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Effects of chemical reaction and space porosity on MHD mixed convective flow in a vertical asymmetric channel with peristalsis

S. Srinivas*, R. Muthuraj

Fluid Dynamics Division, School of Advanced Sciences, VIT University, Vellore - 632 014, India

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ABSTRACT

This work is aimed at describing MHD mixed convective heat and mass transfer peristaltic flow through a vertical porous space in the presence of a chemical reaction. The flow is examined in a wave frame of reference moving with the velocity of the wave. The channel asymmetry is produced by choosing the peristaltic wave train on the walls to have different amplitude and phase. The momentum, energy and concentration equation have been linearized under long-wavelength approximation. Expressions for dimensionless stream function, temperature and concentration field are constructed. The features of the fluid flow, heat and mass transfer characteristics are analyzed by plotting graphs and discussed in detail.

1. Introduction

Peristaltic flows are generated by the propagation of waves along the flexible walls of the channel or tube. These occur widely in many biological and biomedical systems. In physiology, this plays an indispensable role in various situations such as urine transport from kidneys to bladder through the ureter, chyme movement in the gastrointestinal tract, transport of spermatozoa in the ductus efferents of the male reproductive tracts, movements of ovum in the female fallopian tube and circulation of blood in the small blood vessels. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, blood pumps in heart lung machine and transport of corrosive fluids where the contact of the fluid with the machinery parts is prohibited. Peristaltic transport of a toxic liquid is used in nuclear industry to avoid contamination of the outside environment. Such flows are extensively studied in various geometries by using different assumptions of large wave length, small amplitude ratio, small wave number, creeping flow, etc. At present a wealth of literature on this topic dealing with the peristalsis in viscous and non-Newtonian fluid is available (see Refs. [1–12] and several references therein).

Heat transfer in biological tissues involves complicated processes such as heat conduction in tissues, heat convection due to blood flow through the pores of tissues, as well as radiation heat transfer between surface and its environment and there is also mass transfer in organisms. Research interest in flow as well as heat transfer phenomena in a channel/tube has increased substantially in recent years due to developments in the electronic industry, microfabrication technologies, biomedical engineering, etc. The interaction between peristalsis and heat transfer has been investigated recently, where the thermodynamic aspects of blood become significant in processes like oxygenation and hemodialysis [13–15]. Some recent interesting contributions pertaining to heat transfer aspects of peristaltic transport are cited in Refs. [13–25].

Very few investigations have been made to study the combined effects of heat and mass transfer in peristaltic literature [26–30]. Srinivas and Kothandapani [26] have analyzed the influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls. Nadeem et al. [27] have presented a mathematical model to understand

* Corresponding author. Tel.: +91 416 2202514; fax: +91 416 2243092.
E-mail address: srinusuripeddi@hotmail.com (S. Srinivas).

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the influence of heat and mass transfer on peristaltic flow of a third order fluid in a diverging tube. Hayat and Hina [28] have studied the influence of wall properties on the MHD peristaltic flow of a Maxwell fluid with heat and mass transfer. Eldabe et al. [29] have analyzed the problem of peristaltic transport of a non-Newtonian fluid with variable viscosity in the presence of heat and mass transfer and mixed diffusion flow between a vertical wall that deforms in the shape of a traveling wave and a parallel flat wall. Nadeem and Akbar [30] have discussed the influence of radially varying MHD on the peristaltic flow in an annulus with heat and mass transfer. Recently, Srinivas and Muthuraj [31] have examined the problem of MHD flow in a vertical wavy porous space in the presence of a temperature-dependent heat source with slip-flow boundary condition. More recently, Muthuraj and Srinivas [32] have investigated the problem of mixed convective heat and mass transfer in a vertical wavy channel through porous medium with traveling thermal waves.

To the best of our knowledge, the influence of MHD mixed convective heat and mass transfer analysis on peristaltic flow with chemical reaction has not been studied before. Therefore the main goal here is to construct a mathematical model to understand the effect of heat and mass transfer on MHD peristaltic flow of a Newtonian fluid, in a vertical asymmetric channel filled with porous medium, in the presence of chemical reaction. The features of the flow and heat and mass transfer characteristics are analyzed by plotting graphs and discussed in detail. This paper is organized as follows: In Section 2, the general equations are first modeled and then problem statement is given under the long-wavelength and low-Reynolds number assumptions. Section 3 includes the analytic solutions for the problem. Section 4 contains numerical results and discussion. The conclusions are summarized in Section 5.

2. Mathematical model

We consider the motion of an incompressible viscous fluid in a two-dimensional vertical channel induced by sinusoidal wave trains propagating with constant speed c along the channel walls

$$H_1 = d_1 + a_1 \cos \frac{2\pi}{\lambda}(X - ct) \quad \ldots \text{ right-hand side wall,}$$
$$H_2 = -d_2 - b_1 \cos\left(\frac{2\pi}{\lambda}(X - ct) + \varphi\right) \quad \ldots \text{ left-hand side wall,} \tag{1}$$

where a_1, b_1 are the amplitudes of the waves, λ is the wave length, $d_1 + d_2$ is the width of the channel, the phase difference φ varies in the range $0 \le \varphi \le \pi$, $\varphi = 0$ corresponds to symmetric channel with waves out of phase and $\varphi = \pi$ the waves are in phase, and further a_1, b_1, d_1, d_2 and φ satisfies the condition

$$a_1^2 + b_1^2 + 2a_1b_1\cos\varphi \le (d_1 + d_2)^2\,. \tag{2}$$

It is assumed that the temperature at right-hand side wall is T_1' and concentration is C_1' while the temperature at the left-hand side wall is T_2' and concentration is C_2' (Fig. 1). The continuity, momentum, energy and concentration equations are described by

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{3}$$

$$\rho\left[\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right] = -\frac{\partial P}{\partial X} + \mu\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \sigma B_0^2 U - \frac{\mu\phi^*}{k}U + \rho g\beta_t(T - \overline{T}) + \rho g\beta_c(C - \overline{C}) \tag{4}$$

$$\rho\left[\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right] = -\frac{\partial P}{\partial Y} + \mu\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \frac{\mu\phi^*}{k}V, \tag{5}$$

$$\left[\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y}\right] = \frac{K}{\rho c_p}\left[\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right] \tag{6}$$

$$\left[\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y}\right] = D_m\left[\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right] - k_1 C \tag{7}$$

where U, V are the velocity components in the laboratory frame (X, Y), g is the acceleration due to gravity, T is the temperature of the fluid, C is the concentration of the fluid, $\overline{T}$ is the mean value of T_1' and T_2', B_0 is the transverse magnetic field, $\overline{C}$ is the mean value of C_1' and C_2', k is the permeability of the medium, k_1 is the chemical reaction parameter, σ is the coefficient of electric conductivity, ϕ^* is the porosity of the medium, ρ is the density, μ is the coefficient of viscosity of the fluid, P is the pressure, β_t is the coefficient of thermal expansion, β_c is the coefficient of expansion with concentration, c_p is the specific heat at constant pressure, D_m is the coefficient of mass diffusivity, K is the thermal conductivity of the fluid. In writing the above equations the following assumptions are made: (i) Boussinesq approximation is invoked so that the density variations will be retained only in the buoyancy term and (ii) dissipation function effect is neglected.

We shall carry out this investigation in a coordinate system moving with the wave speed c, in which the boundary shape is stationary. The coordinates and velocities in the laboratory frame (X, Y) and the wave frame (x, y) are related by:

$$x = X - ct, \qquad y = Y, \qquad u = U - c, \qquad v = V, \qquad p(x) = P(X, t) \tag{8}$$

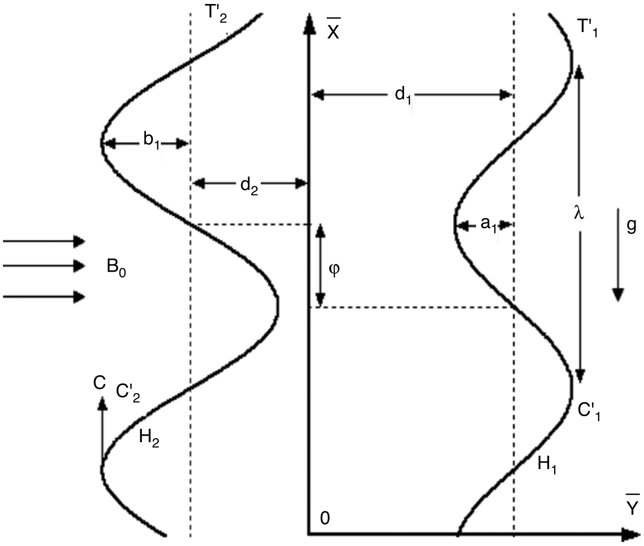

Fig. 1. Schematic diagram of the physical model.

where u, v are the velocity components in the wave frame (x, y), p and P are pressures in wave and fixed frame of references respectively. Introducing the following non-dimensional quantities:

$$\bar{x} = \frac{x}{\lambda}; \quad \bar{y} = \frac{y}{d_1}; \quad \bar{u} = \frac{u}{c}; \quad \bar{v} = \frac{v}{c\delta}; \quad \delta = \frac{d_1}{\lambda}; \quad \bar{p} = \frac{d_1^2 p}{\mu c \lambda}; \quad \bar{t} = \frac{ct}{\lambda}; \quad h_1 = \frac{H_1}{d_1};$$

$$h_2 = \frac{H_2}{d_1}; \quad d = \frac{d_2}{d_1}; \quad a = \frac{a_1}{d_1};$$

$$b = \frac{b_1}{d_1}; \quad \mathrm{Re} = \frac{\rho c d_1}{\mu}; \quad \theta = \frac{T - \overline{T}}{T_1' - \overline{T}}; \quad n = \frac{T_2' - \overline{T}}{T_1' - \overline{T}}; \quad \phi = \frac{C - \overline{C}}{C_1' - \overline{C}}; \quad m = \frac{C_2' - \overline{C}}{C_1' - \overline{C}};$$

$$P_r = \frac{c_p \mu}{K}; \quad S_c = \frac{\mu}{\rho D_m}$$

$$g_t = \frac{\rho g \beta_t \left(T_1' - \overline{T}\right) d_1^2}{\mu c}; \quad g_c = \frac{\rho g \beta_c \left(C_1' - \overline{C}\right) d_1^2}{\mu c}; \quad \gamma = \frac{k_1 d_1^2}{\nu}; \quad D_a = \frac{k}{\phi^* d_1^2}; \quad M^2 = \frac{\sigma B_0^2 d_1^2}{\mu} \tag{9}$$

where Re is the Reynolds number, M is the Hartmann number, D_a is the permeability parameter, g_t is the local Grashof number, g_c is the local mass Grashof number, P_r is the Prandtl number, S_c is the Schmidt number, δ is the dimensionless wave number, and γ is the chemical reaction parameter. In terms of these non-dimensional variables, the basic Eqs. (3)–(7) can be expressed in the non-dimensional form, dropping the bars,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$\mathrm{Re}\delta \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \left(\delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - H^2(u + 1) + g_t \theta + g_c \phi, \tag{11}$$

$$\mathrm{Re}\delta^3 \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \delta^2 \left(\delta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\delta^2}{D_a} v \tag{12}$$

$$\mathrm{Re} P_r \delta \left(u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \left\{ \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right\}, \tag{13}$$

$$\mathrm{Re}\delta \left(u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} \right) = \frac{1}{S_c} \left\{ \delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right\} - \gamma \phi - c_1 \tag{14}$$

where, $c_1 = \frac{k_1 d_1^2 \bar{C}}{\nu (C_1' - \bar{C})}$; $H = \sqrt{M^2 + \frac{1}{D_a}}$.

Introducing the dimensionless stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \tag{15}$$

The compatibility equations which govern the problem in terms of the stream function $\psi(x, y)$ after eliminating the pressure gradient, Eqs. (11)–(14) becomes

$$\mathrm{Re}\delta\left[(\psi_y\psi_{xyy} - \psi_x\psi_{yyy}) + \delta^2(\psi_y\psi_{xxx} - \psi_x\psi_{xxy})\right] = 2\delta^2\psi_{xxyy} + \delta^4\psi_{xxxx} + \psi_{yyyy} - H^2\psi_{yy} - \frac{\delta^2}{D_a}\psi_{xx} + g_t\theta_y + g_c\phi_y \quad (16)$$

$$\mathrm{Re}P_r\delta[\psi_y\theta_x - \psi_x\theta_y] = \delta^2(\theta_{xx} + \theta_{yy}) \quad (17)$$

$$\mathrm{Re}\delta[\psi_y\phi_x - \psi_x\phi_y] = \frac{1}{S_c}(\delta^2\phi_{xx} + \phi_{yy}) - \gamma\phi - c_1. \quad (18)$$

The corresponding boundary conditions are

$$\psi = \frac{q}{2} \qquad \psi_y = -1 \qquad \theta = 1 \qquad \phi = 1 \quad \text{at } y = h_1 \quad (19)$$

$$\psi = -\frac{q}{2} \qquad \psi_y = -1 \qquad \theta = n \qquad \phi = m \quad \text{at } y = h_2. \quad (20)$$

It should be noted that Eq. (11) for the axial pressure gradient becomes

$$\mathrm{Re}\delta\left[\left(\psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)\psi_y\right] = -\frac{\partial p}{\partial x} + \left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_y - H^2(\psi_y + 1) + g_t\theta + g_c\phi. \quad (21)$$

In laboratory frame, the dimensional volume flow rate is

$$Q = \int_{H_2(X,t)}^{H_1(X,t)} U(X, Y, t)\ \mathrm{d}Y \quad (22)$$

in which H_1 and H_2 are function of X and t. The above expression in wave frame becomes

$$q = \int_{h_2}^{h_1} u(x, y)\ \mathrm{d}y, \quad (23)$$

where h_1 and h_2 are functions of x alone. From Eqs. (8), (22) and (23) we can write

$$Q = q + ch_1(x) - ch_2(x). \quad (24)$$

The time-averaged flow over a period T at a fixed position X is

$$\overline{Q} = \frac{1}{T}\int_0^T Q\,\mathrm{d}t. \quad (25)$$

Substituting (24) into (25) and integrating, we get

$$\overline{Q} = q + cd_1 + cd_2. \quad (26)$$

If we find the dimensionless mean flows Θ, in the laboratory frame, and F, in the wave frame, according to

$$\Theta = \frac{\overline{Q}}{cd_1}, \qquad F = \frac{q}{cd_1}, \quad (27)$$

one finds that Eq. (19) becomes

$$\Theta = F + 1 + d \quad (28)$$

in which

$$F = \int_{h_2}^{h_1} \frac{\partial\psi}{\partial y}\mathrm{d}y. \quad (29)$$

We note that $h_1(x)$ and $h_2(x)$ represent the dimensionless form of the surfaces of the peristaltic walls

$$h_1(x) = 1 + a\cos(2\pi x), \qquad h_2(x) = -d - b\cos(2\pi x + \varphi) \quad (30)$$

where a, b, d and φ satisfies the relation Ref. [2]

$$a^2 + b^2 + 2ab\cos\varphi \leq (1 + d)^2. \quad (31)$$

3. Solutions

We seek perturbation solution in terms of the small parameter δ as follows:

$$f = f_0 + \delta f_1 + \delta^2 f_2 + \cdots$$

where f represents any flow variable.

i.e.

$$\psi = \psi_0 + \delta\psi_1 + \delta^2\psi_2 + \cdots$$
$$\theta = \theta_0 + \delta\theta_1 + \delta^2\theta_2 + \cdots$$
$$\phi = \phi_0 + \delta\phi_1 + \delta^2\phi_2 + \cdots$$
$$p = p_0 + \delta p_1 + \delta^2 p_2 + \cdots. \quad (32)$$

Substituting Eq. (32) in Eqs. (16)–(18), collecting the coefficients of various powers of δ, we get

The zeroth order equations are

$$\psi_{0yyyy} - H^2\psi_{0yy} + g_t\theta_{0y} + g_c\phi_{0y} = 0 \quad (33)$$
$$\theta_{0yy} = 0 \quad (34)$$
$$\phi_{0yy} - \gamma S_c\phi_0 - S_c c_1 = 0 \quad (35)$$
$$p_{0x} = \psi_{0yyy} - H^2(\psi_{0y} + 1) + g_t\theta_0 + g_c\phi_0. \quad (36)$$

The corresponding dimensionless boundary conditions in the wave frame are

$$\psi_0 = \frac{q}{2} \qquad \psi_{0y} = -1 \qquad \theta_0 = 1 \qquad \phi_0 = 1 \quad \text{at } y = h_1 \quad (37)$$
$$\psi_0 = -\frac{q}{2} \qquad \psi_{0y} = -1 \qquad \theta_0 = n \qquad \phi_0 = m \quad \text{at } y = h_2. \quad (38)$$

The first order equations are

$$\mathrm{Re}[\psi_{0y}\psi_{0xyy} - \psi_{0x}\psi_{0yyy}] = \psi_{1yyyy} - H^2\psi_{1yy} + g_t\theta_{1y} + g_c\phi_{1y} \quad (39)$$
$$\mathrm{Re}[\psi_{0y}\theta_{0x} - \psi_{0x}\theta_{0y}] = \theta_{1yy} \quad (40)$$
$$\mathrm{Re}[\psi_{0y}\phi_{0x} - \psi_{0x}\phi_{0y}] = \frac{1}{S_c}\theta_{1yy} - \gamma\phi_1 \quad (41)$$
$$p_{1x} = \psi_{1yyy} - H^2\psi_{1y} + g_t\theta_1 + g_c\phi_1 - \mathrm{Re}[\psi_{0y}\psi_{0xy} - \psi_{0x}\psi_{0yy}]. \quad (42)$$

The corresponding dimensionless boundary conditions in the wave frame are

$$\psi_1 = 0 \qquad \psi_{1y} = 0 \qquad \theta_1 = 0 \qquad \phi_1 = 0 \quad \text{at } y = h_1 \quad (43)$$
$$\psi_1 = 0 \qquad \psi_{1y} = 0 \qquad \theta_1 = 0 \qquad \phi_1 = 0 \quad \text{at } y = h_2. \quad (44)$$

Solving the Eqs. (33)–(35) with boundary conditions (37)–(38) and the Eqs. (39)–(41) with boundary conditions (43)–(44), we get

Zeroth order solution:

$$\theta_0 = Ay + B \quad (45)$$
$$\phi_0 = A_1\cosh\alpha y + B_1\sinh\alpha y - \frac{c_1 S_c}{\alpha^2} \quad (46)$$
$$\psi_0 = A_2 + B_2 y + C_2\cosh Hy + D_2\sinh Hy + T_4y^2 + T_5\sinh\alpha y + T_6\cosh\alpha y \quad (47)$$
$$p_{0x} = T_{141} + T_{142}y + T_{143}\cosh\alpha y + T_{144}\sinh\alpha y. \quad (48)$$

First order solution:

$$\begin{aligned}\theta_1 &= A_3 + B_3y + T_{28}y^2 + T_{29}y^3 + T_{30}y^4 + T_{31}\sinh Hy + T_{32}\cosh Hy + T_{33}\sinh\alpha y + T_{34}\cosh\alpha y \\ &\quad + T_{35}y\sinh Hy + T_{36}y\cosh Hy + T_{37}y\sinh\alpha y + T_{38}y\cosh\alpha y \end{aligned} \quad (49)$$

$$\begin{aligned}\phi_1 &= A_4\cosh\alpha y + B_4\sinh\alpha y + (T_{54} + T_{58})y\sinh\alpha y + (T_{55} + T_{59})y\cosh\alpha y + T_{56}\sinh 2\alpha y \\ &\quad + T_{57}\cosh 2\alpha y + T_{58}y^2\sinh\alpha y + T_{59}y^2\cosh\alpha y + T_{60}\sinh(H+\alpha)y + T_{61}\sinh(H-\alpha)y \\ &\quad + T_{62}\cosh(H+\alpha)y + T_{63}\cosh(H-\alpha)y + T_{64}\end{aligned} \quad (50)$$

$$\psi_1 = A_5 + B_5y + (C_5 + T_{106})\cosh Hy + (D_5 + T_{105})\sinh Hy + T_{107}y\sinh Hy + T_{108}y\cosh Hy$$

$$+T_{109}y^2\sinh Hy+T_{110}y^2\cosh Hy+T_{111}\cosh\alpha y+T_{112}\sinh\alpha y+T_{113}y\cosh\alpha y+T_{114}y\sinh\alpha y$$
$$+T_{115}\sinh(H+\alpha)y+T_{116}\sinh(H-\alpha)y+T_{117}\cosh(H+\alpha)y+T_{118}\cosh(H-\alpha)y+T_{119}\cosh 2\alpha y$$
$$+T_{120}\sinh 2\alpha y+T_{121}y^2\cosh\alpha y+T_{122}y^2\sinh\alpha y+T_{123}y^3+T_{124}y^2+T_{125}y+T_{126} \quad (51)$$

$$p_{1x}=T_{180}+T_{181}y+T_{178}y^2+T_{177}y^3+T_{176}y^4+T_{182}\sinh Hy+T_{183}\cosh Hy+T_{184}y\sinh Hy$$
$$+T_{185}y\cosh Hy+T_{186}\sinh\alpha y+T_{187}\cosh\alpha y+T_{188}y\sinh\alpha y+T_{189}y\cosh\alpha y+T_{168}y^2\sinh\alpha y$$
$$+T_{169}y^2\cosh\alpha y+T_{170}\sinh 2\alpha y+T_{171}\cosh 2\alpha y+T_{190}\sinh(H+\alpha)y+T_{191}\sinh(H-\alpha)y$$
$$+T_{192}\cosh(H+\alpha)y+T_{193}\cosh(H-\alpha)y \quad (52)$$

where $\alpha=\sqrt{\gamma S_c}$.

The non-dimensional expression for the pressure rise per wavelength is given as follows:

$$\Delta p_\lambda=\int_0^1\left(\frac{\partial p}{\partial x}\right)\mathrm{d}x. \quad (53)$$

The frictional forces at $y=h_1$ and $y=h_2$ denoted by $F_{\lambda 1}$ and $F_{\lambda 2}$ respectively are given as follows.

$$F_{\lambda 1}=\int_0^1 -h_1^2\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)\mathrm{d}x \quad (54)$$

$$F_{\lambda 2}=\int_0^1 -h_2^2\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)\mathrm{d}x. \quad (55)$$

The coefficient of heat transfer at the wall is given by

$$Z=h_{2x}\theta_{0y}+\delta\left(\theta_{0x}+h_{2x}\theta_{1y}\right). \quad (56)$$

The shearing stress acting on the (left and right) wall is defined as

$$\bar{\tau}=\frac{\sigma_{xy}\left\{1-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right\}+\left(\sigma_{yy}-\sigma_{xx}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\quad \text{at } y=H_2(x) \text{ and } H_1(x) \quad (57)$$

where $\sigma_{xy},\sigma_{yy},\sigma_{xx}$ are the usual stress components.

The non-dimensional shear stress reduces to

$$\tau=H^2(C_2\cosh Hy+D_2\sinh Hy)+2T_4+\alpha^2(T_5\sinh\alpha y+T_6\cosh\alpha y)$$
$$+\delta([H^2(C_5+T_{106})+2(HT_{107}+T_{110})]\cosh Hy+[H^2(D_5+T_{107})+2(HT_{108}+T_{109})]\sinh Hy$$
$$+[H^2T_{107}+4HT_{110}]y\sinh Hy+[H^2T_{108}+4HT_{109}]y\cosh Hy+H^2T_{109}y^2\sinh Hy$$
$$+H^2T_{110}y^2\cosh Hy+[\alpha^2T_{111}+2(\alpha T_{114}+T_{121})]\cosh\alpha y+[\alpha^2T_{112}+2(\alpha T_{113}+T_{122})]\sinh\alpha y$$
$$+[\alpha^2T_{114}+4\alpha T_{121}]y\sinh\alpha y+[\alpha^2T_{113}+4\alpha T_{122}]y\cosh\alpha y+\alpha^2T_{121}y^2\cosh\alpha y+\alpha^2T_{122}y^2\sinh\alpha y$$
$$+4\alpha^2T_{119}\cosh 2\alpha y+4\alpha^2T_{120}\sinh 2\alpha y+T_{115}(H+\alpha)^2\sinh(H+\alpha)y+T_{116}(H-\alpha)^2\sinh(H-\alpha)y$$
$$+T_{117}(H+\alpha)^2\cosh(H+\alpha)y+T_{118}(H-\alpha)^2\cosh(H-\alpha)y+6T_{123}y+2T_{124}). \quad (58)$$

4. Results and discussion

This section provides the behavior of parameters involved in the expressions of flow, heat and mass transfer characteristics. In particular, the influence of Hartmann number (M), permeability parameter (D_a), Schmidt number (S_c), Prandtl number (P_r), Grashof number (g_t), local mass Grashof number (g_c), chemical reaction parameter (γ), Reynolds number (Re), dimensionless flow rate (Θ), mean half width of the channel (d) and phase angle (φ) are examined and are shown graphically in Figs. 2–10. Fig. 2 depicts the effects of M, D_a, g_t, S_c, γ and φ on velocity field. It is apparent from Fig. 2(a) that increasing M, leads to fall in the velocity. Physically speaking, the effect of increasing magnetic field strength dampens the velocity. The effect of permeability parameter on the velocity is displayed in Fig. 2(b). It is clear that the D_a increases, the velocity leads to enhance. An increasing D_a means reduce the drag force and hence cause the flow velocity to increase (as noted in Ref. [22]). Increasing Grashof number means an increase of the buoyancy force, which supports the motion, which is shown in Fig. 2(c). Fig. 2(d) shows the influence of S_c on velocity distribution. The values chosen for S_c are 0.5, 0.6, 0.78, 1 and 2, which corresponds to Hydrogen gas, water vapor, ammonia, carbon dioxide at 25 °C, and ethyl benzene in air, respectively. It is observed that increasing Schmidt number lead to reduce the fluid velocity. Fig. 2(e) illustrates that with an increase of chemical reaction parameter (γ), the velocity field decreases. The opposite result to that is shown in Fig. 2(f) if S_c is replaced by φ.

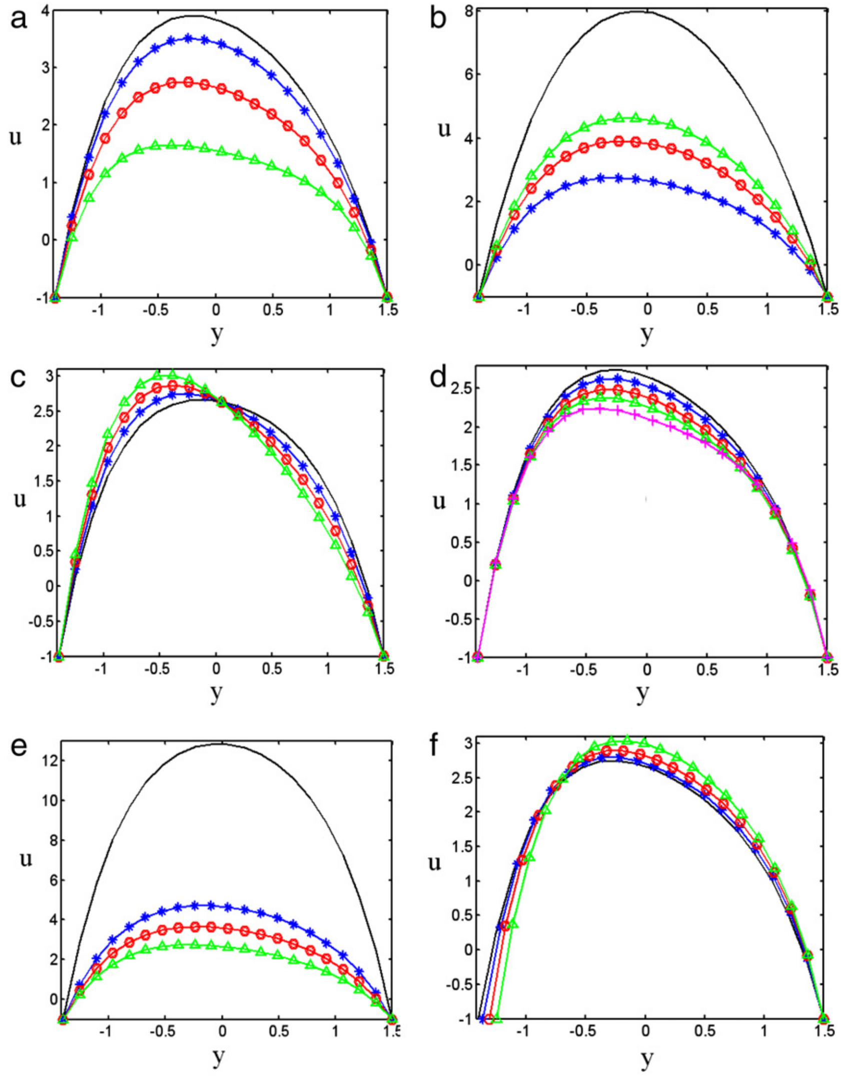

Fig. 2. Velocity distribution $a = 0.5, b = 0.3, d = 1.1, g_c = 5, \mathrm{Re} = 1, m = 2, q = 1, n = 2, c_1 = 1, P_r = 0.71, \delta = 0.01$ (a) $(-)$ $M = 0$, $(*)$ $M = 0.5$, $(\circ)$ $M = 1$, $(\hat{})$ $M = 2, g_t = 5, D_a = 0.5, \gamma = 0.5, \varphi = 0, S_c = 0.5$ (b) $(-)$ $D_a = \infty$, $(*)$ $D_a = 0.5$, $(\circ)$ $D_a = 1$, $(\hat{})$ $D_a = 1.5, g_t = 5, M = 1, \gamma = 0.5, \varphi = 0, S_c = 0.5$ (c) $(-)$ $g_t = 0$, $(*)$ $g_t = 5$, $(\circ)$ $g_t = 10$, $(\hat{})$ $g_t = 15, D_a = 0.5, M = 2, \gamma = 0.5, \varphi = 0, S_c = 0.5$ (d) $(-)$ $S_c = 0.5$, $(*)$ $S_c = 0.6$, $(\circ)$ $S_c = 0.78$, $(\hat{})$ $S_c = 1$, $(+)$ $S_c = 2, g_t = 5, D_a = 0.5, \varphi = 0, M = 1, \gamma = 0.5$ (e) $(-)$ $\gamma = 0$, $(*)$ $\gamma = 0.1$, $(\circ)$ $\gamma = 0.2$, $(\hat{})$ $\gamma = 0.5, g_t = 5, D_a = 0.5, M = 1, \varphi = 0, S_c = 0.5$. (f) $(-)$ $\varphi = 0$, $(*)$ $\varphi = \pi/8$, $(\circ)$ $\varphi = \pi/6$, $(\hat{})$ $\varphi = \pi/3, g_t = 5, D_a = 0.5, M = 1, \gamma = 0.5, S_c = 0.5$.

The effects of g_t, S_c, γ, M and D_a on the dimensionless pressure drop (Δp_λ) against the time-averaged flux (Θ) are illustrated in Fig. 3. The graph is sectored so that the upper right-hand quadrant (I) denotes the region of the peristaltic pumping ($\Theta > 0$ and $\Delta p_\lambda > 0$). Quadrant (II) is designated as augmented flow when $\Theta > 0$ and $\Delta p_\lambda < 0$. Quadrant (IV) such that $\Theta < 0$ and $\Delta p_\lambda > 0$ is called retrograde or backward pumping. It shows that there is a linear relation between Δp_λ and Θ. Further, it is observed that peristaltic pumping region increases with an increase of g_t, γ, M and D_a while it decreases with increasing S_c. Two figures have been made to see the behavior of frictional forces under the presence of $g_t S_c, \gamma$ and M at the channel walls. In Fig. 4 we have plotted the frictional force at the wall $y = h_1$versus dimensionless average volume flow rate Θ for different values of g_t and S_c. The effects of these parameters on the frictional force are quite opposite to that of pumping characteristics. Fig. 5 shows that the variations of γ and M on frictional force (at right wall $y = h_2$ of the channel) versus flow rate Θ. It is clear that the influence of γ and M on $F_{\lambda 2}$ is similar to that of g_t and S_c on $F_{\lambda 1}$.

Fig. 6 is made to see the effects of M, φ, γ, q and D_a on axial pressure gradient $(\mathrm{d}p/\mathrm{d}x)$. It displays the variations of the axial pressure gradient $(\mathrm{d}p/\mathrm{d}x)$ over one wave length $x \in [0, 1]$. Fig. 6(a) illustrates the influence of M on $\mathrm{d}p/\mathrm{d}x$. It is observed that in the wide part of the channel $x \in [0, 0.36]$ and $x \in [0.67, 1]$ the pressure gradient is small, that is, the flow can easily pass without the imposition of large pressure gradient. However, in the narrow part of the channel, $x \in [0.36, 0.67]$ a much larger pressure gradient is needed to maintain the same flux to pass it. Further, it is also found that the pressure gradient increases by increasing M. Fig. 6(b) is made to see the variation of phase difference φ on $\mathrm{d}p/\mathrm{d}x$. It is found that $\mathrm{d}p/\mathrm{d}x$

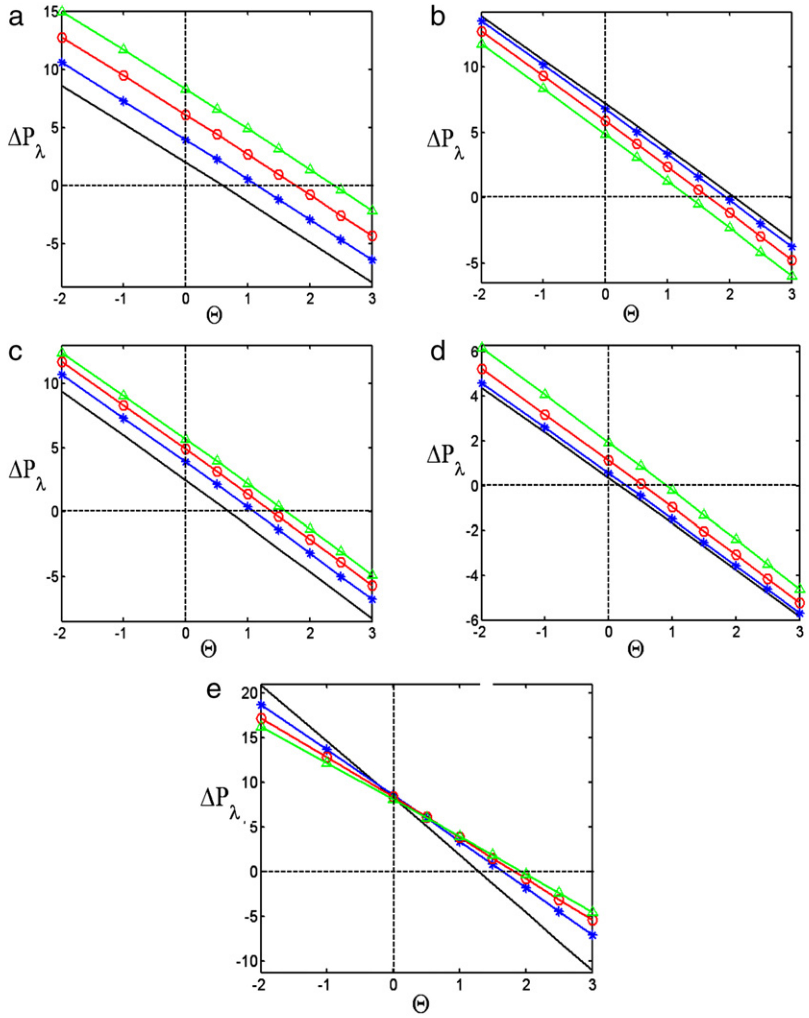

Fig. 3. Pressure drop ($a = 0.5$, $b = 0.3$, $d = 1$, $m = 2$, $n = 2$, $q = 1$, $P_r = 0.71$, $\mathrm{Re} = 2$, $g_c = 5$, $c_1 = 1$, $M = 2$, $\delta = 0.1$, $\varphi = 0$) (a) (−) $g_t = 0$, (∗) $g_t = 2$, (○) $g_t = 4$, (^) $g_t = 6$, $S_c = 0.5$, $D_a = 0.5$, $\gamma = 0.5$ (b) (−) $S_c = 0.5$, (∗) $S_c = 0.6$, (○) $S_c = 0.78$, (^) $S_c = 1$, $\gamma = 0.5$, $D_a = 0.5$, $g_t = 5$ (c) (−) $\gamma = 0.2$, (∗) $\gamma = 0.25$, (○) $\gamma = 0.3$, (^) $\gamma = 0.35$, $S_c = 0.5$, $D_a = 0.5$, $g_t = 5$ (d) (−) $M = 0$, (∗) $M = 0.2$, (○) $M = 0.4$, (^) $M = 0.6$, $S_c = 0.5$, $\gamma = 0.5$, $D_a = 0.5$, $g_t = 5$ (e) (−) $D_a = 0.1$, (∗) $D_a = 0.15$, (○) $D_a = 0.2$, (^) $D_a = 0.25$, $S_c = 0.5$, $\gamma = 0.5$, $g_t = 5$.

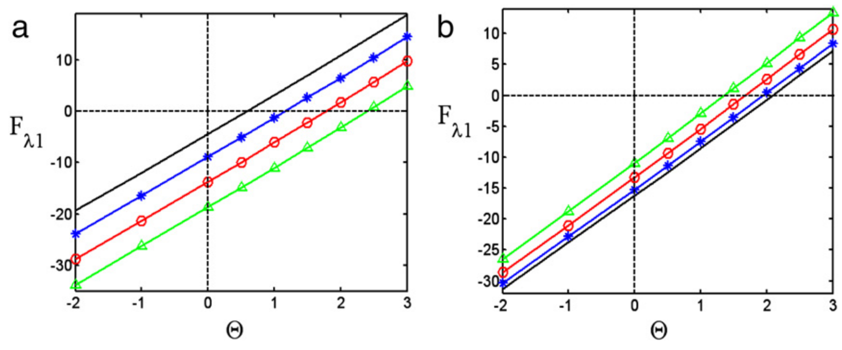

Fig. 4. Frictional forces at the wall $y = h_1$ ($a = 0.5$, $b = 0.3$, $d = 1$, $m = 2$, $n = 2$, $q = 1$, $P_r = 0.71$, $\mathrm{Re} = 2$, $g_c = 5$, $c_1 = 1$, $\gamma = 0.5$, $D_a = 0.5$, $M = 2$, $\delta = 0.1$, $\varphi = 0$) (a) (−) $g_t = 0$, (∗) $g_t = 2$, (○) $g_t = 4$, (^) $g_t = 6$, $S_c = 0.5$ (b) (−) $S_c = 0.5$, (∗) $S_c = 0.6$, (○) $S_c = 0.78$, (^) $S_c = 1$, $g_t = 5$.

decreases both in wider and narrow parts of the channel. Moreover, the narrow region in the channel is shifting to the left with an increase in φ. The results presented in Fig. 6(c) shows the disturbance $\mathrm{d}p/\mathrm{d}x$ for various values of γ. It depicts that by

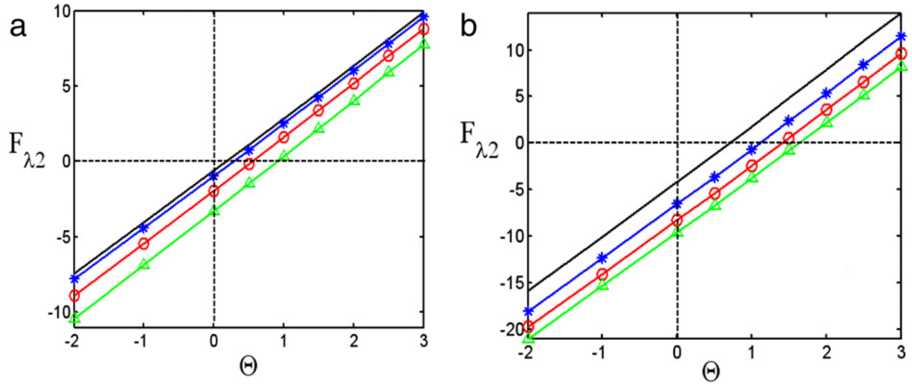

Fig. 5. Frictional forces at the wall $y = h_2$ ($a = 0.5, b = 0.3, d = 1, m = 2, n = 2, q = 1, P_r = 0.71, \mathrm{Re} = 2, g_t = 5, S_c = 0.5, g_c = 5, D_a = 0.5, c_1 = 1, \delta = 0.1, \varphi = 0$) (a) (−) $\gamma = 0.2$, (∗) $\gamma = 0.25$, (∘) $\gamma = 0.3$, (^) $\gamma = 0.35$, $M = 2$ (b) (−) $M = 0$, (∗) $M = 0.2$, (∘) $M = 0.4$, (^) $M = 0.6$, $\gamma = 0.5$.

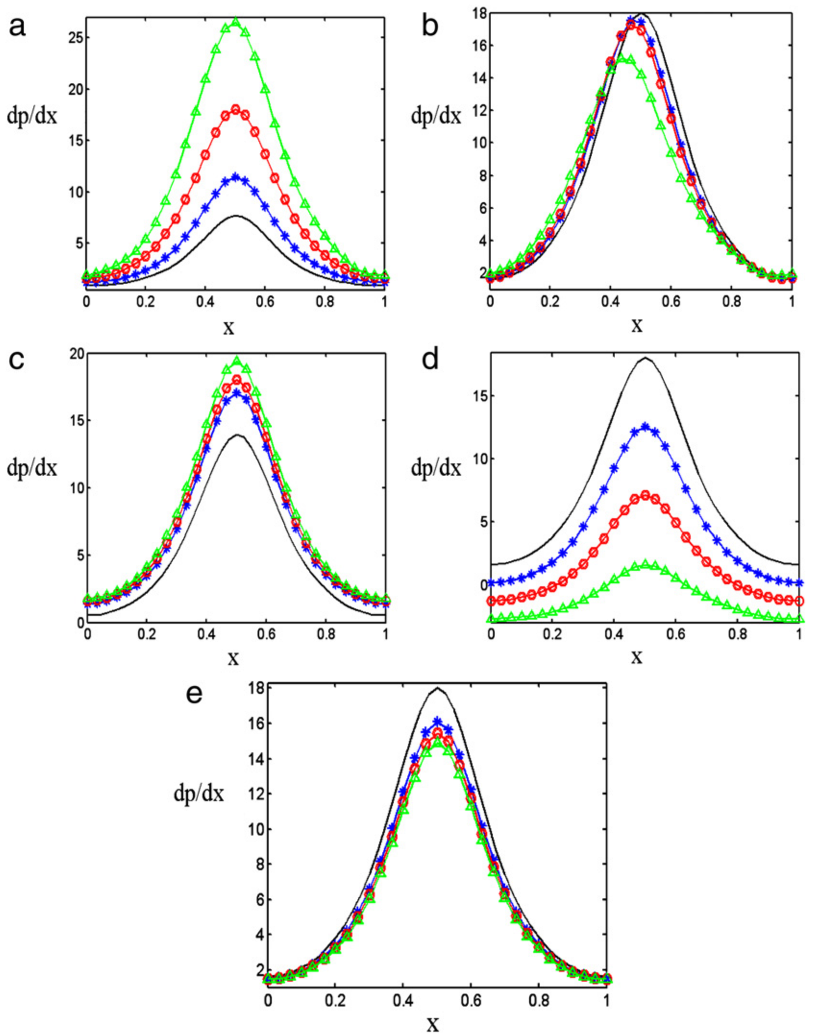

Fig. 6. Pressure gradient ($a = 0.5, b = 0.3, m = 2, n = 2, P_r = 0.71, \mathrm{Re} = 1, S_c = 0.5, g_c = 1, g_t = 1, c_1 = 1, \delta = 0.01, d = 1.1$) (a) (−) $M = 0$, (∗) $M = 1$, (∘) $M = 2$, (^) $M = 3$, $q = -3$, $\gamma = 0.5$, $D_a = 0.5$, $\varphi = 0$ (b) (−) $\varphi = 0$, (∗) $\varphi = \pi/8$, (∘) $\varphi = \pi/6$, (^) $\varphi = \pi/3$, $q = -3$, $D_a = 0.5$, $d = 1$, $\gamma = 0.5$ (c) (−) $\gamma = 0.1$, (∗) $\gamma = 0.3$, (∘) $\gamma = 0.5$, (^) $\gamma = 0.7$, $q = -3$, $d = 1$, $D_a = 0.5$, $\varphi = 0$ (d) (−) $q = -3$, (∗) $q = -2.5$, (∘) $q = -2$, (^) $q = -1.5$, $\gamma = 0.5$, $D_a = 0.5$, $\varphi = 0$ (e) (−) $D_a = 0.5$, (∗) $D_a = 1$, (∘) $D_a = 1.5$, (^) $D_a = 2$, $\gamma = 0.5$, $q = -3$, $\varphi = 0$.

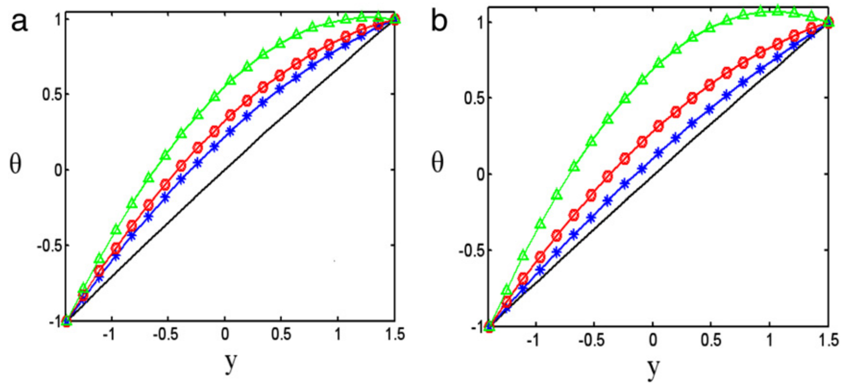

Fig. 7. Temperature distribution ($a = 0.5, b = 0.3, d = 1.1, m = -1, n = -1, q = -1, \gamma = 0.5, \text{Re} = 1, \delta = 0.01, M = 2, c_1 = 1, g_t = 5, g_c = 5, S_c = 0.5, D_a = 0.5$) (a) (−) $P_r = 0.044$, (∗) $P_r = 0.71$, (○) $P_r = 7$, (^) $P_r = 11.4$, $\varphi = 0$ (b) (−) $M = 0$, (∗) $M = 2$, (○) $M = 2.5$, (^) $M = 3$, $P_r = 0.71$.

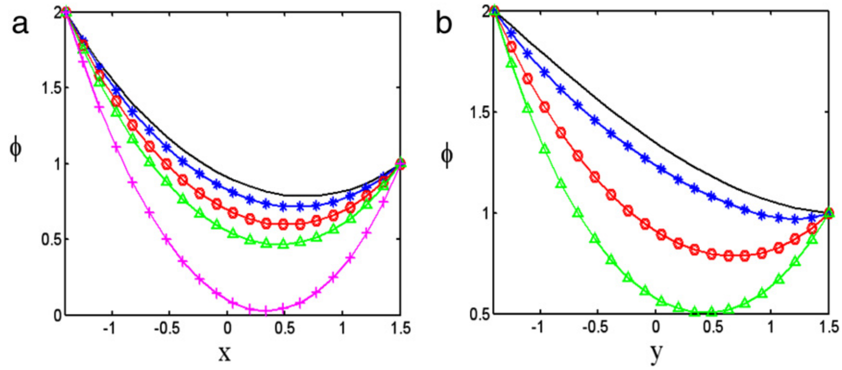

Fig. 8. Concentration distribution ($a = 0.3, b = 0.5, d = 1.1, q = 1, c_1 = 1, m = 2, n = 2, P_r = 0.71, \text{Re} = 1, \gamma = 0.5, g_c = 5, g_t = 5, \delta = 0.01, \varphi = 0$) (a) (−) $S_c = 0.5$, (∗) $S_c = 0.6$, (○) $S_c = 0.78$, (^) $S_c = 1$, (+) $S_c = 2$, $\gamma = 0.5$. (b) (−) $\gamma = -0.5$, (∗) $\gamma = 0.1$, (○) $\gamma = 0.5$, (^) $\gamma = 1.5$, $S_c = 0.5$.

increasing γ, $\mathrm{d}p/\mathrm{d}x$ increases throughout the channel. Fig. 6(d) shows the variation of q on $\mathrm{d}p/\mathrm{d}x$. It depicts that increasing q lead to decrease the pressure gradient. The similar effect can be noticed if q is replaced by D_a (see the Fig. 6(e)).

Fig. 7 depicts the temperature profiles for various values of P_r and M. In Fig. 7(a), we note that increasing P_r (i.e., $P_r = 0.044$, 0.71, 7 and 11, which corresponds to mercury, air, water and water at 4 °C, respectively) leads to increase the fluid temperature. It is also found that the temperature profile is linear for lower value of P_r while it becomes parabolic in nature for higher values of P_r. The behavior of the fluid temperature with changing M is shown in Fig. 7(b). This shows that temperature increases with an increase of M. The aim of Fig. 8 is to examine the fluid concentration for different values of S_c and γ. Fig. 8(a) is prepared to see the influence of S_c on concentration field. It shows that, there is decrease in the concentration distribution with increasing S_c. Similar effects can be found for the behavior of concentration distribution for different values of chemical reaction parameter, which is shown in Fig. 8(b). Fig. 9 is prepared to study the role of different values of γ, Re, D_a and S_c on Shear stress distribution. We notice that stress is in oscillatory behavior, which may be due to peristalsis. Further, we observe that, when $x < 0$ shear stress increases with increasing Re while it decreases with increasing γ, D_a and S_c but this behavior is reversed, when $x > 0$. The effects of g_c, P_r, S_c and φ on coefficient of heat transfer is analyzed through Fig. 10. From this figure, we observe that the absolute value of heat transfer coefficient increases by increasing g_c, P_r, S_c and φ.

5. Conclusion

The problem of MHD mixed convective heat and mass transfer peristaltic flow, through a vertical asymmetric channel with porous medium, in the presence of a chemical reaction has been analyzed. The momentum, energy and concentration equations have been linearized under long-wavelength approximation. Analytical solutions have been developed for stream function, temperature, concentration and heat transfer coefficient. The features of the flow, heat and mass transfer characteristics are analyzed by plotting graphs and discussed in detail. The main findings are summarized as follows:

- The axial pressure gradient increases with an increase in M and γ while it decreases with an increase of φ, q and D_a.
- Pumping rate increases with the increase of g_t, γ, M and D_a.
- Increasing S_c and γ leads to decrease the fluid concentration.

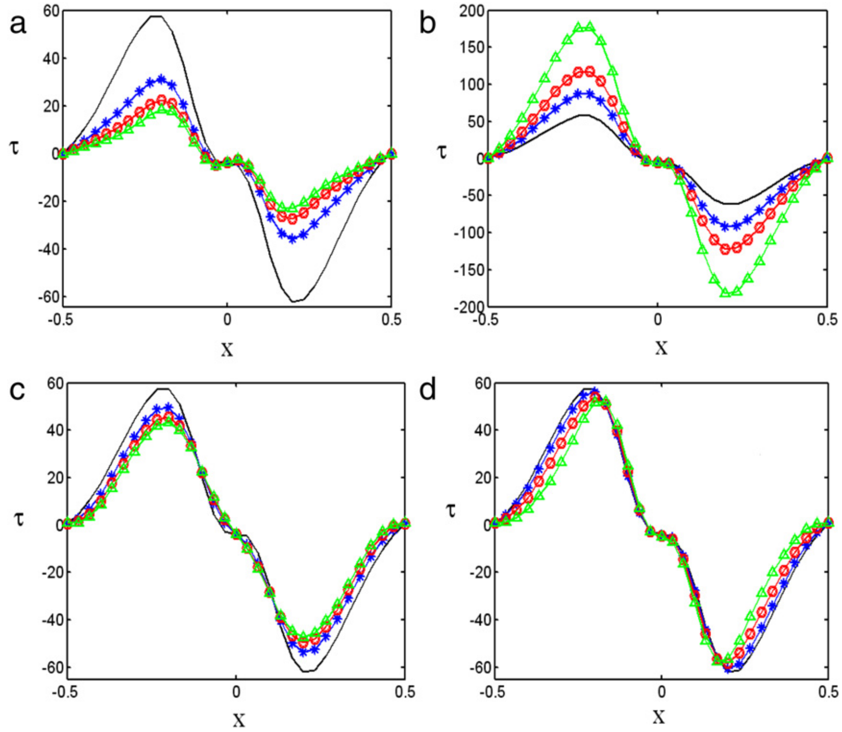

Fig. 9. Shear stress distribution ($a = 0.5, b = 0.3, d = 1.1, m = 1, n = 1, q = -1, c_1 = 1, P_r = 0.71, M = 2, g_c = 5, g_t = 5, \delta = 0.2, \varphi = 0$) (a) (−) $\gamma = 0.1$, (∗) $\gamma = 0.2$, (∘) $\gamma = 0.3$, (^) $\gamma = 0.4$, Re = 1, $S_c = 0.5, D_a = 0.5$ (b) (−) Re = 1, (∗) Re = 3, (∘) Re = 5, (^) Re = 7, $\gamma = 0.1, S_c = 0.5, D_a = 0.5$ (c) (−) $D_a = 0.5$, (∗) $D_a = 0.7$, (∘) $D_a = 0.9$, (^) $D_a = 1.1$, Re = 1, $\gamma = 0.1, S_c = 0.5$ (d) (−) $S_c = 0.5$, (∗) $S_c = 0.6$, (∘) $S_c = 0.78$, (^) $S_c = 1$, Re = 1, $\gamma = 0.1, D_a = 0.5$.

- Heat transfer coefficient (z) increases with an increase of g_c, P_r, S_c and φ.
- The results of the hydrodynamic case for a non-porous space in the absence of chemical reaction can be captured as a limiting case of our analysis by taking $M, \gamma \to 0$ and $D_a \to \infty$.

Acknowledgments

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Appendix

$$A = 1 - Bh_1; \quad B = \frac{1-n}{h_1 - h_2}; \quad A_1 = \frac{1 + \frac{c_1 S_c}{\alpha^2} - B_1 \sinh \alpha h_1}{\cosh \alpha h_1}; \quad B_1 = \frac{\left(1 + \frac{c_1 S_c}{\alpha^2}\right) \cosh \alpha h_2 - (1+m) \cosh \alpha h_1}{\sinh \alpha (h_1 - h_2)};$$

$$D_2 = \frac{T_{13} T_{14} - T_{11} T_{16}}{T_{12} T_{14} - T_{11} T_{15}}; \quad C_2 = \frac{T_{13} T_{15} - T_{12} T_{15}}{T_{11} T_{15} - T_{12} T_{14}}; \quad B_2 = T_9 - HC_2 \sinh Hh_1 - HD_2 \cosh Hh_1;$$

$$A_2 = T_7 - B_2 h_1 - C_2 \cosh Hh_1 - D_2 \sinh Hh_1; \quad A_3 = T_{127} - B_3 h_1; \quad B_3 = \frac{T_{127} - T_{128}}{h_1 - h_2};$$

$$A_4 = \frac{T_{129} - B_4 \sinh \alpha h_1}{\cosh \alpha h_1}; \quad B_4 = \frac{T_{129} \cosh \alpha h_2 - T_{130} \cosh \alpha h_1}{\sinh \alpha (h_1 - h_2)}; \quad D_5 = \frac{T_{137} T_{138} - T_{135} T_{140}}{T_{136} T_{138} - T_{135} T_{139}};$$

$$C_5 = \frac{T_{140} - D_5 T_{139}}{T_{138}}; \quad B_5 = T_{133} - HC_5 \sinh Hh_1 - HD_5 \cosh Hh_1;$$

$$A_5 = T_{131} - B_5 h_1 - C_5 \cosh Hh_1 - D_5 \sinh Hh_1; \quad A_x = -(B_x h_1 + B h_{1x}); \quad B_x = -\frac{1 - n(h_{1x} - h_{2x})}{(h_1 - h_2)^2};$$

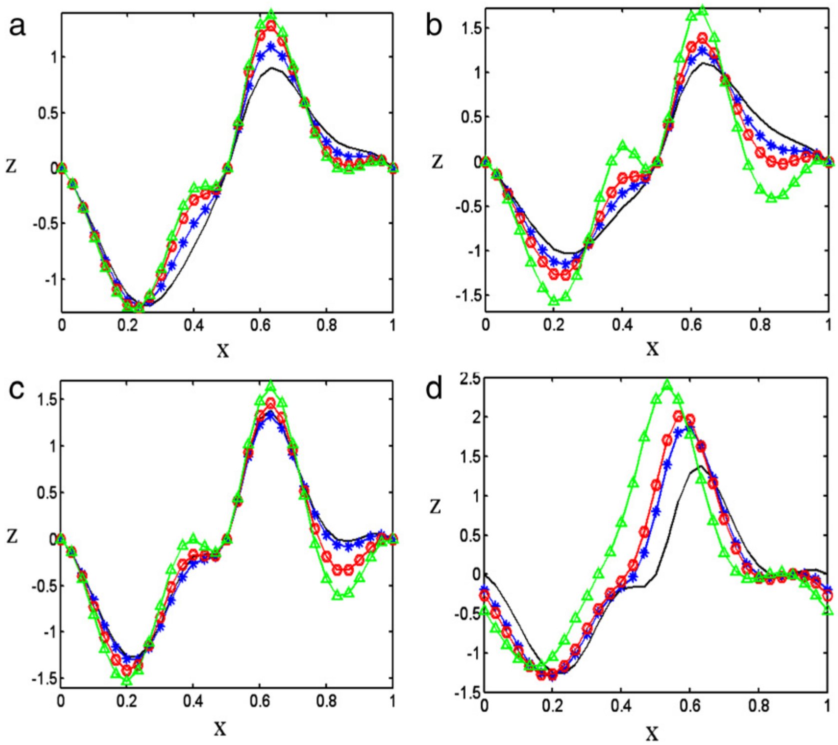

Fig. 10. Coefficient of heat transfer ($a = 0.5$, $b = 0.3$, $d = 1.1$, $m = 2$, $n = 2$, $M = 0$, $D_a = 0.5$, $\mathrm{Re} = 7$, $\gamma = 0.1$, $g_t = 5$, $\delta = 0.01$) (a) (−) $g_c = 0$, (∗) $g_c = 2$, (∘) $g_c = 4$, (^) $g_c = 6$, $\varphi = 0$, $P_r = 7$, $S_c = 0.5$ (b) (−) $P_r = 3$, (∗) $P_r = 5$, (∘) $P_r = 7$, (^) $P_r = 11.4$, $\varphi = 0$, $g_c = 5$, $S_c = 0.5$ (c) (−) $S_c = 0.5$, (∗) $S_c = 1$, (∘) $S_c = 2$, (^) $S_c = 3$, $\varphi = 0$, $g_c = 5$, $P_r = 7$ (d) (−) $\varphi = 0$, (∗) $\varphi = \pi/8$, (∘) $\varphi = \pi/6$, (^) $\varphi = \pi/3$, $P_r = 7$, $g_c = 5$, $S_c = 0.5$.

$$A_{1x} = \frac{\left[\cosh\alpha h_1\left(-[B_1\alpha h_{1x}\cosh\alpha h_1 + B_{1x}\sinh\alpha h_1]\right) - \left(1+\frac{c_1S_c}{\alpha^2} - B_1\sinh\alpha h_1\right)\alpha h_{1x}\sinh\alpha h_1\right]}{\cosh^2\alpha h_1};$$

$$B_{1x} = \frac{\begin{pmatrix}\sinh\alpha(h_1-h_2)\left[\left(1+\dfrac{c_1S_c}{\alpha^2}\right)\alpha h_{2x}\sinh\alpha h_2 - (1+m)\alpha h_{1x}\sinh\alpha h_1\right]\\ -\left[\left(1+\dfrac{c_1S_c}{\alpha^2}\right)\cosh\alpha h_2 - (1+m)\cosh\alpha h_1\right]\alpha(h_{1x}-h_{2x})\cosh\alpha(h_1-h_2)\end{pmatrix}}{\sinh^2\alpha(h_1-h_2)};$$

$$D_{2x} = \frac{\begin{pmatrix}(T_{12}T_{14}-T_{11}T_{15})(T_{13x}T_{14}+T_{13}T_{14x}-[T_{11x}T_{16}+T_{11}T_{16x}])\\ -(T_{13}T_{14}-T_{11}T_{16})(T_{12}T_{14x}+T_{12x}T_{14}-[T_{11x}T_{15}+T_{11}T_{15x}])\end{pmatrix}}{(T_{12}T_{14}-T_{11}T_{15})^2};$$

$$C_{2x} = \frac{\begin{pmatrix}(T_{11}T_{15}-T_{12}T_{14})\,(T_{13}T_{15x}+T_{13x}T_{15}-[T_{12x}T_{15}+T_{12}T_{15x}])\\ -(T_{13}T_{15}-T_{12}T_{15})\,(T_{11x}T_{15}+T_{11}T_{15x}-[T_{12x}T_{14}+T_{12}T_{14x}])\end{pmatrix}}{(T_{11}T_{15}-T_{12}T_{14})^2};$$

$$B_{2x} = T_{9x} - H[C_{2x}\sinh Hh_1 + C_2Hh_{1x}\cosh Hh_1] - H[D_{2x}\cosh Hh_1 + D_2Hh_{1x}\sinh Hh_1]$$

$$A_{2x} = T_{7x} - (B_2h_{1x} + B_{2x}h_1) - (C_{2x}\cosh Hh_1 + C_2Hh_{1x}\sinh Hh_1) - (D_{2x}\sinh Hh_1 + D_2Hh_{1x}\cosh Hh_1);$$

$$T_{1x} = -g_tB_x;\quad T_{2x} = -g_c\alpha A_{1x};\quad T_{3x} = -g_c\alpha B_{1x};\quad T_{4x} = -\frac{T_{1x}}{2H^2};\quad T_{5x} = \frac{T_{2x}}{\alpha^2(\alpha^2-H^2)};\quad T_{6x} = \frac{T_{3x}}{\alpha^2(\alpha^2-H^2)};$$

$$T_{7x} = -2T_4h_1h_{1x} + (T_5\alpha h_{1x}\cosh\alpha h_1 + T_{5x}\sinh\alpha h_1) + (T_6\alpha h_{1x}\sinh\alpha h_1 + T_{6x}\cosh\alpha h_1);$$

$$T_{8x} = -2T_4h_2h_{2x} + (T_5\alpha h_{2x}\cosh\alpha h_2 + T_{5x}\sinh\alpha h_2) + (T_6\alpha h_{2x}\sinh\alpha h_2 + T_{6x}\cosh\alpha h_2);$$

$$T_{9x} = -2(T_4h_{1x} + T_{4x}h_1) + \alpha(T_5\alpha h_{1x}\sinh\alpha h_1 + T_{5x}\cosh\alpha h_1) + \alpha(T_6\alpha h_{1x}\cosh\alpha h_1 + T_{6x}\sinh\alpha h_1);$$

$$T_{10x} = -2(T_4h_{2x} + T_{4x}h_2) + \alpha(T_5\alpha h_{2x}\sinh\alpha h_2 + T_{5x}\cosh\alpha h_2) + \alpha(T_6\alpha h_{2x}\cosh\alpha h_2 + T_{6x}\sinh\alpha h_2);$$

$$T_{11x} = Hh_{1x}\sinh Hh_1 - Hh_2\sinh Hh_2 - H[(h_1-h_2)Hh_{1x}\cosh Hh_1 + (h_{1x}-h_{2x})\sinh Hh_1];$$

$$T_{12x} = Hh_{1x}\cosh Hh_1 - Hh_{2x}\cosh Hh_2 - H[(h_1-h_2)Hh_{1x}\sinh Hh_1 + (h_{1x}-h_{2x})\cosh Hh_1];$$

$T_{13x} = T_{7x} - T_{8x} - [T_{9x}(h_1 - h_2) + T_9(h_{1x} - h_{2x})]$;
$T_{14x} = Hh_{1x}\sinh Hh_1 - Hh_{2x}\sinh Hh_2 - H[(h_1 - h_2)Hh_{2x}\cosh Hh_2 + (h_{1x} - h_{2x})\sinh Hh_2]$;
$T_{15x} = Hh_{1x}\cosh Hh_1 - Hh_{2x}\cosh Hh_2 - H[(h_1 - h_2)Hh_{2x}\sinh Hh_2 + (h_{1x} - h_{2x})\cosh Hh_2]$;
$T_{16x} = T_{7x} - T_{8x} - [T_{10x}(h_1 - h_2) + T_{10}(h_{1x} - h_{2x})]$; $T_1 = -g_t B$; $T_2 = -g_c\alpha A_1$; $T_3 = -g_c\alpha B_1$;
$T_4 = -\frac{T_1}{2H^2}$; $T_5 = \frac{T_2}{\alpha^2(\alpha^2 - H^2)}$; $T_6 = \frac{T_3}{\alpha^2(\alpha^2 - H^2)}$; $T_7 = \frac{q}{2} - T_4 h_1^2 + T_5\sinh\alpha h_1 + T_6\cosh\alpha h_1$;
$T_8 = -\frac{q}{2} - T_4 h_2^2 + T_5\sinh\alpha h_2 + T_6\cosh\alpha h_2$; $T_9 = -1 - 2T_4 h_1 + \alpha T_5\cosh\alpha h_1 + \alpha T_6\sinh\alpha h_1$;
$T_{10} = -1 - 2T_4 h_2 + \alpha T_5\cosh\alpha h_2 + \alpha T_6\sinh\alpha h_2$; $T_{11} = \cosh Hh_1 - \cosh Hh_2 - H(h_1 - h_2)\sinh Hh_1$;
$T_{12} = \sinh Hh_1 - \sinh Hh_2 - H(h_1 - h_2)\cosh Hh_1$; $T_{13} = T_7 - T_8 - T_9(h_1 - h_2)$;
$T_{14} = \cosh Hh_1 - \cosh Hh_2 - H(h_1 - h_2)\sinh Hh_2$; $T_{15} = \sinh Hh_1 - \sinh Hh_2 - H(h_1 - h_2)\cosh Hh_2$;
$T_{16} = T_7 - T_8 - T_{10}(h_1 - h_2)$; $T_{17} = \mathrm{Re}P_r(A_x B_2 - BA_{2x})$; $T_{18} = \mathrm{Re}P_r(B_x B_2 - BB_{2x} + 2T_4 A_x)$;
$T_{19} = 2\mathrm{Re}P_r T_4 B_x$; $T_{20} = \mathrm{Re}P_r(HA_x C_2 - BD_{2x})$; $T_{21} = \mathrm{Re}P_r(HA_x D_2 - BC_{2x})$; $T_{22} = \mathrm{Re}P_r\alpha T_6 A_x$;
$T_{23} = \mathrm{Re}P_r\alpha T_5 A_x$; $T_{24} = \mathrm{Re}P_r HC_2 B_x$; $T_{25} = \mathrm{Re}P_r HD_2 B_x$; $T_{26} = \mathrm{Re}P_r\alpha T_6 B_x$;
$T_{27} = \mathrm{Re}P_r\alpha T_5 B_x$; $T_{28} = \frac{T_{17}}{2}$; $T_{29} = \frac{T_{18}}{6}$; $T_{30} = \frac{T_{19}}{12}$;
$T_{31} = \frac{T_{20}}{H^2} - \frac{2T_{25}}{H^3}$; $T_{32} = \frac{T_{21}}{H^2} - \frac{2T_{24}}{H^3}$; $T_{33} = \frac{T_{22}}{\alpha^2} - \frac{2T_{27}}{\alpha^3}$; $T_{34} = \frac{T_{23}}{\alpha^2} - \frac{2T_{26}}{\alpha^3}$; $T_{35} = \frac{T_{24}}{H^2}$; $T_{36} = \frac{T_{25}}{H^2}$;
$T_{37} = \frac{T_{26}}{\alpha^2}$; $T_{38} = \frac{T_{27}}{\alpha^2}$; $T_{39} = \frac{\mathrm{Re}S_c\alpha}{2}(T_5 A_{1x} - T_6 B_{1x})$; $T_{40} = \mathrm{Re}S_c(B_2 B_{1x} - \alpha A_1 A_{2x})$;
$T_{41} = \mathrm{Re}S_c(B_2 A_{1x} - \alpha B_1 A_{2x})$; $T_{42} = \frac{\alpha\mathrm{Re}S_c}{2}(T_5 B_{1x} + T_6\alpha A_{1x})$; $T_{43} = \frac{\alpha\mathrm{Re}S_c}{2}(T_5 A_{1x} + T_6\alpha B_{1x})$;
$T_{44} = \mathrm{Re}S_c(2T_4 A_{1x} - \alpha B_1 B_{2x})$; $T_{45} = \mathrm{Re}S_c(2T_4 B_{1x} - \alpha A_1 B_{2x})$; $T_{46} = \mathrm{Re}S_c(HC_2 A_{1x} - \alpha B_1 D_{2x})$;
$T_{47} = \mathrm{Re}S_c(HC_2 B_{1x} - \alpha A_1 D_{2x})$; $T_{48} = \mathrm{Re}S_c(HD_2 A_{1x} - \alpha B_1 C_{2x})$; $T_{49} = \mathrm{Re}S_c(HD_2 B_{1x} - \alpha A_1 C_{2x})$;
$T_{50} = \frac{(T_{46} + T_{49})}{2}$; $T_{51} = \frac{(T_{46} - T_{49})}{2}$; $T_{52} = \frac{(T_{47} + T_{48})}{2}$; $T_{53} = \frac{(T_{48} - T_{47})}{2}$; $T_{54} = \frac{T_{40}}{2\alpha}$; $T_{55} = \frac{T_{41}}{2\alpha}$;
$T_{56} = \frac{T_{42}}{3\alpha^2}$; $T_{57} = \frac{T_{43}}{3\alpha^2}$; $T_{58} = \frac{T_{45}}{2\alpha}$; $T_{59} = \frac{T_{44}}{2\alpha}$; $T_{60} = \frac{T_{50}}{(H+\alpha)^2 - \alpha^2}$; $T_{61} = \frac{T_{51}}{(H-\alpha)^2 - \alpha^2}$;
$T_{62} = \frac{T_{52}}{(H+\alpha)^2 - \alpha^2}$; $T_{63} = \frac{T_{53}}{(H-\alpha)^2 - \alpha^2}$; $T_{64} = -\frac{T_{39}}{\alpha^2}$; $T_{65} = \mathrm{Re}(H^2 B_2 D_{2x} - H^3 C_2 A_{2x})$;
$T_{66} = \mathrm{Re}(H^2 B_2 C_{2x} - H^3 D_2 A_{2x})$; $T_{67} = \mathrm{Re}(2T_4 H^2 C_{2x} - H^3 D_2 B_{2x})$; $T_{68} = \mathrm{Re}(2T_4 H^2 D_{2x} - H^3 C_2 B_{2x})$;
$T_{69} = -\mathrm{Re}\alpha^3 T_5 A_{2x}$; $T_{70} = -\mathrm{Re}\alpha^3 T_6 A_{2x}$; $T_{71} = -\mathrm{Re}\alpha^3 T_5 B_{2x}$; $T_{72} = -\mathrm{Re}\alpha^3 T_6 B_{2x}$;
$T_{73} = \frac{\mathrm{Re}}{2}\left[(T_5 D_{2x} + T_6 C_{2x})(H^2\alpha - \alpha^3)\right]$; $T_{74} = \frac{\mathrm{Re}}{2}\left[(T_5 D_{2x} - T_6 C_{2x})(H^2\alpha - \alpha^3)\right]$;
$T_{75} = \frac{\mathrm{Re}}{2}\left[(T_5 C_{2x} + T_6 D_{2x})(H^2\alpha - \alpha^3)\right]$; $T_{76} = \frac{\mathrm{Re}}{2}\left[(T_5 C_{2x} - T_6 D_{2x})(H^2\alpha - \alpha^3)\right]$;
$T_{77} = HT_{31} + T_{36}$; $T_{78} = HT_{32} + T_{35}$; $T_{79} = \alpha T_{33} + T_{38}$; $T_{80} = \alpha T_{34} + T_{37}$; $T_{81} = \alpha A_4 + T_{54} + T_{58}$;
$T_{82} = \alpha B_4 + T_{55} + T_{59}$; $T_{83} = 2T_{59} + (T_{54} + T_{58})\alpha$; $T_{84} = 2T_{58} + (T_{55} + T_{59})\alpha$; $T_{85} = T_{65} - g_t T_{78}$;
$T_{86} = T_{66} - g_t T_{77}$; $T_{87} = T_{67} - g_t HT_{35}$; $T_{88} = T_{68} - Hg_t T_{36}$; $T_{89} = T_{69} - g_t T_{79} - g_c T_{82}$;
$T_{90} = T_{70} - g_t T_{80} - g_c T_{81}$; $T_{91} = T_{71} - g_t\alpha T_{37} - g_c T_{83}$; $T_{92} = T_{72} - g_t\alpha T_{38} - g_c T_{84}$; $T_{93} = T_{73} - g_c T_{62}(H+\alpha)$;
$T_{94} = T_{74} - g_c T_{63}(H-\alpha)$; $T_{95} = T_{75} - g_c T_{60}(H+\alpha)$; $T_{96} = T_{76} - g_c T_{61}(H-\alpha)$; $T_{97} = -2\alpha g_c T_{56}$;
$T_{98} = -2\alpha g_c T_{57}$; $T_{99} = -\alpha g_c T_{58}$; $T_{100} = -\alpha g_c T_{59}$; $T_{101} = -\alpha g_t T_{30}$; $T_{102} = -3g_t T_{29}$; $T_{103} = -2g_t T_{28}$;
$T_{104} = -g_t B_3$; $T_{105} = \frac{T_{85}}{2H}$; $T_{106} = \frac{T_{86}}{2H}$; $T_{107} = \frac{T_{87}}{2H}$; $T_{108} = \frac{T_{88}}{2H}$; $T_{109} = \frac{T_{88}}{2H}$; $T_{110} = \frac{T_{87}}{2H}$;
$T_{111} = \frac{T_{89}}{\alpha^2 - H^2} + \frac{2(\alpha T_{91} - T_{99})}{(\alpha^2 - H^2)^2} + \frac{8\alpha^2 T_{99}}{(\alpha^2 - H^2)^3}$; $T_{112} = \frac{T_{90}}{\alpha^2 - H^2} + \frac{2(\alpha T_{92} - T_{100})}{(\alpha^2 - H^2)^2} + \frac{8\alpha^2 T_{100}}{(\alpha^2 - H^2)^3}$;
$T_{113} = \frac{T_{91}}{\alpha^2 - H^2} - \frac{4\alpha T_{100}}{(\alpha^2 - H^2)^2}$; $T_{114} = \frac{T_{92}}{\alpha^2 - H^2} - \frac{4\alpha T_{99}}{(\alpha^2 - H^2)^2}$; $T_{115} = \frac{T_{93}}{(H+\alpha)^2 - H^2}$; $T_{116} = \frac{T_{94}}{(H-\alpha)^2 - H^2}$;
$T_{117} = \frac{T_{95}}{(H+\alpha)^2 - H^2}$; $T_{118} = \frac{T_{96}}{(H-\alpha)^2 - H^2}$; $T_{119} = \frac{T_{97}}{4\alpha^2 - H^2}$; $T_{120} = \frac{T_{98}}{4\alpha^2 - H^2}$; $T_{121} = \frac{T_{99}}{\alpha^2 - H^2}$;

$$T_{122}=\frac{T_{100}}{\alpha^2-H^2};\quad T_{123}=-\frac{T_{101}}{H^2};\quad T_{124}=-\frac{T_{102}}{H^2};\quad T_{125}=-\frac{6T_{101}}{H^4}-\frac{T_{103}}{H^2};\quad T_{126}=-\frac{T_{104}}{H^2}-\frac{2T_{102}}{H^4};$$

$$\begin{aligned}T_{127} &= -(T_{28}h_1^2+T_{29}h_1^3+T_{30}h_1^4+T_{31}\sinh Hh_1+T_{32}\cosh Hh_1+T_{33}\sinh\alpha h_1+T_{34}\cosh\alpha h_1\\&\quad +T_{35}h_1\sinh Hh_1+T_{36}h_1\cosh Hh_1+T_{37}h_1\sinh\alpha h_1+T_{38}h_1\cosh\alpha h_1)\end{aligned}$$

$$\begin{aligned}T_{128} &= -(T_{28}h_2^2+T_{29}h_2^3+T_{30}h_2^4+T_{31}\sinh Hh_2+T_{32}\cosh Hh_2+T_{33}\sinh\alpha h_2+T_{34}\cosh\alpha h_2\\&\quad +T_{35}h_2\sinh Hh_2+T_{36}h_2\cosh Hh_2+T_{37}h_2\sinh\alpha h_2+T_{38}h_2\cosh\alpha h_2)\end{aligned}$$

$$\begin{aligned}T_{129} &= -[(T_{54}+T_{58})h_1\sinh\alpha h_1+(T_{55}+T_{59})h_1\cosh\alpha h_1+T_{56}\sinh 2\alpha h_1+T_{57}\cosh 2\alpha h_1\\&\quad +T_{58}h_1^2\sinh\alpha h_1+T_{59}h_1^2\cosh\alpha h_1+T_{60}\sinh(H+\alpha)h_1+T_{61}\sinh(H-\alpha)h_1\\&\quad +T_{62}\cosh(H+\alpha)h_1+T_{63}\cosh(H-\alpha)h_1+T_{64}]\end{aligned}$$

$$\begin{aligned}T_{130} &= -[(T_{54}+T_{58})h_2\sinh\alpha h_2+(T_{55}+T_{59})h_2\cosh\alpha h_2+T_{56}\sinh 2\alpha h_2+T_{57}\cosh 2\alpha h_2\\&\quad +T_{58}h_2^2\sinh\alpha h_2+T_{59}h_2^2\cosh\alpha h_2+T_{60}\sinh(H+\alpha)h_2+T_{61}\sinh(H-\alpha)h_2\\&\quad +T_{62}\cosh(H+\alpha)h_2+T_{63}\cosh(H-\alpha)h_2+T_{64}]\end{aligned}$$

$$\begin{aligned}T_{131} &= -[T_{106}\cosh Hh_1+T_{105}\sinh Hh_1+T_{107}h_1\sinh Hh_1+T_{108}h_1\cosh Hh_1+T_{109}h_1^2\sinh Hh_1\\&\quad +T_{110}h_1^2\cosh Hh_1+T_{111}\cosh\alpha h_1+T_{112}\sinh\alpha h_1+T_{113}h_1\cosh\alpha h_1+T_{114}h_1\sinh\alpha h_1\\&\quad +T_{115}\sinh(H+\alpha)h_1+T_{116}\sinh(H-\alpha)h_1+T_{117}\cosh(H+\alpha)h_1+T_{118}\cosh(H-\alpha)h_1\\&\quad +T_{119}\cosh 2\alpha h_1+T_{120}\sinh 2\alpha h_1+T_{121}h_1^2\cosh\alpha h_1+T_{122}h_1^2\sinh\alpha h_1+T_{123}h_1^3\\&\quad +T_{124}h_1^2+T_{125}h_1+T_{126}]\end{aligned}$$

$$\begin{aligned}T_{132} &= -[T_{106}\cosh Hh_2+T_{105}\sinh Hh_2+T_{107}h_2\sinh Hh_2+T_{108}h_2\cosh Hh_2+T_{109}h_2^2\sinh Hh_2\\&\quad +T_{110}h_2^2\cosh Hh_2+T_{111}\cosh\alpha h_1+T_{112}\sinh\alpha h_2+T_{113}h_2\cosh\alpha h_2+T_{114}h_2\sinh\alpha h_2\\&\quad +T_{115}\sinh(H+\alpha)h_2+T_{116}\sinh(H-\alpha)h_2+T_{117}\cosh(H+\alpha)h_2+T_{118}\cosh(H-\alpha)h_2\\&\quad +T_{119}\cosh 2\alpha h_2+T_{120}\sinh 2\alpha h_2+T_{121}h_2^2\cosh\alpha h_2+T_{122}h_2^2\sinh\alpha h_2+T_{123}h_2^3\\&\quad +T_{124}h_2^2+T_{125}h_2+T_{126}]\end{aligned}$$

$$\begin{aligned}T_{133} &= -[(HT_{105}+T_{108})\cosh Hh_1+(HT_{106}+T_{107})\sinh Hh_1+(T_{108}+2T_{109})h_1\sinh Hh_1\\&\quad +(T_{107}+2T_{110})h_1\cosh Hh_1+T_{110}h_1^2\sinh Hh_1+T_{109}h_1^2\cosh Hh_1+(\alpha T_{112}+T_{113})\cosh\alpha h_1\\&\quad +(\alpha T_{111}+T_{114})\sinh\alpha h_1+(T_{114}+2T_{121})h_1\cosh\alpha h_1+(T_{113}+2T_{122})h_1\sinh\alpha h_1\\&\quad +T_{115}(H+\alpha)\cosh(H+\alpha)h_1+T_{116}(H-\alpha)\cosh(H-\alpha)h_1+T_{117}(H+\alpha)\sinh(H+\alpha)h_1\\&\quad +T_{118}(H-\alpha)\sinh(H-\alpha)h_1+2\alpha T_{119}\sinh 2\alpha h_1+2\alpha T_{120}\cosh 2\alpha h_1+\alpha T_{121}h_1^2\sinh\alpha h_1\\&\quad +\alpha T_{122}h_1^2\cosh\alpha h_1+3T_{123}h_1^2+2T_{124}h_1+T_{125}]\end{aligned}$$

$$\begin{aligned}T_{134} &= -[(HT_{105}+T_{108})\cosh Hh_2+(HT_{106}+T_{107})\sinh Hh_2+(T_{108}+2T_{109})h_2\sinh Hh_2\\&\quad +(T_{107}+2T_{110})h_2\cosh Hh_1+T_{110}h_2^2\sinh Hh_2+T_{109}h_2^2\cosh Hh_2+(\alpha T_{112}+T_{113})\cosh\alpha h_2\\&\quad +(\alpha T_{111}+T_{114})\sinh\alpha h_2+(T_{114}+2T_{121})h_2\cosh\alpha h_2+(T_{113}+2T_{122})h_2\sinh\alpha h_2\\&\quad +T_{115}(H+\alpha)\cosh(H+\alpha)h_2+T_{116}(H-\alpha)\cosh(H-\alpha)h_2+T_{117}(H+\alpha)\sinh(H+\alpha)h_2\\&\quad +T_{118}(H-\alpha)\sinh(H-\alpha)h_2+2\alpha T_{119}\sinh 2\alpha h_2+2\alpha T_{120}\cosh 2\alpha h_2+\alpha T_{121}h_2^2\sinh\alpha h_2\\&\quad +\alpha T_{122}h_2^2\cosh\alpha h_2+3T_{123}h_2^2+2T_{124}h_2+T_{125}]\end{aligned}$$

$$T_{135}=\cosh Hh_1-\cosh Hh_2-H(h_1-h_2)\sinh Hh_1;\quad T_{136}=\sinh Hh_1-\sinh Hh_2-H(h_1-h_2)\cosh Hh_1;$$

$$T_{137}=T_{131}-T_{132}-T_{133}(h_1-h_2);\quad T_{138}=\cosh Hh_1-\cosh Hh_2-H(h_1-h_2)\sinh Hh_2;$$

$$T_{139}=\sinh Hh_1-\sinh Hh_2-H(h_1-h_2)\cosh Hh_2;\quad T_{140}=T_{131}-T_{132}-T_{134}(h_1-h_2);$$

$$T_{141}=Ag_t-\frac{g_c c_1 S_c}{\alpha^2}-H^2B_2-H^2;\quad T_{142}=Bg_t-2H^2T_4;\quad T_{143}=(\alpha^2-H^2)\alpha T_5+g_cA_1$$

$$T_{144}=(\alpha^2-H^2)\alpha T_6+g_cB_1;\quad T_{145}=2\mathrm{Re}[T_4A_{2x}+H^2(C_2C_{2x}-D_2D_{2x})];\quad T_{146}=2\mathrm{Re}T_4B_{2x};$$

$$T_{147}=\mathrm{Re}[2T_4C_{2x}+H^2C_2A_{2x}-HB_2D_{2x}];\quad T_{148}=\mathrm{Re}[2T_4D_{2x}+H^2D_2A_{2x}-HB_2C_{2x}];$$

$$T_{149}=\mathrm{Re}[B_{2x}H^2C_2-2HT_4D_{2x}];\quad T_{150}=\mathrm{Re}[B_{2x}H^2D_2-2HT_4C_{2x}];\quad T_{151}=\mathrm{Re}A_{2x}\alpha^2T_5;$$

$$T_{152}=\mathrm{Re}A_{2x}\alpha^2T_6;\quad T_{153}=\mathrm{Re}B_{2x}\alpha^2T_5;\quad T_{154}=\mathrm{Re}B_{2x}\alpha^2T_6;\quad T_{155}=\mathrm{Re}[(\alpha^2-H\alpha)(D_{2x}T_6+C_{2x}T_5)];$$

$$T_{156}=\mathrm{Re}[(\alpha^2+H\alpha)(D_{2x}T_6-C_{2x}T_5)];\quad T_{157}=\mathrm{Re}[(\alpha^2-H\alpha)(C_{2x}T_6+D_{2x}T_5)];$$

$$T_{158}=\mathrm{Re}[(\alpha^2+H\alpha)(C_{2x}T_6-D_{2x}T_5)];\quad T_{159}=6T_{123}-H^2(B_5+T_{125})+g_cT_{64}+g_tA_3;$$

$$T_{160}=H^3(C_5+T_{106})+H^2(2T_{107}-HC_5)+H(6T_{110}+T_{106})+g_tT_{31};$$

$$T_{161}=H^3(D_5+T_{106})+H^2(2T_{108}-HD_5)+H(6T_{109}+T_{105})+g_tT_{32};$$

$$T_{162}=g_tT_{35}+4H^2T_{109};\quad T_{163}=g_tT_{36}+4H^2T_{110};$$

$$T_{164}=\alpha^3T_{111}-H^2(\alpha T_{111}+T_{114})+3\alpha^2T_{114}+6\alpha T_{121}+g_tT_{33}+B_4g_c;$$

$$T_{165}=\alpha^3T_{112}-H^2(\alpha T_{112}+T_{113})+3\alpha^2T_{113}+6\alpha T_{122}+g_tT_{34}+A_4g_c;$$

$$T_{166} = \alpha^3 T_{114} - H^2(\alpha T_{114} + 2T_{121}) + 6\alpha^2 T_{121} + g_t T_{38} + (T_{55} + T_{59}) g_c$$
$$T_{167} = \alpha^3 T_{113} - H^2(\alpha T_{114} + 2T_{122}) + 6\alpha^2 T_{122} + g_t T_{37} + (T_{55} + T_{58}) g_c;$$
$$T_{168} = \alpha^3 T_{121} - H^2 \alpha T_{121} + g_c T_{58}; \quad T_{169} = \alpha^3 T_{122} - H^2 \alpha T_{122} + g_c T_{59}; \quad T_{170} = (8\alpha^3 - 2\alpha) T_{119} + g_c T_{56};$$
$$T_{171} = (8\alpha^3 - 2\alpha) T_{120} + g_c T_{57}; \quad T_{172} = [(H+\alpha)^3 - H^2(H+\alpha)] T_{117} + g_c T_{60};$$
$$T_{173} = [(H-\alpha)^3 - H^2(H-\alpha)] T_{118} + g_c T_{61};$$
$$T_{174} = [(H+\alpha)^3 - H^2(H+\alpha)] T_{115} + g_c T_{62}; \quad T_{175} = [(H-\alpha)^3 - H^2(H-\alpha)] T_{116} + g_c T_{63};$$
$$T_{176} = g_t T_{30}; \quad T_{177} = g_t T_{29}; \quad T_{178} = g_t T_{28} - 3T_{123}; \quad T_{179} = g_t B_3 - 2H^2 T_{124};$$
$$T_{180} = T_{145} + T_{159}; \quad T_{181} = T_{146} + T_{179}; \quad T_{182} = T_{148} + T_{160}; \quad T_{183} = T_{147} + T_{161}; \quad T_{184} = T_{160} + T_{162};$$
$$T_{185} = T_{149} + T_{163}; \quad T_{186} = T_{151} + T_{164}; \quad T_{187} = T_{152} + T_{165}; \quad T_{188} = T_{153} + T_{167}; \quad T_{189} = T_{154} + T_{166};$$
$$T_{190} = T_{155} + T_{172}; \quad T_{191} = T_{156} + T_{173}; \quad T_{192} = T_{157} + T_{174}; \quad T_{193} = T_{158} + T_{175}.$$

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