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Determination of rolling tyre modal parameters using Finite Element techniques and Operational Modal Analysis

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ABSTRACT

In order to address various noise generation mechanisms and noise propagation phenomena of a tyre, it is necessary to study the tyre dynamic behaviour in terms of modal parameters. This paper enumerates a novel method of finding the modal parameters of a rolling tyre using an Explicit Finite Element Analysis and Operational Modal Analysis (OMA). ABAQUS Explicit, a commercial Finite Element (FE) software code has been used to simulate the experiment, a tyre rolling over a semi-circular straight and inclined cleat. The acceleration responses obtained from these simulations are used as input to the OMA. LMS test lab has been used for carrying out the Operational Modal Analysis. The modal results are compared with the published results of Kindt [22] and validated. Also, the modal results obtained from OMA are compared with FE modal results of stationary unloaded tyre, stationary loaded tyre and Steady State Transport rolling tyre.

1. Introduction

Tyre noise is an important source in automotive NVH. The mechanism of noise generation, experimental procedure and theoretical underpinning are well documented [1]. Tyre/road noise generation, amplification and reduction phenomena are basically divided into two groups: vibrational and aerodynamical [1]. When a tyre is excited at the treadband, structural waves such as bending waves, shear waves and longitudinal waves propagate along the circumference. The frequencies of these waves are below 3000 Hz [2,3]. The tyre dynamic behaviour is determined by the propagation and interaction characteristics of these structural waves in the tyre. Though various noise generation mechanisms and their propagation phenomena are well addressed in the literature, the relative importance of these mechanisms under different operating conditions are not yet fully understood [1]. Tyre/road interaction is the main source for tyre vibrations. Modal frequencies, damping and mode shapes describe the dynamic behaviour of a tyre and form the basis for one of the important mechanisms of noise generation. Hence, there is a need to understand and evaluate the mode shapes of a rolling tyre.

Traditionally, these quantities are found from Experimental Modal Analysis (EMA) and/or FE analysis of a non-rolling tyre with the basic assumption of system linearity and reciprocity, obtained from the estimation of Frequency Response Functions (FRFs) for the known dynamic excitation force. But, for a rolling tyre, it is difficult to measure the excitation force at the tyre road interface. Hence, Operational Modal Analysis (OMA) is preferred, as this technique does not require input excitation force. Operational Modal Analysis makes use of the crosspower sum of the output responses with a suitable reference, for modal parameter extraction [4].

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Zegelaar [5] showed that the modes of vibration of a tyre strongly depend on tyre construction, rotational velocity, spindle boundary conditions and contact patch boundary conditions. Yam et al. [6] have obtained three dimensional mode shapes (radial, tangential and lateral directional modal parameters) of a tyre using EMA. Lauwagie et al. [7] have done a comparative study of modal parameter extraction procedure viz., OMA and EMA, either individually or in combination, in a small hydraulic crane modal analysis study.

The modal results of transient vibration of rolling tyre make significant contributions to tyre/road interaction noise prediction. Tyre modal data are used directly in Boundary Element Method and Infinite Element Method based tyre models. Nakajima et al. [8] have used the tyre modal results to define the surface vibration boundary conditions in order to predict air borne noise fields. Constant et al. [9] have studied the vibro-acoustic interaction between tyre and car subsystems and have identified the main suspension parts affecting the structure borne interior noise. Operational deflection shapes of tyre wheel subsystem were used by Kido and Ueyama [10] to quantify the force transmitted through suspension sub-system to the vehicle body; further they addressed the structure borne interior noise of the vehicle.

Several techniques have been developed by researchers to measure rotating tyre vibrations. Burroughs and Dugan [11] used embedded accelerometers, which were placed into the tread block to measure the tyre vibration responses, as it rotates through the contact patch. Though this technique gave good results, the embedded accelerometer frequently failed due to high stress in the contact patch zone. Contrary to the direct measurement techniques, contactless measurement technique was used by Kindt et al. [12,13] to measure the rotating tyre vibration. They used Laser Doppler Vibrometer to measure the vibration velocity normal to the rotating tyre surface. The spindle forces and moments were measured by multi-axial wheel hub dynamometer with an inbuilt encoder and a circular cleat in a tyre-on-tyre arrangement, to have an equivalent scenario of tyre rolling over straight road with straight semi-circular cleat.

Narasimha Rao et al. [14] have used a FE tyre model based on a hyperelastic material that included all the reinforcements, to study cornering, braking and cornering cum braking dynamic behaviour of tyre. This approach is adapted to develop a FE model of tyre for the current research work of structure borne interior and exterior tyre/road interaction noise.

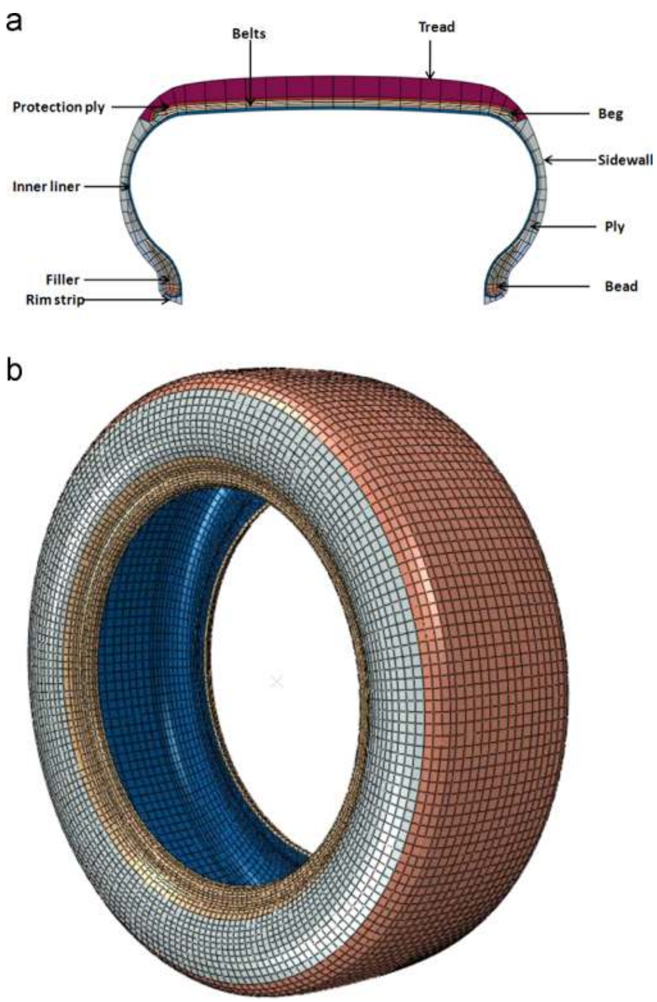

Fig. 1. (a) Section details of 205/55R16 radial passenger car tyre and (b) tyre FE model.

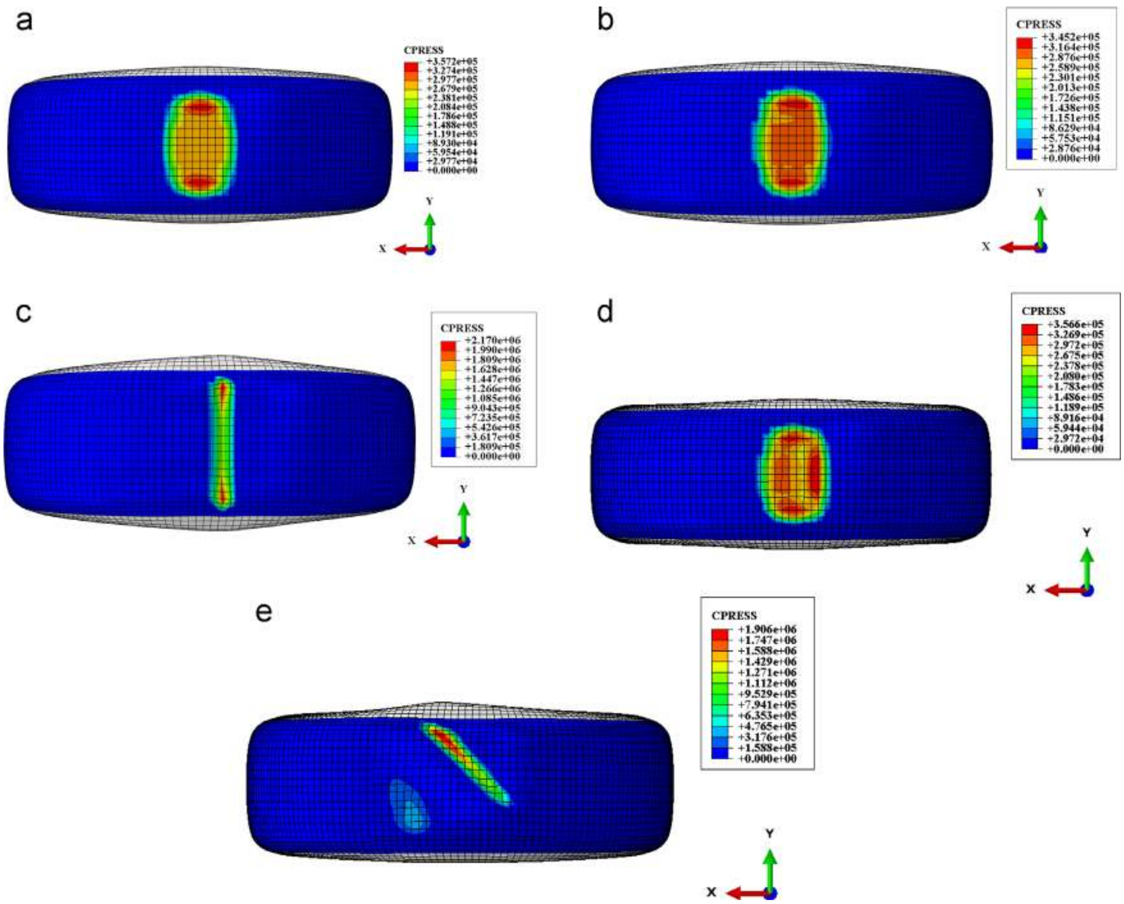

Fig. 2. (a) Contact pressure of stationary loaded tyre, (b) contact pressure of rolling tyre just before straight cleat, (c) contact pressure during straight cleat impact, (d) contact pressure of rolling tyre just before 45° inclined cleat and (e) contact pressure during 45° inclined cleat impact.

Determination of mode shapes of a rolling tyre is a challenging problem. Experiments are difficult to perform. Apart from the novel work of Kindt et al. [12,13] there has been no other experimental work to determine the mode shape of a rolling tyre.

Finite Element Analysis has been a popular technique to determine the mode shapes of a stationary tyre, both as a free body and under load with contact. Recently, Steady State Transport algorithms have been used as a basis for extracting mode shapes [15]. Contact condition has been used for tyre road contact. Contact definition for mode shape determination is a practical approach, but has theoretical limitation. This can easily be seen in the mode shapes published and in this work as well.

Lastly, there have been several attempts to understand and determine modal parameters analytically [16,17]. Here again, these models are limited by the inability to solve the actual loaded tyre, rolling on a road.

Hence, it is felt that a new approach is required to capture the mode shape. In this paper we replace the difficult experimental determination of deformation of a rolling tyre by numerical experiments, using an Explicit Finite Element Analysis. A pneumatic tyre is made to roll over a cleat, and the deformations captured using a commercial explicit Finite Element code ABAQUS [18]. These deformations are used as inputs to a commercial Operational Modal Analysis code from LMS [19] that uses the popular polyreference least square complex exponential frequency domain approach [20].

The Finite Element procedure and further processing with Operational Modal Analysis are discussed in Section 2. In Section 3, obtained rolling tyre modal parameters are compared with experimental results reported by Kindt et al. Also to make an assessment of this procedure, the results obtained, is compared with the mode shapes and resonance frequencies of stationary unloaded, stationary loaded and Steady State Transport rolling tyre. Finally, the conclusions are highlighted in Section 4.

2. Procedure for extraction of mode shape

2.1. Explicit Finite Element Analysis and Operational Modal Analysis

Passenger car radial tyre of size 205/55R16 has been chosen to construct a Finite Element model. The size is the same as that used by Kindt et al. [30] in their study. Fig. 1(a) and (b) shows the details of the tyre section and a smooth full tyre model that has been developed. The colour group of the tyre section describes the various components and constructional details of the tyre. The tyre includes two belts and one ply and one protection ply. The details of material and reinforcement definition are given in Appendix A.

Explicit dynamic rolling of this model over a semi-circular cleat of 25 mm radius has been carried out for the following operating conditions: inflation pressure of 250 kPa, static preloading of 3300 N and speed of 28.3 kmph to have an equivalent numerical experiment as that of tyre on tyre experimental arrangement of Kindt et al. [12]. The minimum cleat dimension of 25 mm is chosen so as to provide a higher energy level to the rolling tyre.

The contact conditions can be invoked by considering the two bodies to be deformable or one to be deformable and the other to be rigid. The contact is defined between two surfaces. Here, the contact is between the deformable tread, defined through its outer surface and the rigid road surface defined through an analytical expression. The definition is through surface interaction. During inflation and loading, the coefficient of friction is assigned to be zero and in the next step during rolling its value is one. In case of tyre rolling over inclined cleat contact pair is defined between tread outer surface and road discrete rigid surface that is created based on the outer surface definition of the road elements.

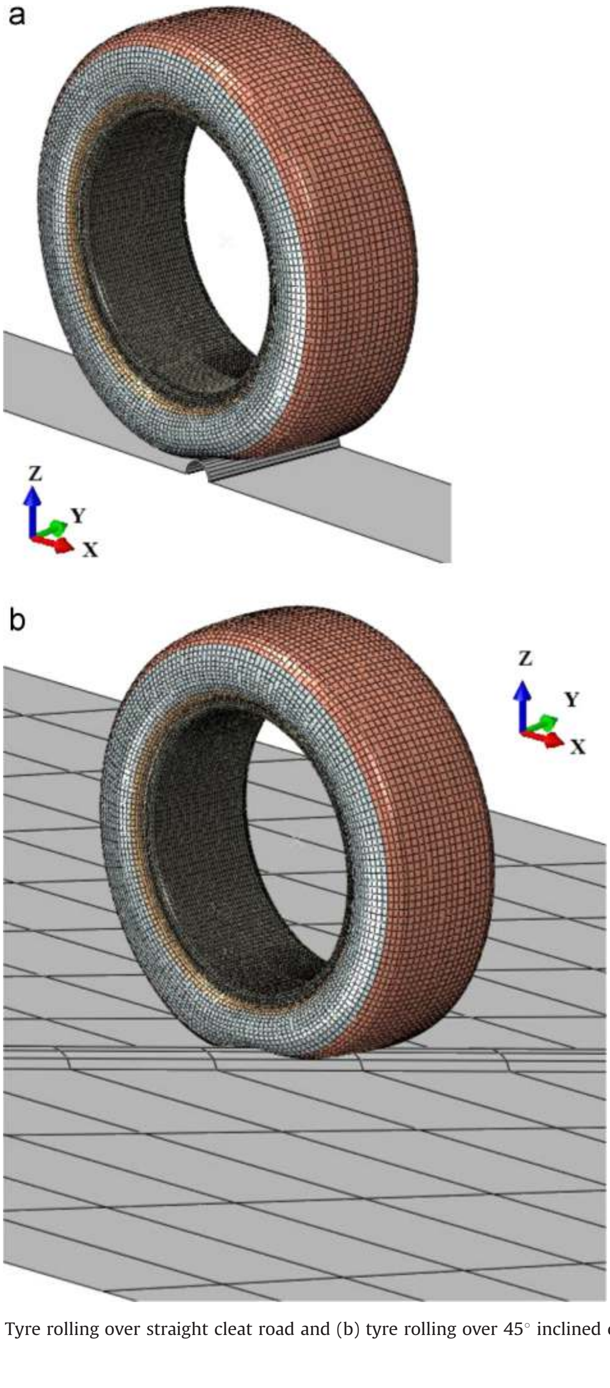

Fig. 3. (a) Tyre rolling over straight cleat road and (b) tyre rolling over 45° inclined cleat road.

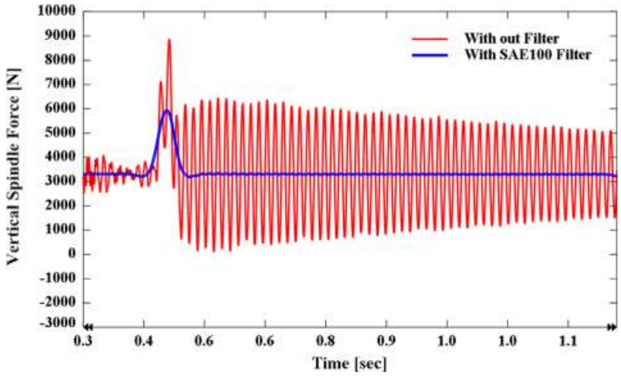

Fig. 4. Vertical spindle force due to straight cleat impact excitation.

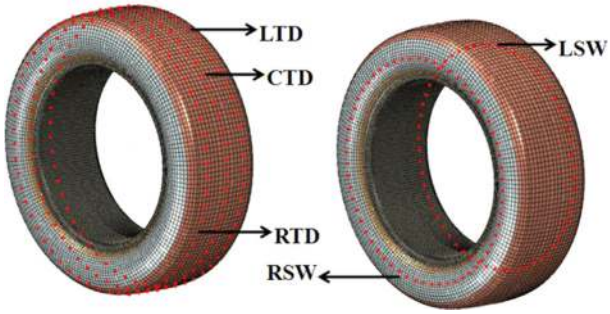

Fig. 5. Acceleration measurement nodes of Finite Element tyre model.

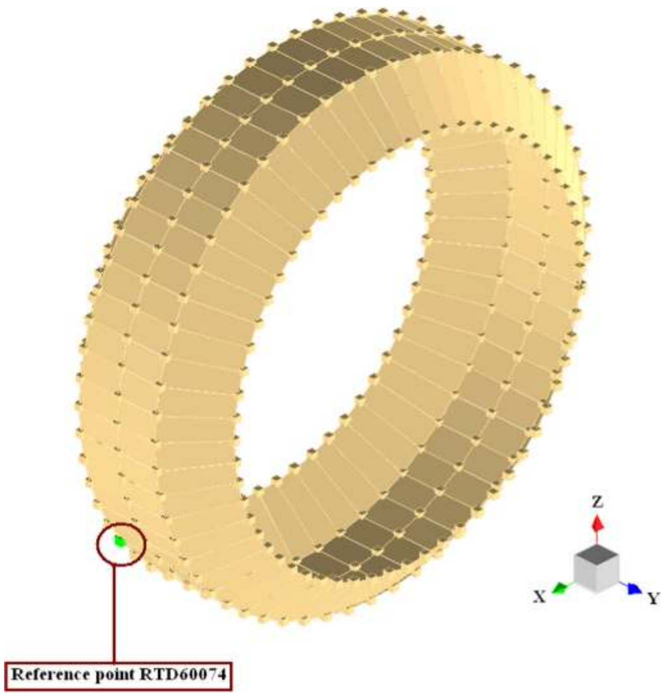

Fig. 6. Modal model geometry for Operational Modal Analysis.

First, the tyre foot print analysis is carried out and its shape is compared with the well-established contact pressure distribution in the literature [34,21,14]. The tyre foot print analysis represents inflation and loading steps of the FE model. During the explicit analysis, inflation and loading steps are carried out simultaneously. Loading is by displacement boundary condition applied to the road. The contact pressure at the end of this step shows an increase in the shoulder region of the tyre, shown in Fig. 2(a) and in the next step explicit rolling of the tyre has been carried out. The contact pressures are observed before and at the time of cleat impact across the treadband. Fig. 2(b) shows that the pressure distribution is symmetric across the contact patch that ensures flat contact of tyre during straight rolling as it approaches the cleat. The line contact shown in Fig. 2(c) ensures impulse excitation across the treadband, which is responsible for flat excitation spectrum. Analogue plots of Fig. 2(b) and (c), are described in (d) and (e) respectively for 45° inclined cleat simulation.

Fig. 3(a) and (b) represents the simulation trials for the same operating conditions, over straight and 45° inclined cleats. The 45° inclined cleat is chosen to have equal excitation in longitudinal and lateral direction of the rolling tyre. The rim is modelled by rigid element outer surface definition and road modelled as rigid surface. The focus of the current research work is to predict structure borne vehicle interior noise due to tyre road interaction. One of the main noise source for this noise is the low frequency (<500 Hz) belt vibrations that contribute to the typical wheel rim dynamics in this region that transfer road excitation to the spindle. The contribution of the tread block vibration is at the mid frequency (500–800 Hz) air borne interior noise and high frequency (800–1500 Hz) exterior noise. Also tread pattern grooves in the tyre leads to other noise generation mechanisms such as pipe resonance, horn effect and so on. Hence smooth tyre is considered to eliminate the influence of tread pattern vibrations and tread groove resonances over the whole tyre vibrations. The spindle force and tyre structural vibration responses are obtained from the simulation results.

During explicit rolling of tyre, the tread nodes are subjected to repeated impact as they come into contact with the road. This includes high frequency numerical noise in the vertical force at the spindle. Such oscillations are common in explicit analysis. Abaqus SAE100 Filter has been used to remove this noise. Fig. 4 compares the variation of spindle force with and without the filter. It is clear that the peak at 0.5 s is due to the impulse excitation of the cleat. The acceleration and vertical spindle force

Table 1
Correlation study for selection of reference point.

Reference	Response	Correlation (%)	Error (%)
RTD60074x	CTD9041x	91.58	9.21
	CTD9041y	66.39	50.62
	CTD9041z	92.76	7.84
RTD60074y	CTD9041x	70.19	42.48
	CTD9041y	32.48	207.90
	CTD9041z	68.22	46.59
RTD60074z	CTD9041x	90.78	10.19
	CTD9041y	80.45	24.31
	CTD9041z	95.36	4.86

Table 2
Comparison of circumferential and cross sectional modes and the effect of loading and rolling over natural frequencies of a tyre modal frequencies are in Hz.

Modes	FEA unloaded	FEA loaded	SST rolling	OMA rolling
Mode(1,0)	71.29	56.71	56.84	74.18
		76.57	76.30	84.90
Mode(2,0)	99.93	92.68	91.10	98.51
		108.89	105.13	113.24
Mode(3,0)	127.53	122.58	–	109.37
		138.71	119.74	125.40
Mode(4,0)	155.53	149.75	132.74	141.39
		167.72	145.84	159.27
Mode(5,0)	183.06	185.95	–	160.25
		195.28	161.52	176.94
Mode(6,0)	209.02	209.94	–	191.55
		219.62	177.74	202.80
Mode(7,0)	232.22	231.39	189.83	215.26
		239.36	198.13	223.08
Mode (2,1)	93.4	95.46	99.16	88.88[a]
		80.96	78.22	80.45[a]
Mode (3,1)	138.82	138.24	–	135.36[a]
		127.76	120.33	119.96[a]
Mode (4,1)	170.14	168.98	150.45	–
		160.90	142.62	–
Mode (5,1)	193.78	192.09	183.67	–
		184.05	163.30	–

'–' not identified modes.
[a] Obtained from inclined cleat test.

measurements are made with a sampling frequency of 2048 Hz. The acceleration responses in *x*, *y* and *z* directions with respect to the fixed global Cartesian references and time are obtained. The measurements points are shown in Fig. 5. RTD, CTD and LTD represent right, centre and left tread points and RSW and LSW represent right and left sidewall measurement points respectively. A wave can be represented with sufficient geometrical resolution if six or more points are measured per wavelength [22]. In the frequency range of interest (<500 Hz), in order to get higher circumferential mode number, a total of 300 points in five circles around the circumference of the tyre are considered for acceleration response measurements (60 nodes on each circle of the tyre). These output responses in x, y and z direction result in 900 signals. They are considered for further signal processing.

Secondly, these output responses are communicated to LMS Test Lab software, as if they have been acquired from 300 tri-axial accelerometers through a data acquisition system. Fig. 6 shows the modal model geometry used for modal extraction where all these 900 signals are mapped to this model. As the energy content of the travelling wave is more in the leading edge than the trailing edge, three sets of reference signals are chosen at the leading edge node point RTD60074; Table 1 represents the correlation of these reference signals with the response signals at the top of the tyre (CTD9041) and it is observed that vertical and longitudinal acceleration responses have good correlation with other measurement points and hence the point (RTD60074) responses are considered to be reference signals for crosspower sum calculation. It is also confirmed that this point selection gives good cross correlation with other response signals in vertical and longitudinal directions. Representative data are shown in Table 1. The amplitude and the phase of the crosspower sum are shown in Fig. 7 as a function of frequency upto 500 Hz.

The polyreference least square complex exponential frequency domain algorithm [23,24] of LMS Test Lab software is used for modal extraction. The stabilisation curve fit for crosspower sum of all spectrums is shown in Fig. 8(a). The agreement between the operational and synthesised crosspower of the chosen reference with the response for the physical modes obtained is good and comparison of operational and synthesised crosspower between the representative reference and response signal is shown in Fig. 8(b). It validates the modal frequency extraction procedure.

Using Automatic Modal Parameter Selection (AMPS) tool, the physical modes are picked from the stabilisation curve from which the modal frequencies, damping and mode shapes are obtained. In this procedure, the FE simulation model does not include the viscoelastic material property and the damping values obtained is not the realistic values of the tyre as reported by Kindt et al. [13] however, the mode shapes and resonance frequencies closely match their results.

2.2. Steady State Transport analysis of rolling tyre

Generally FE Modal analysis is carried out in order to validate the modal parameters obtained from EMA or vice versa. These methods were developed based on the important assumption that the system is linear, time invariant, observable and obeys reciprocity principle [25]. These results are useful to understand the standing wave pattern depicted by the mode shapes. Linear perturbation analysis of unloaded tyre gives repeated eigen values, called double poles. To understand the effect of loading, an eigen value analysis is performed on the inflated, loaded tyre which is fully constrained at wheel spindle interface of the rim. Loading of the tyre is achieved by displacing the road reference node towards the tyre. The established contact between the tyre tread surface and road surface induce large deformations at the treadband. The modal frequencies obtained are no more double poles; they split into two single poles due to loading. However, these large deformations along with the hyperelastic material definition, introduces non-linearity into the system. Also, it is questionable to use contact algorithms for eigen value extraction.

A relative kinematic theory was developed by Nackenhorst [26] based on Arbitrary Lagrangian Eulerian (ALE) description. This theory is applied to solve large deformation rolling contact problem of elastic bodies using FE analysis [31,32]. Readers can refer the paper published by Nackenhorst for theoretical foundation of ALE and application of FE method to solve rolling contact problem of elastic bodies [27]. In this study, the commercial Finite Element code ABAQUS

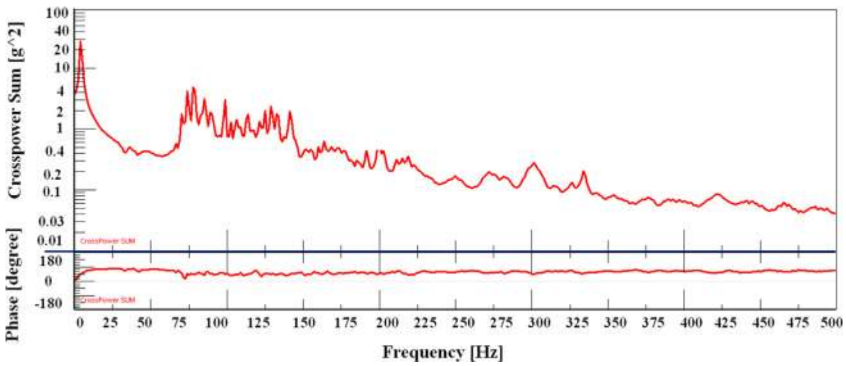

Fig. 7. Crosspower sum.

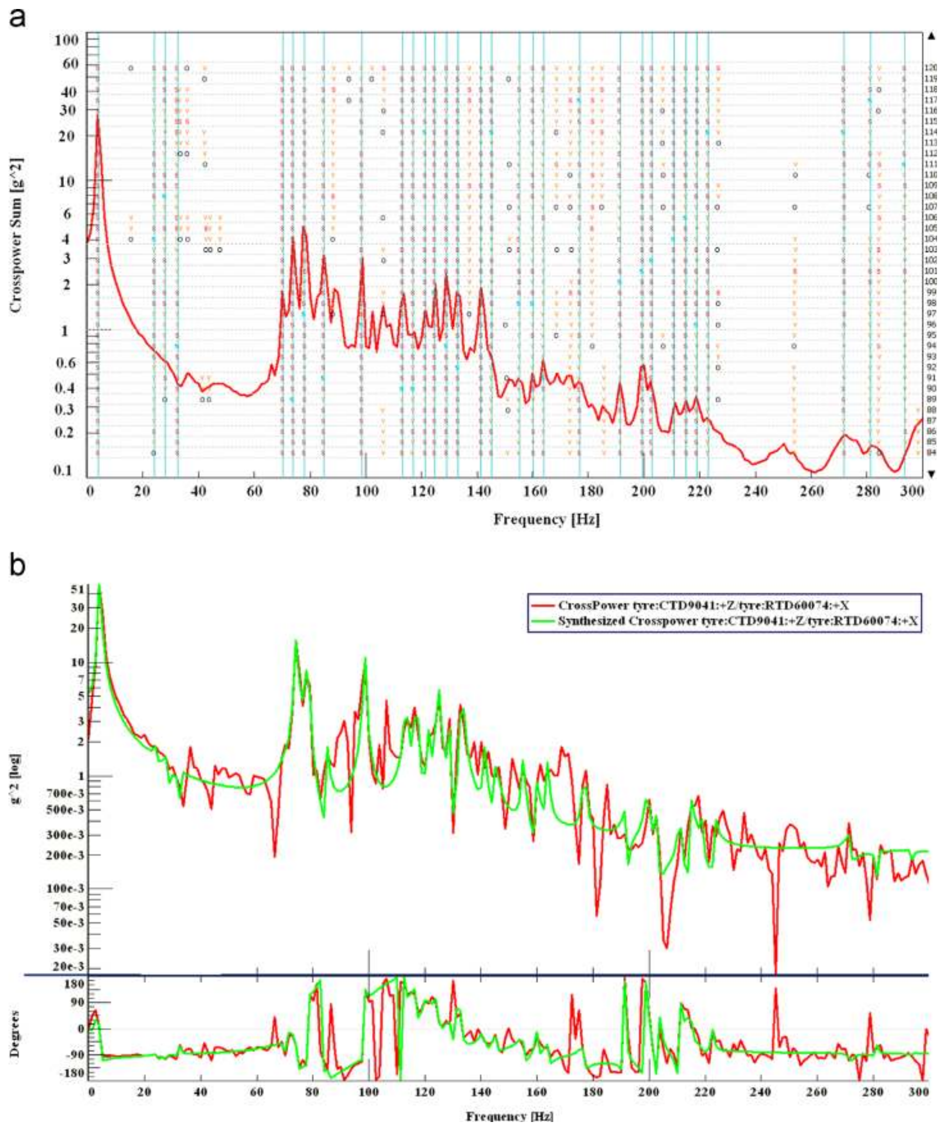

Fig. 8. (a) Stabilisation diagram and pole selection by AMPS and (b) comparison of operational and synthesised crosspower.

has been used for the Steady State Transport analysis. The frictional and inertia effects are also accounted for in this analysis [28].

The Steady State Transport rolling analysis requires both rotational spinning velocity and translating ground velocity of the tyre. First, free rolling angular velocity is determined. A free rolling tyre generally travels farther in one revolution than the value determined by its centre height, but it is less than the value obtained from the radius of the unloaded tyre. The free rolling solution refers to an equilibrium solution for a rolling tyre that has zero torque applied around the axis of the tyre. An equilibrium solution with a non-zero torque is referred to either as braking or as traction depending on the sense of the torque. If the resultant torque opposes the direction of angular velocity of free rolling solution the state is referred to as braking and if the resultant torque acts in the same direction as the angular velocity of free rolling solution it is referred to as traction. Full braking or traction occurs when all the contact points between the tyre and the road are in a state of slip [18]. Hence, from the unloaded and loaded tyre radius, the angular velocities for braking state and traction state are determined for the ground velocity of 28.3 kmph.

Then for the fixed ground velocity, a series of steady state solutions are obtained between the state of braking and the state of traction, by gradually increasing the angular velocity. Thus the free rolling angular velocity is found out to be 25.3 rad/s, corresponding to the zero tangential reaction force at the contact interface. Finally, the Steady State Transport analysis is performed, and complex eigen frequencies are extracted for a free rolling tyre, using Lanczos solver for the frequency range up to 300 Hz [29].

Table 3
Comparison of experimental and FEA-OMA rolling tyre results resonance frequencies in Hz.

Modes	Kindt et al. [30] Experiment – OMA	Finite Element – OMA	Percentage deviation
(1,0)hor.	–	84.90	–
(1,0)vert.	81.83	74.18	9.35
(2,0)0	102.98	98.51	4.34
(2,0)extr.	113.94	113.24	0.61
(3,0)0	124.09	109.37	11.86
(3,0)extr.	139.33	125.40	10.00
(4,0)0	151.76	141.39	6.83
(4,0)extr.	167.52	159.27	4.92
(5,0)0	182.70	160.25	12.29
(5,0)extr.	198.88	176.94	11.03
(2,1)0[a]	96.09	88.88[a]	7.50
(2,1)extr.[a]	91.29	80.45[a]	11.87
(3,1)0[a]	–	135.36[a]	–
(3,1)extr.[a]	–	119.96[a]	–

'–' not identified modes.
[a] Obtained from inclined cleat test.

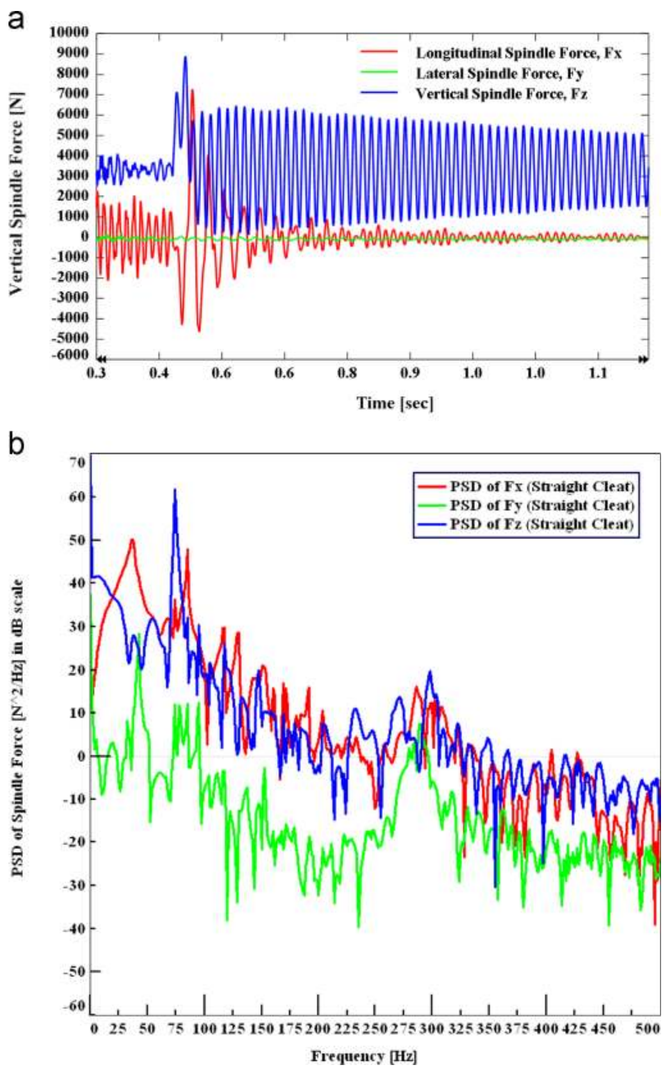

Fig. 9. (a) Straight cleat impact excitation and (b) PSD of spindle force.

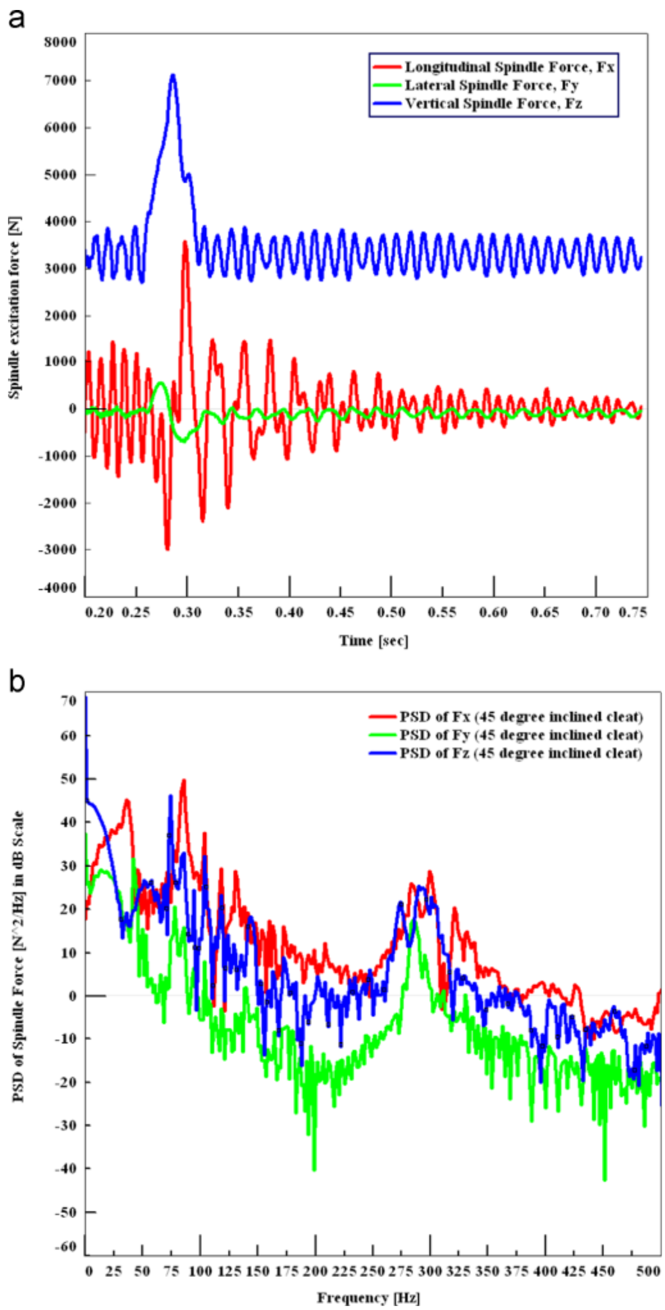

Fig. 10. (a) 45° Inclined cleat impact excitation and (b) PSD of spindle force.

3. Validation of results

3.1. *Description of structural tyre model of Kindt et al. [22]*

Kindt et al. [22] have developed a Finite Element structural tyre model based on flexible ring on an elastic foundation for the prediction of structure borne interior vehicle noise that validate upto 300 Hz (i.e. below (n,2)modes). This model includes wheel, treadband ring, and tyre sidewall and air cavity sub models. However, this model is not a complete fundamental FE tyre model and the characterisation of the sidewall is done through an analytical approach. Isotropic material property is used for the treadband ring. The mass of sidewall is neglected and is represented by spring elements. The equivalent sidewall mass is divided into two halves and added to the treadband and wheel rim. The radial, tangential and axial stiffness parameters of sidewall spring are determined from the natural frequencies of (1,0) mode, torsional mode and axial mode respectively. These modes were obtained from experimental modal analysis. The Young's modulus was

Fig. 11. Operational modal parameters obtained from straight cleat impact test.

tuned by adjusting its value in order to reduce the error on the predicted natural frequencies of the model. Also, when they have considered orthotropic material model definition and found it has a higher influence on cross sectional modes (n,1), compared to the circumferential modes (n,0). The difference between the resonance frequencies of the model with isotropic and orthotropic treadband becomes larger as the index 'n' increases. They used the results obtained from this tyre model with isotropic material definition to compare the experimental modal analysis results of unloaded and loaded tyre to study dynamic behaviour of loaded tyre. However the rolling tyre modal results of the model were not reported. They determined the rolling tyre modal parameters from the cleat impact test from their state of the art, experimental set up of tyre-on-tyre arrangement followed by Operational Modal Analysis. They have observed circumferential modes up to (5,0) and only one cross sectional mode (2,1). The experimental results reported by them have been taken to compare the rolling tyre modal parameters obtained from the procedure reported in this paper.

Fig. 12. Operational modal parameters obtained from 45° inclined cleat impact test.

Table 4
Frequency of tyre rolling speed as function of speed.

Rolling frequency (Hz)	Vehicle Speed					
	28.3 kmph		50 kmph		70 kmph	
	Rolling frequency	Mode captured from OMA	Rolling frequency	Mode captured from OMA	Calculated	Mode captured from OMA
	3.93	4	6.95	7.01	9.78	9.88

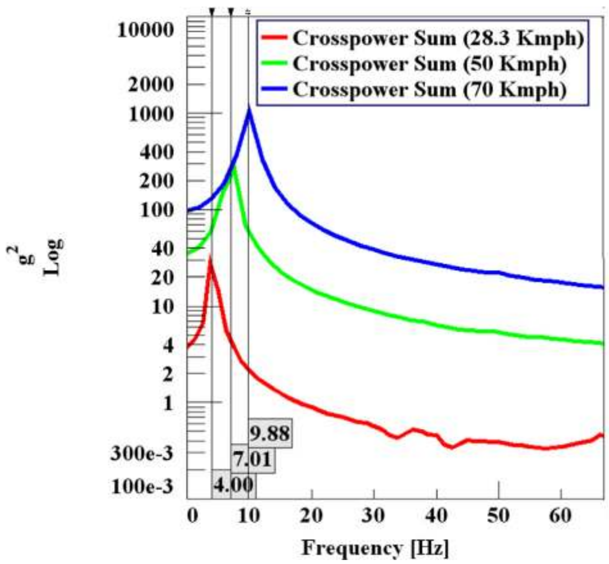

Fig. 13. Variation of rolling frequency as function of vehicle speed.

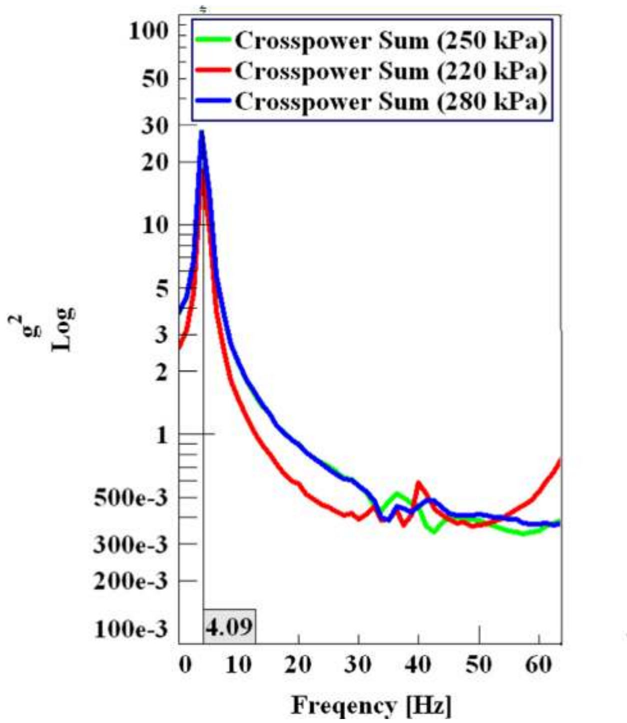

Fig. 14. Variation of rolling frequency as function of inflation pressure.

3.2. *Comparison of rolling tyre modal results*

In this work, the tyre considered is very similar to that used by Kindt et al. [30], though the exact tyre used is not available. As the tyre does not have tread blocks and the construction of the tires in this range is standard and may not vary drastically. Table 3 represents the comparison of determined rolling tyre modal frequencies with the experimental results reported by Kindt et al. [30]. From the straight cleat simulation only circumferential modes are obtained and from inclined cleat simulation, in addition to the circumferential modes, cross sectional modes (2,1) and (3,1) are also obtained. Table 3 shows a comparison between the experimental results and the results obtained from the procedure followed in this paper. A deviation of the current simulation results from the experiment is within 0.6–12%. This clearly shows that the values obtained by the current procedure compares very well with the published experimental results.

Table 5
Operational modal frequencies for speed of 28.3 kmph as function of inflation pressure.

Mode shape (OMA results)	Inflation pressure		
	220 kPa	250 kPa	280 kPa
Mode(1,0)vert.	71.05	74.18	76.93
Mode(1,0)horiz.	81.65	84.89	88.09
Mode(2,0)0	94.87	98.51	104.70
Mode(2,0)extr	–	113.24	–
Mode(3,0)0	107.14	109.37	116.66
Mode(3,0)extr	119.32	125.40	130.59
Mode(4,0)0	123.53	141.39	151.27
Mode(4,0)extr	148.72	159.27	163.41

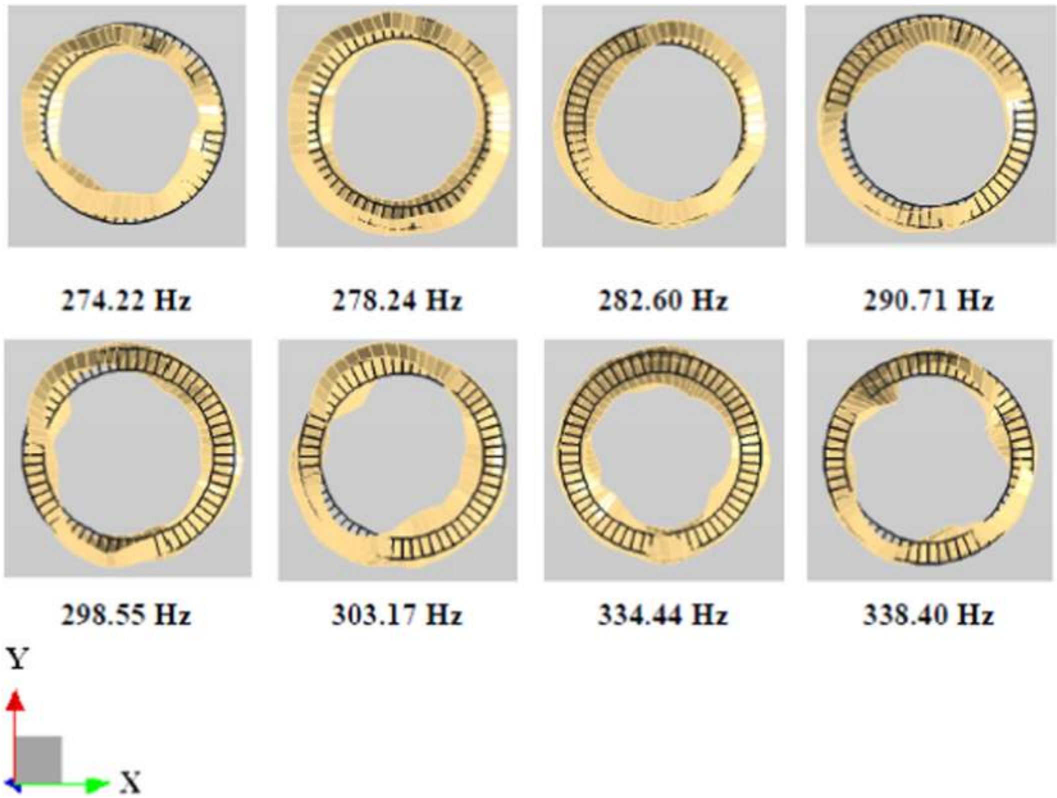

Fig. 15. Cross sectional bending modes due to 45° inclined cleat impact test.

4. Results and discussion

It is necessary to excite the rolling tyre sufficiently in order to invoke complete modal behaviour. Fig. 9(a) and (b) shows the spindle force excitation and power spectral density functions in longitudinal, lateral and vertical directions. Straight cleat impact simulation excites adequately all the circumferential bending modes of the tyre belt vibration. As the tyre rolls over the cleat, the observed excitation is neither longitudinal nor vertical, but a combination of both. The longitudinal and vertical spindle force excitation energy is significant till 300 Hz. As observed by Kindt et al. [22], the oscillations of the spindle forces in the longitudinal direction are damped. The frequency of 32.54 corresponds to the torsional tyre mode. It can also be noted that circumferential modes, upto (7,0) are excited. Also, it is clear that no significant excitation takes place in the lateral direction during the impact. Hence, another trial with 45° inclined cleat has been carried out to achieve lateral excitation.

Fig. 10(a) (spindle force excitation) and (b) (power spectral density functions) shows significant lateral excitation, offered by an inclined cleat. In this case, as seen from Fig. 10(b), some of the cross sectional bending modes of the tyre are obtained in the frequency ranges between 270 Hz and 340 Hz. Also cross sectional rotational modes (2,1)0, (2,1)extr, (3,1)0, and(3,1)

Fig. 16. Mode shape comparison among various tyre conditions.

extr are excited. However, circumferential modes (4, 0)0,(6,0)extr and (7,0)0 that are identified from straight cleat test, are not excited.

Figs. 11 and 12 show mode shapes and the corresponding resonance frequencies obtained from these straight and inclined cleat tests. Modes are represented by (n,m); 'n' refer to circumferential index and 'm' refer to axial bending index. A mode was identified at 4 Hz and this mode has never been reported in the literature. Interestingly, this frequency coincided with the rolling frequency of the tyre. In order to test whether the identified mode changes with speed, numerical tests were done by varying the speed of the tyre. The corresponding frequencies are shown in Table 4. Fig. 13 shows comparison of crosspower sum for different rolling frequencies captured during Operational Modal Analysis. These results clearly show that the first 'mode' captured is due to the speed of the tyre. The inflation pressure was also varied and the corresponding results, shown in Fig. 14, confirm that this frequency is independent of inflation pressure as well. On the other hand, as shown in Table 5, all other modes are a function of inflation pressure. The (1,0) vertical mode at 74 Hz indicates that the belt behaves as a rigid surface and moves in the vertical direction in the wheel plane [22]. The (2,0) mode at 100 Hz is the first deformation belt mode.

Fig. 15 shows the cross sectional bending modes (n,2), obtained from the inclined cleat test. Kindt et al. [22,30] have reported that in general, the start of cross sectional bending modes is approximately 300 Hz. The actual frequency, at which the first (n,2) mode appears, needless to say, depends on the tyre. However, for most tyres the onset of this mode appears in the frequency range of the (10,0) mode [22].

The first $(n,2)$ mode is seen at 274 Hz. The modal parameter results except for the damping values are similar to the findings of Kindt et al. [30]. It is observed from their results that sidewall dynamic behaviour depends on the tread dynamic behaviour up to 300 Hz.

4.1. Comparison of mode shapes obtained by different methods

The results obtained from Operational Modal Analysis are compared with the mode shapes obtained from stationary unloaded tyre, stationary loaded tyre, Steady State Transport rolling tyre. Fig. 16 summaries the circumferential bending modes of the tyre model at different conditions. It is observed that, except the unloaded stationary tyre mode shapes, for other cases two different frequencies exist for the same mode shape. This is due to the effect of loading and rolling of the tyre.

Comparison of only the $(n,0)$ modes (circumferential bending modes) is shown in Fig. 16. These modes are the predominant modes and are excited during rolling of the tyre. These modes are obtained by straight cleat impact test simulation, which excites the belt along its entire width. Moreover, $(n,1)$ modes (cross sectional modes) might not have been excited. However, for other procedures, in addition to the circumferential bending modes, cross sectional tilting and bending modes viz., $(n,1)$ and $(n,2)$ modes are also obtained. Though these modes give a complete idea about the modal behaviour of the tyre, in reality, during operational conditions all these modes may or may not be excited.

The effect of loading and rolling makes all the circumferential mode number 'n' larger than zero, to split into two resonance frequencies and that refers to (ω_{n1}) backward travelling wave, travelling in the opposite direction to the tyre rotation and (ω_{n2}) forward travelling wave that travels in the same direction as of the tyre rotation. In this modal simulation, travelling waves are observed with respect to a fixed reference system, where in $(\omega_{n1} < \omega_{n2})$ is observed and this effect is known as Doppler shift. In addition to the difference in eigenvalues marked by Doppler shift, frequency veering is contributed by the Coriolis effect. It is evidently seen even in case of an observation of unloaded rotating tyre in the co-rotating and stationary reference frame. The Coriolis

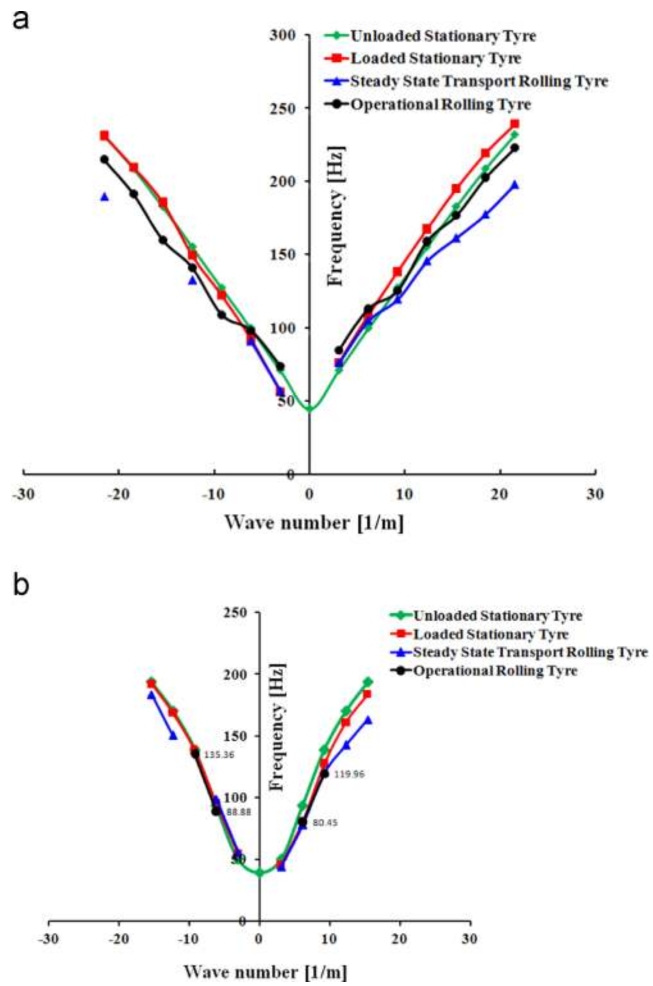

Fig. 17. (a) Comparison of circumferential modes dispersion curves and (b) comparison of cross sectional modes dispersion curves.

effect contributes to the frequency veering [22]. Table 2 gives the effect of loading and rolling of the tyre with respect to their unloaded double poles.

Fig. 17(a) and (b) shows the dispersion curves of circumferential and cross sectional modes respectively, for different conditions of the tyre modal results obtained from FE modal analysis and OMA. The dispersion curve of unloaded stationary tyre is symmetric due to double poles ($\omega_{n1}=\omega_{n2}$). The loading of the tyre causes splitting of real valued eigen modes into forward and backward waves travelling with a complex valued eigen modes. The shift in natural frequency of forward travelling wave is more than the shift in backward travelling wave compared to a standing wave natural frequency.

In the similar way, the procedure followed in this paper also describe the effect of rolling on the tyre modal behaviour and predicts the clear splitting of standing wave into two travelling waves as reported by Kindt et al. [30].

5. Conclusion

Integration of vibration results from cleat impact simulation of a FE rolling tyre model, with OMA technique has been accomplished successfully. Circumferential bending modes are obtained from straight cleat impact simulation, and cross sectional bending modes are obtained additionally from 45° inclined cleat simulations. Modal frequencies extracted from OMA shows clearly the splitting of real eigen modes of an unloaded stationary tyre into two opposite travelling complex eigen mode waves. The results are compared with the published experimental results and a good comparison has been noted. This established procedure can be applied to study dynamic behaviour of tyre under various operation conditions to study the vehicle interior and exterior noise due to tyre/road interaction and hence proposed.

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Appendix A

Yeoh model [35,18] has been used to define strain energy potential. Table A1 gives the material constants used for hyperelastic material definition for various components of the tyre. Rebar elements are defined for both the belts and ply for reinforcement assignment. Nonlinear geometric effect is included in the simulation.

Table A1
Hyperelastic material definition in SI units.

Components	**Density**	**Yeoh model material constants**			
		C10	**C20**	**C30**	**D1**
Inner liner	1050	3.14470e5	−1.10385e5	2.65448e4	1.58998e−7
Ply	1050	3.72454e5	−9.69403e4	2.43385e4	1.34244e−7
wing tip	1050	2.98750e5	−8.25992e4	1.90295e4	1.67364e−7
Protection ply	1050	8.96732e5	−2.80203e4	7.88071e4	5.57580e−8
Belt	1050	8.96732e5	−2.80203e4	7.88071e4	5.57580e−8
Beg	1050	1.21927e6	−4.85253e5	1.96638e5	4.10082e−8
Tread	750	6.16047e5	−1.90709e5	4.75049e4	8.11627e−8
Sidewall	1050	4.87666e5	−1.41343e5	3.86106e4	1.02529e−7
Rimstrip	1050	1.13364e6	−4.43953e5	1.18935e5	4.41059e−8
Filler	1050	8.76048e5	−2.93303e5	7.93587e4	5.70745e−8
		Young's modulus		**Poisson's ratio**	
Bead	7800	2.06399e11		0.3	
Reinforcement definition for continuum elements					
Ply	1070	4.715258e9		0.49	
Protection ply	940.5	2.19652e9		0.49	
Belt	7800	1.74652e11	0.3		
Reinforcement definition of geometry					
	Cross sectional area of rebar (m^2)	**Spacing between rebar (m)**		**Orientation angle (deg)**	
Ply	3.52565e−7	9.056e−4		−1	
Belt1	1.41196e−7	1.581e−3		60	
Belt2	1.41196e−7	1.593e−3		−60	
Protection ply 1	2.29031e−7	1.016e−3		90	

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