Bright and Dark Bragg Solitons in a Fiber Bragg Grating

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***Abstract*—In this paper, we investigate the nonlinear pulse propagation through the fiber Bragg grating structure where the pulse dynamics is governed by the nonlinear-coupled mode (NLCM) equations. Using the multiple scale analysis, we reduce the NLCM equations into the perturbed nonlinear Schrödinger (PNLS) type equation. To construct the bright and dark Bragg solitons in the upper and lower branches of the dispersion curve, we solve the PNLS equation using the coupled amplitude-phase method.**

***Index Terms*—Coupled-mode equation, fiber Bragg grating (FBG), nonlinear wave propagation, soliton.**

I. Introduction

IT IS WELL KNOWN that the light propagating through a periodic medium gives rise to the so-called the stop band wherein the wave vector is imaginary. It physically means that the wave amplitude decays exponentially within the stop band with respect to the distance [1]. After the invention of the laser, there has been much interest in propagating nonlinear pulses through the periodic medium such as a fiber Bragg grating (FBG), which is a periodic variation of the refractive index of the fiber core along the length of the fiber. In the FBG, we also have the stop band known as a photonic bandgap (PBG) that does not allow the propagation of light pulses when the Bragg condition is satisfied. The studies on FBGs have attracted great attention for many years [2]–[18]. The pulse propagation through the FBG is described by the nonlinear-coupled mode (NLCM) equations which are nonintegrable in general. Therefore, the analytical solutions of the NLCM equations are not solitons but solitary waves that can propagate through FBG without changing their shape. Gap solitons are obtained from the NLCM equations and their spectra lie within the photonic bandgap structure. There is another class of solitons called Bragg solitons whose frequencies fall close to, but outside, the band edge of the photonic bandgap. These are obtained from the approximated nonlinear Schrödinger (NLS) equation that results from reducing the NLCM equations using the multiple scale analysis. Generally speaking, the gap solitons are the special class of Bragg solitons. For the first time, Chen and Mills [2] have analyzed the properties of these gap solitons in nonlinear periodic structure. Thereafter, Sipe and Winful [3], Christodoulides and Joseph [4], Aceves and Wabnitz [5], and Winful *et al.* [6] have obtained the analytical solutions for the Bragg solitons. These solitons in FBGs have been extensively reviewed in [7]–[9].

Recently conducted experiments have provided strong evidence for the existence of both theoretically predicted gap solitons and Bragg solitons in FBGs. For the first time, Eggleton *et al.* [10], [11] examined nonlinear pulse propagation through Bragg gratings at frequencies outside the bandgap (where the grating is transmissive but highly dispersive) and succesfully demonstrated the propagation of Bragg solitons. Taverner *et al.* [12], [13] reported the first observation of gap soliton generation in a Bragg grating at frequencies within the photonic bandgap.

The researchers recently have realized the potential applications of these solitons in periodic structures for all-optical switching [14], pulse compression [15], [16], limiting [17], [18], and logic operations [14], [18].

So far, we have discussed the pulse propagation in a one-dimensional (1-D) periodic medium, which consists of positive (or negative) Kerr coefficients only. Recently, Pelinovsky *et al.* [17] have used alternating layers with Kerr coefficients of opposite signs, i.e., both positive and negative Kerr coefficients along the pulse propagation direction in optical gratings. However, this structure possesses high local nonlinearity inside each individual layer. The method of compensating with the Kerr nonlinearities is termed as nonlinearity management of the refractive optical gratings. Nowadays, nonlinearity management is widely employed in the fabrication of structures based on second-harmonic generation to achieve quasi-phase matching (QPM) of the nonlinear interactions [19].

In this paper, we aim to investigate both bright and dark Bragg solitons in such a 1-D periodic structure consisting of alternating layers of nonlinear materials with opposite Kerr coefficients. Therefore, we reduce the NLCM equations into a well-known PNLS equation that describes the nonlinear pulse propagation at the edges of the PBG structure. The solitons existing near the PBG edge are called Bragg solitons. Now we wish to investigate these solitons by solving the PNLS equation through a coupled amplitude-phase method. The rest of the paper is arranged as follows. In Section II, we present the theoretical background of NLCM equations. In Section III, we use the multiple scale analysis to reduce the NLCM equations into a PNLS equation. In

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Section IV, we solve the PNLS equation using a coupled amplitude-phase method, and this ultimately shows the existence of bright and dark Bragg solitons.

II. Theoretical Model

The nonlinear pulse propagation in a periodic nonlinear structure consisting of N alternating layers with different linear refractive indices and different Kerr nonlinearities is governed by the NLCM equations of the form [17]

$$\begin{aligned} &i\frac{\partial A_+}{\partial z} + i\frac{\partial A_+}{\partial t} + \kappa A_- + \alpha\left(|A_+|^2 + 2|A_-|^2\right)A_+ \\ &\quad + \beta\left[\left(2|A_+|^2 + |A_-|^2\right)A_- + A_+^2 A_-^*\right] = 0 \\ &-i\frac{\partial A_-}{\partial z} + i\frac{\partial A_-}{\partial t} + \kappa A_+ + \alpha\left(|A_-|^2 + 2|A_+|^2\right)A_- \\ &\quad + \beta\left[\left(2|A_-|^2 + |A_+|^2\right)A_+ + A_-^2 A_+^*\right] = 0 \end{aligned} \tag{1}$$

where A_+ and A_- are the slowly varying amplitudes of forward and backward propagating waves. The term $\kappa(= n_{ok})$ is the variance of the linear refractive index or strength of the linear grating, $\alpha(= n_{\rm nl})$ is the average Kerr nonlinearity across the structure and $\beta(= n_{2k})$ is the variance of the Kerr nonlinearity between the layers. Pelinovsky *et al.* [17] and Brzozowski *et al.* [18] extensively investigated the above equations in order to study the all-optical limiting in the nonlinear periodic structures.

Before embarking into the discussion of NLCM equations, first we briefly discuss the pulse propagation in the linear regime wherein the nonlinear effects α and β become zero. Now, we discuss the dispersion relation associated with the linear coupled mode (LCM) equations as follows. In the linear case, the solutions to the LCM equations are given by

$$A_\pm = U_\pm e^{i(k_z z - \Omega t)} + c.c \tag{2}$$

where k_z and Ω satisfy the following dispersion relation $\Omega^2 = \kappa^2 + k_z^2$. In this relation, when we impose the condition $k_z = 0$, we get $\Omega = \pm\kappa$. Under this condition, the linear problem has the solution of the form

$$\begin{pmatrix} A_+ \\ A_- \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-i\kappa t} + c.c. \tag{3}$$

It is seen that the above solution satisfies the relation

$$L\begin{pmatrix} A_+ \\ A_- \end{pmatrix} = 0.$$

Here L represents the operator and is given by

$$L = \begin{pmatrix} i\frac{\partial}{\partial t} & \kappa \\ \kappa & i\frac{\partial}{\partial t} \end{pmatrix}.$$

When we introduce the nonlinearity into the system, linear coupled-mode equations change to nonlinear coupled-mode equations. The construction of bright and dark soliton solutions in the upper and lower branches of the dispersion curve follows from the multiple scale analysis, which is discussed below.

III. Multiple Scale Analysis

In FBGs, so far, mainly two theoretical approaches have been developed to describe the nonlinear pulse propagation. The first one is based on the NLCM equations that describe a coupling between forward and backward traveling modes. It is the usual practice to describe the pulse propagation through a periodic medium using NLCM equations when the refractive-index modulation is weak, i.e., $\alpha P_0 \gg \kappa$. In general, these NLCM equations are nonintegrable and are applicable anywhere in the PBG structure. However, in a few cases, NLCM equations have analytical solutions representing the solitary wave solutions. The most general form of the solitary wave solutions to the NLCM equations, for the first time, were derived by the Aceves and Wabnitz [5] and the details follow. The solutions of Aceves *et al.* to the NLCM equations (with "β" being zero) are given by

$$A_\pm = \mu\tilde{A}_\pm \exp[i\eta(\theta)]$$

where the parameter $\tilde{A}_\pm$ are

$$\begin{aligned} \tilde{A}_+ &= \mp\sqrt{\frac{\pm\kappa}{2}}\frac{1}{\Delta}\sin\,\tilde{\delta}\,\exp(\pm i\sigma)\sec h\left(\theta \pm \frac{i\tilde{\delta}}{2}\right) \\ \tilde{A}_- &= -\sqrt{\frac{\pm\kappa}{2}}\Delta\sin\,\tilde{\delta}\,\exp\left(\pm i\sigma\right)\sec h\left(\theta \pm \frac{i\tilde{\delta}}{2}\right) \end{aligned} \tag{4}$$

with

$$\begin{aligned} \theta &= \kappa\gamma(\sin\tilde{\delta})(z - \nu t). \quad \sigma = \kappa\gamma(\cos\tilde{\delta})(\nu z - t). \\ \nu &= \frac{(1 - \Delta^4)}{(1 + \Delta^4)} \quad \gamma = \frac{1}{\sqrt{1 - \nu^2}}. \end{aligned}$$

In (4), γ is the Lorentz factor. Since the expressions for μ and η are well known [5], we do not present them here. Here, the soliton-like solutions for the NLCM equations are considered to be a generalization of the Massive Thirring model (MTM). As a special case, when $\tilde{\delta} = \pi/2$, the most general solutions of Aceves and Wabnitz lead to the slow Bragg solitons, already predicted by Christodulides and Joseph [4].

The second one is based on the NLS equation (Bloch wave analysis). This approach is more general when compared to the first one because the NLCM equations are reduced to an NLS equation under the low intensity limit. That is, when the peak intensity of the pulse is small enough that the nonlinear index change $n_2 I$ is much smaller than the maximum value of δn, i.e, $\alpha P_0 \ll \kappa$. Usually the technique known as multiple scale analysis is adopted to deduce the NLS equation from the NLCM equations [3], [7]. Very recently, under the low intensity limit, Aceves [20] derived a perturbed two-dimensional (2-D) NLS equation to describe the gap soliton bullets in the Kerr-type planar waveguides. Therefore, in this paper, to describe the pulse propagation in FBGs with deep index modulation, we also adopt the multiple scale analysis [7], [20], a technique already followed by Aceves [20] to reduce the NLCM equations into the NLS-type equation. This would also mean that the center frequency of the pulse is being tuned out of the photonic bandgap structure. Now the pulse propagation in FBGs is governed by the NLS equation, which in turn can be easily integrable by the standard Inverse Scattering Transform (IST).

First we find the solution of the above equation in the linear case and then introduce the nonlinearity into the coupled mode equations. In order to introduce the multiple scale analysis, we extend the linear solution to the following form:

$$\begin{pmatrix} A_+ \\ A_- \end{pmatrix} = \varepsilon^{1/2} A(\tau_1, \tau_2, X, Z) \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-i\kappa t} + \varepsilon U_1 + \varepsilon^{3/2} U_2 + \varepsilon^2 U_3 + \cdots \quad (5)$$

where $\tau_1 = \varepsilon t$, $\tau_2 = \varepsilon^2 t$, $X = \varepsilon^{1/2} x$, and $Z = \varepsilon^{1/2} z$. Now, we proceed to solve for (A_+, A_-) for successive orders in ε. Balancing the $O(\varepsilon)$ terms gives

$$LU_1 = -i\frac{\partial A}{\partial z} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i\kappa t}. \quad (6)$$

The solution of the above equation is found to be

$$U_1 = -\frac{i}{2\kappa}\frac{\partial A}{\partial z} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i\kappa t}. \quad (7)$$

Next, we turn to compute the higher order corrections to (A_+, A_-). Balancing the $O(\varepsilon^{3/2})$ terms gives

$$LU_2 = \left(-i\frac{\partial A}{\partial \tau_1} - \frac{1}{2\kappa}\frac{\partial^2 A}{\partial z^2} - (3\alpha - 4\beta)|A|^2 A\right) \times \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-i\kappa t} + \text{c.c.} \quad (8)$$

In order to solve the above equation, the secular terms should be equated to zero and therefore

$$i\frac{\partial A}{\partial \tau_1} + \frac{1}{2\kappa}\frac{\partial^2 A}{\partial z^2} + (3\alpha - 4\beta)|A|^2 A = 0. \quad (9)$$

The above equation represents the well-known NLS equation, which is used to describe the picosecond pulse propagation in the fiber. In FBGs, (9), represents the pulse propagation outside the PBG structure. In (9), the variable "A" represents the amplitude of the envelope associated with the Bloch wave formed by a superposition of A_+ and A_-. For the first time, Sipe and Winful [3] derived this kind of NLS-type equation from the NLCM equations. Sterke and Sipe [21] derived the NLS equation based on the envelope function approach and also presented the soliton solution outside the PBG but within FBG. Without deriving the NLS equation from NLCM equations, Feng and Kneubuhl [22] investigated the formation of new types of solitary wave solutions called gap solitary wave solutions (bright and dark) in the periodic structure. Recently, Iizuka and Wadati [23] used the reductive perturbation method (RPM) and derived a similar type of NLS equation in FBGs. They obtained explicit forms of both bright and dark analytical soliton solutions in FBG which are presented as follows:

$$A = \sqrt{\frac{1}{\kappa r}} C_1 \sec h\left(C_1 z - \frac{C_1 C_2}{\kappa}\tau_1\right) \times \exp i\left\{C_2 z + \frac{1}{2\kappa}\left(C_1^2 - C_2^2\right)\tau_1\right\} \quad (10)$$

$$A = \sqrt{-\frac{1}{\kappa r}} C_1 \frac{1 + \exp(C_3 z - C_4\tau_1 + i\phi)}{1 + \exp(C_3 z - C_4 \tau_1)} \times \exp i\left\{C_2 z - \frac{1}{2\kappa}\left(2C_1^2 + C_2^2\right)\tau_1\right\} \quad (11)$$

where $r = (3\alpha - 4\beta)$. The factors C_1, C_2, C_3, and C_4 are constants. The constant C_4 and $\exp(i\phi)$ are given by

$$C_4 = \left(\frac{C_3}{2\kappa}\right)\left[2C_2 + \sqrt{4C_1^2 - C_3^2}\right]$$

$$\exp(i\phi) = \frac{C_3 + i\sqrt{4C_1^2 - C_3^2}}{C_3 - i\sqrt{4C_1^2 - C_3^2}}.$$

Equations (10) and (11) represent the solutions for the bright and dark Bragg solitons. These solitons were simply referred to as *grating solitons* by Iizuka and Wadati [23]. Further, they showed that these grating solitons become gap solitons when the group velocity becomes zero. Thus, (9) has been extensively investigated by many authors. Aceves [20], in his recent work, considered higher order effects in FBGs and hence derived a perturbed NLS equation to describe gap soliton bullets in planar waveguides. In the present paper, we also consider higher order effects in FBG and derive the perturbed NLS equation. Then, we solve for it by studying the formation of bright and dark Bragg solitons in both upper and lower branches of the dispersion curve in an FBG. In order to see the impact of higher order effects, we continue to balance $O(\varepsilon^2)$ terms, and this gives rise to

$$LU_3 = \left(i\frac{\alpha}{2\kappa}\left(2|A|^2\frac{\partial A}{\partial z} + A^2\frac{\partial A^*}{\partial z}\right) - \frac{i}{2\kappa}\left(-i\frac{\partial A}{\partial \tau_1}\right)\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i\kappa t} + \text{c.c.} \quad (12)$$

From (7), we have

$$\frac{\partial}{\partial z}\left(-i\frac{\partial}{\partial \tau_1}\right)A = \frac{1}{2\kappa}\frac{\partial^3 A}{\partial z^3} + (3\alpha - 4\beta)\left(2|A|^2\frac{\partial A}{\partial z} + A^2\frac{\partial A^*}{\partial z}\right). \quad (13)$$

Therefore, the equation for U_3 can be written as

$$U_3 = -\frac{i}{4\kappa^2}\left[(3\alpha - 4\beta)\left(2|A|^2\frac{\partial A}{\partial z} + A^2\frac{\partial A^*}{\partial z}\right) + \frac{1}{2\kappa}\frac{\partial^3 A}{\partial z^3}\right]\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i\kappa t} + \text{c.c.} \quad (14)$$

Equation (14) represents the perturbation terms that must be added to the NLS equation when we consider the higher order effects in the FBG structure. With this result, the NLS equation changes into PNLS equation, which is presented as follows:

$$i\frac{\partial A}{\partial \tau_1} + \frac{1}{2\kappa}\frac{\partial^2 A}{\partial z^2} + (3\alpha - 4\beta)|A|^2 A + \frac{1}{8\kappa^3}\frac{\partial^3 A}{\partial z^3} + \frac{i}{4\kappa^2}(3\alpha - 4\beta)\left(2|A|^2\frac{\partial A}{\partial z} + A^2\frac{\partial A^*}{\partial z}\right) = 0. \quad (15)$$

The above PNLS equation represents the nonlinear pulse propagation in a periodic medium with higher order effects outside the PBG structure in an FBG. Here, it should be noted that, for the first time, Aceves derived this kind of PNLS equation in his recent work [20]. As we discussed in the introduction, we are interested in analyzing Bragg solitons with higher order effects in a periodic medium. Here, we investigate Bragg solitons at both upper and lower branches of the PBG. It also physically means that we consider both positive (upper branch) and negative (lower branch) Kerr nonlinearities in our investigation and correspondingly they give rise to both bright and dark Bragg solitons.

IV. Bright and Dark Bragg Solitons

As stated above, in this section, we will construct both bright and dark Bragg solitons for the PNLS equation that describes the nonlinear pulse propagation with higher order effects in the periodic medium. Using the coupled amplitude-phase method, we solve the PNLS equation and obtain both bright and dark Bragg solitons. For this purpose, we rewrite (15) in the form

$$i\frac{\partial A}{\partial \tau_1} + a\frac{\partial^2 A}{\partial z^2} + ib\frac{\partial^3 A}{\partial z^3} + c\,|A|^2 A + 2id\left(|A|^2 A\right)_z = 0$$
$$a = \frac{1}{2\kappa},\quad b = \frac{1}{8\kappa^3},\quad c = (3\alpha - 4\beta)$$
$$d = \frac{(3\alpha - 4\beta)}{4\kappa^2} = c\,a^2. \tag{16}$$

The coefficients a and b represent, namely, second- and third-order dispersions. The last term in the above equation accounts for self-steepening which results from including the first derivative of the slowly varying part of the nonlinear polarization.

Now we will solve (16) by the coupled amplitude- phase method, which was introduced in [24]. To start with, we consider the solution of the form

$$A(z,\tau_1) = Q(\tau_1 + \beta_1 z)\exp\left[i\left(kz - \omega\tau_1\right)\right] \tag{17}$$

where the function Q is a real one. The unknown parameters k and ω are directly related to the shifts in the wavenumber and frequency, respectively, while the factor β_1 is the group velocity of the wave. Substituting (17) into (16) and removing the exponential term, we obtain

$$i\beta_1 Q_\chi - kQ + a\left[Q_{\chi\chi} - 2i\omega Q_\chi - Q\omega^2\right]$$
$$+ b\left[iQ_{\chi\chi\chi} + 3\omega Q_{\chi\chi} - 3i\omega^2 Q_\chi - Q\omega^3\right]$$
$$+ cQ^3 + d\left[3iQ^2 Q_\chi + \omega Q^3\right] = 0.$$

Now, separating the real and imaginary parts, we have

$$\beta_1 Q_\chi + bQ_{\chi\chi\chi} + (-2a\omega - 3b\omega^2 + 3dQ^2)Q_\chi = 0 \tag{18}$$
$$-kQ + (a + 3b\omega)Q_{\chi\chi} + (c + d\omega)Q^3 - (a\omega^2 + b\omega^3)Q = 0. \tag{19}$$

Since (18) possesses only third- and first-order derivatives, it can be written in the following form:

$$bQ_{\chi\chi\chi} = \left(-\beta_1 + 2a\omega + 3b\omega^2 - 3dQ^2\right)Q_\chi.$$

Integrating this, we obtain

$$Q_{\chi\chi} = \left(\frac{2a\omega + 3b\omega^2 - \beta_1}{b}\right)Q - \left(\frac{d}{b}\right)Q^3. \tag{20}$$

Writing (19) in the following form

$$Q_{\chi\chi} = \left(\frac{k + a\omega^2 + b\omega^3}{a + 3b\omega}\right)Q - \left(\frac{c + d\omega}{a + 3b\omega}\right)Q^3 \tag{21}$$

it is clear that (20) and (21) can be equivalent only under the following conditions:

$$\left(\frac{2a\omega + 3b\omega^2 - \beta_1}{b}\right) = \left(\frac{k + a\omega^2 + b\omega^3}{a + 3b\omega}\right)$$
$$\left(\frac{d}{b}\right) = \left(\frac{c + d\omega}{a + 3b\omega}\right).$$

From the above relations, we find ω and k as

$$\omega = \frac{cb - da}{2bd} \text{ and}$$
$$k = \frac{\left(2a\omega + 3b\omega^2 - \beta_1\right)(a + 3b\omega) - ab\omega^2 - b^2\omega^3}{b}. \tag{22}$$

Equation (20) can also be written as

$$\left(\frac{dQ}{d\chi}\right)^2 = \left(\frac{2a\omega + 3b\omega^2 - \beta_1}{b}\right)Q^2 - \frac{1}{2}\left(\frac{d}{b}\right)Q^4 + C$$

where C is an arbitrary constant of integration. Further, it can be written as

$$d\chi = \frac{dQ}{\sqrt{\left(\frac{2a\omega + 3b\omega^2 - \beta_1}{b}\right)Q^2 - \frac{1}{2}\left(\frac{d}{b}\right)Q^4 + C}}. \tag{23}$$

From the above equation, it is possible to get the different analytical solutions for different values of the constant of integration C. Among these solutions, we focus our attention on the solutions of bright and dark Bragg solitons. Now, we discuss how the bright soliton is formed outside the PBG but inside the FBG. Thereafter, we apply the same condition to the physical parameters in (23) and finally we obtain the bright soliton solution analytically. To discuss the bright soliton formation, we choose the positive nonlinearity and hence we have the self-focusing effect in an FBG structure. Because of the self-focusing effect, the central frequency of the carrier wave is tuned close to but outside the photonic bandgap of the periodic structure. It physically means that the central frequency is moved to the upper branch of the dispersion curve, where the grating-induced group velocity dispersion (GVD) is anomalous. This anomalous GVD exactly gets balanced with the nonlinearity (self-focusing effect) and, as a result, we have the bright soliton formation outside the PBG but inside the periodic (FBG) structure, which is termed a *bright Bragg soliton*. As we consider positive nonlinearity in the formation of a bright soliton, we choose the cubic nonlinear term as positive and $C = 0$ in the above equation, and obtain the following bright soliton solution:

$$Q_{xx} = \sqrt{\frac{2\left(2a\omega + 3b\omega^2 - \beta\right)}{d}} \times \text{sech}\left(\sqrt{\frac{2a\omega + 3b\omega^2 - \beta}{b}}\right)\chi.$$

Now, substituting the bright soliton envelope in (17), we obtain

$$A = \sqrt{\frac{2\left(2a\omega + 3b\omega^2 - \beta_1\right)}{d}} \times \text{sech}\left(\sqrt{\frac{2a\omega + 3b\omega^2 - \beta_1}{b}}\right)\chi\, e^{i(kz - \omega\tau_1)}. \tag{24}$$

In the above case, the envelope satisfies the bright soliton whose carrier frequencies lie close to but outside the band edge of the PBG and hence we call it a bright Bragg soliton. It should be noted that the existence of solitary waves in the upper branch of the dispersion curve has already been experimentally demonstrated [10].

From the experimental point of view, it is necessary to know the magnitude of the peak power P_0 to excite the bright Bragg soliton. Similarly, the soliton period T_0 turns out to be another important physical parameter that is involved in the formation of a Bragg soliton. Based on [25], from the bright Bragg soliton solution, we calculate the important and interesting physical parameters such as soliton power and pulse width in the form

$$T_0 = \sqrt{\frac{1}{(2a\omega + 3b\omega^2 - \beta_1)}} \tag{25}$$

$$P_0 = \frac{2\left(2a\omega + 3b\omega^2 - \beta_1\right)}{d}. \tag{26}$$

With the known values of the parameters a, b, c, and d in an FBG, we can calculate ω using (22). After calculating the value of ω from (22), for a given T_0, we can easily calculate the value of β_1 from (25). By computing all the values of parameters of (26), we can calculate the power required for generating the bright Bragg soliton.

Similarly, there is another interesting class of solitons called dark solitons and now we discuss the formation of the same in the FBG. Instead of positive nonlinearity, we consider the negative nonlinearity, which gives rise to the self-defocusing effect in the FBG. This self-defocusing effect shifts the central frequency of the carrier wave to the lower branch of the dispersion curve where we have normal GVD. This normal GVD exactly gets balanced with the negative nonlinearity (self-defocusing effect) and, as a result, we get the dark soliton formation outside the PBG but inside the FBG structure. This soliton is referred to as a *dark Bragg soliton*. For analytical purposes, considering the negative nonlinearity, the constant in (23) is chosen in such a way that the value of the expression inside the square root is a perfect square and hence we obtain the dark solitary wave solution as follows:

$$Q = \sqrt{\frac{(2a\omega + 3b\omega^2 - \beta_1)}{d}}\ \tanh\left(\sqrt{\frac{2a\omega + 3b\omega^2 - \beta_1}{2b}}\right)\chi$$

On substituting the dark soliton envelope in (17), we have

$$A = \sqrt{\frac{(2a\omega + 3b\omega^2 - \beta_1)}{d}} \times \tanh\left(\sqrt{\frac{2a\omega + 3b\omega^2 - \beta_1}{2b}}\right)\chi e^{i(kz-\omega\tau_1)}. \tag{27}$$

As stated above, here also the field satisfies the dark soliton solution and their carrier frequencies are close to the band edge, and hence we call the above solution the dark Bragg soliton solution. The main results of this paper are similar to those of Iizuka and Wadati [23].

As has been discussed in the bright soliton case, it is also possible to calculate the power and pulse width for the dark Bragg soliton case and the same is given as follows:

$$T_0 = \sqrt{\frac{2}{(2a\omega + 3b\omega^2 - \beta_1)}} \tag{28}$$

$$P_0 = \frac{\left(2a\omega + 3b\omega^2 - \beta_1\right)}{d}. \tag{29}$$

As discussed in the bright soliton case, by knowing all the parameter values, one can calculate the power required to generate dark Bragg soliton.

V. Conclusion

We have considered the pulse propagation in a 1-D periodic medium that consists of alternating Kerr coefficients with deep index modulation, and hence the governing NLCM equations were reduced to PNLS equation using the multiple scale analysis. The PNLS equation that incorporates both the higher order dispersive effects and self-steepening effects has been derived using the multiple scale analysis. From the PNLS equation, bright and dark Bragg solitary wave solutions have been constructed by a coupled-phase amplitude method. We have also calculated the important and interesting physical parameters such as power and pulse width for both bright and dark Bragg solitons. By knowing all the physical parameters, one can calculate the minimum power required to generate the solitons in the FBG structure.

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