Design of Linear-Quadratic-Regulator for a CSTR process

IOP Conf. Series: Materials Science and Engineering **263** (2017) 052013 doi:10.1088/1757-899X/263/5/052013

Design of Linear-Quadratic-Regulator for a CSTR process

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Abstract-This paper aims at creating a Linear Quadratic Regulator (LQR) for a Continuous Stirred Tank Reactor (CSTR). A CSTR is a common process used in chemical industries. It is a highly non-linear system. Therefore, in order to create the gain feedback controller, the model is linearized. The controller is designed for the linearized model and the concentration and volume of the liquid in the reactor are kept at a constant value as required.

1. Introduciton

A CSTR is a very common process that is used in almost all the chemical and process industries. There are many different kinds of CSTRs available and which can be used according to our requirements, that is, adiabatic and non-isothermal CSTR[6]. This paper deals with the temperature control of non-isothermal CSTR. The main idea of optimal control is to minimize the cost function of the given dynamic system. The LQR is a very reliable and tested method for minimizing the cost function of a system. The resultant of the LQR is used to design a gain feedback controller. The name (LQR) is given from the fact that the dynamics of the system is given in the form of linear differential equations and the cost function is in a quadratic form. The cost function is a representation of the deviation of the controlled variable from the set point. Our work is to minimize the cost function by adding weights as per requirement. The LQR will greatly reduce the effort taken by the humans even though it does certainly include a lot of trial and error and the control engineer can go for many other methods such as controller feedback design via pole placement technique. In this paper we take a normal non isothermal CSTR system and model it. The modeling is followed by the linearization of the system[3,5]. MATLAB is used to perform the LQR operation.

2. Methodology

In a non-isothermal CSTR, there will be an exothermic reaction that will convert one reactant to the product plus some energy. The energy that is produced as a resultant of the reaction will be removed by the coolant medium. In steady state, the heat removed by the coolant medium should be equal to the heat that is produced by the reaction. The increasing temperature of the coolant will increase the temperature of the reactant mixture and thus increase the reactant rate and thus lead to energy that is released. Thus, if our aim is to control the temperature of the product we manipulate the temperature of the coolant.

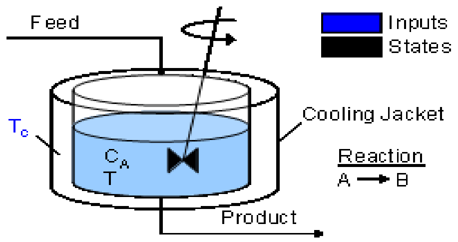

Figure 1. Non isothermal CSTR

2.1 Nonlinear model

$$\frac{dc_a}{dt} = \frac{Q}{V}(c_{ai} - c_a)\text{-}koe^{\frac{-E}{RT}}c_a \quad (1)$$

and

$$\frac{dT}{dt} = \frac{Q}{V}(T_i - T) + jkoe^{\frac{-E}{RT}}c_a \text{-} \frac{UAt}{\rho c_p V}(T_c - T) \quad (2)$$

In this particular model, the volume of the reacting mixture is assumed to be a constant.

The first equation is obtained from the mass balance relations and the second equation is obtained from the energy balance relations. The constants are given by the following:

c_a- Concentration of A in the CSTR (mol/m^3)
Q- Volumetric Flowrate (m^3/s)
V-Volume of the CSTR (m^3)
c_{ai}- Inlet concentration (mol/m^3)
k_o- constant (1/sec)
E- Activation Energy (J/mol)
R- Universal Gas constant (J/molk)
J- Heat of Reaction (J/mol)

U- Overall heat transfer coefficient (W/m^2-K)
A- Area (m^2)
T_i- Inlet Temperature (K)
T- Temperature in the CSTR (K)
T_c- Coolant temperature (K)
ρ- Density of the A-B mixture (Kg/m^3)

2.2 Linearized Model

The linearization of the model is done using taylor series and the higher order terms are neglected. The linearized model of the system is given by,

$$\acute{X}=\begin{bmatrix} -0.01 & 0 \\ 0 & -0.01 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}+\begin{bmatrix} 0 \\ 0.01 \end{bmatrix}[u] \quad (3)$$

$$Y=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (4)$$

X_1 – Concentration (c_a), X_2- Temperature (T)

Table 1: Parameter's values

Parameters	Values
C_a	4(mol/m^3)
Q	1(m^3/s)
V	100(m^3)
c_{ai}	1(mol/m^3)
K_o	.01(1/sec)
E/R	8697 (1/K)
J	10^4(J/mol)
UA	10^4 (W/K)
T_i	289(K)
T_c	289(K)
Rho	100 (Kg/m^3)

2.3 Application of LQR to the given model

For a system at which has a state space model given by,

$\acute{X}=AX+BU$ (5)

The cost function of this system is given by,

$J=\frac{1}{2}\int X^TQX+U^TRU\text{dt}$ (6)
Where Q and R are the weighting matrices,

The control law is given by,
$U=-KX=-R^{-1}B^TPX$ (7)

P can be obtained from,
$PA+A^TP-PBR^{-1}B^TP+Q=0$ (8)
$K=-R^{-1}B^TP$ (9)
The value of *K* can be calculated from the value of *P.*

2.4 Simulation

The quality of the controller designed depends on the Q and R matrices. These can be chosen only by trial and error method and by observation. That is, when we want a higher value for one state variable, we should increase the corresponding variable in the Q matrix and vice versa. Our first requirement is to get the gain matrix K.For that, we first design the Q and R matrices. The Q matrix is given by $\begin{bmatrix}1 & 0\\0 & 2500\end{bmatrix}$and theR matrix is given by $\begin{bmatrix}.01 & 0\\0 & .01\end{bmatrix}$Using the LQR command in MATLAB we got K= $\begin{bmatrix}0 & 499\\0 & 0\end{bmatrix}$. The control law used here is given by,
$U=-KX$ (10)
Using this control law, the following simulink model is created and the K matrix is entered in to it.

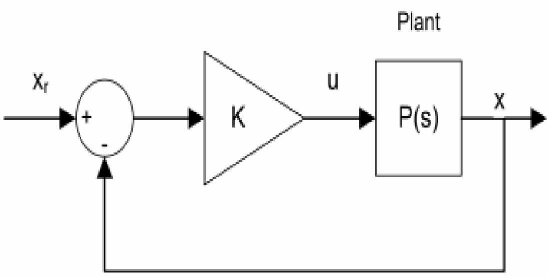

Figure 2. Simulink Block Diagram

IOP Conf. Series: Materials Science and Engineering **263** (2017) 052013 doi:10.1088/1757-899X/263/5/052013

3. Simulation Results

The figure 3 shows the response of temperature and concentration with respect to time. The temperature are given initial value of 289 K respectively. The control signal in this model affects the temperature as shown in Fig.3.

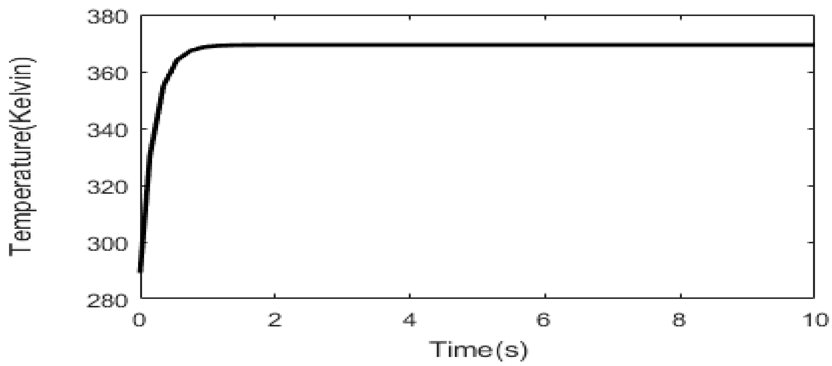

Fig.3. Temperature versus time response

The temperature will settle into the required steady state value without much delay as shown in the figure. This shows that the controller that we have designed is very much apt in controlling the temperature of a non isothermal CSTR. The following figures will illustrate the responses of the concentration and temperature with respect to time.

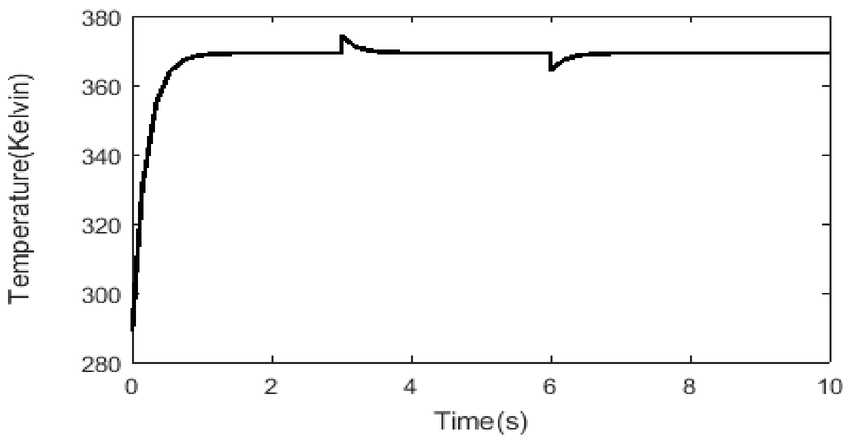

Figure 4. Response of temperature versus time when a step disturbance is given

The system is given with a step disturbance, from a time interval of 3 to 6 seconds with a magnitude of 5. When observing the figure, it can be seen that, when the system is met with a disturbance, the LQR makes the controller parameters change in such a way so as to overcome the disturbance in a matter of seconds.

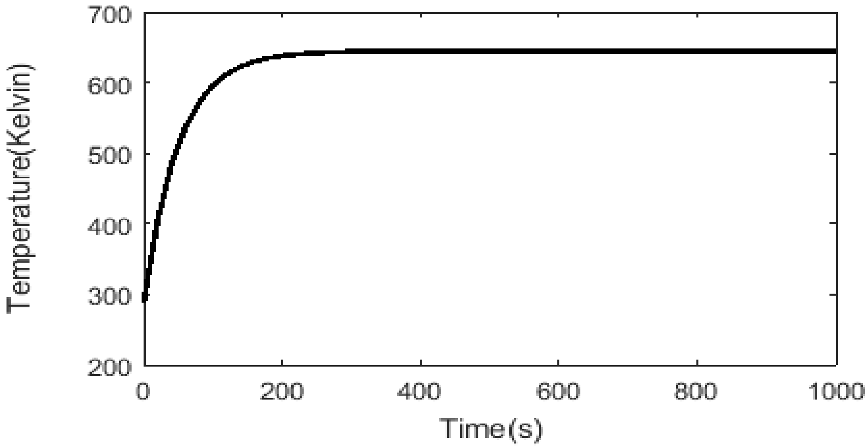

Figure 5. Open loop response of temperature

4. Conclusion

In this paper we have designed a gain feedback controller using LQR for a non-isothermal CSTR. We have linearized the highly nonlinear model of this system. After this LQR is designed with the help of MATLAB. From the outputs obtained, we can conclude that: To get a high value for a particular state variable, we can increase the corresponding value in the Q matrix and vice versa. By adjusting the values of the Q and R matrices we can improve the parameters of the system extensively such that, the steady state error and the settling time are reduced more than that from the usage of other controllers. Thus we can conclude that the LQR is a very good control method till date for any other control methods.

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