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Dual solutions of cross diffusioneffects on MHD Peristaltic flow in a conduit

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Abstract. Thepurpose of this study is to analyze the influence of wall flexibility and cross diffusion on the peristaltic transport of a MHDdissipative fluid in uniform channel in the presence of Joule heating. Dual solutions are obtained for aligned and transverse magnetic field cases. The exact solution is obtained for stream function and velocity. Further,shooting process isemployedtosolvetheenergy and concentration equations.The influence of main parameters on the present flow is explained graphically. The increase in magnetic field decreases the velocity of the fluid. Also Dufour and Soret numbers enhances the temperature while they reduses the concentration field.

Nomenclature:

x, y	Cartesian coordinates	ρ	density of the fluid
u , v	fluid velocities	t	time
p	pressure	C	concentration
μ	viscosity	C_0, C_1	concentrations fluid at lowerand upper walls
d	mean width of the channel	K_T	diffusion ratio
a	amplitude	D_B	mass diffusion coefficient
λ	wavelength	Sr	Soret parameter
c	wave speed	Du	Dufour parameter
c_p	specific heat	τ_0	yield stress
υ	kinematic viscosity	T_0, T_1	temperatures of fluid at lower and upper walls
k_0	thermal conductivity	E_1, E_2, E_3	elasticity parameters
c_s	concentration susceptibility	θ	Non-dimensional temperature
σ	electrical conductivity	ϕ	Non-dimensional concentration
B_0	magnetic field	M	magnetic parameter
T	temperature	Nu	Nusselt number
ε	amplitude ratio	Br	Brinkman number

δ wave number
ψ stream function
Ec Eckert number

1. Introduction

Peristalsis is an importantmechanismoffluid transport which is formed by the propagation of a peristaltic wave on the channel walls. We can observe this natural phenomenon in many physiological systems such as swallowing foodstuff, passage of urine, transport of lymph, blood pumping in blood vessels, ovum transport etc. Also it is useful in pumping some industrial harsh fluids. Further engineers are adapting this principle in preparing different models in pumping michanary. In view of these real applications many authors have investigated the peristaltic transport in various circumstances [1- 4].

The influence of magnetic field,heat and mass transfer on peristalsis has huge applications in industrial and biomedical endineering like drug distribution schemes, canser treatment, reduction of blood loss during operations, design of heat exchengers, solar energy and many others. Few authors have studied the combined effects of heat transfer and magnetohydrodynamics on peristaltic flow. Some of the recent studies are presented in references [5-16]. Very recently, the researchers [17,18] analyzed the convective heat transfer in MHD flows by considering the cross diffusion effects.

Motivated by the above discussed applications and studies, we examinedthe effects of Joul heating, Dufour and Soret numbers on peristaltic transport of a viscous fluid in an elastic channel in the presence of inclined magnetic field. The analytical solutions are presented for stream function and velocity and the energy and concentration equations are solved numerically. The impact of key parameters on the flow characteristics is discussed through graphs.

2. Modelling of the problem

We consider the peristaltic transport of a viscous fluid in a two dimensional uniformchannel.The flow is generated by a peristaltic wave propagating along the elastic walls of the channel with a constant speed c (Fig.1).The Joule heating, diffusionand dissipation effects are considered. The geometry of the channel is given as

$$\bar{h}(\bar{x},\bar{t}) = a\,Sin\frac{2\pi}{\lambda}\left(\bar{x}-c\bar{t}\right)+d \quad (1)$$

As per the assumptions made above, the simplified non-dimensional governing equations and corresponding boundary conditions for the present study are given by (See the references [9-13])

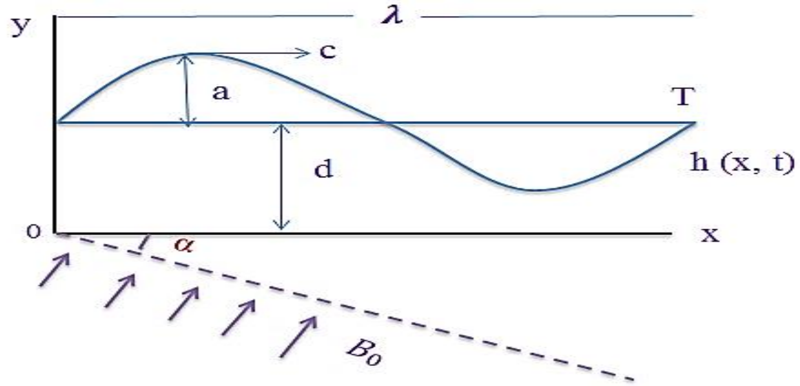

Figure 1. Flow configuration

$$\frac{\partial p}{\partial x} = \frac{\partial^3\psi}{\partial y^3} - M^2\frac{\partial\psi}{\partial y}\cos^2\alpha, \quad (2)$$

$$\frac{\partial p}{\partial y} = 0, \quad (3)$$

$$\frac{\partial^2\theta}{\partial y^2}+Br\left[\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+M^2\cos^2\alpha\left(\frac{\partial\psi}{\partial y}\right)^2\right]+\Pr Du\frac{\partial^2\phi}{\partial y^2}=0, \tag{4}$$

$$\frac{\partial^2\phi}{\partial y^2}+ScSr\frac{\partial^2\theta}{\partial y^2}=0, \tag{5}$$

Appropriate boundary conditions are

$$\psi=0\,,\ \frac{\partial^2\psi}{\partial y^2}=\frac{\partial\theta}{\partial y}=\frac{\partial\phi}{\partial y}=0,\text{ at } y=0 \tag{6}$$

$$\frac{\partial\psi}{\partial y}=0\,,\ \theta=1\,,\ \phi=1\ \text{ at }\ y=h=[\varepsilon\sin 2\pi(x-t)+1] \tag{7}$$

$$\frac{\partial^3\psi}{\partial y^3}-M^2\frac{\partial\psi}{\partial y}\cos^2\alpha-\left(E_3\frac{\partial^2 h}{\partial x\partial t}+E_1\frac{\partial^3 h}{\partial x^3}+E_2\frac{\partial^3 h}{\partial x\partial t^2}\right)=0,\text{ at } y=h \tag{8}$$

The dimensionless parameters used in the above simplified equations are :

$$\left.\begin{aligned}
&x\lambda=\bar{x},\ yd=\bar{y},\ \psi c\,d=\bar{\psi},\ uc=\bar{u},vc=\bar{v},\ p=\frac{d^2\bar{p}}{\mu c\lambda},\tau_{xy}=\frac{d\overline{\tau_{xy}}}{\mu c},t=\frac{c\bar{t}}{\lambda},\ \delta=\frac{d}{\lambda},\\
&\varepsilon=\frac{a}{d},\mathrm{Re}=\frac{\rho cd}{\mu},\theta=\frac{(T-T_0)}{(T_1-T_0)},\phi=\frac{(C-C_0)}{(C_1-C_0)},Sc=\frac{\upsilon}{D_B},Sr=\frac{\rho D_B K_T}{\mu(C_1-C_0)},\\
&M=\sqrt{\frac{\sigma}{\mu}}B_0,\Pr=\frac{\rho\upsilon c_f}{k_0},Ec=\frac{c^2}{c_f(T_1-T_0)},Du=\frac{D_B K_T(C_1-C_0)}{\mu c_p\,\mathrm{c}_s(T_1-T_0)},\\
&E_1=\frac{-\tau d^3}{\lambda^3\mu c},E_2=\frac{m_1 cd^3}{\lambda^3\mu},E_3=\frac{cd^3}{\lambda^2\mu},h=\frac{\bar{h}}{d}=1+\varepsilon\sin 2\pi(x-t),
\end{aligned}\right\} \tag{9}$$

3. Solution of the problem

By differentiating equation (2) with respect to y we obtain

$$\frac{\partial^4\psi}{\partial y^4}-M^2\frac{\partial^2\psi}{\partial y^2}\cos^2\alpha=0 \tag{10}$$

By solving equation (10) with the boundary conditions (6), (7) and (8) we attain the stream function and the velocity as

$$\psi=\frac{C_1}{M^2\cos^2\alpha}\left[\frac{\sinh(M\cos\alpha)y}{M\cos\alpha\cosh(M\cos\alpha)h}-y\right] \tag{11}$$

$$u=\frac{C_1}{M^2\cos^2\alpha}\left[\frac{\cosh(M\cos\alpha)y}{\cosh(M\cos\alpha)h}-1\right] \tag{12}$$

Where

$$C_1=-8\varepsilon\pi^3\left[(E_1+E_2)\cos 2\pi(x-t)-\frac{E_3}{2\pi}\sin 2\pi(x-t)\right]$$

The equations (4) and (5) are coupled and nonlinear therefore these equations are solved numerically by employingR-K-Fehlberg integration schemewith the help of (7), (8) and (11). The Nusselt and Sherwood numbers at the wall are defined by

$$Nu=-\left(\frac{d\theta}{dy}\right)_{at\,y=h},\ Sh=-\left(\frac{d\phi}{dy}\right)_{at\,y=h} \tag{13}$$

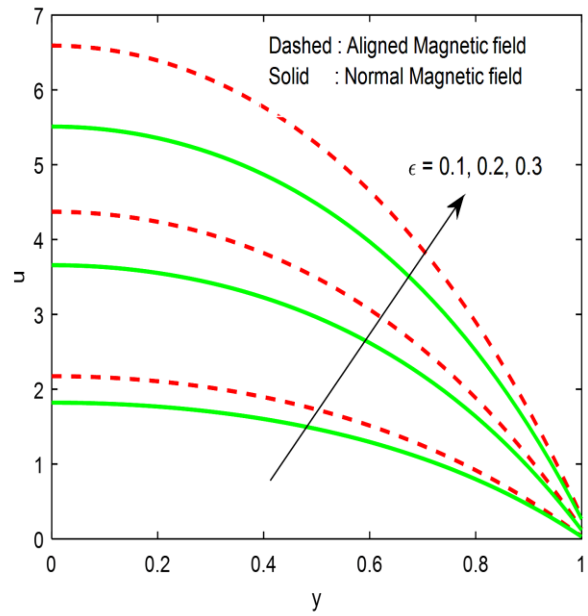

Figure 2. Velocity profiles for ε

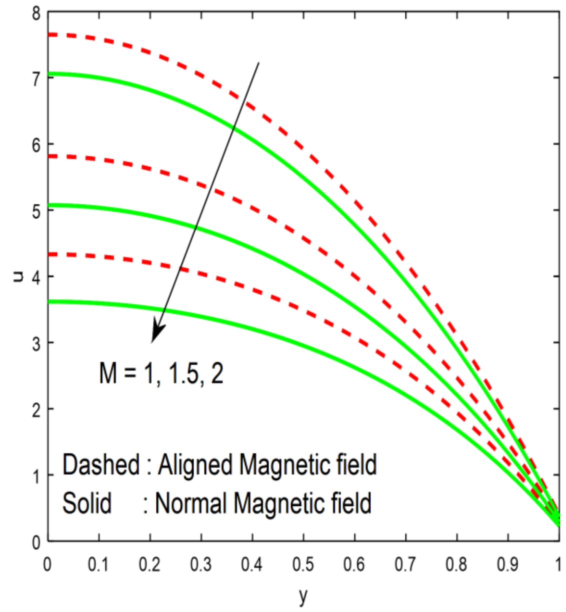

Figure 3. Velocity profiles for M

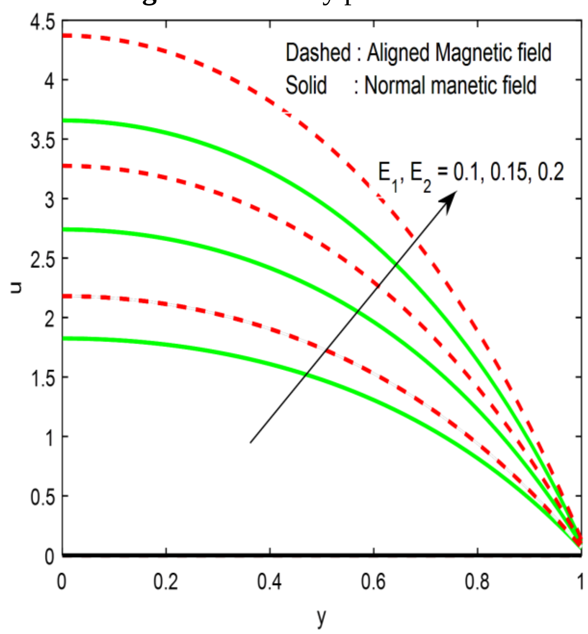

Figure 4. Velocity profiles for E_1, E_2

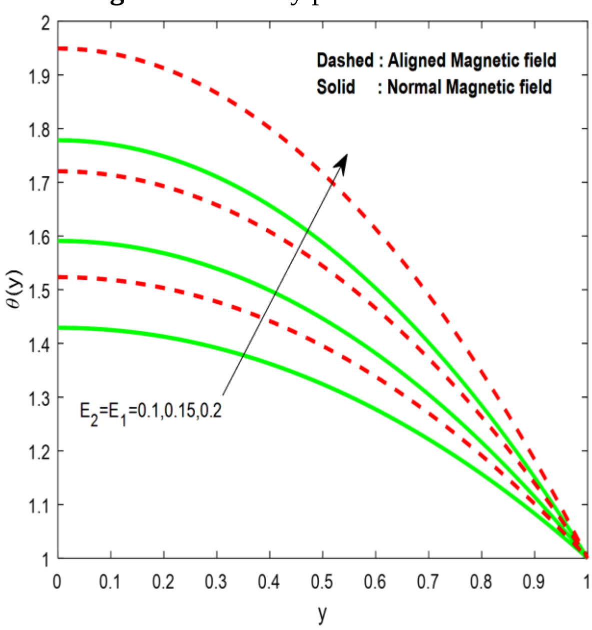

Figure 5. Temperature profiles

4. Results of the problem

The impact of relevant parameters on the flow characteristics are Presentedwiththehelp of graphical and tabularrepresentations.In this section we used the fixed values of various parameters as $x = 0.2$, $\varepsilon = 0.2$, $t = 0.1$, $M = 2$, $E_1 = 0.2$, $E_2 = 0.2$, $E_3 = 0.1$, $Pr = 5$, $Br = 0.2$, $Sc = 0.6$, $Sr = 0.2$, $Du = 0.2$, $\alpha = 0$ (Normal magnetic field) and $\alpha = \pi/6$ (Inclined magnetic field).

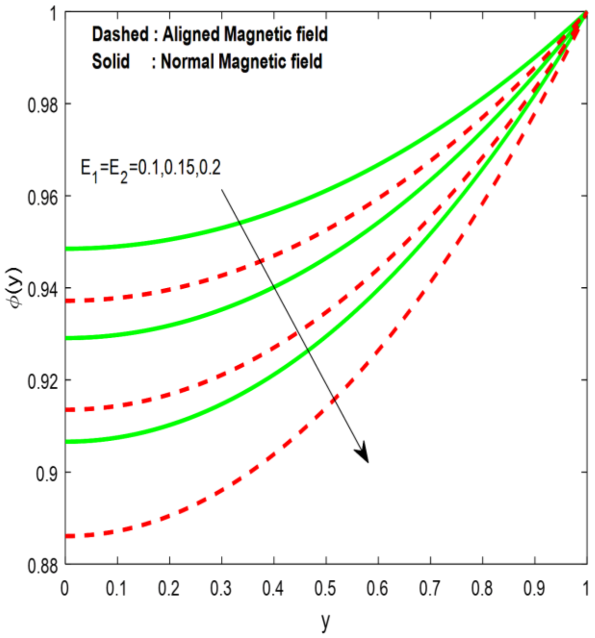

Figure 6. Concentration profiles for E_1, E_2

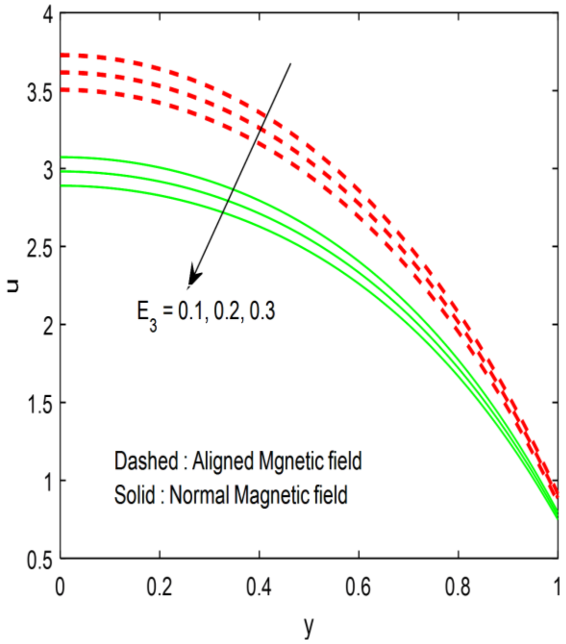

Figure 7. Velocity profiles for E_3

Figures 2 and 3 are plotted to explain the effects of amplitude ratio ε andmagnetic parameter M on velocity. We notice that velocity is an increasing function ε and decreasing function of M .It is clear that the velocity field of the fluid effected by the magneticforce.Figures 4 – 9 are shown that the wall tension (E_1) and mass characterization (E_2) parameters enhances the velocity and temperature while they reduces the concentration. Further, opposite behavior is identified for damping force parameter E_3 .

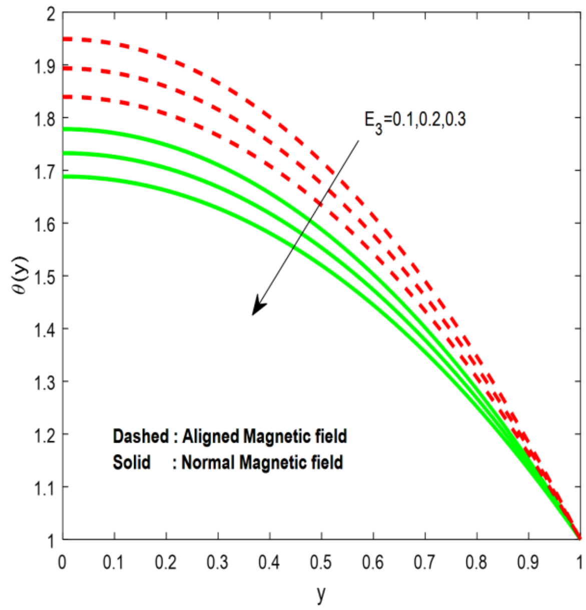

Figure 8. Temperature profiles for E_3

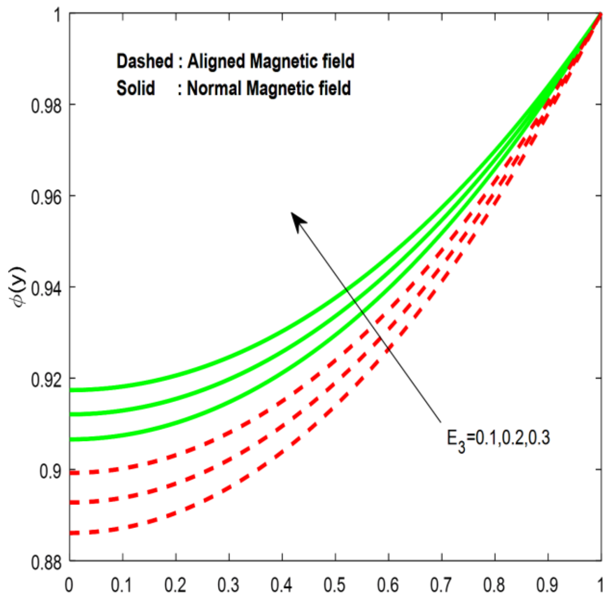

Figure 9. Concentration profiles for E_3

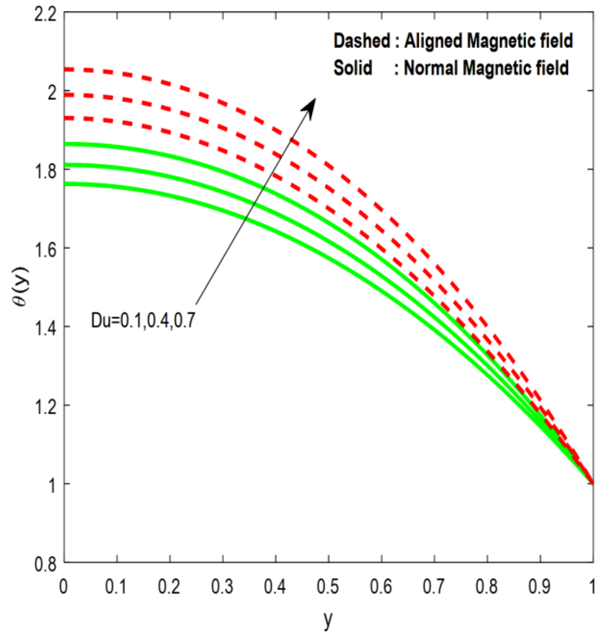

Figure 10. Temperature profiles for *Du*

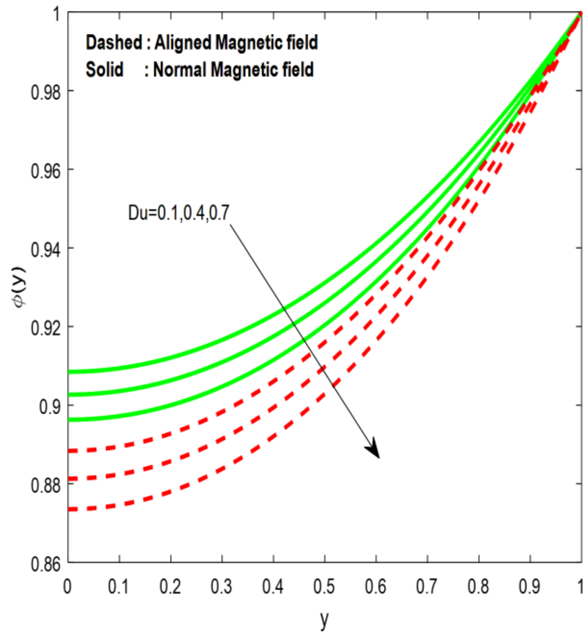

Figure 11. Concentration profilesfor *Du*

Figs. 10-13 illustrates the influence of Dufour and Soret parameters on thermal and concentration fields. It is clear that the rising values of cross diffusion parameters boosts the thermal field and supresses the concentration field. It is interesting to mention that the effect of cross diffusion is high when the magnetic field is transverse. It is noticed from the Figs. 15 and 16 that the increasing values of Brickman number encourages the thermal field and declines the concentration field in both cases.

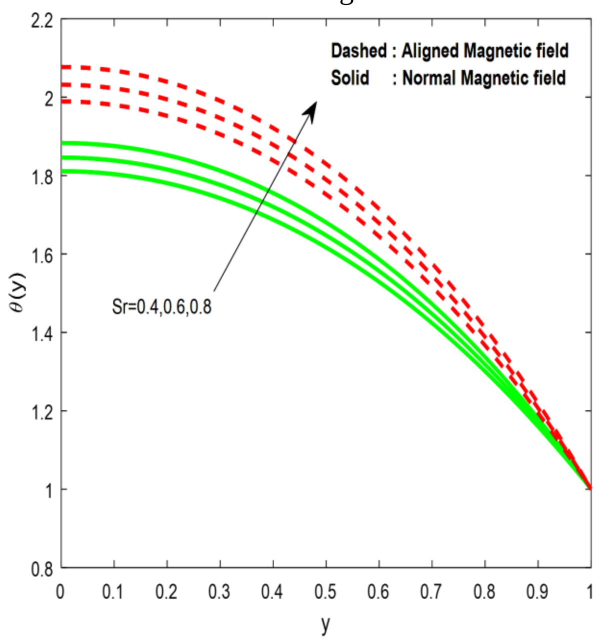

Figure 12. Temperature profiles for *Sr*

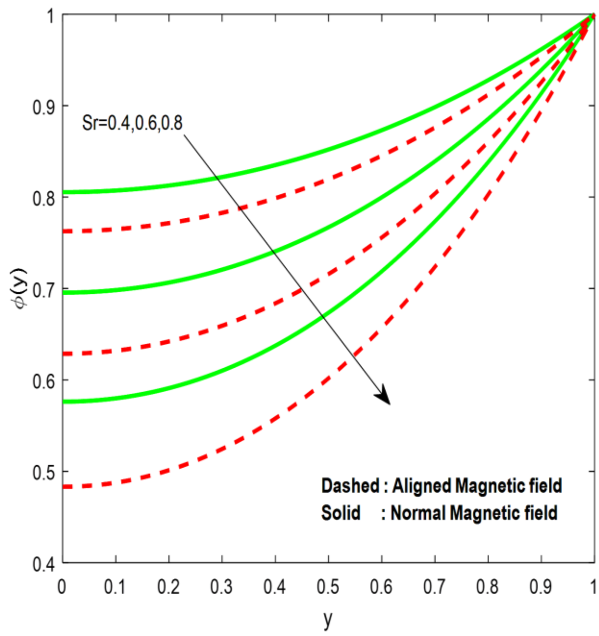

Figure 13. Concentration profilesfor *Sr*

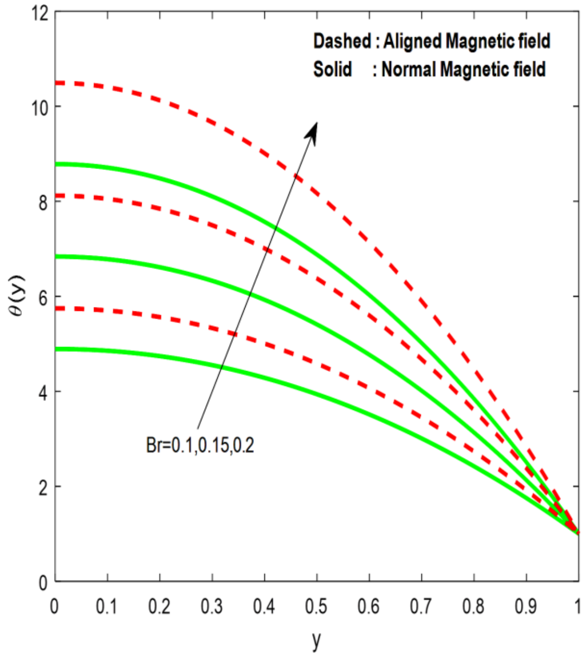

Figure 14. Temperature profiles for Br

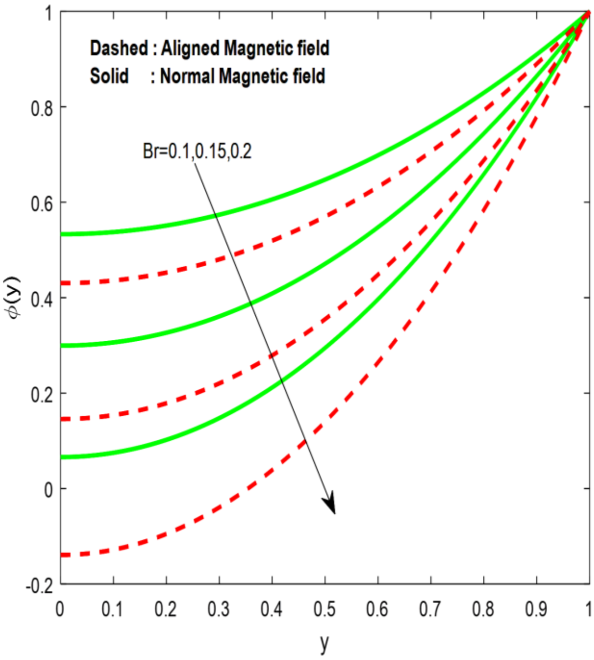

Figure 15. Concentrationprofiles for Br

Table 1 Variations in physical quantities in uniform and non-uniform channel cases

E_1, E_2	E_3	Du	Sr	Br	Aligned magnetic field		Normal magnetic field	
					$-\theta'(h)$	$-\phi'(h)$	$-\theta'(h)$	$-\phi'(h)$
0.1					-0.163415	-0.166791	-0.199292	-0.203410
0.15					-0.173566	-0.176963	-0.211671	-0.215814
0.2					-0.183778	-0.187197	-0.224126	-0.228295
	0.1				-0.144535	-0.147522	-0.176267	-0.179910
	0.2				-0.153513	-0.156518	-0.187216	-0.190881
	0.3				-0.162546	-0.165570	-0.198233	-0.201920
		0.1			-0.186978	-0.191365	-0.228028	-0.233378
		0.4			-0.200235	-0.204717	-0.244195	-0.249661
		0.7			-0.213777	-0.218356	-0.260711	-0.266295
			0.4		-0.768848	-0.787342	-0.937645	-0.960199
			0.6		-0.824784	-0.843735	-1.005861	-1.028972
			0.8		-0.882102	-0.901521	-1.075762	-1.099445
				0.1	-1.634147	-1.667914	-1.992913	-2.034093
				0.15	-1.735654	-1.769628	-2.116705	-2.158137
				0.2	-1.837783	-1.871965	-2.241254	-2.282940

It is evident from the table 1 that the cross diffusion and Brickman number have tendency to reduce the heat and mass transfer rate. The similar trend has been observed for rising valiuesof wall tension (E_1) , mass characterization (E_2) and damping force (E_3)parameters.

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