International Journal of Engineering & Technology, 7 (4.10) (2018) 970-973

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Fuzzy Rough G-Border and Fuzzy Rough G-Exterior in Fuzzy Rough Topological Groups

D.Vidhya[1], T.Yogalakshmi[2]*, V.Visalakshi[3],

[1] *Department of Mathematics, Kalasalingam Academy of Research and Education (Deemed to be University), Krishnan Koil.*
[2] *Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore.*
[3] *Department of Mathematics, SRM Institute of Science and Technology, Kattankulathur,Chennai.*
**Corresponding author E-mail: yogalakshmithangaraj@vit.ac.in.*

Abstract

This paper investigates the concepts of fuzzy rough G-border and fuzzy rough G-exterior. Some interesting properties are established. Also the relationship between them are established.

Keywords: *fuzzy rough G-border and fuzzy rough G-exteriror.*

1. Introduction

Authors [14], [8], [9] and [3] said the concepts of fuzzy sets, applications of fuzzy sets and fuzzy topological spaces. Further various uncertainties that arisein the real world problems are solved by using soft set theory, intuitionistic fuzzy set theory and soft fuzzy set theory etc. Several properties of those theories are discussed in [12] and [13]. Pawlak [7] introduced the concept of rough set. Rough group and rough subgroup was investigated by R. Biswas and S. Nanda [1]. B.P. Mathew and S. J. John [5] were studied the rough topological space. S.Nanda and S. Majmudar [6] analysed the concept of fuzzy rough set. The author [11] discussed and studied the fuzzy rough group and fuzzy rough topological group. The concepts of G-border and G-exterior were studied by Calda, Jafari and Noiri [2]. This paper analyses the concept of fuzzy rough border and fuzzy rough exterior sets. Some interesting properties and characterizations are investigated.

2. Preliminaries

Prosposition: [4]
If A be any fuzzy rough set in Z, $\tilde{0} = (0_L, 0_U)$ be the null fuzzy rough set and $\tilde{1} = (1_L, 1_U)$ be the whole fuzzy rough set in Z, then (i) $\tilde{0} \subset A \subset \tilde{1}$ and (ii) $\bar{\tilde{0}} = \tilde{1}, \bar{\tilde{1}} = \tilde{0}$.

Definition [11]
Any fuzzy rough topology on a rough set X is a family T of fuzzy rough sets in X which satisfies the following conditions:
(i) $\tilde{0}, \tilde{1} \in T$.
(ii) If $A, B \in T$, then $A \cap B \in T$.
(iii) If $A_j \in T$ for all $j \in J$, then $\cup_{j \in J} A_j \in T$.

3. Fuzzy rough G-border and fuzzy rough G-exterior

Definition: 3.1
Any fuzzy rough set ε in (X, T) is said to be fuzzy rough border of ε is defined as

$FRbr(\varepsilon) = \varepsilon \cap FRcl(\varepsilon')$.

Definition: 3.2
Let A be any fuzzy rough topological group in (X, G). Then the fuzzy rough G-border of A is defined and denoted as
$FRGbr(A) = A \cap FRGcl(A')$.

Theorem: 3.3
Let A be any fuzzy rough topological group in (X, G). Then
(i) $FRbr(A) \subseteq FRGbr(A)$
(ii) $FRGbr(A) \subseteq FRGcl(A')$
(iii) $FRGint(FRGbr(A)) \subseteq A$
(iv) $FRGbr(FRGbr(A)) \subseteq FRGbr(A)$

Proof : (i) Since $FRint(A) \subseteq FRGint(A)$
$\Rightarrow A - FRint(A) \subseteq A - FRGint(A)$
$\Rightarrow A \cap FRcl(A') \subseteq A \cap FRGcl(A')$
$\Rightarrow FRbr(A) \subseteq FRGbr(A)$.

(ii) $FRGbr(A) = A \cap FRGcl(A')$
$\subseteq FRGcl(A')$
Hence $FRGbr(A) \subseteq FRGcl(A')$
(iii) $FRGint(FRGbr(A)) = FRGint(A \cap FRGcl(A'))$
$\subseteq A \cap FRGcl(A')$
$= A$
Hence $FRGint(FRGbr(A)) \subseteq A$.
(iv) $FRGbr(FRGbr(A)) = FRGbr(A \cap FRGcl(A'))$
$\subseteq (A \cap FRGcl(A')) \cap (FRGcl(A \cap FRGcl(A'))')$

$\subseteq A \cap FRGcl(A')$

Hence $FRGbr(FRGbr(A)) \subseteq FRGbr(A)$.

Theorem: 3.4

If *A* is ay fuzzy rough open group in (X, G) then $FRGbr(A) \subseteq A$.

Proof:

Since A is a fuzzy rough open group the A' is a fuzzy rough closed group. Now $FRGbr(A) \subseteq A \cap FRGcl(A') = A \cap A' \subseteq A$.

Theorem: 3.5

Two fuzzy rough topological groups L and Q in (X, G) then $FRGbr(L \cup Q) \subseteq FRGr(L) \cup FRGbr(Q)$.

Proof :

$FRGbr(L \cup Q) = (L \cup Q) \cap FRGcl(L \cup Q)'$

$= (L \cup Q) \cap (FRGcl(A' \cap B'))$

$\subseteq (L \cap Q) \cap (FRGcl(L') \cap FRGcl(Q'))$

$= (FRGbr(L) \cap FRGcl(Q')) \cup (FRGbr(Q) \cap FRGcl(L'))$

$= FRGbr(L) \cup FRGbr(Q)$

Hence $FRGbr(L \cup Q) \subseteq FRGr(L) \cup FRGbr(Q)$.

Theorem: 3.6

Let R and S be ant two fuzzy rough topological groups in (X, G). Then $FRGbr(R \cap S) \supseteq FRGbr(R) \cap FRGbr(S)$.

Proof:

$FRGbr(R \cap S) = (R \cap S) \cap FRGcl(R \cap S)'$

$= (R \cap S) \cap (FRGcl(R' \cup S'))$

$= (R \cap S) \cap (FRGcl(A') \cup FRGcl(B'))$

$\supseteq (R \cap FRGcl(R')) \cap (S \cap FRGcl(S'))$

$= FRGbr(R) \cap FRGbr(S)$

Hence $FRGbr(R \cap S) \supseteq FRGbr(R) \cap FRGbr(S)$.

Theorem: 3.7

Any fuzzy rough topological groups A in (X, G) then $FRGbr(A) \subseteq FRGbd(A)$.

Proof:

$FRGbr(A) = A \cap FRGcl(A') \subseteq FRGcl(A) \cap FRGcl(A')$

$= FRGbd(A)$.

Therefore, $FRGbr(A) \subseteq FRGbd(A)$.

Corollary: 3.8

If A is any fuzzy rough closed group in (X, G) then $FRGbr(A) = FRGbd(A)$.

Proof:

Since A is a fuzzy rough closed group, $FRGcl(A) = A$.

Now, $FRGbr(A) = A \cap FRGcl(A')$

$= FRGcl(A) \cap FRGcl(A')$

$= FRGbd(A)$.

Therefore, $FRGbr(A) = FRGbd(A)$.

Definition: 3.9

A fuzzy rough set A in (X, T) is said to be fuzzy rough exterior of A is defined as

$$FRExt(A) = FRint(A')$$

Definition: 3.10

Let A be any fuzzy rough topological group in (X, G). Then the fuzzy rough G-exterior of A is defined as

$$FRGExt(A) = FRGint(A').$$

Theorem: 3.11

Let A be any fuzzy rough topological group in (X, G). Then

(i) $FRExt(A) \subseteq FRGExt(A)$.

(ii) $FRGExt(A) = (FRGcl(A))'$.

(iii) $FRGExt(FRGExt(A)) = FRGint(FRGcl(A))$.

(iv) $FRGExt(\tilde{1}) = \tilde{0}$.

(v) $FRGExt(\tilde{0}) = \tilde{1}$.

(vi) $FRGint(A) \subseteq FRGExt(FRGExt(A))$.

Proof:

(i) Since $FRGcl(A) \subseteq FRcl(A)$,

$\tilde{1} - FRGcl(A) \supseteq \tilde{1} - FRcl(A)$

$\Rightarrow FRGint(A') \supseteq FRint(A')$

By Definition 3.9 and 3.10, $FRExt(A) \subseteq FRGExt(A)$.

(ii) The proof follows from Definition 3.10.

(iii) By Definition 3.10,

$FRGExt(FRGExt(A)) = FRGExt(FRGint(A'))$

$= FRGint(FRGint(A'))'$

$= FRGint(FRGcl(A))$.

Therefore,

$FRGExt(FRGExt(A)) = FRGint(FRGcl(A))$.

(iv) By(ii),

$FRGExt(\tilde{1}) = FRGint(\tilde{1}') = \tilde{0}$.

(v) By (ii),

$FRGExt(\tilde{0}) = FRGint(\tilde{0}') = \tilde{1}$.

(vi) Since $A \subseteq FRGcl(A)$,

$\Rightarrow FRGint(A) \subseteq FRGint(FRGcl(A))$

$\Rightarrow FRGint(A) \subseteq FRGExt(FRGExt(A))$ by (iii).

Theorem: 3.12

Let (X, G) be a fuzzy rough G structure space. Let K and H be any two fuzzy rough topological groups. Then the following conditions hold:

(i) If $K \subseteq H$ then $FRGExt(A) \supseteq FRGExt(B)$.

(ii) $FRGExt(K \cup H) = FRGExt(K) \cap FRGExt(H)$.

(iii) $FRGExt(K \cap H) = FRGExt(K) \cup FRGExt(H)$.

Proof:

(i) Since $K \subseteq H$, $FRGcl(K) \subseteq FRGcl(H)$. Hence the proof is obvious by Definition 3.10.

(ii) $FRGExt(K \cup H) = FRGint(K \cup H)'$

$= FRGint(K' \cap H')$

$= FRGint(K') \cap FRGint(H')$

$= FRGExt(K) \cap FRGExt(H)$.

Therefore, $FRGExt(K \cup H) = FRGExt(K) \cap FRGExt(H)$.

(iii) The proof is similar to (ii).

Theorem: 3.13

Let A be any fuzzy rough topological group in (X, G). Then $FRGExt(A) = A'$ if and only if A is closed.

Proof:

If A is any fuzzy rough closed group then $A = FRGcl(A)$. By Definition 3.10,

$FRGExt(A) = FRGint(A') = (FRGcl(A))' = A'$.

Hence $FRGExt(A) = A'$.

Conversely, $FRGExt(A) = A'$ implies $FRGint(A') = A'$. Hence A' is fuzzy rough open group. Therefore, A is fuzzy rough closed group.

Theorem: 3.14

Let A be any fuzzy rough topological group in (X, G). Then $(FRGbd(A))' = FRGint(A) \cup FRGExt(A)$.

Proof:

Since $FRGbd(A) = FRGcl(A) \cap FRGcl(A')$, then

$(FRGbd(A))' = (FRGcl(A) \cap FRGcl(A'))'$

$= FRGint(A') \cup FRGint(A)$

$= FRGint(A) \cup FRGint(A')$

$= FRGint(A) \cup FRGExt(A)$.

Therefore, $(FRGbd(A))' = FRGint(A) \cup FRGExt(A)$.

Theorem: 3.15

Let A be any fuzzy rough topological group in (X, G). Then $FRGbr(A) = (FRGExt(A))'$.

Proof :

Since $FRGbr(A) = A \cap FRGcl(A')$ then

$FRGbr(A) \subseteq A$

$\subseteq FRGcl(A)$

$= (FRGExt(A))'$.

Hence, $FRGbr(A) = (FRGExt(A))'$.

Remark: 3.16

From Theorem 3.7 and 3.15,

$FRGExt(A) \nsubseteq FRGbr(A) \subseteq FRGbd(A)$.

4. Conclusion

The main Remark 3.16 is also extended in soft set, intuitionistic fuzzy set, fuzzy soft set etc. The concepts which are discussed in this paper have wide applications in network, printing technology and medical image processing.

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