



# FREE CONVECTIVE HEAT TRANSFER OF MHD DISSIPATIVE CARREAU NANOFUID FLOW OVER A STRETCHING SHEET

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## ABSTRACT

Nowadays external magnetic fields are capable of setting the thermal and physical properties of magnetic-nanofluids and regulate the flow and heat transfer characteristics. The strength of the applied magnetic field affects the thermal conductivity of magnetic nanofluids and makes it aeotropic. With this incentive, we investigate the flow and heat transfer of electrically conducting liquid film flow of Carreau nanofluid over a stretching sheet by considering the aligned magnetic field in the presence of space and temperature dependent heat source/sink and viscous dissipation. For this study, we considered kerosene as the base fluid embedded with the silver (Ag) and copper (Cu) nanoparticles. Numerical results are determined by employing Runge-Kutta and Newton's methods. Graphs are exhibited and explained for various parameters of interest. The influence of pertinent parameters on reduced Nusselt number, flow and heat transfer is discussed with the assistance of graphs and tables. It is found that thermal boundary layer of Ag-kerosene nanofluid is highly effective when compared with the Cu-kerosene nanofluid. It is also found that the thermal and momentum boundary layers of Cu-kerosene and Ag-kerosene nanofluids are not uniform.

**Keywords:** MHD, Nanofluid, non-uniform heat source/sink, thermal radiation and free convection.

## 1. INTRODUCTION

Nanofluids are created an enormous interest over the past ten years due to its phenomenal properties and prospective applications. Many researchers have embraced recent uses of slurry made up of water and either silver or copper nanoparticles. Free convective heat transfer in thin film flow of nanofluids often encountered in many industrial and engineering disciplines. This application covers wire and fiber coating, heat exchangers, extrusion process, polymer processing and chemical processing equipment, etc. In the pioneering work of Chen (2003), who studied the power-law fluid film flow of unsteady heat transfer stretching sheet. An analytic solution for the momentum and heat transfer of liquid film flow over a stretching surface was explained by Wang (2006). Chen (2006) and Sajid et al. (2007) discussed the non-Newtonian thin film flow over an unsteady stretching surface taking into the account of viscous dissipation. And found that the fixed value of Prandtl number decreases with the thermal boundary layer.

Two-dimensional liquid film flow over an unsteady stretching sheet was numerically investigated by Dandapat et al. (2008). Heat and mass transfer in MHD non-Newtonian flow were numerically studied by Raju and Sandeep (2016). Dandapat et al. (2007, 2003) analyzed the effect of variable viscosity and thermo-capillarity on the heat transfer of liquid film flow over a stretching sheet. The micropolar film flow over an inclined plate, moving belt and vertical cylinder has numerically explained by Sajid et al. (2009). Dandapat and Chakraborty (2010) and Dandapat and Singh (2011) explained the thin film flow over a non-linear stretching surface with the effect of transverse magnetic field and observed that the raising values of viscosity parameter enhance the velocity field.

The researchers [Abel (2011), Khan et al. (2011), Liu and Megahad (2012), Liu et al. (2013), Vajaravelu et al. (2012)] analyzed the heat transfer characteristics of thin film flows by considering the different

channels. Aziz et al. (2012) examined the effect of thermal radiation and thermocapillarity on the heat transfer thin film flow over a stretching surface. They observed that rising values of thermo capillarity enhances the velocity field. Tawade et al. (2016) presented the unsteady flow and heat transfer of thin film over a stretching surface in the presence of thermal radiation. The Eyring Powell flow and unsteady heat transfer of a laminar liquid film over a stretching sheet were studied by Khader and Megahad (2013) and found that increasing the Prandtl number reduces the temperature field across the thin film.

Anderson et al. (1992, 1996) studied the effect of a power-law fluid caused by thin liquid film on an unsteady stretching surface and found that effect of power-law index is more effective on temperature field. The liquid film flow over an unsteady horizontal stretching sheet was numerically discussed by Santra and Dandapat (2008). Noor and Hashim (2010) investigated the effect of magnetic field and thermocapillarity on an unsteady flow of a liquid film over a stretching sheet. Lin et al. (2015) explained the effect of MHD pseudo-plastic nanofluid flow and heat transfer film flow over a stretching sheet and concluded that raising the value of Hartmann number reduced the velocity profiles.

The study of non-Newtonian fluids has many applications in industry and engineering fields, mainly in crude oil extraction from petroleum manufacturing. The Carreau fluid is also a one of the non-Newtonian fluids. Carreau fluid model is substantial for gooey, high and low shear rates. On account of this headway, it has profited in numerous innovative and assembling streams. The effect of power-law index on unsteady stretching sheet was explained by Chen (2003) and Abbasbandy (2008). Recently, the researchers [Raju et al. (2016), Raju et al. (2016), Sandeep et al. (2016), Babu et al.(2016), Animasun et al. (2016), Sandeep et al.(2016)] studied the heat and mass transfer of MHD flows through different channels. The effect of cross diffusion on MHD bio convection flow over a horizontal surface were discussed by Makinde and Animasaun (2016). Makinde and Animasaun (2016) have presented the MHD nanofluid on bio convection flow of a paraboloid revolution with

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nonlinear thermal radiation and chemical reaction. Sandeep (2016), Ramana Reddy et al. (2017) and Ali et al. (2017) studied the heat transfer behaviour of MHD flows.

To the authors' knowledge no studies have been reported yet on flow and heat transfer of electrically conducting liquid film flow of Carreau nanofluid over a stretching sheet by considering the aligned magnetic field in the presence of space and temperature dependent heat source/sink, viscous dissipation and thermal radiation. For this study, we considered kerosene as the base fluid embedded with the silver (Ag) and copper (Cu) nanoparticles. Numerical results are determined by employing Runge-Kutta and Newton's methods. Graphs are exhibited and explained for various parameters of interest. The influence of pertinent parameters on reduced Nusselt number, flow and heat transfer is discussed with the assistance of graphs and tables

## 2. MATHEMATICAL FORMULATION

Let us consider an unsteady, two-dimensional boundary layer flow of an electrically conducting and heat generating Carreau nanofluid over a stretching sheet bounded by a thin liquid film of uniform thickness  $h(t)$  over a horizontal elastic sheet which emerges from a narrow slit at the origin of the cartesian coordinate system which is schematically represented in Fig.1. The sheet is stretched along the  $x$ -axis with stretching velocity  $U(x, t)$  and  $y$ -axis is normal to it. An inclined magnetic field  $B_0$  is applied to the stretching sheet at an angle  $\gamma$ . The effects of non-uniform heat source/sink, thermal radiation, viscous dissipation and volume fraction are taken into consideration. We assume that the surface temperature  $T_s$  of the stretching sheet varies with respect to distance  $x$ -from the slit as

$$T_s = T_0 - T_{ref} \left( \frac{bx^2}{2\nu_f} \right) (1 - \alpha t)^{-\frac{3}{2}} \quad (1)$$

$$U(x, t) = bx / (1 - \alpha t) \quad (2)$$

The Eqn. (2) is for the sheet velocity  $U(x, t)$  reflects that the elastic sheet, the elastic sheet is fixed at the origin and stretched by applying a force in the positive  $x$ -direction. We used  $\alpha > 0$  because the stretching rate  $b / (1 - \alpha t)$  increases with time. Similarly, Eqn. (2) represents the decrease in sheet temperature from  $T_0$  at the slit in proportion to  $x^2$ .

The constitutive equations for a Carreau fluid is given by

$$\bar{\tau}_{ij} = \eta_0 \left[ 1 + \frac{(n-1)}{2} (\Gamma \bar{\gamma})^2 \right] \bar{\gamma}_{ij} \quad (3)$$

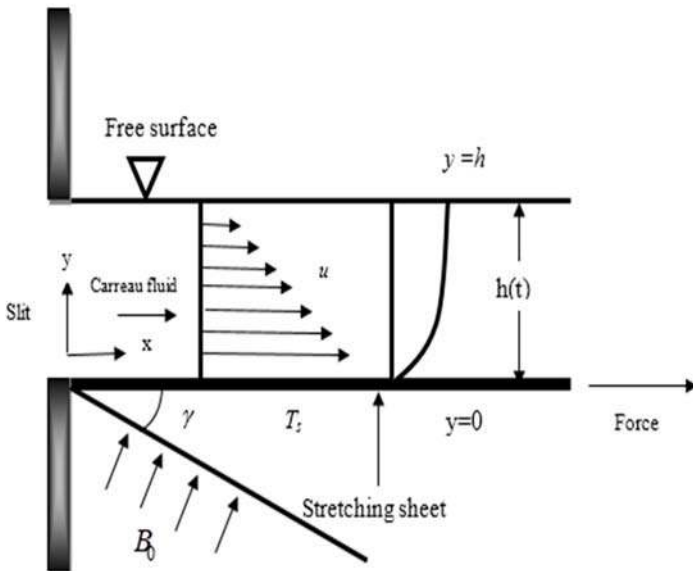


Fig. 1 Flow geometry of the problem

In which  $\bar{\tau}_{ij}$  is the extra stress tensor,  $\eta_0$  is the zero shear rate viscosity,  $\Gamma$  is the time constant,  $n$  is power-law index and  $\bar{\gamma}_{ij}$  is defined as

$$\bar{\gamma} = \sqrt{\frac{1}{2} \sum_i \sum_j \bar{\gamma}_{ij} \bar{\gamma}_{ji}} = \sqrt{\frac{1}{2}} \Pi \quad (4)$$

Here  $\Pi$  is the second invariant strain tensor.

The governing boundary layer equations for momentum and thermal energy with associated boundary conditions are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (5)$$

$$\rho_{nf} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \quad (6)$$

$$= \mu_{nf} \left( 1 + \frac{3(n-1)\Gamma^2}{2} \left( \frac{\partial u}{\partial y} \right)^2 \right) \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 \cos^2 \gamma u, \quad (7)$$

$$(\rho c_p)_{nf} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) \quad (7)$$

$$= k_{nf} \frac{\partial^2 T}{\partial y^2} + \mu_{nf} \left( \frac{\partial u}{\partial y} \right)^2 + q''' ,$$

$$\left. \begin{aligned} u = U_w, \quad v = 0, \quad T = T_s \quad \text{at } y=0, \\ \frac{\partial u}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \text{at } y=h, \\ v = \frac{dh}{dt} \quad \text{as } y = h(t), \end{aligned} \right\} \quad (8)$$

where

$$\left. \begin{aligned} \rho_{nf} &= (1 - \phi) \rho_f + \phi \rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \\ k_{nf} &= k_f \left[ \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \right] \\ (\rho c_p)_{nf} &= (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s \end{aligned} \right\} \quad (9)$$

The non-uniform heat generation/absorption  $q'''$  is taken as

$$q''' = \frac{k_f U_w}{x \nu_f} [A^* (T_s - T_0) f' + B^* (T - T_0)] \quad (10)$$

Let us now introduce the similarity variables as given below

$$\left. \begin{aligned} u &= \frac{bx}{(1 - \alpha t)} f'(\eta), \\ v &= -(b \nu_f)^{-\frac{1}{2}} (1 - \alpha t)^{-\frac{1}{2}} f(\eta), \\ \eta &= (b / \nu_f)^{\frac{1}{2}} (1 - \alpha t)^{-\frac{1}{2}} y, \\ T &= T_0 - T_{ref} (bx^2 / 2\nu_f) (1 - \alpha t)^{-\frac{3}{2}} \theta(\eta), \end{aligned} \right\} \quad (11)$$

Now by using Eqs. (6)- (11), the Eqs. (6)- (8) transformed as

$$f''' \left( 1 + \frac{3(n-1)}{2} W_e f'^2 \right) \quad (12)$$

$$+ B_1 \left\{ B_2 \left( S \left( f' + \frac{\eta}{2} f'' \right) + ff'' - f'^2 \right) - M \cos^2 \gamma f' \right\} = 0, \quad (13)$$

$$B_3 \theta''' + \frac{Ec Pr}{B_1} f''^2 + (A^* f' + B^* \theta)$$

$$- B_4 Pr \left( \frac{S}{2} (\eta \theta' + 3\theta) + 2f' \theta - f \theta' \right) = 0,$$

where,

$$B_1 = (1 - \phi)^{2.5}, \quad B_2 = 1 - \phi + \phi \frac{\rho_s}{\rho_f}, \quad (14)$$

$$B_3 = \frac{k_{nf}}{k_f}, \quad B_4 = 1 - \phi + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f},$$

Corresponding boundary conditions are

$$\left. \begin{aligned} f'(0) = 1, \quad f(0) = 0, \quad \theta(0) = 0, \\ f''(\beta) = 0, \quad \theta'(\beta) = 0, \quad f(\beta) = \frac{S\beta}{2} \end{aligned} \right\}, \quad (15)$$

here  $S = \alpha / b$  is unsteadiness parameter and prime represents differentiation with respect to  $\eta$ . Further,  $\beta$  indicates the value of the similarity variable  $\eta$  at the free surface so that  $\eta$  value gives

$$\beta = \left( \frac{b}{\nu_f(1 - \alpha t)} \right)^{\frac{1}{2}} h, \quad (16)$$

The rate at which film thickness varies can be obtained by differentiating (16) w.r.t,

$$\frac{dh}{dt} = -\frac{\alpha\beta}{2} \left( \frac{\nu_f}{b(1 - \alpha t)} \right)^{\frac{1}{2}}, \quad (17)$$

where

$$\text{Pr} = (\mu c_p)_f / k_f, \quad We^2 = \frac{b^3 x^2 \Gamma^2}{\nu_f(1 - \alpha t)^3}, \quad (18)$$

$$M = \frac{\sigma B_0^2}{\rho_f b}, \quad R = \frac{4\sigma^* T_0^3}{k^* k_f}, \quad S = \frac{\alpha}{b}, \quad Ec = \frac{U_w^2}{c_p(T_s - T_0)},$$

The physical quantities of practical interest in this problem are the skin friction coefficient  $C_f$  and the Nusselt number  $Nu$ , which are given as

$$C_f = \frac{\tau_w}{\rho_f U_w^2}, \quad Nu = \frac{q_w x}{k_f(T_s - T_0)}, \quad (19)$$

where,

$$\tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -\frac{k_{nf}}{k_f} \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad (20)$$

Substituting Eqn. (19) in Eqn. (20), we obtain

$$B_1 C_{fx} \text{Re}_x^{-1/2} = f''(0), \quad (1/B_3) Nu_x \text{Re}_x^{1/2} = -\theta'(0), \quad (21)$$

where,  $\text{Re}_x = U_w x / \nu_f$  is the local Reynolds number.

### 3. RESULTS AND DISCUSSION

Eqs. (15) and (16), subject to the boundary conditions Eq. (18) are solved numerically using Runge-Kutta and Newton's methods. The influence of pertinent parameters namely, magnetic field parameter, unsteadiness parameter, heat source/sink parameter, Eckert number, volume fraction of nanoparticles etc. on the flow and heat transfer of the thin film flow are discussed. For numerical computations, we considered the non-dimensional parameter values as  $S = We = 0.5$ ,  $\gamma = \pi / 4$ ,  $A^* = B^* = 0.2$ ,  $M = 2$ ,  $n = 1.5$ ,  $Ec = 0.1$ . These values are kept as common in entire study except the variations in the respective figures.

Figs. 2 and 3 explain the effect of  $M$  on the velocity and temperature fields respectively. It is observed that increasing values of  $M$  declines the velocity field and enhances the temperature field.

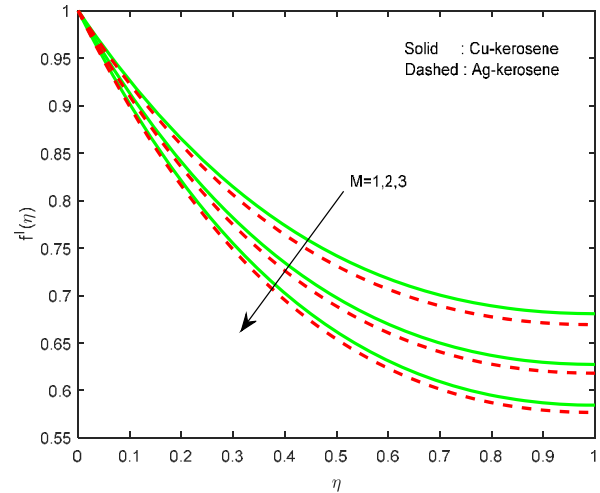


Fig. 2 Velocity profile for different values of  $M$

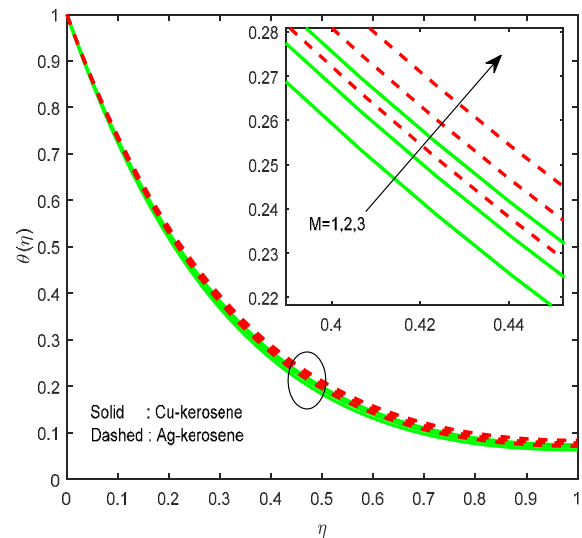


Fig. 3 Temperature profile for different values of  $M$

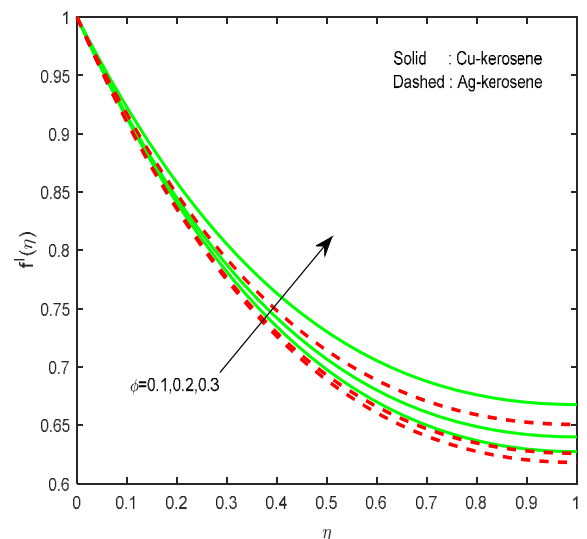
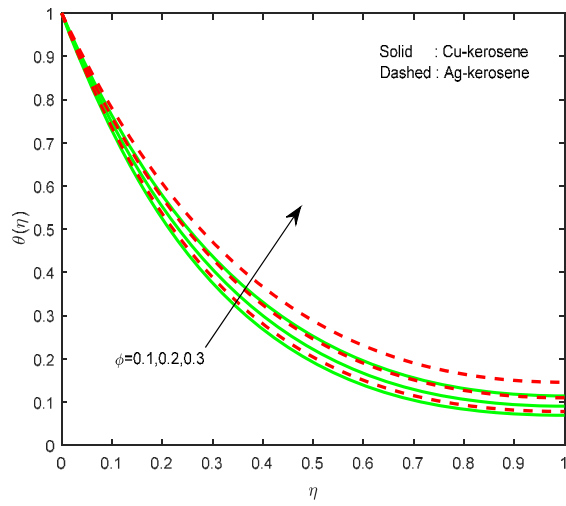
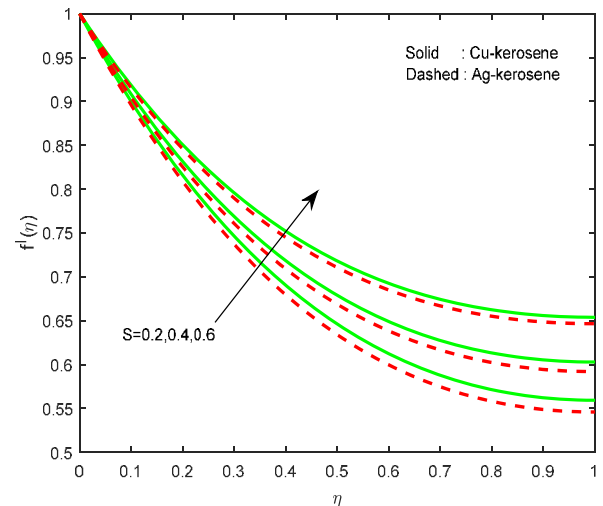


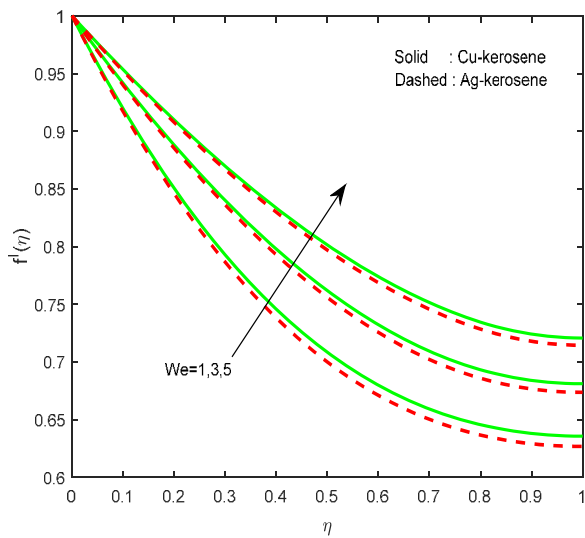
Fig. 4 Velocity profile for different values of  $\phi$



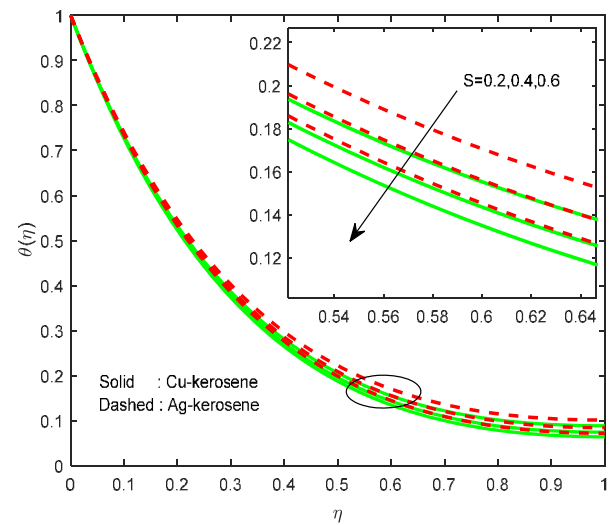
**Fig. 5** Temperature profile for different values of  $\phi$



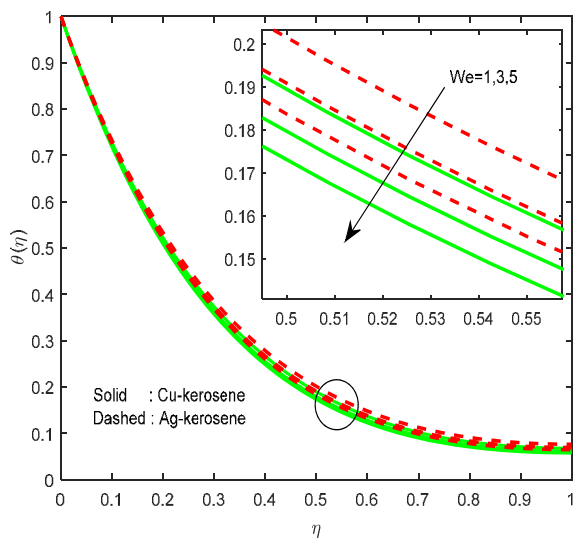
**Fig. 8** Velocity profile for different values of  $S$



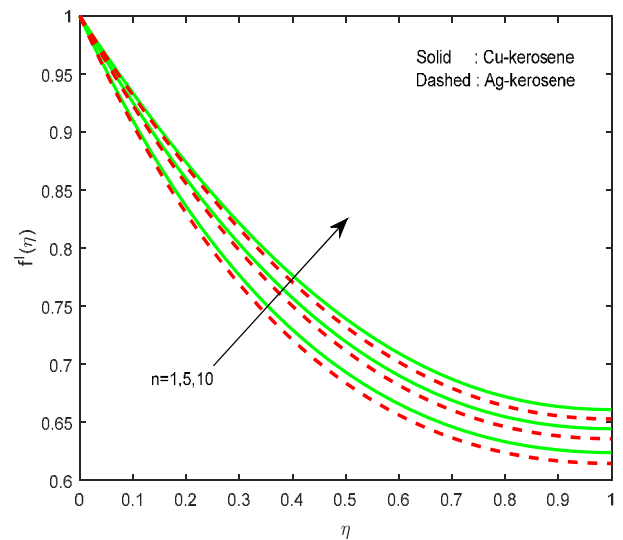
**Fig. 6** Velocity profile for different values of  $We$



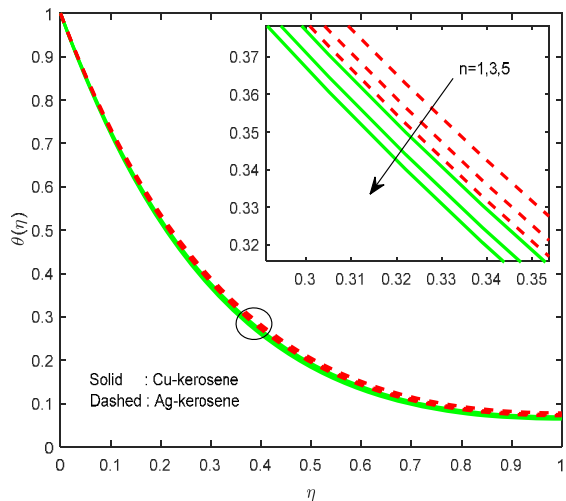
**Fig. 9** Temperature profile for different values of  $S$



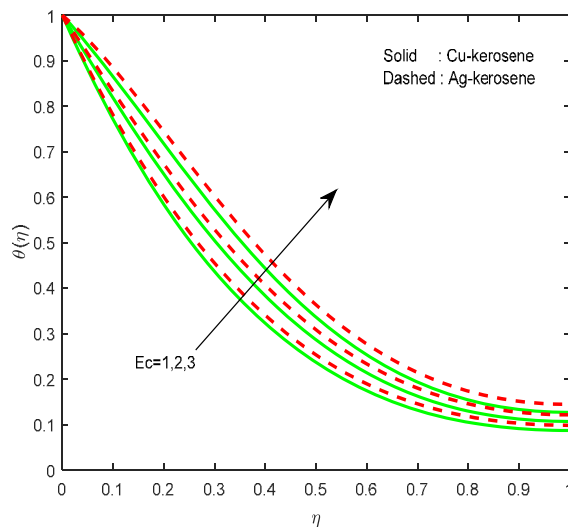
**Fig. 7** Temperature profile for different values of  $We$



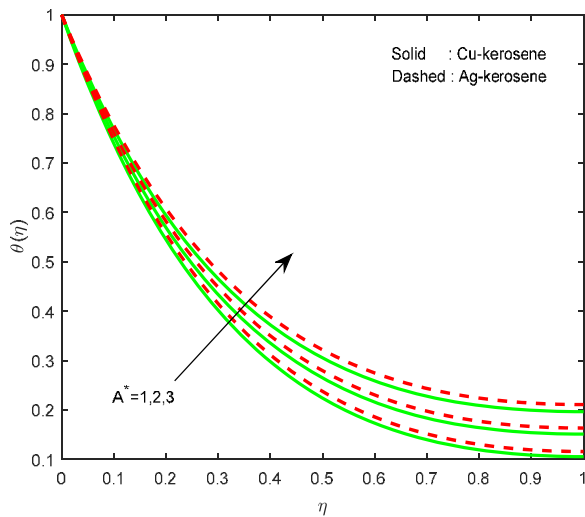
**Fig. 10** Velocity profile for different values of  $n$



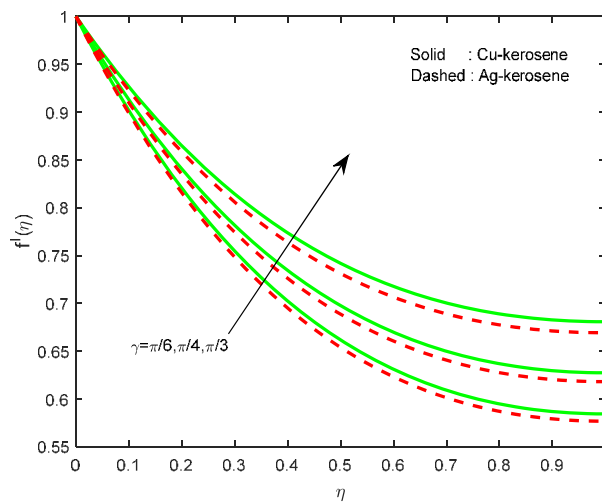
**Fig. 11** Temperature profile for different values of  $n$



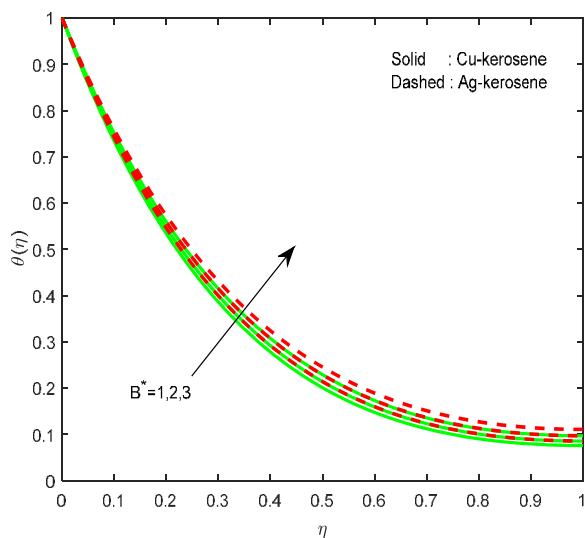
**Fig. 14** Temperature profile for different value of  $Ec$



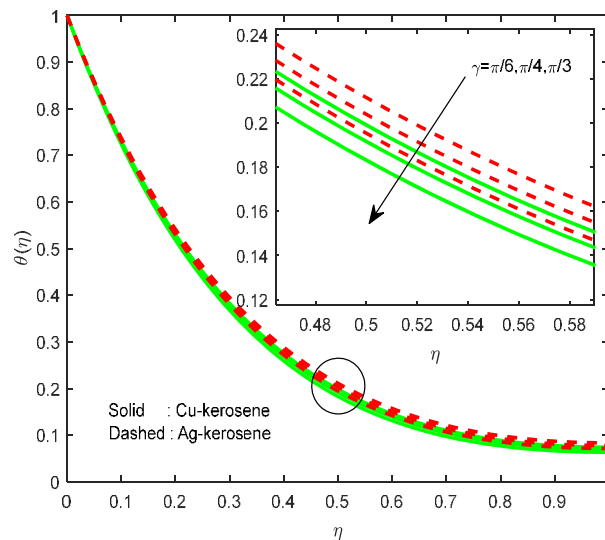
**Fig. 12** Temperature profile for different values of  $A^*$



**Fig. 15** Temperature profile for different value of  $\gamma$



**Fig. 13** Temperature profile for different values of  $B^*$



**Fig. 16** Temperature profile for different value of  $\gamma$

**Table 1** Physical parameter values of  $f''(0)$  and  $-\theta'(0)$  for Cu-kerosene nanofluid

$M$	$\phi$	$We$	$S$	$n$	$A^*$	$Ec$	$\gamma^0$	$f''(0)$	$-\theta'(0)$
1								-0.800673	3.183502
2								-0.951051	3.137925
3								-1.077238	3.097322
	.1							-0.951051	3.137925
	.2							-0.926769	2.900338
	.3							-0.843920	2.683437
		1						-0.865479	3.155764
		3						-0.611938	3.218581
		5						-0.484571	3.252867
			.2					-1.090240	3.094797
			.4					-1.002314	3.125135
			.6					-0.894041	3.149859
				1				-0.995049	3.129665
				5				-0.796797	3.171534
				10				-0.700307	3.195477
					1			-0.951051	3.002623
					2			-0.951051	2.833496
					3			-0.951051	2.664369
						1		-0.951051	2.534955
						2		-0.951051	1.864989
						3		-0.951051	1.195023
							30	-1.077238	3.097322
							45	-0.951051	3.137925
							60	-0.800673	3.183502

Generally, introducing the transverse magnetic field create a drag force due to the Lorentz force and hence the result retarding the velocity field. The effects of  $\phi$  on the velocity and temperature profiles are depicted in Figs. 4 and 5, respectively. The result shows that as the solid volume fraction of the film increases both the velocity and temperature field increases. This is due to the fact that increasing in volume fraction of nano particle enhances the thermal conductivity of the flow. Figs. 6 and 7 demonstrate the influence of  $We$  on the velocity and temperature profiles. It is observed that the velocity increases for increasing values of  $We$  and opposite trend has been observed in temperature field. Physically, higher value of  $We$  will depreciate the viscosity forces of the Carreau fluid.

Figs. 8 and 9 illustrate the influence of unsteadiness parameter on velocity and temperature profiles respectively. It is observed that increasing values of  $S$  boosts the velocity field and declines the temperature field. Generally, unsteadiness parameter  $S$  is controlled by both  $a$  and  $b$ , increasing the value of unsteadiness parameter increases the heat loose by the thin film. Due to this reason we have seen a fall in the temperature field. Figs. 10 and 11 represent the effect of power law index on velocity and temperature fields. We noticed an enhancement in the velocity profile and depreciate the temperature profile. Physically, increase in the power law index made to thicken the liquid film associated with an enhancement of the thermal boundary layer.

Fig.12 and 13 display the influence of non-uniform heat source/sink parameter on the temperature field. It is observed that increasing the non-uniform heat source/sink parameter enhances the temperature fields. The effect of Eckert number on temperature profile is shown in Fig.14. We obtain that increasing values of Eckert number enhance the temperature profiles. Due to the fact that heat energy is saved in the liquid due to the frictional heating. The influence of aligned angle on velocity and temperature profiles is presented in Figs. 15 and 16. We obtained an

interesting result that the enhancement in the value of aligned parameter increases the velocity field and depreciate the temperature field.

Table-1 and 2 show the effect of physical parameters on friction factor and local Nusselt number for Cu-kerosene and Ag- Kerosene nanofluids. It is evident from the tables that increasing values of the magnetic field parameter reduce friction factor and heat transfer rate. A rise in the value of volume fraction of nanoparticles enhances the friction factor and declines the heat transfer rate. An increase in the value of aligned angle parameter enhances both friction factor and heat transfer rate. Weissenberg and unsteadiness parameters have tendency to enhance the heat transfer rate.

**Table 2** Physical parameter values of  $f''(0)$  and  $-\theta'(0)$  for Ag-kerosene nanofluid

$M$	$\phi$	$We$	$S$	$n$	$A$	$Ec$	$\gamma^0$	$f''(0)$	$-\theta'(0)$
1								-0.841593	3.090642
2								-0.987394	3.045404
3								-1.110328	3.005010
	.1							-0.987394	3.045404
	.2							-0.982125	2.739717
	.3							-0.907088	2.473053
		1						-0.894219	3.064925
		3						-0.627126	3.132014
		5						-0.495591	3.168031
			.2					-1.133904	2.982102
			.4					-1.041797	3.026448
			.6					-0.926340	3.063119
				1				-1.036497	3.036220
				5				-0.820912	3.081939
				10				-0.719234	3.107549
					1			-0.987394	2.906601
					2			-0.987394	2.733098
					3			-0.987394	2.559594
						1		-0.987395	2.390294
						2		-0.987395	1.662395
						3		-0.987395	0.934497
							30	-1.110328	3.005010
							45	-0.987394	3.045404
							60	-0.841593	3.090642

#### 4. CONCLUSIONS

This study presents the flow and heat transfer of electrically conducting liquid film flow of Carreau nanofluid over a stretching sheet by considering the aligned magnetic field in the presence of space and temperature dependent heat source/sink and viscous dissipation. For this study, we considered kerosene as the base fluid embedded with the silver (Ag) and copper (Cu) nanoparticles. Numerical results are determined by employing Runge-Kutta and Newton's methods. Graphs are exhibited and explained for various parameters of interest. The conclusions of the present study are as follows:

- Increasing in power-law index and unsteadiness parameter enhances the heat transfer rate.
- Magnetic field parameter have tendency to reduce skin friction coefficient and local Nusselt number.
- Rise in the Weissenberg number depreciate the friction factor and increases the heat transfer rate.
- An increase in aligned angle enhances the skin friction coefficient and local Nusselt number.
- Temperature field of Ag-kerosene nanofluid is effective when compared with the temperature field of Cu-kerosene nanofluid.



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