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Fitting of full Cobb-Douglas and full VRTS cost frontiers by solving goal programming problem

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Abstract. The present research article first defines two popular production functions viz, Cobb-Douglas and VRTS production frontiers and their dual cost functions and then derives their cost limited maximal outputs. This paper tells us that the cost limited maximal output is cost efficient. Here the one side goal programming problem is proposed by which the full Cobb-Douglas cost frontier, full VRTS frontier can be fitted. This paper includes the framing of goal programming by which stochastic cost frontier and stochastic VRTS frontiers are fitted. Hasan et al. [1] used a parameter approach Stochastic Frontier Approach (SFA) to examine the technical efficiency of the Malaysian domestic banks listed in the Kuala Lumpur stock Exchange (KLSE) market over the period 2005-2010. AshkanHassani [2] exposed Cobb-Douglas Production Functions application in construction schedule crashing and project risk analysis related to the duration of construction projects. Nan Jiang [3] applied Stochastic Frontier analysis to a panel of New Zealand dairy forms in 1998/99-2006/2007.

1. Introduction

Applications of goal linear programming for a mathematical economics problem involve properly structured production functions and their dual cost functions. Two popular production functions are considered, (i) the Cobb-Douglas production frontier and (ii) the Variable Returns to Scale production frontier.

Cobb-Douglas production frontier:

$u = A\prod_{i=1}^{n} x_i^{\alpha_i}$ .Its dual cost function as derived by the principles of duality is

$$Q\left(u,\mathrm{p}\right) = Bu^{\frac{1}{\theta}}\prod_{i=1}^{n} p_i^{\alpha_i/\theta}, \theta \le \alpha_i \le 1, \theta = \sum_{i=1}^{n}\alpha_i$$

If returns to scale are constant, $\theta = 1$. Consequently, we obtain $Q(u,\mathrm{p}) = Bu\prod_{i=1}^{n} p_i^{\alpha_i}$ .VRTS production frontier is $u^{\alpha} e^{\theta u} = A\prod_{i=1}^{n} x_i^{\alpha_i}, A > 0, \alpha > 0, 0 \le \alpha_i \le 1, \theta = \sum \alpha_i = 1$. Where θ is unrestricted for sign. Its dual cost function is $Q(u,p) = u^{\alpha} e^{\theta u} B \Pi p_i^{\alpha_i}$.

2. Cost limited maximal output

A producer in the cost expansion, targets cost and requires to know the potential output or revenue. Such an output is called cost limited maximal output. For the Cobb-Douglas and VRTS production frontiers, the expressions for cost limited maximal outputs are derived. The cost limited maximal output of the Cobb-Douglas production frontier is

$\Gamma(p) = \left[\dfrac{C}{B \Pi p_i^{\alpha_i/\theta}} \right]^{\theta}$. Where C is the target cost.

The cost limited maximal output of VRTS production function can be estimated by solving the non-linear equation $\Gamma^{\alpha}(p) e^{\theta\Gamma(p)} B \Pi p_i^{\alpha_i} = C$. Where C is the target cost. It is shown that the cost limited maximal output is cost efficient output, where the cost efficiency is defined as,

$\delta = \dfrac{Q(u,p)}{C}$ .Where $Q(u,p)$: factor minimal cost function and C : observed or target cost.

For the Cobb-Douglas production or cost structure, we obtain $\Gamma(p) = \delta^{-\theta} u$ .Where u : Observer output and θ: Returns to scale parameter. In addition, if returns to scale are constant then $\Gamma(p) = \delta^{-1} u$.

A producer in the process of expansion not only needs to know the cost limited maximal output but also the quantum of resources required. Implementing Shephard's [12] lemma we obtain the following input demand equations:

$$X_i(u,p) = \left(\frac{\alpha_i B}{\theta} \right) u^{1/\theta} p_i^{\frac{\alpha_i}{\theta} - 1} \prod_{j \ne 1} p_j^{\frac{\alpha_i}{\theta}}, \mathrm{i} = 1,2,\ldots\ldots$$

In terms of cost limited maximal output, these input demand equations may be expressed as

$$X_i(\Gamma(p),p) = \left(\frac{\alpha_i B}{\theta} \right) (\Gamma(p))^{\frac{1}{\theta}} p_i^{\frac{\alpha_i}{\theta} - 1} \prod_{j \ne 1} p_j^{\frac{\alpha_i}{\theta}}, \mathrm{i} = 1,2,..$$

If the production or cost structure obeys the VRTS frontier, the cost limited maximal output can be obtained as solution of the equation $\Gamma^{\alpha}(p) e^{\theta\Gamma(p)} = \delta^{-1} u_0^{\alpha} e^{\theta u_0}$.

where u_0 is the observed output and δ is the radial measure of cost efficiency. The potential efficient inputs are obtained making use of the input demand equation,

$$X_i(u,p) = (\alpha_i B) u^{\alpha} e^{\theta u} p_i^{\alpha_i - 1} \prod_{j \ne 1} p_j^{\alpha_i}, i = 1,2,\ldots..$$

In terms of cost limited maximal output these input demand equations takes the form,

$$X_i(\Gamma(p),p) = (\alpha_i B) \Gamma^{\alpha}(p) e^{\theta\Gamma(p)} p_i^{\alpha_i - 1} \prod_{j \ne 1} p_j^{\alpha_i}, i = 1,2,\ldots$$

To estimate the cost limited maximal output and the constituent factor demands knowledge of the parametric estimates of the cost frontiers is necessary. It may be noted that, although the parameters of cost function can be derived from those of the parameters of its dual cost function, in general, no matter what estimation method is chosen, the two sets of estimates do not coincide. We prefer to estimate the cost frontier since its slacks give directly the cost efficiencies, which in turn are used to estimate the cost limited maximal output. Further, one can visualize two frontiers, viz., the full and stochastic cost frontiers. Fitting of a full frontier is possible by postulating and solving suitable linear programming problems. The Cobb-Douglas and the VRTS cost frontiers can be expressed as linear in unknown parameters by taking logarithms.

3. Fitting of full Cobb-Douglas cost frontier

A full Cobb-Douglas cost frontier can be fitted by solving the one side goal programming problem

Min $Z = \sum_{j=1}^{k} \in_j$, Subject to $b + \eta \ln u_j + \sum \beta_j \ln p_{ij} + \in_j = d_j$

$\beta_j = \dfrac{\alpha_j}{\theta}, \sum \beta_j = 1$ and , where b is unrestricted for sign.

The stochastic cost frontier is fitted by solving the following goal programming problem

Min $Z = \sum_{j=1}^{k} \in_j^{+} + \sum_{j=1}^{k} \in_j^{-}$, subject to $b + \eta \ln u_j + \sum \beta_j \ln p_{ij} + \in_j^{+} + \in_j^{-} = d_j$

$\sum \beta_j = 1$ and $\beta_j \geq 0, \eta \geq 0$, where b is unrestricted for sign.

$\in_j^{+}, \in_j^{-} \geq 0$ and $\in_j^{+} . \in_j^{-} = 0$

Fitting full VRTS cost frontier:

The full VRTS frontier can be fitted by solving

Min $\Pi = \sum \in_j$, subject to $\alpha \ln u_i + \theta u_i + \ln B + \sum \alpha_j \ln p_{ij} + \in_j = \ln C_i, i = 1, 2, \ldots, k$

$\alpha \geq 0, b = \ln B$ is unrestricted for sign.

The stochastic VRTS frontier can be fitted by solving, Min $\sum \in_i^{+} + \sum \in_i^{-}$

Subject to $\alpha \ln u_i + \theta u_i + \ln B + \sum \alpha_j \ln p_{ij} + \in_i^{+} - \in_i^{-} = \ln C_i, i = 1, 2, \ldots, k$

$\alpha \geq 0, b = \ln B$ is unrestricted for sign, $\alpha_j \geq 0, \sum \alpha_j = 1$, $\in_i^{+}, \in_i^{-} \geq 0$ and $\in_i^{+} . \in_i^{-} = 0$

The parametric estimates of both the stochastic cost frontiers are robust and equivalent to maximum likelihood estimates, if the underlying errors follow the Laplace distribution. An alternative for parametric production or cost frontier is non-parametric production or cost frontier. The cost limited maximal output, in non-parametric approach, can be estimated by solving the following linear programming problem:

$\Gamma(p) = Max\ u$, Subject to $px \leq C, \sum \alpha_i x_i \leq x, \sum \alpha_i u_i \geq u$ and $\alpha_i \geq 0, x_i \geq 0, u \geq 0$

The optimal solution not only gives cost minimal maximal output, but also the potential optimal inputs

4. Conclusions

In the above research paper by the principle of duality dual cost functions of two properly structured production functions viz Cobb-Douglas and VRTS are derived. In addition to these the GPPs are proposed to fit them. Finally this article gives an estimation method of cost limited maximal output by solving a LPP in non -parametric approach.

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