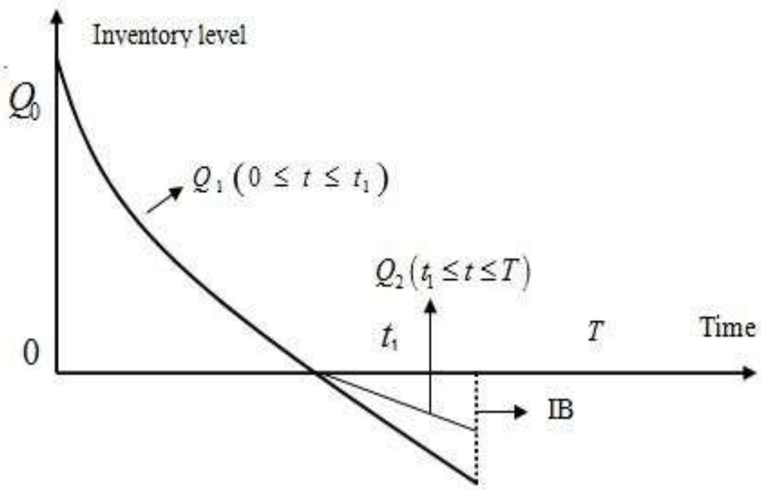

Figure 1. The graphical presentation of inventory level

The instantaneous inventory level $I(t)$ at time $t\ (0 \le t \le t_1 + t_2)$ can be modelled by the following differential equations. During the interval $[0,\ t_1]$, the inventory is depleted due to the combined effects of demand as well as the deterioration and Shortages occur during the period $[\ t_1,\ t_1 + t_2]$. The behavior of inventory in a cycle is depicted in Figure 1. Hence, the differential equation below represents the inventory status is given by

$$\frac{dI_1(t)}{dt} + \alpha\beta t^{\beta-1} I_1(t) = -\left(a + bt + ct^2 + dt^3\right),\ 0 \le t \le t_1 \tag{1}$$

With boundary conditions $I_1(0) = Q_0$ and $I_1(t_1) = 0$,
The solution of the equation (1) is

$$I_1(t) = \begin{bmatrix} a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{d}{4}(t_1^4 - t^4) + \frac{a\alpha t_1^{\beta+1}}{\beta+1} - \frac{a\alpha t^{\beta+1}}{\beta+1} \\ + \frac{b\alpha t_1^{\beta+2}}{\beta+2} - \frac{b\alpha t^{\beta+2}}{\beta+2} + \frac{c\alpha t_1^{\beta+3}}{\beta+3} - \frac{c\alpha t^{\beta+3}}{\beta+3} + \frac{d\alpha t_1^{\beta+4}}{\beta+4} - \frac{d\alpha t^{\beta+4}}{\beta+4} - a\alpha t_1 t^{\beta} \\ + a\alpha t^{\beta+1} - \frac{b\alpha t_1^2 t^{\beta}}{2} + \frac{b\alpha t^{\beta+2}}{2} - \frac{c\alpha t_1^3 t^{\beta}}{3} + \frac{c\alpha t^{\beta+3}}{3} - \frac{d\alpha t_1^4 t^{\beta}}{4} + \frac{d\alpha t^{\beta+4}}{4} \end{bmatrix},\ 0 \le t \le t_1 \tag{2}$$

We get the maximum inventory level is

$$Q_0 = \left[at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{dt_1^4}{4} + \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} + \frac{c\alpha t_1^{\beta+3}}{\beta+3} + \frac{d\alpha t_1^{\beta+4}}{\beta+4} \right] \tag{3}$$

Due to shortage during $[t_1,\, t_1+t_2]$, the demand at time $'t'$ is partially backlogged at rate $e^{-\delta(t_1+t_2-t)}$. The differential equation governing the amount of demand is backlogged.

$$\frac{dI_2(t)}{dt} = -(a+bt+ct^2+dt^3)\ e^{-\delta(t_1+t_2-t)} \qquad t_1 \le t \le t_1+t_2 \tag{4}$$

with boundary condition $I_2(t_1)=0$, the solution of Equation (4) is

$$I_2(t) = \begin{bmatrix} a(t_1-t) + \frac{b}{2}(t_1^2-t^2) + \frac{c}{3}(t_1^3-t^3) + \frac{d}{4}(t_1^4-t^4) \\ -a\delta\left\{ t_1(t_1-t) + t_2(t_1-t) - \frac{1}{2}(t_1^2-t^2) \right\} \\ -b\delta\left\{ \frac{1}{2}t_1(t_1^2-t^2) + \frac{1}{2}t_2(t_1^2-t^2) - \frac{1}{3}(t_1^3-t^3) \right\} \\ -c\delta\left\{ \frac{1}{3}t_1(t_1^3-t^3) + \frac{1}{3}t_2(t_1^3-t^3) - \frac{1}{4}(t_1^4-t^4) \right\} \\ -d\delta\left\{ \frac{1}{4}t_1(t_1^4-t^4) + \frac{1}{4}t_2(t_1^4-t^4) - \frac{1}{5}(t_1^5-t^5) \right\} \end{bmatrix} \quad t_1 \le t \le t_1+t_2 \tag{5}$$

The maximum backlogged inventory BI is obtained at $t=t_1+t_2$ then from equation (5)

$$BI = -I_2(t_1+t_2) = \begin{bmatrix} at_2 + b\left(t_1t_2 + \frac{t_2^2}{2} \right) + c\left(t_1^2t_2 + t_1t_2^2 + \frac{t_2^3}{3} \right) \\ +d\left(t_1^3t_2 + t_1t_2^3 + \frac{3t_1^2t_2^2}{2} + \frac{t_2^4}{4} \right) - a\delta\left\{ \frac{t_2^2}{2} \right\} - b\delta\left\{ \frac{t_1t_2^2}{2} + \frac{t_2^3}{6} \right\} \\ -c\delta\left\{ \frac{t_1^2t_2^2}{2} + \frac{t_1t_2^3}{3} + \frac{t_2^4}{12} \right\} - d\delta\left\{ \frac{t_1^2t_2^3}{2} + \frac{t_1^3t_2^2}{2} + \frac{t_1t_2^4}{4} + \frac{t_2^5}{20} \right\} \end{bmatrix} \tag{6}$$

From equations (3) and (6), we have

$$Q = Q_0 + BI$$

$$Q = \begin{bmatrix} at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{dt_1^4}{4} + \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} + \frac{c\alpha t_1^{\beta+3}}{\beta+3} + \frac{d\alpha t_1^{\beta+4}}{\beta+4} + at_2 + \\ b\left(t_1t_2 + \frac{t_2^2}{2} \right) + c\left(t_1^2t_2 + t_1t_2^2 + \frac{t_2^3}{3} \right) + d\left(t_1^3t_2 + t_1t_2^3 + \frac{3t_1^2t_2^2}{2} + \frac{t_2^4}{4} \right) - a\delta\left\{ \frac{t_2^2}{2} \right\} \\ -b\delta\left\{ \frac{t_1t_2^2}{2} + \frac{t_2^3}{6} \right\} - c\delta\left\{ \frac{t_1^2t_2^2}{2} + \frac{t_1t_2^3}{3} + \frac{t_2^4}{12} \right\} - d\delta\left\{ \frac{t_1^2t_2^3}{2} + \frac{t_1^3t_2^2}{2} + \frac{t_1t_2^4}{4} + \frac{t_2^5}{20} \right\} \end{bmatrix} \tag{7}$$

Cost components:

The cost components per cycle of the total inventory cost are follows

Ordering cost $(OC) = A$

Holding cost $(HC)=\int_0^{t_1}(g+ht)\,I_1(t)\,dt$

$$HC=\left[\begin{array}{l}\frac{agt_1^2}{2}+\frac{bgt_1^3}{3}+\frac{cgt_1^4}{4}+\frac{dgt_1^5}{5}+\frac{ag\alpha t_1^{\beta+2}}{(\beta+2)}+\frac{bg\alpha t_1^{\beta+3}}{(\beta+3)}+\frac{cg\alpha t_1^{\beta+4}}{(\beta+4)}\\+\frac{dg\alpha t_1^{\beta+5}}{(\beta+5)}-\frac{ag\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{bg\alpha t_1^{\beta+3}}{(\beta+1)(\beta+3)}-\frac{cg\alpha t_1^{\beta+4}}{(\beta+1)(\beta+4)}-\frac{dg\alpha t_1^{\beta+5}}{(\beta+1)(\beta+5)}\\+\frac{aht_1^3}{6}+\frac{bht_1^4}{8}+\frac{cht_1^5}{10}+\frac{dht_1^6}{12}+\frac{a\alpha ht_1^{\beta+3}}{2(\beta+3)}+\frac{b\alpha ht_1^{\beta+4}}{2(\beta+4)}+\frac{c\alpha ht_1^{\beta+5}}{2(\beta+5)}\\+\frac{d\alpha ht_1^{\beta+6}}{2(\beta+6)}-\frac{a\alpha ht_1^{\beta+3}}{(\beta+2)(\beta+3)}\frac{b\alpha ht_1^{\beta+4}}{(\beta+2)(\beta+4)}+\frac{c\alpha ht_1^{\beta+5}}{(\beta+2)(\beta+5)}-\frac{d\alpha ht_1^{\beta+6}}{(\beta+2)(\beta+6)}\end{array}\right] \tag{8}$$

The cost of deterioration per cycle (DC) is

$$DC=C_d\left[Q_0-\int_0^{t_1}\left(a+bt+ct^2+dt^3\right)dt\right]$$

$$DC=C_d\left[\frac{a\alpha t_1^{\beta+1}}{\beta+1}+\frac{b\alpha t_1^{\beta+2}}{\beta+2}+\frac{c\alpha t_1^{\beta+3}}{\beta+3}+\frac{d\alpha t_1^{\beta+4}}{\beta+4}\right] \tag{9}$$

The cost of shortage per cycle due to backlogged (SC) is

$$SC=-C_1\int_{t_1}^{t_1+t_2}I_2(t)\,dt\,,\qquad t_1\le t\le t_1+t_2$$

$$=C_1\left[\begin{array}{l}a\left(\frac{t_2^2}{2}\right)+b\left(\frac{t_1t_2^2}{2}+\frac{t_2^3}{6}\right)+c\left(\frac{t_1^2t_2^2}{2}+\frac{t_1t_2^3}{3}+\frac{t_2^4}{12}\right)+\\d\left(\frac{t_1^3t_2^2}{2}+\frac{t_1^2t_2^3}{2}+\frac{t_1t_2^4}{4}+\frac{t_2^5}{20}\right)-a\delta\left(\frac{t_2^3}{3}\right)-b\delta\left(\frac{t_1t_2^3}{3}+\frac{t_2^4}{12}\right)\\-c\delta\left(\frac{t_1^2t_2^3}{3}+\frac{t_1t_2^4}{6}+\frac{t_2^5}{30}\right)-d\delta\left(\frac{t_1^3t_2^3}{3}+\frac{t_1^2t_2^4}{4}+\frac{t_1t_2^5}{10}+\frac{t_2^6}{60}\right)\end{array}\right] \tag{10}$$

The cost due to lost sales (LS) is given by

$$LS=C_2\int_{t_1}^{t_1+t_2}\left(a+bt+ct^2+dt^3\right)\left[1-e^{-\delta(t_1+t_2-t)}\right]dt\quad t_1\le t\le t_1+t_2$$

$$LS=C_2\left[a\delta t_1t_2+\frac{a\delta t_2^2}{2}+\frac{b\delta t_1t_2^2}{2}+\frac{b\delta t_2^3}{6}+\frac{c\delta t_1t_2^3}{3}+\frac{c\delta t_2^4}{12}+\frac{d\delta t_1t_2^4}{4}+\frac{d\delta t_2^5}{20}\right] \tag{11}$$

The purchase cost per cycle (PC) is given by $PC=CQ$

$$PC=C\left[\begin{array}{l}at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}+\frac{dt_1^4}{4}+\frac{a\alpha t_1^{\beta+1}}{\beta+1}+\frac{b\alpha t_1^{\beta+2}}{\beta+2}+\frac{c\alpha t_1^{\beta+3}}{\beta+3}+\frac{d\alpha t_1^{\beta+4}}{\beta+4}+at_2+\\b\left(t_1t_2+\frac{t_2^2}{2}\right)+c\left(t_1^2t_2+t_1t_2^2+\frac{t_2^3}{3}\right)+d\left(t_1^3t_2+t_1t_2^3+\frac{3t_1^2t_2^2}{2}+\frac{t_2^4}{4}\right)-a\delta\left\{\frac{t_2^2}{2}\right\}\\-b\delta\left\{\frac{t_1t_2^2}{2}+\frac{t_2^3}{6}\right\}-c\delta\left\{\frac{t_1^2t_2^2}{2}+\frac{t_1t_2^3}{3}+\frac{t_2^4}{12}\right\}-d\delta\left\{\frac{t_1^2t_2^3}{2}+\frac{t_1^3t_2^2}{2}+\frac{t_1t_2^4}{4}+\frac{t_2^5}{20}\right\}\end{array}\right] \tag{12}$$

The total annual cost per unit time is given by

$$TC = \frac{1}{T}\left[OC + HC + DC + SC + LS + PC\right]$$

$$TC = \frac{1}{t_1 + t_2}\left[\begin{array}{l}
A + \left[\begin{array}{l}
\dfrac{agt_1^{\ 2}}{2} + \dfrac{bgt_1^{\ 3}}{3} + \dfrac{cgt_1^{\ 4}}{4} + \dfrac{dgt_1^{\ 5}}{5} + \dfrac{ag\alpha t_1^{\ \beta+2}}{(\beta+2)} + \dfrac{bg\alpha t_1^{\ \beta+3}}{(\beta+3)} + \dfrac{cg\alpha t_1^{\ \beta+4}}{(\beta+4)} \\
+\dfrac{dg\alpha t_1^{\ \beta+5}}{(\beta+5)} - \dfrac{ag\alpha t_1^{\ \beta+2}}{(\beta+1)(\beta+2)} - \dfrac{bg\alpha t_1^{\ \beta+3}}{(\beta+1)(\beta+3)} - \dfrac{cg\alpha t_1^{\ \beta+4}}{(\beta+1)(\beta+4)} - \dfrac{dg\alpha t_1^{\ \beta+5}}{(\beta+1)(\beta+5)} \\
+\dfrac{aht_1^{\ 3}}{6} + \dfrac{bht_1^{\ 4}}{8} + \dfrac{cht_1^{\ 5}}{10} + \dfrac{dht_1^{\ 6}}{12} + \dfrac{a\alpha ht_1^{\ \beta+3}}{2(\beta+3)} + \dfrac{b\alpha ht_1^{\ \beta+4}}{2(\beta+4)} + \dfrac{c\alpha ht_1^{\ \beta+5}}{2(\beta+5)} \\
+\dfrac{d\alpha ht_1^{\ \beta+6}}{2(\beta+6)} - \dfrac{a\alpha ht_1^{\ \beta+3}}{(\beta+2)(\beta+3)} \dfrac{b\alpha ht_1^{\ \beta+4}}{(\beta+2)(\beta+4)} + \dfrac{c\alpha ht_1^{\ \beta+5}}{(\beta+2)(\beta+5)} - \dfrac{d\alpha ht_1^{\ \beta+6}}{(\beta+2)(\beta+6)}
\end{array}\right] \\
+C_d\left[\dfrac{a\alpha t_1^{\ \beta+1}}{\beta+1} + \dfrac{b\alpha t_1^{\ \beta+2}}{\beta+2} + \dfrac{c\alpha t_1^{\ \beta+3}}{\beta+3} + \dfrac{d\alpha t_1^{\ \beta+4}}{\beta+4}\right] \\
+C_1\left[\begin{array}{l}
a\left(\dfrac{t_2^{\ 2}}{2}\right) + b\left(\dfrac{t_1t_2^{\ 2}}{2} + \dfrac{t_2^{\ 3}}{6}\right) + c\left(\dfrac{t_1^{\ 2}t_2^{\ 2}}{2} + \dfrac{t_1t_2^{\ 3}}{3} + \dfrac{t_2^{\ 4}}{12}\right) + \\
d\left(\dfrac{t_1^{\ 3}t_2^{\ 2}}{2} + \dfrac{t_1^{\ 2}t_2^{\ 3}}{2} + \dfrac{t_1t_2^{\ 4}}{4} + \dfrac{t_2^{\ 5}}{20}\right) - a\delta\left(\dfrac{t_2^{\ 3}}{3}\right) - b\delta\left(\dfrac{t_1t_2^{\ 3}}{3} + \dfrac{t_2^{\ 4}}{12}\right) \\
-c\delta\left(\dfrac{t_1^{\ 2}t_2^{\ 3}}{3} + \dfrac{t_1t_2^{\ 4}}{6} + \dfrac{t_2^{\ 5}}{30}\right) - d\delta\left(\dfrac{t_1^{\ 3}t_2^{\ 3}}{3} + \dfrac{t_1^{\ 2}t_2^{\ 4}}{4} + \dfrac{t_1t_2^{\ 5}}{10} + \dfrac{t_2^{\ 6}}{60}\right)
\end{array}\right] \\
+C_2\left[a\delta t_1t_2 + \dfrac{a\delta t_2^{\ 2}}{2} + \dfrac{b\delta t_1t_2^{\ 2}}{2} + \dfrac{b\delta t_2^{\ 3}}{6} + \dfrac{c\delta t_1t_2^{\ 3}}{3} + \dfrac{c\delta t_2^{\ 4}}{12} + \dfrac{d\delta t_1t_2^{\ 4}}{4} + \dfrac{d\delta t_2^{\ 5}}{20}\right] \\
+C\left[\begin{array}{l}
at_1 + \dfrac{bt_1^{\ 2}}{2} + \dfrac{ct_1^{\ 3}}{3} + \dfrac{dt_1^{\ 4}}{4} + \dfrac{a\alpha t_1^{\ \beta+1}}{\beta+1} + \dfrac{b\alpha t_1^{\ \beta+2}}{\beta+2} + \dfrac{c\alpha t_1^{\ \beta+3}}{\beta+3} + \dfrac{d\alpha t_1^{\ \beta+4}}{\beta+4} + at_2 + \\
b\left(t_1t_2 + \dfrac{t_2^{\ 2}}{2}\right) + c\left(t_1^{\ 2}t_2 + t_1t_2^{\ 2} + \dfrac{t_2^{\ 3}}{3}\right) + d\left(t_1^{\ 3}t_2 + t_1t_2^{\ 3} + \dfrac{3t_1^{\ 2}t_2^{\ 2}}{2} + \dfrac{t_2^{\ 4}}{4}\right) - a\delta\left\{\dfrac{t_2^{\ 2}}{2}\right\} \\
-b\delta\left\{\dfrac{t_1t_2^{\ 2}}{2} + \dfrac{t_2^{\ 3}}{6}\right\} - c\delta\left\{\dfrac{t_1^{\ 2}t_2^{\ 2}}{2} + \dfrac{t_1t_2^{\ 3}}{3} + \dfrac{t_2^{\ 4}}{12}\right\} - d\delta\left\{\dfrac{t_1^{\ 2}t_2^{\ 3}}{2} + \dfrac{t_1^{\ 3}t_2^{\ 2}}{2} + \dfrac{t_1t_2^{\ 4}}{4} + \dfrac{t_2^{\ 5}}{20}\right\}
\end{array}\right]
\end{array}\right] \tag{13}$$

The conditions for minimization of the total cost (TC) per unit time are

$$\frac{\partial TC}{\partial t_1} = 0 \text{ and } \frac{\partial TC}{\partial t_2} = 0 \tag{14}$$

Providing that the optimal values for t_1 and t_2 obtained from equation (14) satisfies the sufficient condition

$$\frac{\partial^2 TC}{\partial t_1^{\ 2}} \times \frac{\partial^2 TC}{\partial t_2^{\ 2}} - \left(\frac{\partial^2 TC}{\partial t_1 \partial t_2}\right)^2 > 0$$

3.2. The special case of completely backlogging

Due to shortage during $[t_1,\ t_1+t_2]$, the demand at time $'t'$ is completely backlogged. Hence, the inventory level can be represented by the following differential equation:

$$\frac{dI_2(t)}{dt} = -(a+bt+ct^2+dt^3)\ , \quad t_1 \le t \le t_1+t_2$$

with the boundary condition $I_2(t_1)=0$

$$I_2(t) = \left[a(t_1-t)+\frac{b}{2}(t_1^{\,2}-t^2)+\frac{c}{3}(t_1^{\,3}-t^3)+\frac{d}{4}(t_1^{\,4}-t^4) \right] \tag{15}$$

The maximum backlogged inventory BI is obtained at $t=t_1+t_2$ then from equation (15)

$$BI = -I_2(t_1+t_2) = \left[at_2 + b\left(t_1t_2+\frac{t_2^{\,2}}{2}\right) + c\left(t_1^{\,2}t_2+t_1t_2^{\,2}+\frac{t_2^{\,3}}{3}\right) + d\left(t_1^{\,3}t_2+t_1t_2^{\,3}+\frac{3t_1^{\,2}t_2^{\,2}}{2}+\frac{t_2^{\,4}}{4}\right)\right] \tag{16}$$

Hence the order quantity during $[0,\ t_1+t_2]$

$$Q = Q_0 + BI$$

$$Q = \left[\begin{array}{l} at_1+\dfrac{bt_1^{\,2}}{2}+\dfrac{ct_1^{\,3}}{3}+\dfrac{dt_1^{\,4}}{4}+\dfrac{a\alpha t_1^{\,\beta+1}}{\beta+1}+\dfrac{b\alpha t_1^{\,\beta+2}}{\beta+2}+\dfrac{c\alpha t_1^{\,\beta+3}}{\beta+3}+\dfrac{d\alpha t_1^{\,\beta+4}}{\beta+4}+ \\ \left[at_2 + b\left(t_1t_2+\dfrac{t_2^{\,2}}{2}\right) + c\left(t_1^{\,2}t_2+t_1t_2^{\,2}+\dfrac{t_2^{\,3}}{3}\right) + d\left(t_1^{\,3}t_2+t_1t_2^{\,3}+\dfrac{3t_1^{\,2}t_2^{\,2}}{2}+\dfrac{t_2^{\,4}}{4}\right)\right] \end{array} \right] \tag{17}$$

The total shortage cost during interval $[t_1,\ t_1+t_2]$ is

$$SC = -C_1 \int_{t_1}^{t_1+t_2} I_2(t)\ dt\ , \quad t_1 \le t \le t_1+t_2$$

$$= C_1\left[a\left(\frac{t_2^{\,2}}{2}\right) + b\left(\frac{t_1t_2^{\,2}}{2}+\frac{t_2^{\,3}}{6}\right) + c\left(\frac{t_1^{\,2}t_2^{\,2}}{2}+\frac{t_1t_2^{\,3}}{3}+\frac{t_2^{\,4}}{12}\right) + d\left(\frac{t_1^{\,3}t_2^{\,2}}{2}+\frac{t_1^{\,2}t_2^{\,3}}{2}+\frac{t_1t_2^{\,4}}{4}+\frac{t_2^{\,5}}{20}\right)\right] \tag{18}$$

Purchase cost per cycle $(PC)=CQ$

$$PC = CQ = \left[\begin{array}{l} a(t_2+t_1)+\dfrac{b}{2}(t_2^{\,2}+t_1^{\,2})+\dfrac{c}{3}(t_2^{\,3}+t_1^{\,3})+\dfrac{d}{4}(t_2^{\,4}+t_1^{\,4})+\dfrac{a\alpha t_1^{\,\beta+1}}{\beta+1} \\ -\dfrac{a\alpha t_d^{\,\beta+1}}{\beta+1}+\dfrac{b\alpha t_1^{\,\beta+2}}{\beta+2}-\dfrac{b\alpha t_d^{\,\beta+2}}{\beta+2}+\dfrac{c\alpha t_1^{\,\beta+3}}{\beta+3}-\dfrac{c\alpha t_d^{\,\beta+3}}{\beta+3} \\ +\dfrac{d\alpha t_1^{\,\beta+4}}{\beta+4}-\dfrac{d\alpha t_d^{\,\beta+4}}{\beta+4}-a\alpha t_1t_d^{\,\beta}+a\alpha t_d^{\,\beta+1}-\dfrac{b\alpha t_1^{\,2}t_d^{\,\beta}}{2} \\ +\dfrac{b\alpha t_d^{\,\beta+2}}{2}-\dfrac{c\alpha t_1^{\,3}t_d^{\,\beta}}{3}+\dfrac{c\alpha t_d^{\,\beta+3}}{3}-\dfrac{d\alpha t_1^{\,4}t_d^{\,\beta}}{4}+\dfrac{d\alpha t_d^{\,\beta+4}}{4} \end{array} \right] \tag{19}$$

Hence, the total relevant cost per unit time is given by

$$TC = \frac{1}{T}\left[OC+HC+DC+SC+PC \right]$$

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$$TC=\frac{1}{t_1+t_2}\left[\begin{array}{l} A+\left[\begin{array}{l}\frac{agt_1^{2}}{2}+\frac{bgt_1^{3}}{3}+\frac{cgt_1^{4}}{4}+\frac{dgt_1^{5}}{5}+\frac{ag\alpha t_1^{\beta+2}}{(\beta+2)}+\frac{bg\alpha t_1^{\beta+3}}{(\beta+3)}+\frac{cg\alpha t_1^{\beta+4}}{(\beta+4)}\\ +\frac{dg\alpha t_1^{\beta+5}}{(\beta+5)}-\frac{ag\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{bg\alpha t_1^{\beta+3}}{(\beta+1)(\beta+3)}-\frac{cg\alpha t_1^{\beta+4}}{(\beta+1)(\beta+4)}-\frac{dg\alpha t_1^{\beta+5}}{(\beta+1)(\beta+5)}\\ +\frac{aht_1^{3}}{6}+\frac{bht_1^{4}}{8}+\frac{cht_1^{5}}{10}+\frac{dht_1^{6}}{12}+\frac{a\alpha ht_1^{\beta+3}}{2(\beta+3)}+\frac{b\alpha ht_1^{\beta+4}}{2(\beta+4)}+\frac{c\alpha ht_1^{\beta+5}}{2(\beta+5)}\\ +\frac{d\alpha ht_1^{\beta+6}}{2(\beta+6)}-\frac{a\alpha ht_1^{\beta+3}}{(\beta+2)(\beta+3)}\frac{b\alpha ht_1^{\beta+4}}{(\beta+2)(\beta+4)}+\frac{c\alpha ht_1^{\beta+5}}{(\beta+2)(\beta+5)}-\frac{d\alpha ht_1^{\beta+6}}{(\beta+2)(\beta+6)}\end{array}\right]\\ +C_d\left[\frac{a\alpha t_1^{\beta+1}}{\beta+1}+\frac{b\alpha t_1^{\beta+2}}{\beta+2}+\frac{c\alpha t_1^{\beta+3}}{\beta+3}+\frac{d\alpha t_1^{\beta+4}}{\beta+4}\right]\\ +C_1\left[\begin{array}{l}a\left(\frac{t_2^{2}}{2}\right)+b\left(\frac{t_1t_2^{2}}{2}+\frac{t_2^{3}}{6}\right)+c\left(\frac{t_1^{2}t_2^{2}}{2}+\frac{t_1t_2^{3}}{3}+\frac{t_2^{4}}{12}\right)+\\ d\left(\frac{t_1^{3}t_2^{2}}{2}+\frac{t_1^{2}t_2^{3}}{2}+\frac{t_1t_2^{4}}{4}+\frac{t_2^{5}}{20}\right)\end{array}\right]\\ +C\left[\begin{array}{l}at_1+\frac{bt_1^{2}}{2}+\frac{ct_1^{3}}{3}+\frac{dt_1^{4}}{4}+\frac{a\alpha t_1^{\beta+1}}{\beta+1}+\frac{b\alpha t_1^{\beta+2}}{\beta+2}+\frac{c\alpha t_1^{\beta+3}}{\beta+3}+\frac{d\alpha t_1^{\beta+4}}{\beta+4}+at_2+\\ b\left(t_1t_2+\frac{t_2^{2}}{2}\right)+c\left(t_1^{2}t_2+t_1t_2^{2}+\frac{t_2^{3}}{3}\right)+d\left(t_1^{3}t_2+t_1t_2^{3}+\frac{3t_1^{2}t_2^{2}}{2}+\frac{t_2^{4}}{4}\right)\end{array}\right]\end{array}\right] \quad (20)$$

The conditions for minimization of the total cost (TC) per unit time are

$\frac{\partial TC}{\partial t_1}=0$ and $\frac{\partial TC}{\partial t_2}=0$ (21)

Providing that the optimal values for t_1 and t_2 obtained from equation (21) satisfies the sufficient condition

$$\frac{\partial^2 TC}{\partial t_1^{2}}\times\frac{\partial^2 TC}{\partial t_2^{2}}-\left(\frac{\partial^2 TC}{\partial t_1\partial t_2}\right)^2>0$$

3.3. Without shortage model

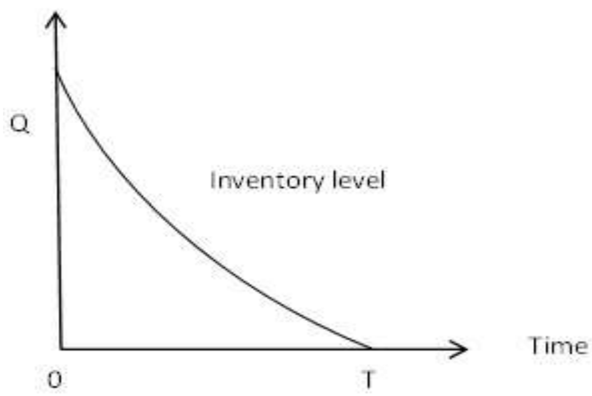

Figure 2. The graphical presentation of inventory level

The decreases of the inventory level occurs due to demand and deterioration during the time $[0,\ T]$. Hence, the differential equation below represents the inventory status is given by

$$\frac{dI_1(t)}{dt}+\theta I_1(t)=-D(t),\ 0\le t\le T$$

Now putting the value of θ in above equation we get,

$$\frac{dI_1(t)}{dt}+\alpha\beta t^{\beta-1}I_1(t)=-\left(a+bt+ct^2+dt^3\right),\ 0\le t\le T \tag{22}$$

with boundary conditions $I_1(0)=Q$ and $I_1(T)=0$,

The solution of the equation (22) is

$$I_1(t)=\left[\begin{array}{l} a(T-t)+\frac{b}{2}(T^2-t^2)+\frac{c}{3}(T^3-t^3)+\frac{d}{4}(T^4-t^4)+\frac{a\alpha T^{\beta+1}}{\beta+1}-\frac{a\alpha t^{\beta+1}}{\beta+1} \\ +\frac{b\alpha T^{\beta+2}}{\beta+2}-\frac{b\alpha t^{\beta+2}}{\beta+2}+\frac{c\alpha T^{\beta+3}}{\beta+3}-\frac{c\alpha t^{\beta+3}}{\beta+3}+\frac{d\alpha T^{\beta+4}}{\beta+4}-\frac{d\alpha t^{\beta+4}}{\beta+4}-a\alpha Tt^{\beta} \\ +a\alpha t^{\beta+1}-\frac{b\alpha T^2t^{\beta}}{2}+\frac{b\alpha t^{\beta+2}}{2}-\frac{c\alpha T^3t^{\beta}}{3}+\frac{c\alpha t^{\beta+3}}{3}-\frac{d\alpha T^4t^{\beta}}{4}+\frac{d\alpha t^{\beta+4}}{4} \end{array}\right],\ 0\le t\le T \tag{23}$$

We get the optimum order quantity is given by

$$Q=\left[aT+\frac{bT^2}{2}+\frac{cT^3}{3}+\frac{dT^4}{4}+\frac{a\alpha T^{\beta+1}}{\beta+1}+\frac{b\alpha T^{\beta+2}}{\beta+2}+\frac{c\alpha T^{\beta+3}}{\beta+3}+\frac{d\alpha T^{\beta+4}}{\beta+4}\right] \tag{24}$$

Cost components

The cost components per cycle of the total inventory cost are follows

Ordering cost $(OC)=A$

Holding cost per cycle $(HC)=\int_0^T (g+ht)\ I_1(t)\ dt$

$$HC=\left[\begin{array}{l} \frac{agT^2}{2}+\frac{bgT^3}{3}+\frac{cgT^4}{4}+\frac{dgT^5}{5}+\frac{ag\alpha T^{\beta+2}}{(\beta+2)}+\frac{bg\alpha T^{\beta+3}}{(\beta+3)}+\frac{cg\alpha T^{\beta+4}}{(\beta+4)} \\ +\frac{dg\alpha T^{\beta+5}}{(\beta+5)}-\frac{ag\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{bg\alpha T^{\beta+3}}{(\beta+1)(\beta+3)}-\frac{cg\alpha T^{\beta+4}}{(\beta+1)(\beta+4)}-\frac{dg\alpha T^{\beta+5}}{(\beta+1)(\beta+5)} \\ +\frac{ahT^3}{6}+\frac{bhT^4}{8}+\frac{chT^5}{10}+\frac{dhT^6}{12}+\frac{\alpha hT^{\beta+3}}{2(\beta+3)}+\frac{b\alpha hT^{\beta+4}}{2(\beta+4)}+\frac{c\alpha hT^{\beta+5}}{2(\beta+5)} \\ +\frac{d\alpha hT^{\beta+6}}{2(\beta+6)}-\frac{a\alpha hT^{\beta+3}}{(\beta+2)(\beta+3)}\frac{b\alpha hT^{\beta+4}}{(\beta+2)(\beta+4)}+\frac{c\alpha hT^{\beta+5}}{(\beta+2)(\beta+5)}-\frac{d\alpha hT^{\beta+6}}{(\beta+2)(\beta+6)} \end{array}\right] \tag{25}$$

The cost of deterioration per cycle (DC) is

$$DC=C_d\left[Q-\int_0^T\left(a+bt+ct^2+dt^3\right)dt\right]$$

$$DC=C_d\left[\frac{a\alpha T^{\beta+1}}{\beta+1}+\frac{b\alpha T^{\beta+2}}{\beta+2}+\frac{c\alpha T^{\beta+3}}{\beta+3}+\frac{d\alpha T^{\beta+4}}{\beta+4}\right] \tag{26}$$

The purchase cost per cycle (PC) is

$PC=CQ$

$$PC = C\left[aT + \frac{bT^2}{2} + \frac{cT^3}{3} + \frac{dT^4}{4} + \frac{a\alpha T^{\beta+1}}{\beta+1} + \frac{b\alpha T^{\beta+2}}{\beta+2} + \frac{c\alpha T^{\beta+3}}{\beta+3} + \frac{d\alpha T^{\beta+4}}{\beta+4} \right] \quad (27)$$

$$TC = \frac{1}{T}\left[OC + HC + DC + PC \right]$$

$$TC = \frac{1}{T}\left[\begin{array}{l} A + HC\left[\begin{array}{l} \frac{agT^2}{2} + \frac{bgT^3}{3} + \frac{cgT^4}{4} + \frac{dgT^5}{5} + \frac{ag\alpha T^{\beta+2}}{(\beta+2)} + \frac{bg\alpha T^{\beta+3}}{(\beta+3)} + \frac{cg\alpha T^{\beta+4}}{(\beta+4)} \\ + \frac{dg\alpha T^{\beta+5}}{(\beta+5)} - \frac{ag\alpha T^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{bg\alpha T^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{cg\alpha T^{\beta+4}}{(\beta+1)(\beta+4)} - \frac{dg\alpha T^{\beta+5}}{(\beta+1)(\beta+5)} \\ + \frac{ahT^3}{6} + \frac{bhT^4}{8} + \frac{chT^5}{10} + \frac{dhT^6}{12} + \frac{a\alpha hT^{\beta+3}}{2(\beta+3)} + \frac{b\alpha hT^{\beta+4}}{2(\beta+4)} + \frac{c\alpha hT^{\beta+5}}{2(\beta+5)} \\ + \frac{d\alpha hT^{\beta+6}}{2(\beta+6)} - \frac{a\alpha hT^{\beta+3}}{(\beta+2)(\beta+3)} \frac{b\alpha hT^{\beta+4}}{(\beta+2)(\beta+4)} + \frac{c\alpha hT^{\beta+5}}{(\beta+2)(\beta+5)} - \frac{d\alpha hT^{\beta+6}}{(\beta+2)(\beta+6)} \end{array}\right] \\ + C_d\left[\frac{a\alpha T^{\beta+1}}{\beta+1} + \frac{b\alpha T^{\beta+2}}{\beta+2} + \frac{c\alpha T^{\beta+3}}{\beta+3} + \frac{d\alpha T^{\beta+4}}{\beta+4} \right] \\ + C\left[aT + \frac{bT^2}{2} + \frac{cT^3}{3} + \frac{dT^4}{4} + \frac{a\alpha T^{\beta+1}}{\beta+1} + \frac{b\alpha T^{\beta+2}}{\beta+2} + \frac{c\alpha T^{\beta+3}}{\beta+3} + \frac{d\alpha T^{\beta+4}}{\beta+4} \right] \end{array}\right] \quad (28)$$

The condition for minimization of the total cost (TC) per unit time is

$$\frac{dTC}{dT} = 0 \quad (29)$$

The condition is also satisfied for the value T from equation (29)

$$\frac{d^2TC}{dT^2} > 0 \text{ for all } T > 0$$

4. Numerical illustrations

Example 1.Partially backlogging model

Let $A = 500,\ a = 5,\ b = 9,\ c = 15,\ d = 20,\ g = 0.9,\ h = 0.4,\ \alpha = 0.5,\ \beta = 2,\ C_1 = 0.8,\ C_2 = 0.5,$ $C = 0.6,\ C_d = 0.2,\ \delta = 0.5.$ We get the optimal values are $t_1 = 0.0532$, $t_2 = 1.4538,\ Q = 63.4161$ and $TC = 385.6440.$

Example 2. Completely backlogging model

Let $A = 500,\ a = 5,\ b = 9,\ c = 15,\ d = 20,\ g = 0.9,\ h = 0.4,\ \alpha = 0.5,\ \beta = 2,\ C_1 = 0.8,\ C_2 = 0.5,$ $C = 0.6,\ C_d = 0.2.$ We get the optimal values are $t_1 = 0.3283,\ t_2 = 1.8413,\ Q = 211.5212$ and $TC = 337.0816.$

Example 3. Without shortage model

Let $A=500,\ a=5,\ b=9,\ c=15,\ d=20,\ g=0.9,\ h=0.4,\ \alpha=0.5,\ \beta=2,\ C=0.6,\ C_d=0.2.$
The optimal solutions are $T=1.6213,\ Q=155.1932$ and $TC=469.3064.$

5. Sensitivity analysis

This section, we now study the effect of changes in the major parameters such as $A,\ a,\ b,\ c,\ d,\ \alpha,\ \beta,\ h,$ g, $C_1,\ C_2,\ C_d,\ C$ *and* δ on $T,\ Q,$ and TC in the EOQ model of the Examples 1, 2 and 3. We change one parameter at a time and keep the other parameter remains unchanged to examine the sensitivity analysis. The results are summarized in Tables 1, 2 and 3 respectively.

Table 1. Sensitivity analysis of the partially backlogging model

Parameters		Optimum	values		
		t_1	t_2	Q	TC
	500	0.0532	1.4538	63.4161	385.6440
A	600	0.0533	1.4823	69.7639	462.0844
	700	0.0534	1.5030	73.5036	542.4440
	800	0.0535	1.5356	76.5736	626.7559
	3	0.0645	1.4436	60.2932	384.1750
a	5	0.0532	1.4538	63.4161	385.6440
	7	0.0455	1.4638	66.7078	386.1143
	9	0.0384	1.4739	70.2081	387.5401
	7	0.0540	1.4140	61.5606	381.7034
b	8	0.0536	1.4339	62.4875	383.1732
	9	0.0532	1.4538	63.4161	385.6440
	10	0.0531	1.4735	64.3563	390.0970
	12	0.0540	1.4249	63.3397	377.8584
c	13	0.0538	1.4349	63.3937	380.3527
	14	0.0535	1.4445	63.4095	382.9589
	15	0.0532	1.4538	63.4161	385.6440
	18	0.0596	1.4370	57.9141	362.4546
d	19	0.0562	1.4418	60.6876	378.7082
	20	0.0532	1.4538	63.4161	385.6440
	21	0.0505	1.4634	66.0970	393.1456
	0.7	0.0530	1.5106	80.1677	352.8869
g	0.8	0.0531	1.5009	71.3675	362.9681
	0.9	0.0532	1.4538	63.4161	385.6440
	1.0	0.0534	0.9642	56.5808	451.5986
	0.4	0.0532	1.4538	63.4161	385.6440
h	0.6	0.0533	0.9863	33.2002	528.6654
	0.8	0.0536	0.7489	25.1297	675.0084
	1.0	0.0541	0.3724	18.3196	1255.4
	0.4	0.0537	1.5055	68.2256	376.3233
α	0.5	0.0532	1.4538	63.4161	385.6440

	0.6	0.0531	1.4008	58.8471	396.1553
	0.7	0.0528	1.3460	54.4788	408.1601
β	2	0.0532	1.4538	63.4161	385.6440
	3	0.0507	1.0133	32.5185	514.3264
	4	0.0505	1.0112	31.5797	523.7526
	5	0.0503	1.0100	31.0173	543.3986
C_1	0.6	0.0385	1.4790	65.5919	368.6346
	0.7	0.0457	1.4612	64.7230	375.2839
	0.8	0.0532	1.4538	63.4161	385.6440
	0.9	0.0610	1.4410	62.6937	398.5820
C_2	0.4	0.0526	1.5227	64.4996	384.1346
	0.5	0.0532	1.4538	63.4161	385.6440
	0.6	0.0547	1.4377	62.4115	386.3026
	0.7	0.0559	1.4250	61.2112	387.9645
C_d	0.1	0.0530	1.4759	65.4366	380.5661
	0.2	0.0532	1.4538	63.4161	385.6440
	0.3	0.0535	1.4315	61.4497	390.9426
	0.4	0.0537	1.4089	59.5102	396.5343
C	0.5	0.0517	1.5458	72.1778	365.2568
	0.6	0.0532	1.4538	63.4161	385.6440
	0.7	0.0547	1.3692	56.3104	406.7431
	0.8	0.0562	1.2908	50.4771	428.6863
δ	0.4	0.0414	1.8852	120.6759	333.8138
	0.5	0.0532	1.4538	63.4161	385.6440
	0.6	0.0665	1.1876	43.0946	446.7000
	0.7	0.0824	1.0047	33.9506	506.8823

5.1 Observations:

The following observations are based on the results of Table-1

1) When parameters A increases, t_1, t_2, Q and TC increases.
2) When parameters a, b, c and d are increases, t_1 is decreases while t_2 ,Q and TC increases.
3) When parameters α and β are increases, t_1, t_2 and Q decreases while TC increases.
4) When the value of the model parameter C_1 and C_2 increases, t_1 and TC increases while t_2 and Q decreases.
5) When parameter g, h, C, δ and C_d are increases, t_1 and TC increases while t_2 and Q decreases.

Table 2. Sensitivity analysis of the completely backlogging model

Parameters		Optimum values			
		t_1	t_2	Q	TC
	500	0.3283	1.8413	211.5212	337.0816

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A	600	0.3609	1.9041	242.2979	382.1038
	700	0.3938	1.9552	272.4117	425.4143
	800	0.4273	1.9970	301.9924	467.3100
	3	0.3300	1.7321	206.6395	334.2240
a	5	0.3283	1.8413	211.5212	337.0816
	7	0.3141	1.8833	216.3196	339.8388
	9	0.3020	1.9430	220.8863	342.5250
	7	0.3404	1.8275	209.0469	334.4820
b	8	0.3345	1.8393	210.3056	335.7837
	9	0.3283	1.8413	211.5212	337.0816
	10	0.3121	1.8536	212.8180	338.3760
	12	0.3510	1.8156	208.3298	332.0964
c	13	0.3433	1.8242	209.0882	333.7751
	14	0.3357	1.8328	210.8176	335.4366
	15	0.3283	1.8413	211.5212	337.0816
	18	0.3571	1.8208	209.7261	332.0695
d	19	0.3422	1.8361	210.6193	334.6118
	20	0.3283	1.8413	211.5212	337.0816
	21	0.3154	1.8565	212.4847	339.4837
	0.7	0.2439	1.9278	214.3377	248.3508
g	0.8	0.2763	1.8941	212.5894	260.1756
	0.9	0.3283	1.8413	211.5212	337.0816
	1.0	0.4925	0.8767	210.8569	437.0286
	0.5	0.3283	1.8413	211.5212	337.0816
h	0.7	0.3398	1.8321	211.3563	357.0654
	0.8	0.3421	1.8301	211.2370	365.0516
	1.0	0.3549	1.8293	211.0983	371.0395
	0.4	0.4470	1.9246	213.3963	335.1096
α	0.5	0.3283	1.8413	211.5212	337.0816
	0.6	0.2553	1.8035	210.5574	339.0647
	0.7	0.2087	1.7896	209.9856	341.0621
	2	0.3283	1.8413	211.5212	337.0816
β	3	0.3036	1.6165	211.5204	338.8140
	4	0.2942	1.5359	210.4937	339.3393
	5	0.2798	1.4903	209.5872	340.3501
	0.6	0.2195	2.0193	231.6483	327.0375
C_1	0.7	0.2697	1.9327	220.6760	332.1738
	0.8	0.3283	1.8413	211.5212	337.0816
	0.9	0.3976	1.7426	204.1045	341.8069
	0.1	0.3022	1.8571	212.7540	336.5305

C_d	0.2	0.3283	1.8413	211.5212	337.0816
	0.3	0.3455	1.8351	211.0396	337.7024
	0.4	0.3671	1.8245	210.8573	338.3598
	0.5	0.3191	1.8594	227.1118	327.1102
C	0.6	0.3283	1.8413	211.5212	337.0816
	0.7	0.3328	1.8224	198.1813	346.6142
	0.8	0.3411	1.8032	186.5460	355.7622

5.2 Observations:

The following observations are based on the results of Table-2

1) When the value of the model parameters A increases, t_1, t_2, Q and TC increases.
2) When the value of the model parameters a, b, c and d are increases, t_1 is decreases while t_2, Q and TC increases.
3) When parameters α and β are increases, t_1, t_2 and Q decreases while TC increases.
4) When parameter C_1 increases, t_1 and TC increases while t_2 and Q decreases.
5) When parameter g, h, C and C_d are increases, t_1 and TC increases while t_2 and Q decreases.

Table 3. Sensitivity analysis of the without shortage model

Parameters		Optimum	values	
		T	Q	TC
	500	1.6213	155.1932	469.3064
A	600	1.6895	171.0763	529.6909
	700	1.7491	186.0204	587.8372
	800	1.8022	200.2129	644.1425
	3	1.6262	150.3956	464.3530
a	5	1.6213	155.1932	469.3064
	7	1.6164	159.9458	474.2475
	9	1.6116	164.6766	479.1763
	7	1.6301	151.8576	464.3536
b	8	1.6257	153.5371	466.8350
	9	1.6213	155.1932	469.3064
	10	1.6169	156.8258	471.7678
	12	1.6434	151.7124	460.9469
c	13	1.6359	152.8927	463.7612
	14	1.6285	154.0451	466.5474
	15	1.6213	155.1932	469.3064
	18	1.6456	154.3331	462.6698
d	19	1.6332	154.7617	466.0253
	20	1.6213	155.1932	469.3064
	21	1.6099	155.6354	472.5176

g	0.7	1.5917	166.8083	454.0684
	0.8	1.6155	160.6847	461.8448
	0.9	1.6213	155.1932	469.3064
	1.0	1.6488	150.2264	476.4856
h	0.4	1.6213	155.1932	469.3064
	0.6	1.6572	147.7173	476.4740
	0.8	1.6976	141.4703	483.0756
	1.0	1.7316	136.1641	489.2136
α	0.4	1.6328	159.7197	459.6346
	0.5	1.6213	155.1932	469.3064
	0.6	1.6098	151.3362	478.9100
	0.7	1.5984	149.1772	488.4456
β	2	1.6213	155.1932	469.3064
	3	1.5219	152.1764	485.8965
	4	1.4023	150.2220	505.6042
	5	1.2996	142.1630	522.7303
C_d	0.1	1.6149	156.0002	464.4055
	0.2	1.6213	155.1932	469.3064
	0.3	1.6377	154.3896	474.2005
	0.4	1.6441	153.5894	479.0877
C	0.5	1.6068	158.6924	459.6728
	0.6	1.6213	155.1932	469.3064
	0.7	1.6463	151.8673	478.8196
	0.8	1.6617	148.6865	488.2175

The following observations are based on the results of Table-3

5.3 Observations

1) When the value of the model parameters A increases, T, Q and TC increases.
2) When parameters a, b, c and d are increases, T is decreases while Q and TC increases.
3) When parameters α and β are increases, T and Q decreases while TC increases.
4) When parameter g, h, C and C_d are increases, T and TC increases while Q decreases.

Three dimensional graphs are shown in the following Figures 3, 4 and 5

Figure 3 shows that in partially backlogging model, while increasing the Backlogging rate (δ) we obtain that the Order quantity (Q) decreases and the total cost increases.

Figure 4 shows that in completely backlogging model, while increasing Ordering cost (A) we obtain that the Order quantity (Q) and the total cost are increases.

Figure 5 shows that in without shortage model, while increasing Ordering cost (A) we obtain that the Order quantity (Q) and the total cost are increases.

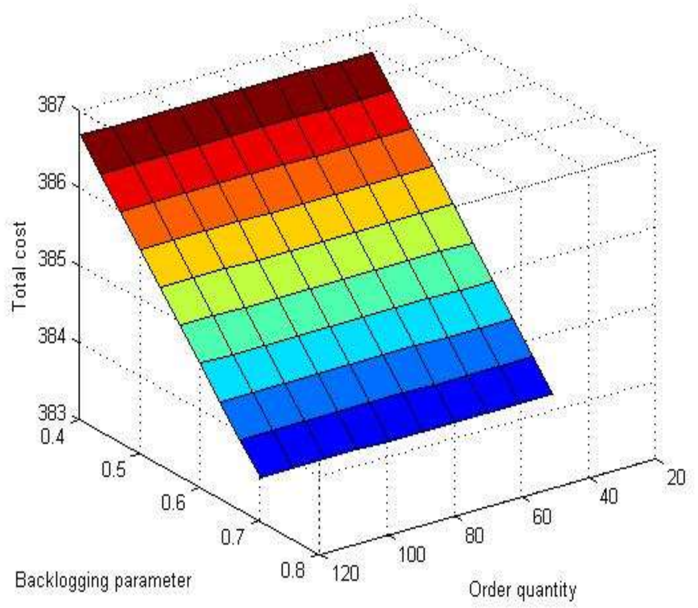

Figure 3. Total cost versus Q and δ

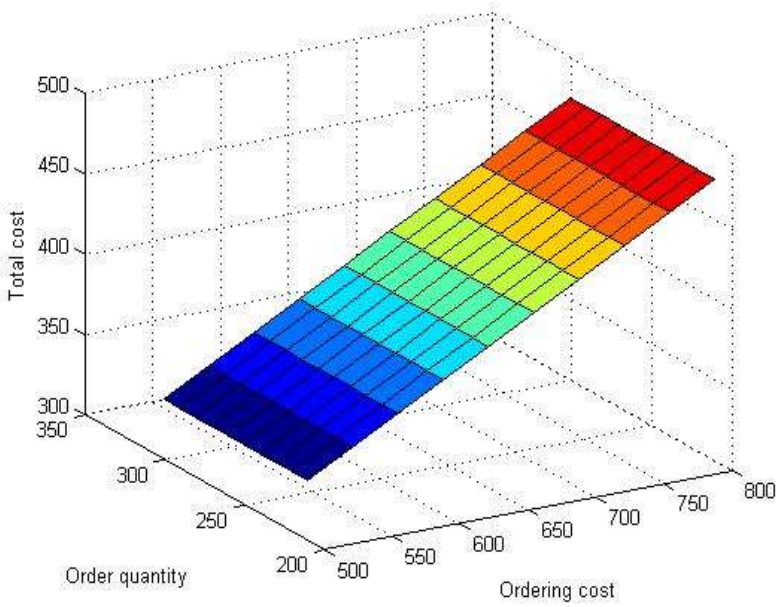

Figure 4. Total cost versus Q and A

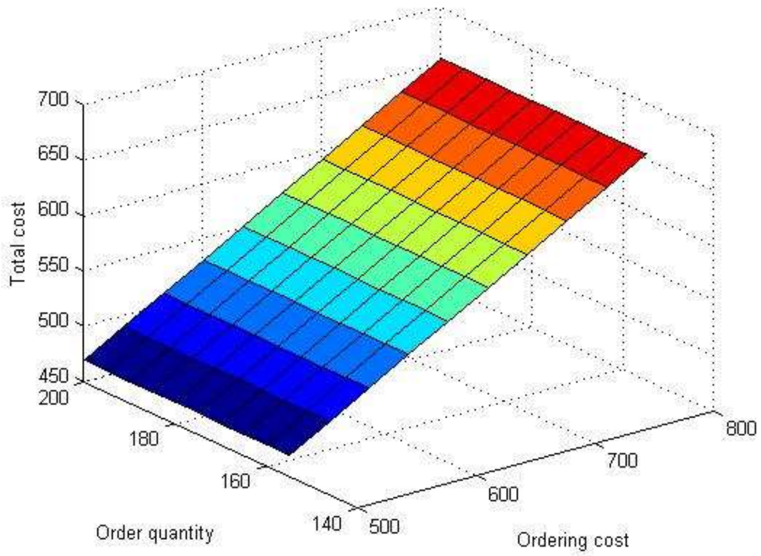

Figure 5. Total cost versus Q and A

6. Conclusion

In this article, we developed an economic order quantity (EOQ) model for deteriorating items with time-dependent demand and two-parameter weibull distribution for deteriorating items. Shortages are allowed in the inventory system are complete and partial backlogging model. The numerical examples have been given to illustrate it. Finally, the sensitivity analysis shows the changes in the values of different parameters. Our research result implies that, total cost per unit time of partial backlogging is more than the complete backlogging which is clear from the numerical examples and sensitivity analysis. In future we extend the study further the proposed model can be incorporate into more realistic assumptions, such as stock-dependent demand, price-dependent demand, price discount, quantity discount.

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