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ERRATUM

Erratum to: Time optimal control of an additional food provided predator–prey system with applications to pest management and biological conservation

P. D. N. Srinivasu · B. S. R. V. Prasad

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The proof of the Theorem 5 in Srinivasu and Prasad (2009) is incomplete as the hypothesis of the theorem does not always imply the inequality (37). It is possible that $1 - \xi + \alpha_{\min}\xi \geq 0$. Below we fill this gap by presenting the remaining proof of Theorem 5 when the parameters satisfy

$$1 - \xi + \alpha_{\min}\xi \geq 0 \tag{E.1}$$

To prove this part we make use of the properties of the zero solution of the linear system (21), (22) which governs the co-state variables along the optimal path. This linear system can be conveniently written in a matrix form as

$$\begin{pmatrix} \frac{d\lambda}{dt} \\ \frac{d\mu}{dt} \end{pmatrix} = \begin{pmatrix} -a_1(t) & -b_1(t) \\ a_2(t) & -b_2(t) \end{pmatrix} \begin{pmatrix} \lambda(t) \\ \mu(t) \end{pmatrix} \tag{E.2}$$

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P. D. N. Srinivasu (✉) · B. S. R. V. Prasad
Department of Mathematics, Andhra University,
Visakhapatnam 530 003, India
e-mail: pdns@rediffmail.com

B. S. R. V. Prasad
e-mail: srvprasad_bh@hotmail.com

where

$$a_1(t) = 1 - \frac{2x(t)}{\gamma} - \frac{(1+\alpha(t)\xi)y(t)}{(1+\alpha(t)\xi + x(t))^2}, \tag{E.3}$$

$$b_1(t) = \frac{\beta(1+\alpha(t)\xi - \xi)y(t)}{(1+\alpha(t)\xi + x(t))^2}, \tag{E.4}$$

$$a_2(t) = \frac{x(t)}{1+\alpha(t)\xi + x(t)}, \tag{E.5}$$

$$b_2(t) = \frac{\beta(x(t)+\xi)}{1+\alpha(t)\xi + x(t)} - \delta \tag{E.6}$$

Observe here that $a_2(t) > 0$. From the assumption (E.1) we have $b_1(t) \geq 0$. If $\alpha(t) = \alpha_{\min}$ then from the hypothesis it follows that $b_2(t) > 0$. Here $a_1(t)$ can be either positive or negative depending on the values of the parameters and the state variables.

The characteristic equation associated with the system (E.2) is

$$m^2 + (a_1(t) + b_2(t))m + (a_1(t)b_2(t) + a_2(t)b_1(t)) = 0. \tag{E.7}$$

The system (E.2) essentially admits (0, 0) as its equilibrium. By studying the qualitative behavior of the system (E.2) based on the properties of the functions $a_1(t)+b_2(t)$ and $a_1(t)b_2(t) + a_2(t)b_1(t)$, we can assess the behavior of the equilibrium solution $(0, 0)$. From Theorem 4 in Srinivasu and Prasad (2009), if we assume the value of λ at the terminal time T to be positive, from the continuity of $a_1(t) + b_2(t)$ and $a_1(t)b_2(t) + a_2(t)b_1(t)$, it implies that there exists a left neighborhood of T, say $[a, T]$, in which we have $\lambda(t) > 0$ and $\mu(t) < 0$. The proof would be complete if we can show that $a = 0$. Below we shall show that it is possible to choose the initial values for the costate variables in such a way that these variables do not change their sign in $[0, T]$, as a consequence the switching function also does not change its sign along the optimal path.

Since $b_1(t) \geq 0$, the discriminant of (E.2) can change its sign and consequently, the path of the system (E.2) initiating in the fourth quadrant of $\lambda\mu$-space may leave that quadrant as time progresses. Note that at the terminal time T, we have $x(T) = 0$ and $y(T) > 1 + \alpha_{\min}\xi$. Thus the zero solution of the system (21), (22) behaves like a saddle in the vicinity of the terminal time. Therefore, it is always possible to choose the initial value for the costate variable μ sufficiently far from 0 on the negative μ-axis (with $\lambda(0) > 0$ so chosen to make the associated Hamiltonian take the value -1 at $t = 0$) so that by the time the co-state gets closer to positive λ-axis it is influenced by the the saddle nature of the zero solution. Therefore, the solutions initiating in the fourth quadrant will not leave that quadrant. Therefore we have $\sigma(t) > 0$ for all $t \in [0, T]$ and hence $\alpha(t) = \alpha_{\min}$ for all $t \in [0, T]$.

Reference

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