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AN INNOVATIVE EXACT METHOD FOR SOLVING FULLY INTERVAL INTEGER TRANSPORTATION PROBLEMS

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Abstract: A new method namely, the mid-width method, is proposed herein for finding the optimal interval solution to an interval biomedical transportation problem in which shipping cost, supply and demand parameters are real intervals. The mid-width method is an exact method and is developed on two independent transportation problems which are obtained from a fully integer transportation problem. A numerical example in the field of pharmaceutical logistics is presented for understanding the solution procedure of the suggested method. Furthermore, the proposed method is extended to fuzzy transportation problems.

Ke**ywords:** Transportation problem, Real intervals, Mid-width method, Optimal interval solution, Triangular fuzzy numbers.

1. Introduction

Solving the interval transportation problem, researchers have divided the problem into two sub-problems namely, upper and lower level. Firstly, the upper level problem is solved and after that, the lower level problem with upper bound constraints on the decision variables is solved. This concept motivated us to develop the proposed methodology.

Several well-organized techniques for solving transportation problems with the assumption of precise source, destination parameter, and penalty factors were established. In real-life models, these conditions may not always be fulfilled. To deal with imprecise coefficients in transportation problems, many researchers [4-6,8-10,18,20] have proposed fuzzy and interval programming techniques for solving them. A new method viz., the fuzzy technique is used to solve an interval transportation problem, by considering the right bound and the midpoint of the intervals was proposed by Das et al.[6]. Sengupta and Pal [18] presented a new fuzzy orientation method for solving interval transportation problems by considering the midpoint and width of the interval in the objective function. Palmer et al. [11] demonstrated an efficient method for solving optimal stopping problems with a probabilistic constraint, in to which they have optimized the expected cumulative cost, but constrained by an upper bound on the probability that the cost exceeds a specified threshold. A new method called, the separation method was developed to solve integer interval transportation problems based on the zero-point method [13] by Pandian and Natarajan [14].

Cuenca Mira et al. [1] studied the multi-objective optimization problems. For this they were concerned with the parametric decomposition theorem. Chou et al. [2] determined that the disease evolution was a semi-Markov process, for finding the optimal timing to initiate a medical treatment. Ramesh, Kumar and Murugesan [16] recommended a fuzzy interval method to find an optimal solution for fuzzy transportation problems. A determined solution method for solving transportation problems considers the unit cost of transportation from a source to a destination as a rough integer interval was discussed by Subhakanta, Dash and Mohanty [19]. Recently, Akilbasha et al. [3] proposed a new method namely, the split and separation method for finding an optimal solution for integer transportation problems with a rough environment. An advanced method namely, the slice-sum method is used for determining an optimal solution, to fully rough interval integer transportation problems and has been developed by Pandian et al. for pharmaceutical sciences [15]. Singh et al. [17] discussed the multi-objective geometry optimization of a gas cyclone; there they characterized the gas cyclone into two performance parameters, namely the Euler and Stokes numbers.

This paper is structured as follows: In Section 2., some basic definitions and results were related to real intervals and fuzzy numbers are presented. A fully interval transportation problem is discussed, and a theorem that connects a relation between an optimal solution of a fully interval integer transportation problem and a pair of its induced transportation problems is derived in Section 3. In Section 4., a new method namely, the mid-width method is used for solving fully interval integer transportation problem is proposed, and a numerical example is given for understanding the solution procedure of the proposed method. In Section 5., the developed new method is extended to fully fuzzy transportation problems and finally, the conclusion is given in Section 6.

2. Preliminaries

We need the following definitions and results related to real intervals and fuzzy numbers which can be found in [7,12,14,18,21].

Let D denote the set of all closed bounded intervals on the real line R. That is, $D = \{[a,b]: a \le b, a$ and b are in $R\}$.

Definition 2.1: Let $A = [a,b]$ and $B = [c,d]$ be in D. Then,

(i) $A \oplus B = [a+c,\ b+d]$ and

(ii) $A \otimes B = [p,q]$,

Where $p = \text{minimum}\{ac, ad, bc, bd\}$ and $q = \text{maximum}\{ac, ad, bc, bd\}$.

Definition 2.2: Let $A = [a,b]$ and $B = [c,d]$ be in D. Then,

(i) $A \le B$ if and only if $a \le c$ and $b \le d$ and

(ii) $A = B$ if and only if $a = c$ and $b = d$.

Definition 2.3: Let $A = [a,b]$ be in D. Then, A is said to be positive denoted by $A \ge 0$ if $a \ge 0$.

Definition 2.4: Let $A=[a,b]$ in D. Then, A is said to be integer if a and b are integers.

Now, the mid-value of an interval $A=[a,b]$, $m(A)$ is defined as $m(A)=\dfrac{a+b}{2}$ and the half width-value of an interval $A=[a,b]$, $w(A)$ is defined as $w(A)=\dfrac{b-a}{2}$.

Definition 2.5: Let $A=[a,b]$ and $B=[c,d]$ be in D. Then,

(i) $A\approx B$ if and only if $m(A)=m(B)$

(ii) $A\succ B$ if and only if $m(A)>m(B)$ and

(iii) $A\prec B$ if and only if $m(A)<m(B)$.

Definition 2.6: Let $A=[a,b]$ and $B=[c,d]$ be in D. Then,

(i) $A\le B$ if and only if $m(A)\le m(B)$ and $w(A)\le w(B)$

(ii) $A\ge B$ if and only if $m(A)\ge m(B)$ and $w(A)\ge w(B)$ and

(iii) $A=B$ if and only if $m(A)=m(B)$ and $w(A)=w(B)$.

Now, from the definitions of mid-value and half-width of an interval number, we establish the following results.

Result 2.1: If $m(A)=m_1$ and $w(A)=w_1$, then $A=[m_1-w_1,\ m_1+w_1]$

Result 2.2: Let $A=[a,b]$ and $B=[c,d]$ be in D and p and q are real numbers. Then,

(i) $m(pA+qB)=pm(A)+qm(B)$ and (ii) $w(pA+qB)=|p|w(A)+|q|w(B)$.

Definition 2.7: Let A be a classical set and $\mu_A(x)$ be a membership function from A to [0,1]. A fuzzy set A^* with the membership function $\mu_A(x)$ is defined by

$A^*=\{(x,\mu_A(x)):x\in A \text{ and } \mu_A(x)\in[0,1]\}$.

Definition 2.8: A real fuzzy number $\tilde{a}=(a_1,a_2,a_3,a_4)$ is a fuzzy subset from the real line R with the membership function $\mu_{\tilde{a}}(x)$ satisfying the following conditions**.**

(i) $\mu_{\tilde{a}}(x)$ is a continuous mapping from R to the closed interval [0, 1],

(ii) $\mu_{\tilde{a}}(x)$ =0 for every $a\in(-\infty,\ a_1]$,

(iii) $\mu_{\tilde{a}}(x)$ is strictly increasing and continuous on $[a_1,\ a_2]$,

(iv) $\mu_{\tilde{a}}(x)$ = 1 for every $a\in[a_2,\ a_3]$,

(v) $\mu_{\tilde{a}}(x)$ is strictly decreasing and continuous on $[a_3,\ a_4]$ and

(vi) $\mu_{\tilde{a}}(x) = 0$ for every $a \in [a_4, +\infty)$.

Definition 2.9: A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2 and a_3 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below

$\mu_{\tilde{a}}(x) = 0$ if $x \le a_1$; $\mu_{\tilde{a}}(x) = \dfrac{(x - a_1)}{(a_2 - a_1)}$ if $a_1 \le x \le a_2$;

$\mu_{\tilde{a}}(x) = \dfrac{(a_3 - x)}{(a_3 - a_2)}$ if $a_2 \le x \le a_3$ and $\mu_{\tilde{a}}(x) = 0$ if $x \ge a_3$.

Let $F(R)$ be a set of all triangular fuzzy numbers over R, a set of real numbers.

Definition 2.10: Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be in $F(R)$. Then

(i) $\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ and

(ii) $\tilde{a} \otimes \tilde{b} = (t_1, t_2, t_3)$

Where $t_1 = \text{minimum}\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$**;** $t_2 = a_2b_2$ and $t_3 = \text{maximum}\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$**.**

Definition 2.11: Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be in $F(R)$. Then,

(i) $\tilde{a}$ and $\tilde{b}$ are said to be equal if $a_{\text{i}} = b_{\text{i}}$, i =1,2,3, and

(ii) $\tilde{a}$ is said to be less than or equal $\tilde{b}$ if $a_{\text{i}} \le b_{\text{i}}$, i =1,2,3.

Definition 2.12: Let $\tilde{a} = (a_1, a_2, a_3)$ be in $F(R)$. $\tilde{a}$ is said to be positive if $a_1 \ge 0$.

Definition 2.13: Let $\tilde{a} = (a_1, a_2, a_3)$ be in $F(R)$. $\tilde{a}$ is said to be integer if $a_{\text{i}} \ge 0$, i =1,2,3 are integers.

3. Fully Interval Integer Transportation Problems

Consider the following fully interval integer transportation problem (P):

(P) Minimize $Z = [z_1, z_2] = \sum_{\text{i}=1}^{\text{m}} \sum_{\text{j}=1}^{\text{n}} [c_{\text{ij}}, d_{\text{ij}}] \otimes [x_{\text{ij}}, y_{\text{ij}}]$

Subject to

$$\sum_{\text{j}=1}^{\text{n}} [x_{\text{ij}}, y_{\text{ij}}] = [a_{\text{i}}, p_{\text{i}}], \ \text{i} = 1,2,\ldots,\text{m} \quad (1)$$

$$\sum_{\text{i}=1}^{\text{m}} [x_{\text{ij}}, y_{\text{ij}}] = [b_{\text{j}}, q_{\text{j}}], \ \text{j} = 1,2,\ldots,\text{n} \quad (2)$$

$$x_{ij} \geq 0,\ y_{ij} \geq 0,\ i=1,2,...,m \text{ and } j=1,2,...,n \text{ and are integers} \quad (3)$$

Where c_{ij} and d_{ij} are positive real numbers for all i and j, a_i and p_i are positive real numbers for all i and b_j and q_j are positive real numbers for all j.

Definition 3.1: The set $\{[x_{ij}, y_{ij}]$, for all $i=1,2,...,m$ and $j=1,2,...,n\}$ is said to be a feasible solution of (P) if they satisfy the equations (1), (2) and (3).

Definition 3.2: A feasible solution $\{[x_{ij}, y_{ij}]$, for $i=1,2,...,m$ and $j=1,2,...,n\}$ of the problem (P) is said to be an optimal solution of (P) if

$$\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}]\otimes[x_{ij}, y_{ij}] \leq \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}]\otimes[u_{ij}, v_{ij}],$$

for all feasible $\{[u_{ij}, v_{ij}]$ for $i=1,2,...,m$ and $j=1,2,...,n\}$.

Now, let be $m(Z)=m([z_1,z_2])$; $w(Z)=w([z_1,z_2])$; $m([c_{ij},d_{ij}])=t_{ij}$; $w([c_{ij},d_{ij}])=s_{ij}$; $m([x_{ij},y_{ij}])=m_{ij}$; $w([x_{ij},y_{ij}])=w_{ij}$; $m([a_i,p_i])=u_i$; $w([a_i,p_i])=v_i$; $m([b_j,q_j])=k_j$ and $w([b_j,q_j])=h_j$.

Now, the following theorem which finds a relation between an optimal solution of a fully interval integer transportation problem and a pair of induced transportation problems is established and also, is used in the proposed method.

Theorem 3.1: If the set $\{m_{ij}^{\circ}$, for all i and j$\}$ is an optimal solution of the mid-value transportation problem (M) of (P) where

(M) Minimize $m(Z)=\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}m_{ij}$

Subject to

$$\sum_{j=1}^{n} m_{ij}=u_i,\ i=1,2,...,m;$$

$$\sum_{i=1}^{m} m_{ij}=k_j,\ j=1,2,...,n;$$

$$m_{ij}\geq 0,\ i=1,2,...,m \text{ and } j=1,2,...,n$$

and the set $\{w_{ij}^{\circ}$, for all i and j$\}$ is an optimal solution of the half-width transportation problem (W) of (P) where

(W) Minimize $w(Z) = \sum_{i=1}^{m} \sum_{j=1}^{n} s_{ij} w_{ij}$

Subject to

$\sum_{j=1}^{n} w_{ij} = v_i$, $i = 1,2,...,m$;

$\sum_{i=1}^{m} w_{ij} = h_j$, $j = 1,2,...,n$;

$w_{ij} \geq 0$, $i = 1,2,...,m$ and $j = 1,2,...,n$,

then the set of intervals $\{[m_{ij}^{\circ} - w_{ij}^{\circ}, m_{ij}^{\circ} + w_{ij}^{\circ}]$, for all i and j $\}$ is an optimal solution to the problem (P) provided $[m_{ij}^{\circ} - w_{ij}^{\circ}, m_{ij}^{\circ} + w_{ij}^{\circ}]$, for all $i = 1,2,...,m$ and $j = 1,2,...,n$ are integers.

Proof: Now, since $\{m_{ij}^{\circ}$, for all i and j $\}$ and $\{w_{ij}^{\circ}$, for all i and j $\}$ are feasible solutions to the problems (M) and (W) respectively and from the equality conditions of two intervals, we can conclude that the set of intervals $\{[m_{ij}^{\circ} - w_{ij}^{\circ}, m_{ij}^{\circ} + w_{ij}^{\circ}]$, for all i and j$\}$ is a feasible solution to the problem (P).

Let the set $\{[x_{ij}, y_{ij}]$, for all $i = 1,2,...,m$ and $j = 1,2,...,n\}$ be a feasible solution to the problem (P).

Then, by the equality relation conditions, we can conclude that the set $\{m([x_{ij}, y_{ij}])$, for all $i = 1,2,...,m$ and $j = 1,2,...,n\}$ is a feasible solution to the problem (M) and the set $\{w([x_{ij}, y_{ij}])$, for all $i = 1,2,...,m$ and $j = 1,2,...,n\}$ is a feasible solution to the problem (W).

Now, since the set $\{m_{ij}^{\circ}$, for all i and j$\}$ and the set $\{w_{ij}^{\circ}$, for all i and j$\}$ are optimal solution to the problem (M) and the problem (W) respectively, it implies that

$$\sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} m([x_{ij}, y_{ij}]) \geq \sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} m_{ij}^{\circ} \tag{4}$$

and

$$\sum_{i=1}^{m} \sum_{j=1}^{n} s_{ij} w([x_{ij}, y_{ij}]) \geq \sum_{i=1}^{m} \sum_{j=1}^{n} s_{ij} w_{ij}^{\circ}. \tag{5}$$

Now, using (4), we have

$$m(Z) = m\left(\sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]\right) = \sum_{i=1}^{m} \sum_{j=1}^{n} m([c_{ij}, d_{ij}]) m([x_{ij}, y_{ij}]) \geq \sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} m_{ij}^{\circ}. \tag{6}$$

Now, using (5), we get

$$w(Z) = w\left(\sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]\right) = \sum_{i=1}^{m} \sum_{j=1}^{n} w([c_{ij}, d_{ij}]) w([x_{ij}, y_{ij}]) \geq \sum_{i=1}^{m} \sum_{j=1}^{n} s_{ij} w_{ij}^{\circ}. \tag{7}$$

Now, from (6) and (7), we can obtain that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij},d_{ij}]\otimes[x_{ij},y_{ij}] \geq \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij},d_{ij}]\otimes[m_{ij}^{\circ}-w_{ij}^{\circ},m_{ij}^{\circ}+w_{ij}^{\circ}].$$

This implies that the set of intervals $\{[m_{ij}^{\circ}-w_{ij}^{\circ},m_{ij}^{\circ}+w_{ij}^{\circ}]$, for all i and j} is an optimal solution of the problem (P).

Hence, the theorem is proved.

4. Mid-Width Method

We, now introduce a new algorithm namely, mid-width method for finding an optimal solution to a fully interval integer transportation problem (P).

The proposed method proceeds as follows.

Step 1. Construct two independent transportation problems called, mid-value transportation problem (M) and half-width transportation problem (W) from the given problem (P).

Step 2. Solve the problem (M) using a transportation algorithm. Let $\{m_{ij}^{\circ}$, for all i and j} be an optimal solution of the problem (M).

Step 3. Solve the problem (W) using any transportation algorithm. Let $\{w_{ij}^{\circ}$, for all i and j} be an optimal solution of the problem (W).

Step 4. The optimal solution of the given problem (P) is $\{[m_{ij}^{\circ}-w_{ij}^{\circ},m_{ij}^{\circ}+w_{ij}^{\circ}]$, for all i and j} if $[m_{ij}^{\circ}-w_{ij}^{\circ},m_{ij}^{\circ}+w_{ij}^{\circ}]$, for all i and j are integers (by the Theorem 3.1).

The mid-width method for solving fully integer transportation problem is illustrated by the following example.

Example 4.1: A pharmaceutical company produces a product in its three factories F1, F2 and F3. The product will be sent to four destinations D1, D2, D3 and D4 from the three factories. Determine a shipping plan for the company from three factories to four destinations such that the total shipping cost should be minimum using the following numerical data obtained from the company:

The minimum supply from F1, F2 and F3 are 7, 17 and 16 respectively and the maximum supply from F1, F2 and F3 are 9, 21 and 18 respectively. The minimum demand for D1, D2, D3 and D4 are 10, 2, 13 and 15 respectively and the maximum demand for D1, D2, D3 and D4 are 12, 4, 15 and 17 respectively.

The unit shipping cost range from each supply point to each demand point is given below:

	D1	D2	D3	D4
F1	[1,2]	[1,3]	[5,9]	[4,8]
F2	[1,2]	[7,10]	[2,6]	[3,5]
F3	[7,9]	[7,11]	[3,5]	[5,7]

Table 1: Unit shipping cost range from supply points to demand points

The given problem can be modeled as an interval integer transportation problem as follows:

	D1	D2	D3	D4	Supply
F1	[1,2]	[1,3]	[5,9]	[4,8]	[7,9]
F2	[1,2]	[7,10]	[2,6]	[3,5]	[17,21]
F3	[7,9]	[7,11]	[3,5]	[5,7]	[16,18]
Demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

Table 2: 3X4 fully interval integer transportation problem

Now, by the Step 1., the mid-value transportation problem (M) of the given problem is given below:

	D1	D2	D3	D4	Supply
F1	1.5	2	7	6	8
F2	1.5	8.5	4	4	19
F3	8	9	4	6	17
Demand	11	3	14	16	44

Table 3: Mid-value transportation problem (M) of the problem (P)

Now, by the Step 2., an optimal solution to the problem (M) is

$$m^{\circ}_{11} = 5;\ m^{\circ}_{12} = 3;\ m^{\circ}_{21} = 6;\ m^{\circ}_{24} = 13;\ m^{\circ}_{33} = 14 \text{ and } m^{\circ}_{34} = 3.$$

Now, by the Step 1., the half-width transportation problem (W) of the given problem is given below:

	D1	D2	D3	D4	Supply
F1	0.5	1	2	3.5	1
F2	0.5	1.5	2	1	2
F3	1	1	1	1	1
Demand	1	1	1	1	4

Table 4: Half-width transportation problem (W) of the problem (P)

Now, by the Step 3., an optimal solution to the problem (W) is $w^{\circ}_{12} = 1;\ w^{\circ}_{21} = 1;\ w^{\circ}_{24} = 1$ and $w^{\circ}_{33} = 1$.

Now, by the Step 4., an optimal solution to the given transportation problem is given below: $[x_{11}, y_{11}] = [5,5]$; $[x_{12}, y_{12}] = [2,4]$; $[x_{21}, y_{21}] = [5,7]$; $[x_{24}, y_{24}] = [12,14]$; $[x_{33}, y_{33}] = [13,15]$ and $[x_{34}, y_{34}] = [3,3]$ with minimum interval transportation cost [102,202].

5. Fully Fuzzy Integer Transportation Problems

Consider the following fuzzy integer transportation problem (FP) where

(FP) Minimize $\tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$

Subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, \ i = 1,2,...,m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, \ j = 1,2,...,n$$

$\tilde{x}_{ij} \geq \tilde{0}$, $i = 1,2,...,m$ and $j = 1,2,...,n$ and are integers

where the decision variables $\tilde{x}_{ij}$, for all i and j are triangular fuzzy numbers and parameters $\tilde{c}_{ij}$, $\tilde{a}_i$ and $\tilde{b}_j$ are positive triangular fuzzy numbers for all i and j.

A triangular fuzzy number (a,b,c) can be represented as an interval number as follows.

$$(a,b,c) = [a + (b-a)\alpha, c - (c-b)\alpha] \ ; \ 0 \leq \alpha \leq 1. \tag{8}$$

Using the relation (8), the given fully fuzzy transportation problem (FP) can be made into a fully interval transportation problem (ITP). Using the mid-width method, an optimal interval solution to the interval transportation problem is determined. Then, using the relation (8), we can obtain an optimal fuzzy solution to the given fully fuzzy transportation problem (FP). The solving procedure is demonstrated using the following numerical example.

Example 5.1: A pharmaceutical company has three plants at three locations S1, S2 and S3 which supply to three warehouses D1, D2 and D3. Monthly plant capacities are (1,6,11), (2,3,4) and (3,4,5) triangular fuzzy units respectively. Monthly warehouse requirements are (3,7,11), (1,3,5) and (2,3,4) triangular fuzzy units respectively. Determine an optimal distribution for the company in order to minimize the total shipping cost given that the unit transportation costs in triangular fuzzy parameters are given below:

	D1	D2	D3

S1	(1,2,3)	(10,11,12)	(4,7,10)
S2	(0,1,2)	(1,6,11)	(0,1,2)
S3	(1,5,9)	(5,15,25)	(3,9,15)

Table 5: Unit transportation costs in triangular fuzzy parameters

The given problem can be modeled as a fuzzy transportation problem as follows:

	D1	D2	D3	Supply
S1	(1,2,3)	(10,11,12)	(4,7,10)	(1,6,11)
S2	(0,1,2)	(1,6,11)	(0,1,2)	(2,3,4)
S3	(1,5,9)	(5,15,25)	(3,9,15)	(3,4,5)
Demand	(3,7,11)	(1,3,5)	(2,3,4)	(6,13,20)

Table 6: 3X3 fully fuzzy transportation problem

Now, the interval form of the given fully fuzzy transportation problem (ITP) is given as follows:

	D1	D2	D3	Supply
S1	$[1+\alpha, 3-\alpha]$	$[10+\alpha, 12-\alpha]$	$[4+3\alpha, 10-3\alpha]$	$[1+5\alpha, 11-5\alpha]$
S2	$[\alpha, 2-\alpha]$	$[1+5\alpha, 11-5\alpha]$	$[\alpha, 2-\alpha]$	$[2+\alpha, 4-\alpha]$
S3	$[1+4\alpha, 9-4\alpha]$	$[5+10\alpha, 25-10\alpha]$	$[3+6\alpha, 15-6\alpha]$	$[3+\alpha, 5-\alpha]$
Demand	$[3+4\alpha, 11-4\alpha]$	$[1+2\alpha, 5-2\alpha]$	$[2+\alpha, 4-\alpha]$	$[6+7\alpha, 20-7\alpha]$

Table 7: Interval form of the given fully fuzzy transportation problem (ITP)

Now, by the Step 1., the mid-value transportation problem (M) of the problem (ITP) is given below:

	D1	D2	D3	Supply
S1	2	11	7	6
S2	1	6	1	3
S3	5	15	9	4
Demand	7	3	3	13

Table 8: Mid-value transportation problem (M) of the problem (ITP)

Now, by the Step 2., an optimal solution to the problem (M) is $m_{11}^{\circ} = 3;\ m_{12}^{\circ} = 3;\ m_{23}^{\circ} = 3$ and $m_{31}^{\circ} = 4$.

Now, by the Step 1., the half-width transportation problem (W) of the problem (ITP) is given below:

	D1	D2	D3	Supply
S1	$1-\alpha$	$1-\alpha$	$3-3\alpha$	$5-5\alpha$

S2	$1-\alpha$	$5-5\alpha$	$1-\alpha$	$1-\alpha$
S3	$4-4\alpha$	$10-10\alpha$	$6-6\alpha$	$1-\alpha$
Demand	$4-4\alpha$	$2-2\alpha$	$1-\alpha$	$7-7\alpha$

Table 9: Half-width transportation problem (W) of the problem (ITP)

Now, by the Step 3., an optimal solution to the problem (W) is $w_{11}^{\circ}=3-3\alpha;\ w_{12}^{\circ}=2-2\alpha;\ w_{23}^{\circ}=1-\alpha$ and $w_{31}^{\circ}=1-\alpha.$

Now, by the Step 4., an optimal solution to the transportation problem (ITP) is given below:
$[x_{11}^{L},x_{11}^{U}]=[3\alpha,6-3\alpha];\ [x_{12}^{L},x_{12}^{U}]=[1+2\alpha,5-2\alpha]$; $[x_{11}^{L},x_{11}^{U}]=[2+\alpha,4-\alpha]$ and $[x_{31}^{L},x_{31}^{U}]=[3+\alpha,5-\alpha]$

Now, by using the relation (8), we obtain that an optimal solution to the given fuzzy transportation problem is given below:
$\tilde{x}_{11}=(0,3,6);\ \tilde{x}_{12}=(1,3,5)$; $\tilde{x}_{23}=(2,3,4)$ and $\tilde{x}_{31}=(3,4,5)$ with minimum fuzzy transportation cost (19,86,183).

Remark 5.1: The mid-width method can be also used to solve fuzzy transportation problems having trapezoidal fuzzy number parameters since the proposed method is initiated from real interval number.

6. Conclusion

A new method namely, mid-width method for computing an optimal solution to fully integer transportation problems has been proposed in this paper. The proposed method is based on two independent transportation problems which are obtained from the given interval transportation problem and not based on decision variable upper bounded conditions and also, it is an exact method. A numerical example has been presented for demonstrating the solution procedure of the proposed method. Further, the proposed method is extended to fuzzy transportation problems. Logistics can be seen as a link between the manufacturing and marketing operations of a company. The mid-width method is potentially a significant tool for decision makers when they are handling various types of pharmaceutical logistic problems having interval or fuzzy parameters.

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