An optimal EOQ inventory model for non-instantaneous deteriorating items with ramp type demand rate, time dependent holding cost and shortages

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An optimal EOQ inventory model for non-instantaneous deteriorating items with ramp type demand rate, time dependent holding cost and shortages

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Abstract. An economic order quantity inventory model for non-instantaneous deteriorating items has been developed with ramp type demand rate, time dependent deterioration rate and time dependent holding cost. Shortages in inventory are allowed in this model, which is partially backlogged. The main objective of this model is to develop an optimal replenishment policy that minimizes the total average inventory cost. Numerical examples are used to illustrate the developed model.

1. Introduction

Products which losses its utility or its marginal rate that decreases the usefulness from the original one. This phenomenon is termed as deterioration. Also it may be defined as damage, decay, dryness, spoilage and vaporization. Examples: fashion goods, foods, electronic items, chemicals, etc.
In real life situation, most of the products have a period of maintaining quality or the original condition. During that time lag, there was no deterioration takes place. (E.g. vegetables, fruits, meat, fish and so on). This phenomenon is defined as "non-instantaneous deterioration".

A partial backlogging and stock dependent inventory model [10] is considered for non-instantaneous deteriorating items. Optimal replenishment policies for non-instantaneous deteriorating items [1] are proposed with stock dependent demand. An EOQ model for non-instantaneous deteriorating items [2] [11] are developed with trade credits. An inventory model for non-instantaneous deteriorating items [3] is derived with stock dependent demand under inflation. Inventory models for non-instantaneous deteriorating items [4], [5] are developed with partial backlogging, permissible delay in payments, inflation and selling price-dependent demand and customer returns. An EOQ models for non-instantaneous deteriorating items [6] is developed with cubic demand rate, time dependent holding cost and allowed permissible delay in payments. An EOQ model for items [7] is considered with three parameter weibull distribution deterioration, ramp type demand and shortages. A partial backlogging inventory model deteriorating items [8] is considered with ramp type demand. A deterministic model for non-instantaneous deteriorating items [9] with ramp type demand rate and shortages under permissible delay in payments.

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1.1 Assumptions

1. Demand of the product is ramp type function of time.

   i.e. $D(t) = \begin{cases} at, t < \mu \\ a\mu, t \geq \mu \end{cases}$ at any time $t \geq 0,\ a > 0, \mu > 0$

   where 'a' stands for the initial demand and 'μ' is a fixed point in time.
2. The deterioration rate is assumed as time dependent, i.e. $\theta(t) = \theta t$
3. During the shortage period, the backlogging rate is is dependent on the customers waiting time for the next replenishment.

   i.e., for the negative inventory the backlogging rate is defined by

   $B(t) = \frac{1}{1+\delta(T-t)}$; $0 < \delta < 1$, δ denotes the backlogging parameter .
4. A finite planning horizon is assumed.
5. Holding cost per unit is also assumed as time dependent, $i.e. H(t) = ht\ , h > 0.$
6. Replenishment rate is instantaneous and infinite .
7. The time gap between placing an order and its actual arrival is zero.
8. A finite planning horizon is assumed.

1.2 Notations

$I(t)$: Stock level at any time t, $0 \leq t \leq T$
Q_1 : Stock level at time $t = 0$.
Q_2 : Stock level at time $t = T$.
c_1 : Unit purchasing cost of an item
c_2 : Unit shortage cost of an item
c_3 : Unit lost sale cost of an item
T : Time interval between two successive orders.
TC : Total average inventory cost per unit cycle.
A : Ordering cost per unit order is known and constant.
t_d : Time lag in which the product parades no deterioration.
μ : Parameter of ramp type demand function.

2. Mathematical formulation and solution of the model

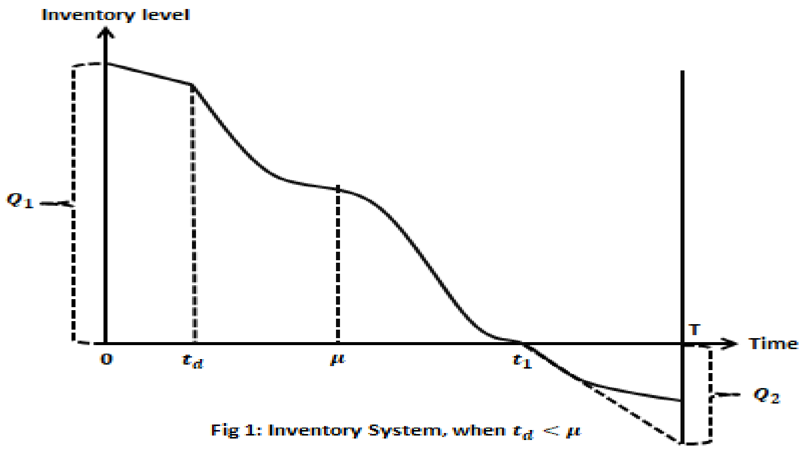

Fig 1: Inventory System, when $t_d < \mu$

Stock levels $I_1(t)$, $I_2(t)$, $I_3(t)$ and $I_4(t)$ at any time 't' in the intervals $[0, t_d]$, $[t_d, \mu]$, $[\mu, t_1]$ and $[t_1, T]$ are represented by the following differential equations

$$\frac{dI_1(t)}{dt} = -at, \quad 0 \le t \le t_d \tag{1}$$

with boundary condition $I_1(0) = Q_1$.

$$\frac{dI_2(t)}{dt} + \theta t\, I_2(t) = -at, \; t_d \le t \le \mu \tag{2}$$

$$\frac{dI_3(t)}{dt} + \theta t\, I_3(t) = -a\mu, \; \mu \le t \le t_1 \tag{3}$$

with boundary conditions $I_2(\mu) = I_3(\mu)$ and $I_3(t_1) = 0$.

$$\frac{dI_4(t)}{dt} = -\frac{a\mu}{1+\delta(T-t)}, \; t_1 \le t \le T \tag{4}$$

with boundary condition $I_4(t_1) = 0$.

Now, solutions of equations (1) to (4) are as follows:

$$I_1(t) = Q_1 - \frac{at^2}{2}, 0 \le t \le t_d \tag{5}$$

$$I_2(t) = a\left[\mu t_1 - \frac{\mu^3}{2} + \frac{\mu\theta(t_1^3-\mu^3)}{6} + \frac{\theta\mu^5}{8} - \frac{t^2}{2} + \frac{\theta t^4}{8} - \frac{\mu t_1\theta t^2}{2} + \frac{\theta\mu^3 t^2}{4}\right], t_d \le t \le \mu \tag{6}$$

$$I_3(t) = a\mu\left[t_1 - t + \frac{\theta(t_1^3-t^3)}{6} + \frac{(t_1-t)\theta t^2}{4}\right], \mu \le t \le t_1 \tag{7}$$

$$I_4(t) = \frac{a\mu}{\delta} log\left[\frac{1+\delta(T-t)}{1+\delta(T-t_1)}\right], t_1 \le t \le T \tag{8}$$

Due to continuity of stock level at $t = t_d$, it follows from equation (5) and (6) , we can obtain initial inventory level as

$$Q_1 = a\left[\mu t_1 - \frac{\mu^3}{2} + \frac{\mu\theta\left(t_1^3-\mu^3\right)}{6} + \frac{\theta\mu^5}{8} + \frac{\theta t_d^4}{8} - \frac{\theta\mu t_1 t_d^2}{2} + \frac{\theta\mu^3 t_d^2}{4}\right]$$

With boundary condition $I_4(T) = -Q_2$, we can obtain the negative inventory as

$$I_4(T) = -Q_2 = -\frac{a\mu}{\delta}log[1+\delta(T-t_1)]$$

$$\text{Total Inventory}, Q = Q_1 + Q_2 \tag{9}$$

(i) Ordering Cost $OC = \frac{A}{T}$

(ii) Holding Cost

$$HC = \frac{1}{T}\int_0^T ht\, I(t)dt$$

$$HC = \frac{h}{T}\int_0^{t_d} tI_1(t)dt + \frac{h}{T}\int_{t_d}^{\mu} t\, I_2(t)dt + \frac{h}{T}\int_{\mu}^{t_1} tI_3(t)dt$$

$$HC = HC_1 + HC_2 + HC_3$$

where

$$HC_1 = \frac{ha}{T}\left[-\frac{t_1^4}{8} + \frac{t_1^2}{2}\left\{\mu t_1 - \frac{\mu^3}{2} + \frac{\mu\theta\left(t_1^3-\mu^3\right)}{6} + \frac{\theta\mu^5}{8} + \frac{\theta t_d^4}{8} - \frac{\theta\mu t_1 t_d^2}{2} + \frac{\theta\mu^3 t_d^2}{4}\right\}\right]$$

$$HC_2 = \frac{ha}{T}\left[\begin{array}{c} \frac{\mu^3 t_1}{2} - \frac{\mu^5}{4} + \frac{\theta\mu^3\left(t_1^3-\mu^3\right)}{12} + \frac{\theta\mu^7}{16} - \frac{\mu^4}{8} + \frac{\theta\mu^6}{48} \\ -\frac{\mu^5 t_1\theta}{8} + \frac{\mu^7\theta}{16} - \frac{\mu t_d^2 t_1}{2} + \frac{\mu^3 t_d^2}{4} - \frac{\mu\theta\left(t_1^3-\mu^3\right)t_d^2}{12} - \frac{\theta\mu^5 t_d^2}{16} \\ +\frac{t_d^4}{8} - \frac{\theta t_d^6}{48} + \frac{\mu t_1\theta t_d^4}{8} - \frac{\mu^3\theta t_d^4}{16} \end{array}\right]$$

$$HC_3 = \frac{ha\mu}{T}\left[\frac{t_1^3}{6} + \frac{\theta t_1^5}{40} - \frac{\mu^2 t_1}{2} + \frac{\mu^3}{3} - \frac{\theta\mu^2 t_1^3}{12} - \frac{\theta\mu^4 t_1}{8} + \frac{2\theta\mu^5}{15}\right]$$

(iii) Deterioration Cost

$$DC = \frac{C_1}{T}\left[Q_1 - \left(\int_{t_d}^{\mu} D(t)dt + \int_{\mu}^{t_1} D(t)dt\right)\right]$$

$$DC = \frac{C_1}{T}\left[-\frac{\mu^3}{2} + \frac{\mu\theta\left(t_1^3-\mu^3\right)}{6} + \frac{\theta\mu^5}{8} + \frac{\theta t_d^4}{8} - \frac{\mu\theta t_1 t_d^2}{2} + \frac{\theta\mu^3 t_d^2}{4} + \frac{\mu^2}{2}\right]$$

(iv) Shortage Cost

$$SC = \frac{C_2}{T}\int_{t_1}^{T}[I_4(t)]\,dt$$

$$SC = \frac{C_2 a\mu}{T\delta^2}\left[-log\left(1+\delta(T-t_1)\right) + \delta(T-t_1)\right]$$

(v) Cost due to lost sales

$$CLS = \frac{C_3 a\mu}{T\delta}\int_{t_1}^{T}\left[1 - \frac{1}{1+\delta(T-t)}\right]dt$$

$$CLS = \frac{C_3 a\mu}{T\delta}\left[-log[1+\delta(T-t_1)] + \delta(T-t_1)\right]$$

Total average inventory cost, $TC = OC + HC + DC + SC + CLS$

$$TC = \begin{bmatrix} \frac{A}{T} + \frac{ha}{T}\left[-\frac{t_1^4}{8} + \frac{t_1^2}{2}\left\{\mu t_1 - \frac{\mu^3}{2} + \frac{\mu\theta\left(t_1^3-\mu^3\right)}{6} + \frac{\theta\mu^5}{8} + \frac{\theta t_d^4}{8} - \frac{\theta\mu t_1 t_d^2}{2} + \frac{\theta\mu^3 t_d^2}{4}\right\}\right] \\ + \frac{ha}{T}\begin{bmatrix} \frac{\mu^3 t_1}{2} - \frac{\mu^5}{4} + \frac{\theta\mu^3\left(t_1^3-\mu^3\right)}{12} + \frac{\theta\mu^7}{16} - \frac{\mu^4}{8} + \frac{\theta\mu^6}{48} \\ -\frac{\mu^5 t_1\theta}{8} + \frac{\mu^7\theta}{16} - \frac{\mu t_d^2 t_1}{2} + \frac{\mu^3 t_d^2}{4} - \frac{\mu\theta\left(t_1^3-\mu^3\right)t_d^2}{12} - \frac{\theta\mu^5 t_d^2}{16} \\ + \frac{t_d^4}{8} - \frac{\theta t_d^6}{48} + \frac{\mu t_1\theta t_d^4}{8} - \frac{\mu^3\theta t_d^4}{16} \end{bmatrix} \\ + \frac{ha\mu}{T}\left[\frac{t_1^3}{6} + \frac{\theta t_1^5}{40} - \frac{\mu^2 t_1}{2} + \frac{\mu^3}{3} - \frac{\theta\mu^2 t_1^3}{12} - \frac{\theta\mu^4 t_1}{8} + \frac{2\theta\mu^5}{15}\right] \\ + \frac{C_1}{T}\left[-\frac{\mu^3}{2} + \frac{\mu\theta\left(t_1^3-\mu^3\right)}{6} + \frac{\theta\mu^5}{8} + \frac{\theta t_d^4}{8} - \frac{\mu\theta t_1 t_d^2}{2} + \frac{\theta\mu^3 t_d^2}{4} + \frac{\mu^2}{2}\right] \\ + \frac{C_2 a\mu}{T\delta^2}\left[-log\left(1+\delta(T-t_1)\right) + \delta(T-t_1)\right] \\ + \frac{C_3 a\mu}{T\delta}\left[-log[1+\delta(T-t_1)] + \delta(T-t_1)\right] \end{bmatrix} \tag{10}$$

Our objective is to minimize the total variable inventory cost per unit time

Necessary conditions for total variable inventory cost to be minimized are

(i) $\frac{d(TC)}{dt_1} = 0$ and (ii) $\frac{d^2(TC)}{dt_1^2} > 0$

$$\frac{d(TC)}{dt_1} = \begin{bmatrix} \frac{ha}{T}\left[-\frac{t_1^3}{2} + \frac{3\mu t_1^2}{2} - \frac{\mu^3 t_1}{2} + \frac{5\mu\theta t_1^4}{12} - \frac{\mu^3\theta t_1}{6} + \frac{\theta\mu^5 t_1}{8} + \frac{\theta t_1 t_d^4}{8} - \frac{3\theta\mu t_1^2 t_d^2}{4} + \frac{\theta\mu^3 t_1 t_d^2}{4}\right] \\ + \frac{ha}{T}\left[\frac{\mu^3}{2} + \frac{\theta\mu^3 t_1^2}{4} - \frac{\mu^5\theta}{8} - \frac{\mu t_d^2}{2} - \frac{\mu\theta t_1^2 t_d^2}{4} + \frac{\mu\theta t_d^4}{8}\right] \\ + \frac{ha\mu}{T}\left[\frac{t_1^2}{2} + \frac{\theta t_1^4}{8} - \frac{\mu^2}{2} - \frac{\theta\mu^2 t_1^2}{4} - \frac{\theta\mu^4}{8}\right] \\ + \frac{C_1}{T}\left[+\frac{\mu\theta t_1^2}{2} - \frac{\mu\theta t_d^2}{2}\right] + \frac{C_2 a\mu}{T}\left[\left(\frac{t_1-T}{1+\delta(T-t_1)}\right)\right] + \frac{C_3 a\mu}{T}\left[\left(\frac{\delta(t_1-T)}{1+\delta(T-t_1)}\right)\right] \end{bmatrix}$$

$$\frac{d^2(TC)}{dt_1^2} = \begin{bmatrix} \frac{ha}{T}\left[-\frac{3t_1^2}{2} + 3\mu t_1 - \frac{\mu^3}{2} + \frac{5\mu\theta t_1^3}{3} - \frac{\mu^3\theta}{6} + \frac{\theta\mu^5}{8} + \frac{\theta t_d^4}{8} - \frac{3\theta\mu t_1 t_d^2}{2} + \frac{\theta\mu^3 t_d^2}{4}\right] \\ + \frac{ha}{T}\left[\frac{\theta\mu^3 t_1}{2} - \frac{\mu\theta t_1 t_d^2}{2}\right] + \frac{ha\mu}{T}\left[t_1 + \frac{\theta t_1^3}{2} - \frac{\theta\mu^2 t_1}{2}\right] + \frac{C_1}{T}[\mu\theta t_1] \\ + \frac{(C_2+C_3\delta)a\mu}{T}\left[\left(\frac{1}{1+\delta(T-t_1)^2}\right)\right] \end{bmatrix}$$

3. Numerical analysis

Model 1. Non-instantaneous deteriorating items with partial backlogging

Let $A = 200,\ h = 0.03,\ a = 800, t_d = 0.5$ week, $\mu = 1$ week, $T = 20$weeks, $\theta = 0.01,\ \delta = 0.5$, $C_1 = 20, C_2 = 25, C_3 = 70.$
The optimum solutions are $t_1^* = 14.5498$ weeks and $TC^* = Rs.18{,}187$ per unit cycle and $Q^* = 17{,}437\ units\ per\ unit\ cycle$

Model 2. Non-instantaneous deteriorating items with complete backlogging

Let $A = 200$, $h = 0.03$, $a = 800, t_d = 0.5\, week$, $\mu = 1\, week$, $T = 20$weeks, $\theta = 0.01$, $\delta = 1$, $C_1 = 20, C_2 = 25, C_3 = 70$.
The optimum solutions are $t_1^* = 12.7471$ weeks and $TC^* = Rs. 16{,}970$ per unit cycle and $Q^* = 14{,}25$ units per unit cycle

Model 3. Instantaneous deteriorating items with partial backlogging

Let $A = 200$, $h = 0.03$, $a = 800$,, $\mu = 1$ week, $T = 20$ weeks, $\theta = 0.01$, $\delta = 0.5$, $C_1 = 20, C_2 = 25, C_3 = 70$.
The optimum solutions are $t_1^* = 14.5339$ weeks and $TC^* = Rs. 18{,}241$ per unit cycle and $Q^* = 17{,}428$ units per unit cycle

Model 4. Instantaneous deteriorating items with complete backlogging

Let $A = 200$, $h = 0.03$, $a = 800$, $\mu = 1$ week, $T = 20$ weeks, $\theta = 0.01$, $\delta = 1$, $C_1 = 20, C_2 = 25, C_3 = 70$.
The optimum solutions are $t_1^* = 12.7270$ weeks and $TC^* = Rs. 17{,}016$ per unit cycle and $Q^* = 14{,}220$ units per unit cycle

3.1 Observations:

The following results are observed from numerical examples:

1. The optimum total average cost in Model 1 is Rs.54 less than that of Model 3 and optimum order quantity in Model 1 is 9 units more than that of Model 3.

2. The optimum total average cost in Model 2 is Rs.46 less than that of Model 4 and optimum order quantity in Model 2 is 15 units more than that of Model 4.

3. The optimum total average cost in Model 1 is Rs.1217 more than that of Model 2 and optimum order quantity in Model 1 is 3202 units more than that of Model 2.

4. The optimum total average cost in Model 3 is Rs.1225 more than that of Model 4 and optimum order quantity in Model 3 is 3208 units more than that of Model 4.

4. Conclusion

In this paper, an optimal economic order quantity inventory model for non-instantaneous deteriorating items is developed with ramp type demand rate, time dependent deterioration rate. Time dependent holding cost has seemed to be a realistic assumption in some real world business problems, so time dependent holding is assumed in this model. Few more models are extended from the original model and compared with each other. Numerical examples are also provided to illustrate the proposed model.

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