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Analysis of bulk queueing system with single service and single vacation

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Abstract. A batch arrival queueing system with two types of service pattern with single vacation is considered. If the queue length is more than or equal to the minimum of batch size 'a', the bulk service can be provided otherwise the single service or single vacation is provided. After completing the bulk service, if the queue length is less than 'a' then the server leaves for single vacation. After completing the single vacation, if the queue length is greater than or equal to 'a' then the server will provide the bulk service, otherwise, the server goes to single service. The queue size distribution of the developed model is obtained and the performance measures of the developed queueing model are established. The particular cases of the proposed queueing model are also discussed.

1. Introduction

Queueing theory was introduced by Erlang [4]. Single server priority queue was proposed by Miller [15]. Chaudhry and Templeton [2] explained detail study of bulk queueing models. Queueing models with the server vacations model have been examined by various authors. Stochastic Models in Queueing Theory introduced by Medhi [14]. In vacation queueing model, the server utilizes the idle time for the different purpose. A literature survey on the server vacation models was proposed by Ke et al. [10]. A literature survey on the server vacation models and priority systems was proposed by Takagi [20]. Supplementary variables technique (SVT) proposed for convert a non-Markovian model into Markovian models. The SVT was first initiated by Cox [1] and then followed by Lee [12]. Krishna Reddy et al. [18] analyzed bulk service queueing system with N-policy.

Jeyakumar and Senthilnathan [7] studied the behaviour of the queueing system with server breakdown without interruption. Haridass and Arumuganathan [5] analyzed a queueing system under a restricted admissibility policy. Madhu Jain and Anamika Jain [8] proposed a priority queueing model to analyze the reliability characteristics. Ke [9] proposed the batch arrival queueing system under various vacation policies. Wang et al. [22] studied the different kinds of system performance measures for the T policy. Sanjeet Singh and Naveen Kapil [19] studied a single removable and non-reliable server queueing system under steady-state conditions.

Takine and Sengupta [21] worked on the queue-length and the waiting time distribution of a single-server queue when it is stopped for a short period. Maurya [13] studied when the arrivals and service are in batches with two states. Haridass and Nithya [6] analyzed a batch arrival bulk service (BABS) queueing system with server failure and vacation intrusion. Pukazhenthil and Ezhilvanan [17] presented a vacation queueing model with bulk service rule. Kumar et al. [11] investigated the state



dependent, not able to be trusted or believed bulk queueing model with two phase of service and m phase of repair.

This paper is systematized as follows: the Mathematical model is discussed in Section 2. In Section 3, notations used are presented. A queue size distribution for the developed queueing model is obtained in Section 4. The probability generating function (PGF) of the queue size distribution is established in Section 5. In Section 6, the performance measures for the developed queueing models are determined and the particular cases of the model are discussed. The conclusion is given in Section 7.

2. Mathematical Model

A batch arrival queueing system with two types of service pattern with single vacation is considered. The server begins the service at least 'a' customers are waiting in the queue. If the queue length reaches a threshold value 'a', then the server chooses batch service with a minimum of 'a' customers and maximum of 'b' customers. After completing the batch service, if the queue length is greater than or equal to the minimum of batch size 'a' then the server will continue the batch service according to Neuts [16] general bulk service rule otherwise the server leaves for single vacation. After completing the single vacation, if the queue length is greater than or equal to 'a' then the server will provide the batch service, otherwise, the server goes to single service. After completing the single service, if the queue length is greater than or equal to 'a' then the server will provide batch service, otherwise, the server goes to single vacation. The proposed model is represented schematically in Figure 2.1.

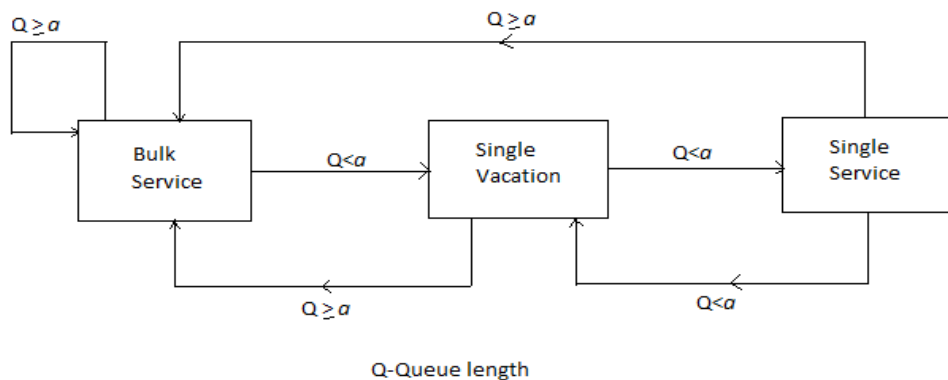


Figure 2.1 Schematic representation of the queueing model.

3. Notation

- λ Poisson Arrival rate
- Y Group size random variable of the arrival
- g_k $\Pr(Y=k)$
- $H_q(t)$ Number of customers waiting for service at time t
- $H_s(t)$ Number of customers under the service at time t

Let $L(x)[l(x)]\{\tilde{L}(\theta)\}(L^0(t))$ denotes the Cumulative Distribution Function(CDF) [Probability Density Function(PDF)] {Laplace-Stieltjes transform (LST)}(remaining time) of single service. Let $M(x)[m(x)]\{\tilde{M}(\theta)\}(M^0(t))$ denotes the CDF[PDF]{LST}(remaining time) of batch service. Let $N(x)[n(x)]\{\tilde{N}(\theta)\}(N^0(t))$ denotes the CDF[PDF]{LST} (remaining time) of the vacation time.

$$C(t) = \begin{cases} [0] - \text{if the server is on single service} \\ [1] - \text{if the server is on batch service} \\ [2] - \text{if the server is on single vacation} \end{cases}$$

Now, the state probabilities are established as follows

$$T_{1j}(x, t)\delta t = \Pr\{H_s(t) = 1, H_q(t) = j, x \leq L^0(t) \leq x + \delta t, C(t) = 0\}, j < a - 1$$

$$G_{ij}(x, t)\delta t = \Pr\{H_s(t) = i, H_q(t) = j, x \leq M^0(t) \leq x + \delta t, C(t) = 1\}, a \leq i \leq b, j \geq 0$$

$$D_j(x, t)\delta t = \Pr\{H_q(t) = j, x \leq N^0(t) \leq x + \delta t, C(t) = 2\}, 0 \leq j \leq a - 1$$

4. Queue size distributions

Now, from the above state probabilities, considering all possibilities and using SVT, we get the following steady state system difference-differential equations for the queueing models.

$$-T_{10}'(x) = -\lambda T_{10}(x) + D_0(0)l(x) \quad (1)$$

$$-T_{1j}'(x) = -\lambda T_{1j}(x) + D_j(0)l(x) + \lambda \sum_{k=1}^j T_{1j-k}(x)g_k, j = 1, 2, \dots, a - 1 \quad (2)$$

$$-T_{1j}'(x) = -\lambda T_{1j}(x) + \lambda \sum_{k=1}^j T_{1j-k}(x)g_k, j \geq a \quad (3)$$

$$-G_{i0}'(x) = -\lambda G_{i0}(x) + \sum_{m=a}^b G_{mi}(0)m(x) + D_0(0)m(x) + T_0(0)m(x), a \leq i \leq b \quad (4)$$

$$-G_{ij}'(x) = -\lambda G_{ij}(x) + \lambda \sum_{k=1}^j G_{i(j-k)}g_k, a \leq i \leq b - 1, j \geq 1 \quad (5)$$

$$-G_{bj}'(x) = -\lambda G_{bj}(x) + \sum_{m=a}^b G_{mb+j}(0)m(x) + \lambda \sum_{k=1}^j G_{b(j-k)}g_k m(x) + D_{b+j}(0)m(x) + T_{b+j}(0)m(x), j \geq 1 \quad (6)$$

$$-D_0'(x) = -\lambda D_0(x) + \sum_{m=a}^b G_{m0}(0)n(x) + T_{10}(0)n(x) \quad (7)$$

$$-D_j'(x) = -\lambda D_j(x) + \sum_{k=1}^j D_{(j-k)}(x)\lambda g_k + \sum_{m=a}^b G_{mj}(0)n(x) + T_{1j}(0)n(x), 1 \leq j \leq a - 1 \quad (8)$$

LST of the above equations from (1) to (8) computes as follows

$$\theta \tilde{T}_{10}(\theta) - T_{10}(0) = \lambda \tilde{T}_{10}(\theta) - D_0(0)\tilde{L}(\theta) \quad (9)$$

$$\theta \tilde{T}_{1j}(\theta) - \tilde{T}_{1j}(0) = \lambda \tilde{T}_{1j}(\theta) - D_j(0)\tilde{L}(\theta) - \lambda \sum_{k=1}^j \tilde{T}_{1j-k}(\theta)g_k, j = 1, 2, \dots, a - 1 \quad (10)$$

$$\theta \tilde{T}_{1j}(\theta) - T_{1j}(0) = \lambda \tilde{T}_{1j}(\theta) - \lambda \sum_{k=1}^j \tilde{T}_{1j-k}(\theta)g_k, j \geq a \quad (11)$$

$$\theta \tilde{G}_{i0}(\theta) - G_{i0}(0) = \lambda \tilde{G}_{i0}(\theta) - \sum_{m=a}^b G_{mi}(0)\tilde{M}(\theta) - T_i(0)\tilde{M}(\theta) - D_i(0)\tilde{M}(\theta), a \leq i \leq b \quad (12)$$

$$\theta \tilde{G}_{ij}(\theta) - G_{ij}(0) = \lambda \tilde{G}_{ij}(\theta) - \lambda \sum_{k=1}^j \tilde{G}_{i(j-k)}(\theta) g_k, a \leq i \leq b-1, j \geq 1 \quad (13)$$

$$\theta \tilde{G}_{bj}(\theta) - G_{bj}(0) = \lambda \tilde{G}_{bj}(\theta) - \sum_{m=a}^b G_{mb+j}(0) \tilde{M}(\theta) - \lambda \sum_{k=1}^j \tilde{G}_{b(j-k)}(\theta) g_k - D_{b+j}(0) \tilde{M}(\theta) - T_{b+j}(0) \tilde{M}(\theta), j \geq 1 \quad (14)$$

$$\theta \tilde{D}_0(\theta) - D_0(0) = \lambda \tilde{D}_0(\theta) - \sum_{m=a}^b G_{m0}(0) \tilde{N}(\theta) - T_{10}(0) \tilde{N}(\theta) \quad (15)$$

$$\theta \tilde{D}_j(\theta) - D_j(0) = \lambda \tilde{D}_j(\theta) - \sum_{k=1}^j \tilde{D}_{(j-k)}(\theta) \lambda g_k - \sum_{m=a}^b G_{mj}(0) \tilde{N}(\theta) - T_{1j}(0) \tilde{N}(\theta), 1 \leq j \leq a-1 \quad (16)$$

In order to find the system size distribution, we define the following PGF:

$$\begin{aligned} \tilde{T}_1(z, \theta) &= \sum_{j=0}^{\infty} T_{1j}(\theta) z^j & \text{and} & \quad T_1(z, 0) = \sum_{j=0}^{\infty} T_{1j}(0) z^j \\ \tilde{G}_i(z, \theta) &= \sum_{j=0}^{\infty} G_{ij}(\theta) z^j & \text{and} & \quad G_i(z, 0) = \sum_{j=0}^{\infty} G_{ij}(0) z^j, a \leq i \leq b \\ \tilde{D}(z, \theta) &= \sum_{j=0}^{\infty} D_j(\theta) z^j & \text{and} & \quad D(z, 0) = \sum_{j=0}^{\infty} D_j(0) z^j \end{aligned} \quad (17)$$

Now, from equations (9), (10), (11) and (17), we obtain

$$(\theta - \lambda + \lambda Y(z)) \tilde{T}_1(z, \theta) = T_1(z, 0) - \tilde{L}(\theta) \left[\sum_{j=0}^{a-1} D_j(0) z^j \right] \quad (18)$$

Now, from equations (12), (13) and (17), we find

$$(\theta - \lambda + \lambda Y(z)) \tilde{G}_i(z, \theta) = G_i(z, 0) - \tilde{M}(\theta) \left\{ \sum_{m=a}^b G_{mi}(0) + T_i(0) + D_i(0) \right\}, a \leq i \leq b-1 \quad (19)$$

Now, from equations (12), (14) and (17), we establish

$$\begin{aligned} z^b (\theta - \lambda + \lambda Y(z)) \tilde{G}_b(z, \theta) &= z^b G_b(z, 0) \\ &- \tilde{M}(\theta) \left\{ \left[\sum_{m=a}^{b-1} [G_m(z, 0)] + G_b(z, 0) \right] - \sum_{j=0}^{b-1} G_{mj}(0) z^j \right\} \\ &+ \left\{ T_1(z, 0) - \sum_{j=0}^{b-1} T_{1j}(0) z^j \right\} + \left\{ D(z, 0) - \sum_{j=0}^{b-1} D_j(0) z^j \right\} \end{aligned} \quad (20)$$

Now, from equations (15), (16) and (17), we have

$$(\theta - \lambda + \lambda Y(z)) \tilde{D}(z, \theta) = D(z, 0) - \tilde{N}(\theta) \sum_{j=0}^{a-1} \left\{ \sum_{m=a}^b G_{mj}(0) + T_{1j}(0) \right\} z^j \quad (21)$$

Now, substituting $\theta = \lambda - \lambda Y(z)$ in equations (18)-(21), we conclude that

$$T_1(z, 0) = \tilde{L}(\lambda - \lambda Y(z)) \sum_{j=0}^{a-1} D_j(0) z^j \quad (22)$$

$$G_i(z, 0) = \tilde{M}(\lambda - \lambda Y(z)) \left\{ \sum_{m=a}^b G_{mi}(0) + T_{1i}(0) + D_i(0) \right\}, a \leq i \leq b-1 \quad (23)$$

$$G_b(z, 0) = \frac{\tilde{M}(\lambda - \lambda Y(z)) f(z)}{\left[z^b - \tilde{M}(\lambda - \lambda Y(z)) \right]} \quad (24)$$

$$\text{where } f(z) = \sum_{m=a}^{b-1} G_m(z, 0) + D(z, 0) + T(z, 0) - \sum_{j=0}^{b-1} \left\{ G_{mj}(0) + T_{1j}(0) + D_j(0) \right\} z^j$$

$$D(z, 0) = \tilde{N}(\lambda - \lambda Y(z)) \sum_{j=0}^{a-1} \left\{ \sum_{m=a}^b G_{mj}(0) + T_{1j}(0) \right\} z^j \quad (25)$$

Now, using (18) and (22), we have

$$\tilde{T}_1(z, \theta) = \frac{\left(\tilde{L}(\lambda - \lambda Y(z)) - \tilde{L}(\theta) \right) \sum_{j=0}^{a-1} D_j(0) z^j}{(\theta - \lambda + \lambda Y(z))} \quad (26)$$

Now, using (19) and (23), we obtain

$$\tilde{G}_i(z, \theta) = \frac{\left(\tilde{M}(\lambda - \lambda Y(z)) - \tilde{M}(\theta) \right) \left\{ \sum_{m=a}^b G_{mi}(0) + T_{1i}(0) + D_i(0) \right\}}{(\theta - \lambda + \lambda Y(z))}, a \leq i \leq b-1 \quad (27)$$

Now, using (20) and (24), we find

$$\tilde{G}_b(z, \theta) = \frac{\left(\tilde{M}(\lambda - \lambda Y(z)) - \tilde{M}(\theta) \right) f(z)}{\left[z^b - \tilde{M}(\lambda - \lambda Y(z)) \right] (\theta - \lambda + \lambda Y(z))} \quad (28)$$

$$\text{where } f(z) = \sum_{m=a}^{b-1} G_m(z, 0) + D(z, 0) + T(z, 0) - \sum_{j=0}^{b-1} \left\{ G_{mj}(0) + T_{1j}(0) + D_j(0) \right\} z^j$$

Now, using (21) and (25), we determine

$$\tilde{D}(z, \theta) = \frac{\left(\tilde{N}(\lambda - \lambda Y(z)) - \tilde{N}(\theta) \right) \sum_{j=0}^{a-1} \left\{ \sum_{m=a}^b G_{mj}(0) + T_{1j}(0) \right\} z^j}{(\theta - \lambda + \lambda Y(z))} \quad (29)$$

5. PGF of queue size

Now, the PGF of the queue size at an arbitrary time epoch is obtained as,

$$P(z) = \sum_{i=1}^{b-1} \tilde{G}_i(z, 0) + \tilde{G}_b(z, 0) + \tilde{T}_1(z, 0) + \tilde{D}(z, 0) \quad (30)$$

Now, using the equations (26)-(29) with $\theta=0$ in (30), we conclude that

$$\text{Substitute } e_i = \sum_{m=a}^b G_{mj}(0) + T_{1j}(0) \text{ and } f_i = T_{1j}(0)$$

$$P(z) = \frac{\left(\tilde{M}(\lambda - \lambda Y(z)) - 1\right) \sum_{i=a}^{b-1} (e_i + f_i)(z^b - z^i) + \left(\tilde{L}(\lambda - \lambda Y(z)) - 1\right)(z^b - 1) \sum_{i=0}^{a-1} f_i z^i + \left(\tilde{N}(\lambda - \lambda Y(z)) - 1\right)(z^b - 1) \sum_{i=0}^{a-1} e_i z^i}{(-\lambda + \lambda Y(z))(z^b - \tilde{M}(\lambda - \lambda Y(z)))} \tag{31}$$

Equation (39) involving b unknowns e_0, e_1, \dots, e_{b-1} and f_0, f_1, \dots, f_{b-1} . Using the following result we can express f_i in terms of e_i . Now, the above equation contains only ‘ b ’ unknown’s e_0, e_1, \dots, e_{b-1} .

Results: $f_j = \sum_{i=0}^n \alpha_i e_{j-i}, n = 0, 1, 2, \dots, a-1$ where α_i is the probability of i customers arrive during avocation.

6. Performance measures

6.1 Expected queue length (EQL)

The EQL is obtained by differentiating the above equation(31) and put $z = 1$ is represented by

$$\lim_{z \rightarrow 1} P'(z) = E(Q) = \frac{(2b\lambda E(X) - 2B1^2) \left\{ \begin{aligned} & \left[3bS2^2 + 3bS1'' + 3b(b-1)S1 + 6biS1 \right] \sum_{i=0}^{a-1} f_i + [3bV2^2 + 3bV1'' + 3b(b-1)V1] \\ & + 6biV1 \sum_{i=0}^{a-1} e_i + [3B2^2 + 3B1''] \sum_{i=0}^{a-1} (e_i + f_i)(b-i) \\ & - 3B1 \sum_{i=0}^{a-1} [b(b-1) - i(i-1)](e_i + f_i) \end{aligned} \right\}}{12[2b\lambda E(X) - B1^2]^2}$$

Where

$$\begin{aligned} \lambda E(B)E(X) &= B1, \lambda E(V)E(X) = V1, \lambda E(S)E(X) = S1, \lambda^2 E(B^2)[E(X)]^2 = B2^2, \\ \lambda^2 E(V^2)[E(X)]^2 &= V2^2, \lambda^2 E(S^2)[E(X)]^2 = S2^2, \lambda E(B)E(X)X''(1) = B1'', \\ \lambda E(V)E(X)X''(1) &= V1'', \lambda E(S)E(X)X''(1) = S1'', \lambda^2 E(B)[E(X)]^2 = B1^2, \\ \lambda^3 E(B^3)[E(X)]^3 &= B2^3, \lambda^2 E(B)E(X)X''(1) = B1''^2 \end{aligned}$$

6.2 Expected length of idle period

Let I be the Random Variable (RV) of ‘idle period’. Let U be a RV defined by

$$U = \begin{cases} 0, & \text{if the queue length is at least 'a' after the vacation.} \\ 1, & \text{if the queue length is less than 'a' after the single service.} \end{cases}$$

The expected length of idle period, $E(I)$ is given by

$$E(I) = E(V)P(U = 0) + [E(V) + E(I)]P(U = 1)$$

where $E(V)$ is the mean vacation time

From equation (25), we have

$$D(z, 0) = \tilde{N}(\lambda - \lambda Y(z)) \sum_{j=0}^{a-1} e_j z^j = \sum_{j=0}^{\infty} \alpha_j z^j \sum_{j=0}^{a-1} e_j z^j$$

That is,

$$\sum_{j=0}^{\infty} f_j z^j = \sum_{j=0}^{a-1} \left[\sum_{i=0}^n e_{j-i} \alpha_i z^j \right] + \sum_{j=a}^{\infty} \left[\sum_{i=0}^{a-1} e_i \alpha_{j-i} z^j \right]$$

Equating the coefficient of z^n ($n=0,1,2,\dots,a-1$) on both sides, we get

$$P(U=0) = 1 - \sum_{j=0}^{a-1} f_j z^j = 1 - \sum_{j=0}^{a-1} \sum_{i=0}^n e_{j-i} \alpha_i z^j$$

Now, $E(I)$ is given as

$$E(I) = \frac{E(V)}{P(U=0)} = \frac{E(V)}{1 - \sum_{j=0}^{a-1} \sum_{i=0}^n e_{j-i} \alpha_i z^j}$$

6.3 Expected length of busy period

Let B be the RV of “busy period”. Let J be RV defined by

$$J = \begin{cases} 0, & \text{if the queue length is less than } a \text{ after single vacation} \\ 1, & \text{if the queue length is at least } a \text{ after the batch service} \end{cases}$$

$$E(B) = \frac{E(S)}{P(J=0)} = \frac{E(S)}{\sum_{j=0}^{a-1} (e_j + f_j)}$$

Where $E(S)$ is the mean service time

6.4 Probability that the server is on vacation, single service and batch service

Now, from the equation (29), we have

$$P(V) = E(V) \sum_{j=0}^{a-1} e_j$$

Now, from the equation (26), we have

$$P(S) = E(S) \sum_{j=0}^{a-1} f_j$$

Now, from the equation (27) and (28), we have

$$P(B) = \lim_{Z \rightarrow 1} \left(\sum_{i=a}^{b-1} \tilde{G}_i(z, 0) + G_b(z, 0) \right) = E(B) \sum_{j=a}^{b-1} (e_j + f_j) + \frac{E(B)}{b(1-\rho)} f'(1)$$

$$\text{where } f'(1) = \lambda E(X) E(B) \sum_{j=a}^{b-1} (e_j + f_j) z^j + \lambda E(X) E(V) \sum_{j=0}^{a-1} e_j z^j + \lambda E(X) E(S) \sum_{j=0}^{a-1} f_j z^j - \sum_{j=0}^{b-1} (e_j + f_j) z^j, \rho = \frac{\lambda E(X) E(B)}{b}$$

Particular Cases

In this section, some of the existing models are deduced as a particular case of the proposed model.

Case-1

When $a=b=1$ (“no bulk service”), the equation (31) reduces to

$$P(z) = \frac{(\tilde{L}(\lambda - \lambda Y(z)) - 1)(z - 1)f_0 + (\tilde{N}(\lambda - \lambda Y(z)) - 1)(z - 1)e_0}{(-\lambda + \lambda Y(z))(z - \tilde{M}(\lambda - \lambda Y(z)))}$$

Which coincides with the results of Choudhury [3].

Case-2

When $\tilde{L}(\lambda - \lambda Y(z)) = 1$ (“no single service”), the equation (31) reduces to

$$P(z) = \frac{(\tilde{M}(\lambda - \lambda Y(z)) - 1)(z^b - 1) \sum_{i=a}^{b-1} (e_i + f_i) z^i + (\tilde{N}(\lambda - \lambda Y(z)) - 1)(z^b - 1) \sum_{i=0}^{a-1} e_i z^i}{(-\lambda + \lambda Y(z))(z^b - \tilde{M}(\lambda - \lambda Y(z)))}$$

Which coincides with the results of Krishna Reddy et al.[18].

7. Conclusion

The batch arrival queueing systems with two types of service pattern with single vacation have been studied in this paper. In the proposed model, if the queue length is more than or equal to the minimum of batch size ‘a’, the bulk service can be provided otherwise the single service or single vacation is provided. So, the service will not be stopped during service except the vacation period. We have established the system size distribution of the proposed model and the performance measures of the given queueing model by using PGF technique.

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