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Influences of chemical reaction and wall properties on MHD Peristaltic transport of a Dusty fluid with Heat and Mass transfer

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Abstract The influence of elasticity of flexible walls on peristaltic transport of a dusty fluid with heat and mass transfer in a horizontal channel in the presence of chemical reaction has been investigated under long wavelength approximation. Expressions have been constructed for stream function, temperature and concentration by using perturbation technique. The effects of various parameters on heat and mass transfer characteristics of the flow are discussed through graphs.

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1. Introduction

The physical mechanism of the flow induced by the traveling wave can be well understood and is known as the so-called peristaltic transport mechanism. Peristaltic pumping is also used in medical instruments such as the heart–lung machine. Investigation on peristaltic transport of non-Newtonian fluids is of utmost importance owing to its wide range of applications in engineering and biology. In particular, the study of such fluids has applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubricants, colloidal and suspension solutions. Further, such analysis may serve for the intrauterine fluid motion in a

sagittal cross section of the uterus under cancer therapy and drug analysis (see Refs. [1–14]). In view of these applications, many analytical, numerical and experimental attempts have been made to understand peristalsis in different situations for Newtonian and non-Newtonian fluids. Later, Hayat and Noreen [10] have investigated the influence of an induced magnetic field on the peristaltic flow of an incompressible fourth grade fluid in a symmetric channel with heat transfer using long wavelength, low Reynolds number and small Deborah number assumptions. The study of the peristaltic transport of viscoelastic non-Newtonian fluids with fractional Maxwell model in a channel was discussed by Tripathi [11]. Srinivas and Muthuraj [12] have investigated the effects of chemical reaction and space porosity on MHD mixed convective peristaltic flow in a vertical asymmetric channel. Alla et al. [13] have examined the effects of both rotation and magnetic field of a micro polar fluid through a porous medium induced by sinusoidal peristaltic waves traveling down the channel walls. The peristaltic transport of a Williamson nanofluid in a

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tapered asymmetric channel under the action of a thermal radiation parameter was examined by Kothandapani and Prakash [14].

However, the wall properties are essential to be taken into consideration in various real situations. Therefore, some authors have developed theoretical models to describe peristaltic flow in channel/tube with wall properties [15–26]. Srinivas and Kothandapani [19] have examined the effects of heat and mass transfer on peristaltic transport in a porous space with compliant walls. Ali et al. [20] have analyzed the peristaltic motion of a non-Newtonian fluid in a channel having compliant boundaries. Hayat and Hina [21] have investigated the effects of heat and mass transfer on the MHD peristaltic flow of a Maxwell fluid in a planar channel with compliant walls. Mekheimer and Abdel-Wahab [22] have studied the effects of compliant wall properties on the flow of a Newtonian viscous compressible fluid when the wave propagating (surface acoustic wave) along the walls in a confined parallel-plane microchannel is conducted by considering the slip velocity. Muthuraj and Srinivas [23] have discussed the MHD peristaltic flow of a Newtonian fluid through porous space in a vertical channel with compliant walls under the assumptions of long wavelength and low Reynolds number. The influence of heat and mass transfer on the peristaltic transport of Johnson–Segalman fluid in a curved channel with flexible walls was investigated by Hina et al. [24]. Eldabe et al. [25] have analyzed the effect of wall properties on the peristaltic transport of a dusty fluid through porous channel with heat and mass transfer by using perturbation technique for small geometric parameter. They have also discussed the peristaltic motion of a power-law model, with heat and mass transfer through an asymmetric channel [26]. Hayat et al. [27] have examined the effect of elasticity of the flexible walls on the peristaltic flow of a power-law fluid with heat transfer.

The flow of a MHD fluid through a channel in the presence of a transverse magnetic field is encountered in a variety of applications such as magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters. In particular, the magnetohydrodynamic flows of non-Newtonian fluids are of great interest in magneto-therapy. Due to the importance of MHD flows, many studies have been carried out examining the effects of magnetic field on hydrodynamic flow in various configurations [28–32]. Moreover, two phase flows in which solid spherical particles are distributed in a clean fluid have attracted the interest of a number of researchers due to its practical applications such as petroleum industry, purification of crude oil, and physiological flows (see Refs. [33–45]). Saffman [33] pioneered the study of the fluid – particle system. They have discussed the convective stability of the particulate poiseuille flow with the assumption that the solid phase is distributed homogeneously. Later, the effects of dependence on temperature of the viscosity and electric conductivity, Reynolds number and particle concentration on the unsteady MHD flow and heat transfer of a dusty, electrically conducting fluid between parallel plates in the presence of an external uniform magnetic field have been investigated by Eguia et al. [41]. Hakan Erol [42] has studied the propagation of time harmonic waves in prestressed, anisotropic elastic tubes filled with viscous fluid containing dusty particles and the fluid is assumed to be incompressible and Newtonian. Ramesh et al. [43] have analyzed the steady two-dimensional MHD flow of a dusty fluid near the stagnation point over a permeable stretching

sheet with the effect of non-uniform source/sink. Pavithra and Gireesha [44] have examined the boundary layer flow and heat transfer of a dusty fluid over an exponentially stretching surface in the presence of viscous dissipation and internal heat generation/absorption using Runge–Kutta method. Most recently, the problem of a steady two-dimensional magnetohydrodynamic (MHD) flow of a dusty fluid over a stretching hollow cylinder was analyzed by Rakesh et al. [45].

To the best of our knowledge, no investigation has been made on the heat and mass transfer effects on peristaltic transport of MHD dusty fluid in a horizontal channel with compliant walls in the presence of chemical reaction. Therefore, in this paper, we have extended the results of Eldabe et al. [25] for peristaltic transport of a MHD dusty fluid through a two-dimensional horizontal channel with combined effects of uniform magnetic field and chemical reaction. Analytical solutions of the momentum, heat and concentration equations are obtained by using perturbation technique for both fluid and solid particles. The features of flow, heat and mass transfer characteristics are presented and discussed graphically. The present paper is organized in the following fashion. The problem is formulated in Section 2. Section 3 deals with the solution of the problem under long wavelength assumption. Results and discussion are given in Section 4. The conclusions have been summarized in Section 5.

2. Formulation of the problem

Consider the laminar flow of an incompressible fluid that contains small solid particles, whose number density N_0 (assumed to be constant) is large enough to define average properties of the dust particles at a point through a symmetrical two-dimensional channel. Choose the Cartesian coordinates (x, y) , where x is along the walls and y is perpendicular to it (see Fig. 1). The geometry of the wall surface is described by

$$\eta = d + a \sin\left(\frac{2\pi}{\lambda}(x - ct)\right) \quad (1)$$

where d is the half width of the channel, ‘ a ’ is the amplitude of the wave, λ is the wavelength, t is the time, and c is the wave velocity.

The governing equation of motion of the flexible wall may be expressed as [21]

$$L(\eta) = p - p_0 \quad (2)$$

$$L = -T^* \frac{\partial^2}{\partial x^2} + m^* \frac{\partial^2}{\partial t^2} + d^* \frac{\partial}{\partial t} + B \frac{\partial^4}{\partial x^4} + K^* \quad (3)$$

where p_0 is the pressure on the outside of the wall, T^* is the elastic tension in the membrane, m^* is the mass per unit area, d^* is the coefficient of viscous damping, B is the flexural rigidity of the plate, and K^* is the spring stiffness. The momentum, energy and concentration equations are

$$\rho \left(\frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial x} + v_f \frac{\partial u_f}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u_f}{\partial x^2} + \frac{\partial^2 u_f}{\partial y^2} \right) + KN_0(u_s - u_f) - \sigma B_0^2 u_f \quad (4)$$

$$\rho \left(\frac{\partial v_f}{\partial t} + u_f \frac{\partial v_f}{\partial x} + v_f \frac{\partial v_f}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v_f}{\partial x^2} + \frac{\partial^2 v_f}{\partial y^2} \right) + KN_0(v_s - v_f) \quad (5)$$

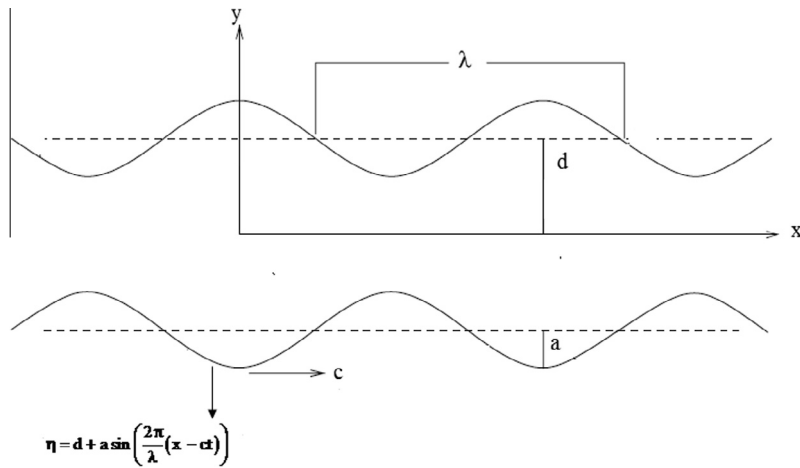


Figure 1 Flow geometry of the problem.

$$\rho c_p \left(\frac{\partial T_f}{\partial t} + u_f \frac{\partial T_f}{\partial x} + v_f \frac{\partial T_f}{\partial y} \right) = \frac{N_0 c_p}{\tau_T} (T_s - T_f) + \frac{N_0}{\tau_v} [(u_s - u_f)^2 + (v_s - v_f)^2] + k_c \left(\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right) \quad (6)$$

$$\frac{\partial C_f}{\partial t} + u_f \frac{\partial C_f}{\partial x} + v_f \frac{\partial C_f}{\partial y} = D_f \left(\frac{\partial^2 C_f}{\partial x^2} + \frac{\partial^2 C_f}{\partial y^2} \right) - k_1 C_f \quad (7)$$

$$\frac{\partial u_s}{\partial t} + u_s \frac{\partial u_s}{\partial x} + v_s \frac{\partial u_s}{\partial y} = \frac{K}{m} (u_f - u_s) - \sigma B_0^2 u_s \quad (8)$$

$$\frac{\partial v_s}{\partial t} + u_s \frac{\partial v_s}{\partial x} + v_s \frac{\partial v_s}{\partial y} = \frac{K}{m} (v_f - v_s) \quad (9)$$

$$N_0 c_m \left(\frac{\partial T_s}{\partial t} + u_s \frac{\partial T_s}{\partial x} + v_s \frac{\partial T_s}{\partial y} \right) = \frac{-N_0 c_p}{\tau_T} (T_s - T_f) \quad (10)$$

$$\frac{\partial C_s}{\partial t} + u_s \frac{\partial C_s}{\partial x} + v_s \frac{\partial C_s}{\partial y} = D_s \left(\frac{\partial^2 C_s}{\partial x^2} + \frac{\partial^2 C_s}{\partial y^2} \right) - k_1 C_s \quad (11)$$

with the boundary conditions

$$u_f = u_s = 0, \quad T_f = T_{f1}, \quad T_s = T_{s1}, \quad C_f = C_{f1} \quad \text{and} \quad C_s = C_{s1} \quad \text{at } y = -\eta$$

$$u_f = u_s = 0, \quad T_f = T_{f2}, \quad T_s = T_{s2}, \quad C_f = C_{f2} \quad \text{and} \quad C_s = C_{s2} \quad \text{at } y = \eta \quad (13)$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial x} L(\eta) = \mu \left[\frac{\partial^2 u_f}{\partial x^2} + \frac{\partial^2 u_f}{\partial y^2} \right] - \rho \left[\frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial x} + v_f \frac{\partial u_f}{\partial y} \right] + KN_0 (u_s - u_f) - \sigma B_0^2 u_f \quad (14)$$

where σ is the electrical conductivity of the fluid, B_0 is the applied magnetic field, ρ is the density of the fluid and K is (resistance coefficient for the dust particles) a constant, T_f is the temperature of the fluid, c_p is the specific heat of the fluid,

C_f is the concentration of the fluid, k_c is the thermal conductivity of the fluid, D_f is the coefficient of mass diffusivity of the fluid particles, D_s is the coefficient of mass diffusivity of the solid particles, c_m is the specific heat of solid particles, T_s is the temperature of solid particles, and τ_T is the thermal relaxation time of the solid particles.

$$u_f = \frac{\partial \psi_f}{\partial y}, \quad v_f = -\frac{\partial \psi_f}{\partial x}, \quad u_s = \frac{\partial \psi_s}{\partial y}, \quad v_s = -\frac{\partial \psi_s}{\partial x} \quad (15)$$

$$x^* = \frac{x}{\lambda}, \quad y^* = \frac{y}{d}, \quad \eta^* = \frac{\eta}{d}, \quad t^* = \frac{vt}{\lambda d}, \quad \psi_f^* = \frac{\psi_f}{v},$$

$$\psi_s^* = \frac{m\psi_s}{kd^2}, \quad \theta_f = \frac{T_f - T_{f1}}{T_{f2} - T_{f1}}, \quad \theta_s = \frac{T_s - T_{s1}}{T_{s2} - T_{s1}},$$

$$\phi_f = \frac{C_f - C_{f1}}{C_{f2} - C_{f1}}, \quad \phi_s = \frac{C_s - C_{s1}}{C_{s2} - C_{s1}} \quad (16)$$

Introducing (15) and (16) into Eqs. (4)–(14), we obtain the following equations (after dropping primes and eliminating the pressure term)

$$\delta \left[\frac{\partial}{\partial t} \left(\frac{\partial^2 \psi_f}{\partial y^2} + \delta^2 \frac{\partial^2 \psi_f}{\partial x^2} \right) + \frac{\partial \psi_f}{\partial y} \frac{\partial}{\partial x} \left(\frac{\partial^2 \psi_f}{\partial y^2} + \delta^2 \frac{\partial^2 \psi_f}{\partial x^2} \right) - \frac{\partial \psi_f}{\partial x} \frac{\partial}{\partial y} \left(\frac{\partial^2 \psi_f}{\partial y^2} + \delta^2 \frac{\partial^2 \psi_f}{\partial x^2} \right) \right] = \nabla_1^4 \psi_f + P \left[\frac{1}{R} \nabla_1^2 \psi_s - \nabla_1^2 \psi_f \right] - M^2 \frac{\partial^2 \psi_f}{\partial y^2} \quad (17)$$

$$\delta \left[\frac{\partial \theta_f}{\partial t} + \frac{\partial \psi_f}{\partial y} \frac{\partial \theta_f}{\partial x} - \frac{\partial \psi_f}{\partial x} \frac{\partial \theta_f}{\partial y} \right] = \frac{N_0 l_1}{\tau_T} (\theta_s - \theta_f) + \frac{N_0 l_1 E_c}{\tau_v} \left[\left(\frac{1}{R} \frac{\partial \psi_s}{\partial y} - \frac{\partial \psi_f}{\partial y} \right)^2 + \delta^2 \left(\frac{1}{R} \frac{\partial \psi_s}{\partial x} - \frac{\partial \psi_f}{\partial x} \right)^2 \right] + \frac{1}{Pr} \left[\delta^2 \frac{\partial^2 \theta_f}{\partial x^2} + \frac{\partial^2 \theta_f}{\partial y^2} \right] \quad (18)$$

$$\delta \left[\frac{\partial \phi_f}{\partial t} + \frac{\partial \psi_f}{\partial y} \frac{\partial \phi_f}{\partial x} - \frac{\partial \psi_f}{\partial x} \frac{\partial \phi_f}{\partial y} \right] = \frac{1}{Sc_1} \left[\delta^2 \frac{\partial^2 \phi_f}{\partial x^2} + \frac{\partial^2 \phi_f}{\partial y^2} \right] - \gamma \phi_f + K_1 \quad (19)$$

$$\delta \left[R \frac{\partial}{\partial t} (\nabla_1^2 \psi_s) + \frac{\partial \psi_s}{\partial y} \frac{\partial}{\partial x} (\nabla_1^2 \psi_s) - \frac{\partial \psi_s}{\partial x} \frac{\partial}{\partial y} (\nabla_1^2 \psi_s) \right] \\ = R \nabla_1^2 \psi_f - \nabla_1^2 \psi_s - M^2 A \frac{\partial^2 \psi_s}{\partial y^2} \quad (20)$$

$$\delta \left[\frac{\partial \theta_s}{\partial t} + \frac{1}{R} \frac{\partial \psi_s}{\partial y} \frac{\partial \theta_s}{\partial x} - \frac{1}{R} \frac{\partial \psi_s}{\partial x} \frac{\partial \theta_s}{\partial y} \right] = -l_2 (\theta_s - \theta_f) \quad (21)$$

$$\delta \left[\frac{\partial \phi_s}{\partial t} + \frac{1}{R} \frac{\partial \psi_s}{\partial y} \frac{\partial \phi_s}{\partial x} - \frac{1}{R} \frac{\partial \psi_s}{\partial x} \frac{\partial \phi_s}{\partial y} \right] \\ = \frac{1}{S_{c2}} \left[\delta^2 \frac{\partial^2 \phi_s}{\partial x^2} + \frac{\partial^2 \phi_s}{\partial y^2} \right] - \gamma \phi_s + K_2 \quad (22)$$

with the boundary conditions,

$$\frac{\partial \psi_f}{\partial y} = 0, \frac{\partial \psi_s}{\partial y} = 0, \theta_f = 0, \theta_s = 0, \phi_f = 0, \phi_s = 0 \text{ at } y = -\eta \quad (23)$$

$$\frac{\partial \psi_f}{\partial y} = 0, \frac{\partial \psi_s}{\partial y} = 0, \theta_f = 1, \theta_s = 1, \phi_f = 1, \phi_s = 1 \text{ at } y = \eta \quad (24)$$

$$\left[E_1 \frac{\partial^3}{\partial x \partial t^2} + E_2 \frac{\partial^2}{\partial x \partial t} + E_3 \frac{\partial^5}{\partial x^5} + E_4 \frac{\partial^3}{\partial x^3} + E_5 \frac{\partial}{\partial x} \right] \eta \\ = \nabla_1^2 \frac{\partial \psi_f}{\partial y} - \delta \left[\frac{\partial}{\partial t} \frac{\partial \psi_f}{\partial y} + \frac{\partial \psi_f}{\partial y} \frac{\partial}{\partial x} \frac{\partial \psi_f}{\partial y} - \frac{\partial \psi_f}{\partial x} \frac{\partial}{\partial y} \frac{\partial \psi_f}{\partial y} \right] \\ + P \left[\frac{1}{R} \frac{\partial \psi_s}{\partial y} - \frac{\partial \psi_f}{\partial y} \right] - M^2 \frac{\partial \psi_f}{\partial y} \quad (25)$$

where $\nabla_1^2 = \left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$, $\delta = \frac{d}{\lambda}$ is geometric parameter, $P = \frac{KN_0 d^2}{\rho \nu}$, $R = \frac{m \nu}{k d^2}$, $A = \frac{m \mu}{k d^2}$, $l_1 = \frac{d^2}{\mu}$, $l_2 = \frac{c_p d^2}{\tau_T c_m \nu}$ are non dimensional parameters, $M = \sqrt{\frac{\sigma B_0^2 d^2}{\mu}}$ is the Hartmann number, $\gamma = \frac{k_1 d^2}{\nu}$ is the chemical parameter, $K_1 = \frac{-k_1 c_1 d^2}{\nu(c_2 - c_1)}$, $K_2 = \frac{-k_1 c_1 d^2}{\nu(c_2 - c_1)}$, $E_c = \frac{\nu^2}{d^2 c_p (T_2 - T_1)}$ is the Eckert number, $P_r = \frac{\mu c_p}{k_c}$ is the Prandtl number, $S_{c1} = \frac{\nu}{D_f}$ is the Schmidt number of the fluid particles and $S_{c2} = \frac{\nu}{D_s}$ is the Schmidt number of solid Particles, ν is the kinematic coefficient of the viscosity, $E_1 = -\frac{T^* d^4}{\lambda^3 \rho \nu^2}$, $E_2 = \frac{m^* d^2}{\lambda^3 \rho}$, $E_3 = \frac{d^* d^5}{\lambda^2 \rho \nu}$, $E_4 = \frac{B d^4}{\lambda^3 \rho \nu^2}$ and $E_5 = \frac{K^* d^4}{\lambda \rho \nu^2}$ are the non-dimensional elasticity parameters.

3. Method of solution

We seek perturbation solution in terms of small parameter $\delta \ll 1$ as follows:

$$f = f_0 + \delta f_1 + \delta^2 f_2 + \delta^3 f_3 + \dots \quad (26)$$

where 'f' represents any flow variable.

$$\psi_f = \psi_{f0} + \delta \psi_{f1} + \delta^2 \psi_{f2} + \dots \quad (27)$$

$$\psi_s = \psi_{s0} + \delta \psi_{s1} + \delta^2 \psi_{s2} + \dots \quad (28)$$

$$\theta_f = \theta_{f0} + \delta \theta_{f1} + \delta^2 \theta_{f2} + \dots \quad (29)$$

$$\theta_s = \theta_{s0} + \delta \theta_{s1} + \delta^2 \theta_{s2} + \dots \quad (30)$$

$$\phi_f = \phi_{f0} + \delta \phi_{f1} + \delta^2 \phi_{f2} + \dots \quad (31)$$

$$\phi_s = \phi_{s0} + \delta \phi_{s1} + \delta^2 \phi_{s2} + \dots \quad (32)$$

Substituting (27)–(32) in the system of Eqs. (17)–(22) subject to the boundary conditions (23)–(25), we get:

Zeroth order equations:

$$\frac{\partial^4 \psi_{f0}}{\partial y^4} + P \left[\frac{1}{R} \frac{\partial^2 \psi_{s0}}{\partial y^2} - \frac{\partial^2 \psi_{f0}}{\partial y^2} \right] - M^2 \frac{\partial^2 \psi_{f0}}{\partial y^2} = 0 \quad (33)$$

$$\frac{1}{P_r} \left(\frac{\partial^2 \theta_{f0}}{\partial y^2} \right) + \frac{N_0 l_1}{\tau_T} (\theta_{s0} - \theta_{f0}) + \frac{N_0 l_1 E_c}{\tau_v} \left(\frac{1}{R} \frac{\partial \psi_{s0}}{\partial y} - \frac{\partial \psi_{f0}}{\partial y} \right)^2 = 0 \quad (34)$$

$$\frac{1}{S_{c1}} \frac{\partial^2 \phi_{f0}}{\partial y^2} - \gamma \phi_{f0} + K_1 = 0 \quad (35)$$

$$R \frac{\partial^2 \psi_{f0}}{\partial y^2} - \frac{\partial^2 \psi_{s0}}{\partial y^2} - M^2 A \frac{\partial^2 \psi_{s0}}{\partial y^2} = 0 \quad (36)$$

$$l_2 (\theta_{s0} - \theta_{f0}) = 0 \quad (37)$$

$$\frac{1}{S_{c2}} \frac{\partial^2 \phi_{s0}}{\partial y^2} - \gamma \phi_{s0} + K_2 = 0 \quad (38)$$

with the boundary conditions,

$$\frac{\partial \psi_{f0}}{\partial y} = 0, \frac{\partial \psi_{s0}}{\partial y} = 0, \theta_{f0} = 0, \theta_{s0} = 0, \\ \phi_{f0} = 0, \phi_{s0} = 0 \text{ at } y = -\eta \quad (39)$$

$$\frac{\partial \psi_{f0}}{\partial y} = 0, \frac{\partial \psi_{s0}}{\partial y} = 0, \theta_{f0} = 1, \theta_{s0} = 1, \\ \phi_{f0} = 1, \phi_{s0} = 1 \text{ at } y = \eta \quad (40)$$

$$\frac{\partial^3 \psi_{f0}}{\partial y^3} + P \left[\frac{1}{R} \frac{\partial \psi_{s0}}{\partial y} - \frac{\partial \psi_{f0}}{\partial y} \right] - M^2 \frac{\partial \psi_{f0}}{\partial y} \\ = \left[E_1 \frac{\partial^3}{\partial x \partial t^2} + E_2 \frac{\partial^2}{\partial x \partial t} + E_3 \frac{\partial^5}{\partial x^5} + E_4 \frac{\partial^3}{\partial x^3} + E_5 \frac{\partial}{\partial x} \right] \eta \\ \text{at } y = \pm \eta \quad (41)$$

First order equations:

$$\frac{\partial^4 \psi_{f1}}{\partial y^4} + P \left[\frac{1}{R} \frac{\partial^2 \psi_{s1}}{\partial y^2} - \frac{\partial^2 \psi_{f1}}{\partial y^2} \right] - M^2 \frac{\partial^2 \psi_{f1}}{\partial y^2} \\ = \left[\frac{\partial^3 \psi_{f0}}{\partial t \partial y^2} + \frac{\partial \psi_{f0}}{\partial y} \frac{\partial^3 \psi_{f0}}{\partial x \partial y^2} - \frac{\partial \psi_{f0}}{\partial x} \frac{\partial^3 \psi_{f0}}{\partial y^3} \right] \quad (42)$$

$$\frac{\partial \theta_{f0}}{\partial t} + \frac{\partial \psi_{f0}}{\partial y} \frac{\partial \theta_{f0}}{\partial x} - \frac{\partial \psi_{f0}}{\partial x} \frac{\partial \theta_{f0}}{\partial y} \\ + \frac{2N_0 l_1 E_c}{R \tau_v} \left(\frac{\partial \psi_{s0}}{\partial y} \frac{\partial \psi_{f1}}{\partial y} + \frac{\partial \psi_{s1}}{\partial y} \frac{\partial \psi_{f0}}{\partial y} \right) \\ = \frac{1}{P_r} \frac{\partial^2 \theta_{f1}}{\partial y^2} + \frac{N_0 l_1}{\tau_T} (\theta_{s1} - \theta_{f1}) \quad (43)$$

$$\frac{1}{S_{c1}} \frac{\partial^2 \phi_{f1}}{\partial y^2} - \gamma \phi_{f1} = \frac{\partial \phi_{f0}}{\partial t} + \frac{\partial \psi_{f0}}{\partial y} \frac{\partial \phi_{f0}}{\partial x} - \frac{\partial \psi_{f0}}{\partial x} \frac{\partial \phi_{f0}}{\partial y} \quad (44)$$

$$R \frac{\partial^2 \psi_{f1}}{\partial y^2} - \frac{\partial^2 \psi_{s1}}{\partial y^2} - M^2 A \frac{\partial^2 \psi_{s1}}{\partial y^2} = R \frac{\partial^3 \psi_{s0}}{\partial t \partial y^2} + \frac{\partial \psi_{s0}}{\partial y} \frac{\partial^3 \psi_{s0}}{\partial x \partial y^2} - \frac{\partial \psi_{s0}}{\partial x} \frac{\partial^3 \psi_{s0}}{\partial y^3} \quad (45)$$

$$l_2(\theta_{s1} - \theta_{f1}) = \frac{-\partial \theta_{s0}}{\partial t} - \frac{1}{R} \frac{\partial \psi_{s0}}{\partial y} \frac{\partial \theta_{s0}}{\partial x} + \frac{1}{R} \frac{\partial \psi_{s0}}{\partial x} \frac{\partial \theta_{s0}}{\partial y} \quad (46)$$

$$\frac{1}{S_{c2}} \frac{\partial^2 \phi_{s1}}{\partial y^2} - \gamma \phi_{s1} = \frac{\partial \phi_{s0}}{\partial t} + \frac{1}{R} \frac{\partial \psi_{s0}}{\partial y} \frac{\partial \phi_{s0}}{\partial x} - \frac{1}{R} \frac{\partial \psi_{s0}}{\partial x} \frac{\partial \phi_{s0}}{\partial y} \quad (47)$$

with the boundary conditions,

$$\begin{aligned} \frac{\partial \psi_{f1}}{\partial y} = 0, \quad \frac{\partial \psi_{s1}}{\partial y} = 0, \quad \theta_{f1} = 0, \quad \theta_{s1} = 0, \quad \phi_{f1} = 0, \\ \phi_{s1} = 0 \quad \text{at} \quad y = -\eta \end{aligned} \quad (48)$$

$$\begin{aligned} \frac{\partial \psi_{f1}}{\partial y} = 0, \quad \frac{\partial \psi_{s1}}{\partial y} = 0, \quad \theta_{f1} = 0, \quad \theta_{s1} = 0, \quad \phi_{f1} = 0, \\ \phi_{s1} = 0 \quad \text{at} \quad y = \eta \end{aligned} \quad (49)$$

$$\begin{aligned} \frac{\partial^3 \psi_{f1}}{\partial y^3} + P \left[\frac{1}{R} \frac{\partial \psi_{s1}}{\partial y} - \frac{\partial \psi_{f1}}{\partial y} \right] \\ - \left[\frac{\partial^2 \psi_{f0}}{\partial t \partial y} + \frac{\partial \psi_{f0}}{\partial y} \frac{\partial^2 \psi_{f0}}{\partial x \partial y} - \frac{\partial \psi_{f0}}{\partial x} \frac{\partial^2 \psi_{f0}}{\partial y^2} \right] \\ - M^2 \frac{\partial \psi_{f1}}{\partial y} = 0 \quad \text{at} \quad y = \pm \eta \end{aligned} \quad (50)$$

Solving Eqs. (33)–(38) and (42)–(47) together with the boundary conditions (39)–(41) and (48)–(50), we get the expression for stream functions, velocity, temperature and concentration of fluid and solid particles

$$\psi_{f0} = A_3 + B_3 y + A_4 \cosh \alpha_1 y + B_4 \sinh \alpha_1 y \quad (51)$$

$$u_{f0} = B_3 + A_4 \alpha_1 \sinh \alpha_1 y + B_4 \alpha_1 \cosh \alpha_1 y \quad (52)$$

$$\psi_{s0} = T_1 A_4 \cosh \alpha_1 y + T_1 B_4 \sinh \alpha_1 y + E y + F \quad (53)$$

$$u_{s0} = T_1 A_4 \alpha_1 \sinh \alpha_1 y + T_1 B_4 \alpha_1 \cosh \alpha_1 y + E \quad (54)$$

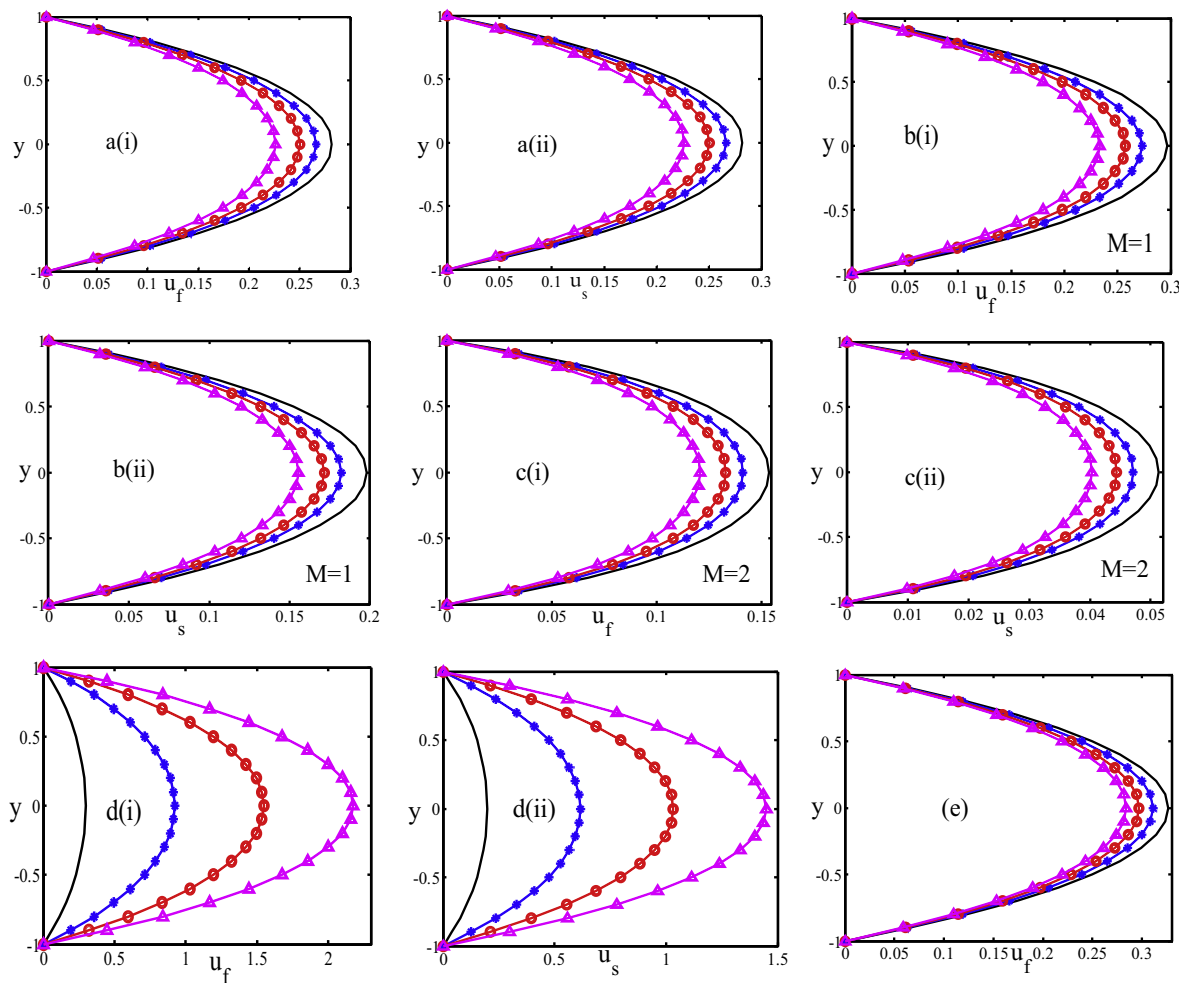


Figure 2 Velocity distribution ($\gamma = 0.5, R = 1, A = 0.5, P_r = 0.71, K_1 = 1, K_2 = 1, E_c = 0.2, S_{c1} = 0.5, E_3 = 0.5, E_5 = 0.2, S_{c2} = 0.5, N_0 = 10, \tau_T = 1, \tau_v = 1, l_1 = 0.1, l_2 = 2, \varepsilon = 0.001$) (–) $E_1 = 0.2$, (*) $E_1 = 0.4$, (O) $E_1 = 0.6$, (\wedge) $E_1 = 0.9, M = 1, E_2 = 0.2, E_4 = 0.1$, (b) (–) $E_2 = 0.2$, (*) $E_2 = 0.5$, (O) $E_2 = 0.7$, (\wedge) $E_2 = 1, E_4 = 0.1, E_1 = 0.01$, (c) (–) $E_2 = 0.2$, (*) $E_2 = 0.5$, (O) $E_2 = 0.7$, (\wedge) $E_2 = 1, E_4 = 0.1, E_1 = 0.01$, (d) (–) $E_4 = 0.1$, (*) $E_4 = 0.3$, (O) $E_4 = 0.5$, (\wedge) $E_4 = 0.7, M = 1, E_2 = 0.2$, (e) (–) $P = 0$, (*) $P = 0.5$, (O) $P = 1$, (\wedge) $P = 1.5, M = 1, E_1 = 0.2, E_2 = 0.5, E_4 = 0.01$.

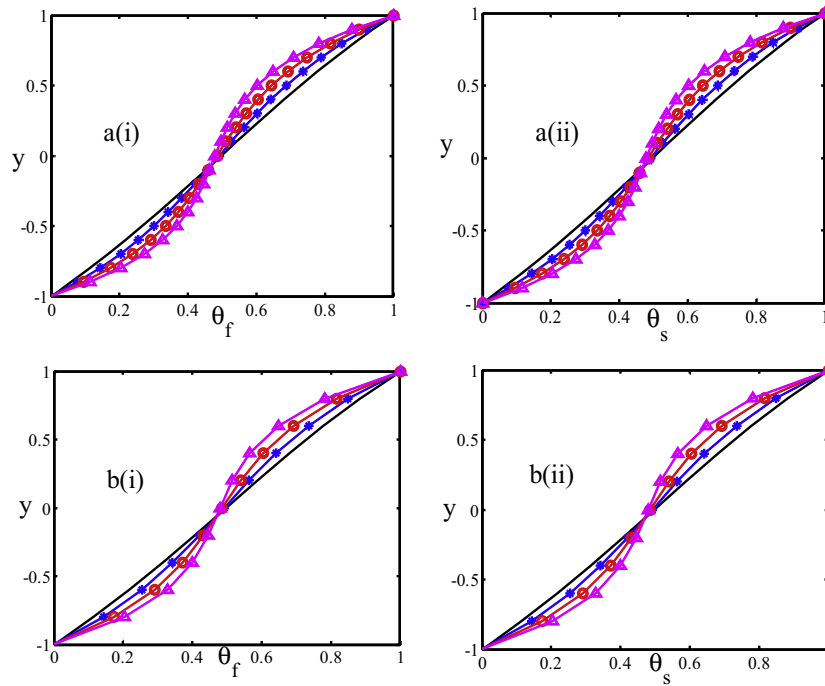


Figure 3 Temperature distribution, ($\gamma = 0.5, R = 1, A = 1, K_1 = 1, K_2 = 1, S_{c1} = 0.5, S_{c2} = 0.5, M = 2, E_1 = 0.01, E_2 = 0.5, E_3 = 0.2, E_4 = 0.1, E_5 = 0.6, t = 1, N_0 = 10, l_1 = 0.1, l_2 = 2, \tau_T = 1, \tau_v = 1, \varepsilon = 0.001$), (a) (–) $E_c = 1$, (*) $E_c = 3$, (○) $E_c = 5$, (∧) $E_c = 7, P_r = 1$, (b) (–) $P_r = 1$, (*) $P_r = 3$, (○) $P_r = 5$, (∧) $P_r = 7, E_c = 1$.

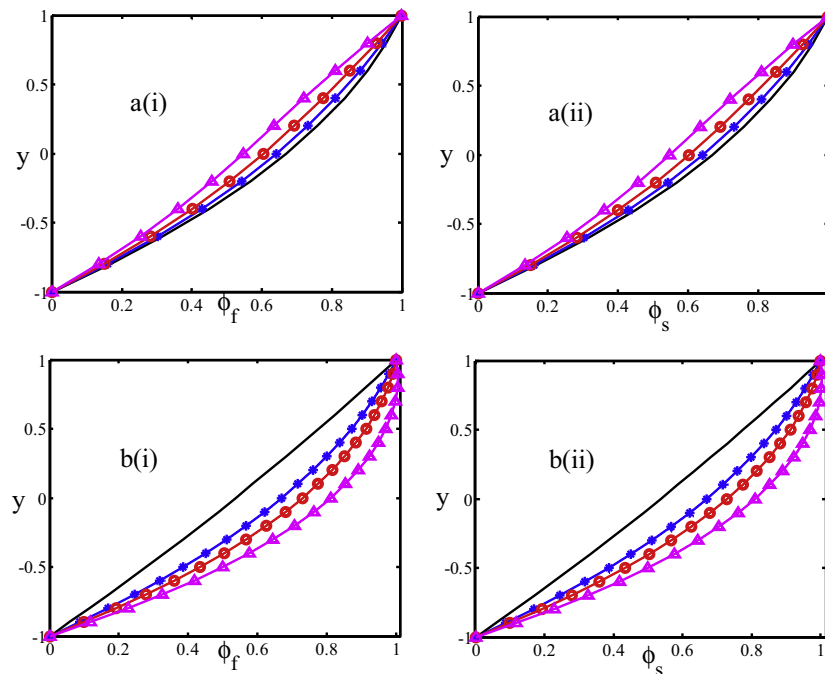


Figure 4 Concentration distribution, ($R = 1, A = 1, K_1 = 1, K_2 = 1, N_0 = 10, M = 2, E_1 = 0.01, E_2 = 0.5, E_3 = 0.2, E_4 = 0.1, E_5 = 0.6, t = 1, l_1 = 0.1, l_2 = 2, E_c = 0.2, P_r = 1, \tau_T = 1, \tau_v = 1, \varepsilon = 0.001$) (a) (–) $\gamma = 0.5$, (*) $\gamma = 0.7$, (○) $\gamma = 1$, (∧) $\gamma = 1.5, S_{c1} = 0.5, S_{c2} = 0.5$, (b) (i) (–) $S_{c1} = 0.1$, (*) $S_{c1} = 0.5$, (○) $S_{c1} = 0.7$, (∧) $S_{c1} = 1, S_{c2} = 0.5, \gamma = 0.5$, (ii) (–) $S_{c2} = 0.1$, (*) $S_{c2} = 0.5$, (○) $S_{c2} = 0.7$, (∧) $S_{c2} = 1, S_{c1} = 0.5, \gamma = 0.5$.

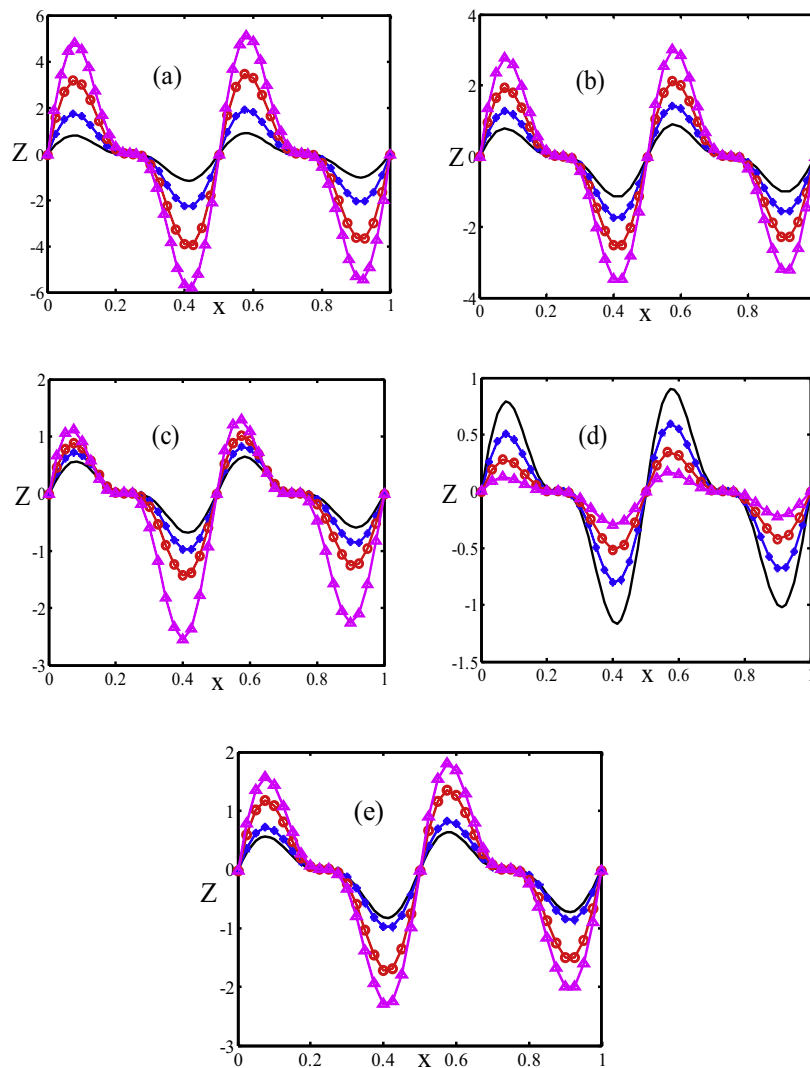


Figure 5 Coefficient of heat transfer rate, ($\gamma = 5$, $R = 1$, $A = 0.2$, $K_1 = 1$, $K_2 = 1$, $S_{c1} = 2$, $S_{c2} = 2$, $M = 0.2$, $t = 1$, $l_1 = 0.1$, $l_2 = 0.1$, $E_c = 1$, $\tau_\tau = 1$, $\tau_v = 1$, $E_5 = 1$, $N_0 = 1$, $\varepsilon = 0.001$), (a) (–) $E_1 = 0.5$, (*) $E_1 = 0.7$, (○) $E_1 = 0.9$, (∧) $E_1 = 1$, $P_r = 1$, $E_2 = 1$, $E_3 = 0.7$, $E_4 = 0.01$, (b) (–) $E_2 = 1$, (*) $E_2 = 1.2$, (○) $E_2 = 1.4$, (∧) $E_2 = 1.6$, $P_r = 1$, $E_3 = 0.7$, $E_4 = 0.01$, $E_1 = 0.5$, (c) (–) $E_3 = 0.1$, (*) $E_3 = 0.5$, (○) $E_3 = 1$, (∧) $E_3 = 2$, $P_r = 1$, $E_2 = 1$, $E_4 = 0.01$, $E_1 = 0.5$, (d) (–) $E_4 = 0.01$, (*) $E_4 = 0.015$, (○) $E_4 = 0.2$, (∧) $E_4 = 0.025$, $P_r = 1$, $E_2 = 1$, $E_3 = 0.7$, $E_1 = 0.5$, (e) (–) $P_r = 0.71$, (*) $P_r = 1$, (○) $P_r = 1.5$, (∧) $P_r = 2$, $E_2 = 1$, $E_3 = 0.7$, $E_4 = 0.01$, $E_1 = 0.5$.

$$\theta_{f0} = A_5 + B_5 y + T_{11} \cosh \alpha_1 y + T_{12} \sinh \alpha_1 y + T_{13} \cosh 2\alpha_1 y + T_{14} \sinh 2\alpha_1 y + T_{15} y^2 \quad (55)$$

$$\theta_{s0} = A_5 + B_5 y + T_{11} \cosh \alpha_1 y + T_{12} \sinh \alpha_1 y + T_{13} \cosh 2\alpha_1 y + T_{14} \sinh 2\alpha_1 y + T_{15} y^2 \quad (56)$$

$$\phi_{f0} = A_1 \cosh \alpha y + B_1 \sinh \alpha y + \frac{K_1 S_{c1}}{\alpha^2} \quad (57)$$

$$\phi_{s0} = A_2 \cosh \beta y + B_2 \sinh \beta y + \frac{K_2 S_{c2}}{\beta^2} \quad (58)$$

$$\psi_{f1} = A_8 + B_8 y + \cosh \alpha_1 y (A_9 + T_{58} y + T_{60} y^2) + \sinh \alpha_1 y (B_9 + T_{57} y + T_{59} y^2) \quad (59)$$

$$u_{f1} = B_8 + \sinh \alpha_1 y (A_9 \alpha_1 + T_{57} + T_{62} y + \alpha_1 T_{60} y^2) + \cosh \alpha_1 y (B_9 \alpha_1 + T_{58} + T_{61} y + \alpha_1 T_{59} y^2) \quad (60)$$

$$\psi_{s1} = \cosh \alpha_1 y (A_9 T_{100} + T_{102} + T_{103} y + T_{106} y^2) + \sinh \alpha_1 y (B_9 T_{100} + T_{104} + T_{101} y + T_{105} y^2) + G y + H \quad (61)$$

$$u_{s1} = \sinh \alpha_1 y (A_9 T_{93} + T_{95} + T_{96} y + T_{99} y^2) + \cosh \alpha_1 y (B_9 T_{93} + T_{97} + T_{94} y + T_{98} y^2) + G \quad (62)$$

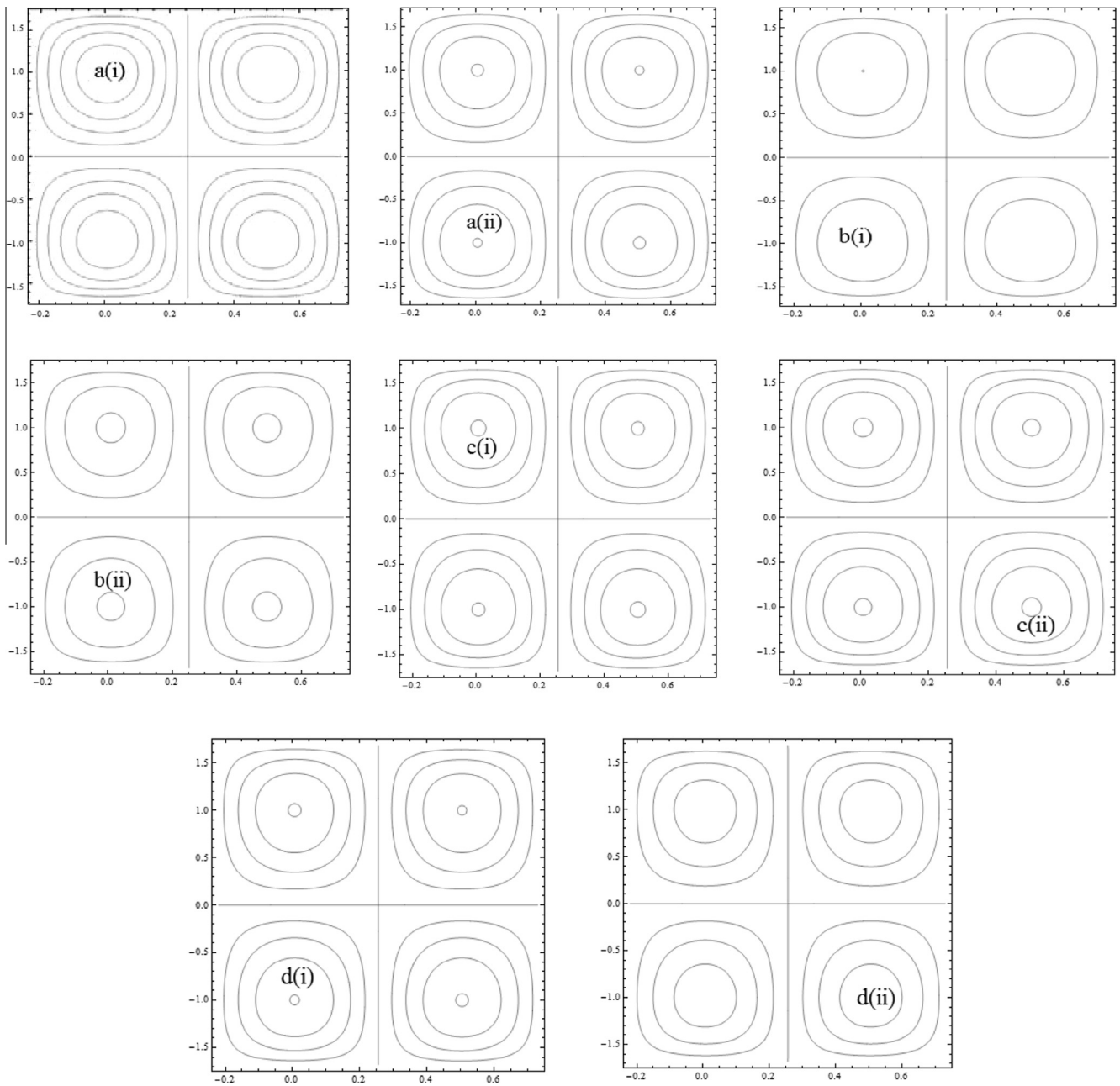


Figure 6 Effects of E_1 , E_4 , E_5 and M on Stream line pattern, ($\gamma = 5$, $R = 1$, $A = 0.2$, $K_1 = 1$, $K_2 = 1$, $S_{c1} = 2$, $S_{c2} = 2$, $M = 0.2$, $t = 1$, $l_1 = 0.1$, $l_2 = 0.1$, $E_c = 1$, $\tau_T = 1$, $\tau_v = 1$, $E_5 = 1$, $N_0 = 1$, $\varepsilon = 0.001$), (a) (i) $E_1 = 0.05$ (ii) $E_1 = 0.1$; (b) (i) $E_4 = 0.05$ (ii) $E_4 = 0.1$; (c) (i) $E_5 = 0.05$ (ii) $E_5 = 0.1$ (d) (i) $M = 0.1$ (ii) $M = 0.5$.

$$\begin{aligned}
 \theta_{fl} = & A_{10} + B_{10}y + \cosh \alpha_1 y (B_8 T_{161} + B_9 T_{162} + G T_{163} \\
 & + T_{164} + T_{172}y + T_{184}y^2) + \sinh \alpha_1 y (B_8 T_{165} \\
 & + G T_{166} + A_9 T_{167} + T_{168} + T_{171}y + T_{183}y^2) \\
 & + \sinh 2\alpha_1 y (T_{170} + B_9 T_{173} + A_9 T_{174} + T_{179}y \\
 & + T_{181}y^2) + \cosh 2\alpha_1 y (A_9 T_{175} + B_9 T_{176} + T_{169} \\
 & + T_{180}y + T_{182}y^2) + T_{177} \sinh 3\alpha_1 y + T_{178} \cosh 3\alpha_1 y \\
 & + T_{185}y^4 + T_{186}y^3 + y^2 (G T_{187} + B_8 T_{188} + T_{189} \\
 & + A_9 T_{190} + B_9 T_{191})
 \end{aligned} \tag{63}$$

$$\begin{aligned}
 \theta_{sl} = & A_{10} + B_{10}y + \cosh \alpha_1 y (B_8 T_{161} + B_9 T_{162} + G T_{163} \\
 & + T_{192} + T_{197}y + T_{203}y^2) + \sinh \alpha_1 y (B_8 T_{165} \\
 & + G T_{166} + A_9 T_{167} + T_{193} + T_{196}y + T_{202}y^2) \\
 & + \cosh 2\alpha_1 y (T_{194} + A_9 T_{175} + B_9 T_{176} + T_{201}y \\
 & + T_{182}y^2) + \sinh 2\alpha_1 y (T_{195} + B_9 T_{173} + A_9 T_{174} \\
 & + T_{200}y + T_{181}y^2) + T_{198} \sinh 3\alpha_1 y + T_{199} \cosh 3\alpha_1 y \\
 & + T_{185}y^4 + T_{186}y^3 + y^2 (G T_{187} + B_8 T_{188} + T_{204} \\
 & + A_9 T_{190} + B_9 T_{191}) + T_{205}y + T_{206}
 \end{aligned} \tag{64}$$

$$\begin{aligned} \phi_{f1} = & \cosh \alpha y (A_6 + T_{26}y - T_{32}y - T_{33}y^2) \\ & + \sinh \alpha y (B_6 + T_{25}y + T_{34}y + T_{31}y^2) \\ & + T_{27} \sinh(\alpha_1 + \alpha)y + T_{28} \sinh(\alpha_1 - \alpha)y \\ & + T_{29} \cosh(\alpha_1 + \alpha)y + T_{30} \cosh(\alpha_1 - \alpha)y \end{aligned} \quad (65)$$

$$\begin{aligned} \phi_{s1} = & \cosh \beta y (A_7 + T_{46}y + S_3y^2) + \sinh \beta y (B_7 + T_{47}y \\ & + S_4y^2) + T_{42} \sinh(\alpha_1 + \beta)y + T_{43} \sinh(\beta - \alpha_1)y \\ & + T_{44} \cosh(\alpha_1 + \beta)y + T_{45} \cosh(\beta - \alpha_1)y \end{aligned} \quad (66)$$

where $\alpha = \sqrt{\gamma S_{c1}}$; $\beta = \sqrt{\gamma S_{c2}}$; $\alpha_1 = \sqrt{P + M^2 - \frac{P}{1+AM^2}}$.

The coefficient of heat transfer rate at the wall is given by

$$Z_f = Z_{f0} + \delta Z_{f1} \quad (67)$$

where $Z_{f0} = \eta_x \theta_{f0y}$; $Z_{f1} = \theta_{f0x} + \eta_x \theta_{f1y}$.

4. Results and discussion

This section presents the graphical results in order to discuss the quantitative effects of the parameters involved in the analysis. In Figs. 2–7, we have shown the effects of Hartmann number (M), Schmidt number (S_c), amplitude parameter (δ) and the elastic parameters E_1, E_2, E_3, E_4 and E_5 . Fig. 2 shows the influences of E_1, E_2, E_4, P and M on velocity field. The effect of different values of E_1 is graphed in Fig. 2a. It shows that the effect of increasing the above-mentioned elastic parameter leads to decrease the velocity throughout the channel. In Fig. 2b and c, fluid velocity is decreasing function with increasing the parameters E_2 and M . This is because of the presence of the transverse magnetic field which creates a resistive force similar to the drag force that acts in the opposite direction of the fluid motion, thus causing the velocity of the fluid to decrease. An opposite result is true for increasing elastic parameter E_4 , which is shown in Fig. 2d. Fig. 2e confirms that, increasing the material parameter P leads to decrease the fluid velocity which means that velocity of non-Newtonian fluid is significantly reduced as compared to Newtonian fluid ($P = 0$).

To see the effects of E_c and P_r on temperature distribution, we have plotted Fig. 3. It is observed that the temperature profiles are linear for lower values of the parameters E_c and P_r while it becomes parabolic for higher values. Further, temperature is gradually enhanced with increasing the

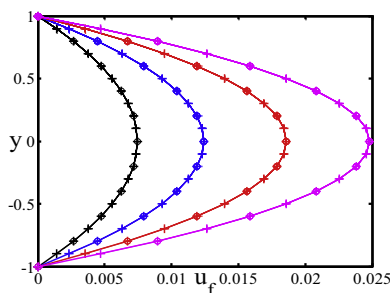


Figure 7 Velocity distribution, ($x = 1, t = 1, l_1 = 0.1, l_2 = 0.1, \tau_T = 1, \tau_v = 1, E_c = 0.1, N_0 = 10, P_r = 0.1, \varepsilon = 0.001, E_3 = 0.05, E_1 = 0.07, (-) E_2 = 0.01, (-) E_2 = 0.05, (-) E_2 = 0.1, (-) E_2 = 0.15, E_4 = 0, E_5 = 0$), \circ Results of Eldabe et al. [25] (Non-porous case), \square Results of the Present paper (when $M, E_4, E_5 \rightarrow 0$).

values of the parameters. The influence of the parameters of γ and S_c on concentration distribution is displayed in Fig. 4. Fig. 4a is plotted for various values of chemical reaction parameter γ . It shows that increasing γ enhances the fluid concentration. The opposite trend is seen for the case when S_c is increased as noted in Fig. 4b. To see the influence of the parameters E_1, E_2, E_3, E_4 and P_r on coefficient of heat transfer (Z), we have displayed in Fig. 5. It shows the oscillatory behavior of heat transfer which may be due to peristalsis. It depicts that maximum amplitude of Z enhanced with increasing E_1, E_2, E_3 , and P_r while the opposite is true for increasing E_4 .

The formation of an internally circulating bolus of fluid by closed streamlines is shown in Fig. 6. This trapped bolus pushed a head along peristaltic waves. The aim of Fig. 6 was to examine the influence of different parameters on trapping. Fig. 6a shows that the trapped bolus decreases with increasing elastic parameter E_1 . The influence of E_4 on trapping is displayed in Fig. 6b. It depicts that streamlines are increased with an increase of E_4 . The quite opposite effect can be noticed when increasing the parameter E_5 , which is shown in Fig. 6c. Fig. 6d shows the opposite effect of Fig. 6b, if E_4 is replaced by magnetic parameter. Furthermore, our results are in good agreement with the results of Eldabe et al. [25] (for non-porous case) by choosing $M \rightarrow 0, E_4 \rightarrow 0, E_5 \rightarrow 0$ (see Fig. 7).

5. Conclusion

In this section, numerical calculations have been performed in order to see the dependence of heat and mass transfer characteristics of MHD peristaltic transport of a dusty fluid through a horizontal channel for both fluid and solid particles. Analytical solutions for the problem are obtained by using perturbation technique for both fluid and solid particles. The features of flow, heat and mass transfer characteristics are presented and discussed graphically. The effects of pertinent parameters on flow, heat and mass transfer characteristics have been studied. Investigation of such analysis is of utmost importance owing to its wide range of applications in engineering and biological systems. In particular, it may serve for the intrauterine fluid motion in a sagittal cross section of the uterus under cancer therapy and drug analysis, the transport of lymph in the lymphatic vessels, and the vasomotion in small blood vessels such as arterioles, venues, and capillaries [1–14]. The main observations of the presented attempt may be summarized as follows:

- The effects of different values of E_1, E_2 and ' M ' lead to suppressing the fluid velocity while the opposite trend can be seen with increasing E_4 .
- Increasing values of E_c and P_r , the fluid temperature profiles are parabolic in nature whereas they are linear for lower values of thermal parameters. Further, temperature is gradually enhanced with increasing these parameters.
- Increasing chemical reaction parameter enhanced the fluid concentration but increasing Schmidt number decreases the concentration of the fluid.
- The trapped bolus decreases and lines are also reduced with an increase in magnetic parameter while the elastic parameter E_4 on trapping is opposite to magnetic parameter.

- The results of hydrodynamics can be obtained as a limiting case of our analysis by taking $M \rightarrow 0$.

Appendix

$$T_1 = \frac{R}{1 + AM^2};$$

$$T_2 = \frac{N_0 l_1 E_c P_r}{\tau_v} \left(\frac{T_1 A_4 \alpha_1^2}{R^2} + A_4 \alpha_1^2 - \frac{2T_1 B_4 B_3 \alpha_1}{R} - \frac{2EB_4 \alpha_1}{R} \right);$$

$$T_3 = \frac{N_0 l_1 E_c P_r}{\tau_v} \left(\frac{T_1 B_4 \alpha_1^2}{R^2} + B_4 \alpha_1^2 - \frac{2T_1 A_4 B_3 \alpha_1}{R} - \frac{2EA_4 \alpha_1}{R} \right);$$

$$T_4 = \frac{N_0 l_1 E_c P_r}{\tau_v} \left(\frac{2T_1 A_4^2 \alpha_1^2}{R} \right); \quad T_5 = \frac{N_0 l_1 E_c P_r}{\tau_v} \left(\frac{4T_1 A_4 B_4 \alpha_1^2}{R} \right);$$

$$T_6 = \frac{N_0 l_1 E_c P_r}{\tau_v} \left(\frac{2T_1 B_4^2 \alpha_1^2}{R} \right); \quad T_7 = \frac{N_0 l_1 E_c P_r}{\tau_v} \left(\frac{2EB_3}{R} \right);$$

$$T_8 = \left(-\frac{T_4}{2} - \frac{T_6}{2} \right); \quad T_9 = \frac{T_5}{2};$$

$$T_{10} = \left(\frac{T_4}{2} - \frac{T_6}{2} - T_7 \right); \quad T_{11} = \left(-\frac{T_2}{\alpha_1^2} \right);$$

$$T_{12} = \left(-\frac{T_3}{\alpha_1^2} \right); \quad T_{13} = \left(-\frac{T_8}{4\alpha_1^2} \right);$$

$$T_{14} = \left(\frac{T_9}{4\alpha_1^2} \right); \quad T_{15} = \left(-\frac{T_{10}}{2} \right);$$

$$T_{16} = \left(\alpha_1^3 + \frac{PT_1 \alpha_1}{R} - P\alpha_1 - M^2 \alpha_1 \right);$$

$$T_{17} = S_{c1} (A_{1t} + B_3 A_{1x});$$

$$T_{18} = S_{c1} (B_{1t} + B_3 B_{1x});$$

$$T_{19} = S_{c1} \left(\frac{A_4 \alpha_1 A_{1x} - B_{4x} B_1 \alpha + B_{1x} B_4 \alpha_1 - A_{4x} A_1 \alpha}{2} \right);$$

$$T_{20} = S_{c1} \left(\frac{A_4 \alpha_1 A_{1x} - B_{4x} B_1 \alpha - B_{1x} B_4 \alpha_1 + A_{4x} A_1 \alpha}{2} \right);$$

$$T_{21} = S_{c1} \left(\frac{A_4 \alpha_1 B_{1x} - B_{4x} A_1 \alpha + A_{1x} B_4 \alpha_1 - A_{4x} B_1 \alpha}{2} \right);$$

$$T_{22} = S_{c1} \left(\frac{B_4 \alpha_1 A_{1x} - A_{4x} B_1 \alpha - B_{1x} A_4 \alpha_1 + B_{4x} A_1 \alpha}{2} \right);$$

$$T_{23} = S_{c1} (B_{3x} B_1 \alpha); \quad T_{24} = S_{c1} (B_{3x} A_1 \alpha);$$

$$T_{25} = \frac{T_{17}}{2\alpha}; \quad T_{26} = \frac{T_{18}}{2\alpha}; \quad T_{27} = \frac{T_{19}}{\alpha_1^2 + 2\alpha\alpha_1};$$

$$T_{28} = \frac{T_{20}}{\alpha_1^2 - 2\alpha\alpha_1}; \quad T_{29} = \frac{T_{21}}{\alpha_1^2 + 2\alpha\alpha_1};$$

$$T_{30} = \frac{T_{22}}{\alpha_1^2 - 2\alpha\alpha_1}; \quad T_{31} = \frac{T_{23}}{4\alpha}; \quad T_{32} = \frac{T_{23}}{4\alpha^2};$$

$$T_{33} = \frac{T_{24}}{4\alpha}; \quad T_{34} = \frac{T_{24}}{4\alpha^2};$$

$$T_{36} = S_{c2} \left(\frac{T_1 A_4 \alpha_1 A_{2x}}{2R} - \frac{T_1 B_{4x} B_2 \beta}{2R} + \frac{T_1 B_4 \alpha_1 B_{2x}}{2R} - \frac{T_1 A_{4x} A_2 \beta}{2R} \right);$$

$$T_{37} = S_{c2} \left(\frac{T_1 A_4 \alpha_1 A_{2x}}{2R} - \frac{T_1 B_{4x} B_2 \beta}{2R} - \frac{T_1 B_4 \alpha_1 B_{2x}}{2R} + \frac{T_1 A_{4x} A_2 \beta}{2R} \right);$$

$$T_{38} = S_{c2} \left(\frac{T_1 A_4 \alpha_1 B_{2x}}{2R} - \frac{T_1 B_{4x} A_2 \beta}{2R} + \frac{T_1 B_4 \alpha_1 A_{2x}}{2R} - \frac{T_1 A_{4x} B_2 \beta}{2R} \right);$$

$$T_{39} = S_{c2} \left(\frac{T_1 A_2 \beta B_{4x}}{2R} - \frac{T_1 B_{2x} A_4 \alpha_1}{2R} + \frac{T_1 B_4 \alpha_1 A_{2x}}{2R} - \frac{T_1 A_{4x} B_2 \beta}{2R} \right);$$

$$T_{40} = S_{c2} \left(B_{2t} - \frac{F_x}{R} A_2 \beta + \frac{E}{R} B_{2x} \right);$$

$$T_{41} = S_{c2} \left(A_{2t} - \frac{F_x}{R} B_2 \beta + \frac{E}{R} A_{2x} \right);$$

$$S_1 = S_{c2} \left(\frac{-E_x}{R} A_2 \beta \right); \quad S_2 = S_{c2} \left(\frac{-E_x}{R} B_2 \beta \right);$$

$$T_{42} = \frac{T_{36}}{\alpha_1^2 + 2\alpha_1 \beta}; \quad T_{43} = \frac{T_{37}}{\alpha_1^2 - 2\alpha_1 \beta}; \quad T_{44} = \frac{T_{38}}{\alpha_1^2 + 2\alpha_1 \beta};$$

$$T_{45} = \frac{T_{39}}{\alpha_1^2 - 2\alpha_1 \beta}; \quad T_{46} = \frac{T_{40}}{2\beta} - \frac{S_2}{4\beta^2}; \quad T_{47} = \frac{T_{41}}{2\beta} - \frac{S_1}{4\beta^2};$$

$$S_3 = \frac{S_1}{4\beta}; \quad S_4 = \frac{S_2}{4\beta};$$

$$T_{49} = (RT_1 A_{4t} \alpha_1^2 + ET_1 A_{4x} \alpha_1^2 - F_x T_1 B_4 \alpha_1^3);$$

$$T_{50} = (RT_1 B_{4t} \alpha_1^2 + ET_1 B_{4x} \alpha_1^2 - F_x T_1 A_4 \alpha_1^3);$$

$$T_{51} = (-E_x T_1 B_4 \alpha_1^3); \quad T_{52} = (-E_x T_1 A_4 \alpha_1^3);$$

$$T_{53} = \left(\frac{P}{R(1 + AM^2)} T_{49} + A_{4t} \alpha_1^2 + B_3 A_{4x} \alpha_1^2 - A_{3x} B_4 \alpha_1^3 \right);$$

$$T_{54} = \left(\frac{P}{R(1 + AM^2)} T_{50} + B_{4t} \alpha_1^2 + B_3 B_{4x} \alpha_1^2 - A_{3x} A_4 \alpha_1^3 \right);$$

$$T_{55} = \left(\frac{P}{R(1 + AM^2)} T_{51} - B_{3x} B_4 \alpha_1^3 \right);$$

$$T_{56} = \left(\frac{P}{R(1 + AM^2)} T_{52} - B_{3x} A_4 \alpha_1^3 \right); \quad T_{57} = \left(\frac{T_{53}}{2\alpha_1^3} - \frac{5T_{56}}{4\alpha_1^4} \right);$$

$$T_{58} = \left(\frac{T_{54}}{2\alpha_1^3} - \frac{5T_{55}}{4\alpha_1^4} \right); \quad T_{59} = \left(\frac{T_{55}}{4\alpha_1^3} \right); \quad T_{60} = \left(\frac{T_{56}}{4\alpha_1^3} \right);$$

$$T_{61} = (\alpha_1 T_{57} + 2T_{60}); \quad T_{62} = (\alpha_1 T_{58} + 2T_{59});$$

$$T_{63} = (\alpha_1 T_{61} + 4\alpha_1 T_{60}); \quad T_{64} = (\alpha_1 T_{57} + T_{61});$$

$$T_{65} = (\alpha_1 T_{62} + 4\alpha_1 T_{59}); \quad T_{66} = (\alpha_1 T_{58} + T_{62});$$

$$T_{67} = (\alpha_1 T_{63} + 2\alpha_1^2 T_{60}); \quad T_{68} = (\alpha_1 T_{64} + T_{63});$$

$$T_{69} = (\alpha_1 T_{65} + 2\alpha_1^2 T_{59}); \quad T_{70} = (\alpha_1 T_{66} + T_{65});$$

$$T_{71} = \left\{ -T_{11} - \frac{1}{R} \left[T_1 B_4 \alpha_1 A_{5x} + ET_{11} + \left(\frac{T_1 B_4 T_{13} \alpha_1 - T_1 A_4 T_{14} \alpha_1}{2} \right) - B_5 T_1 A_{4x} - F_x T_{12} \alpha_1 - T_1 A_{4x} T_{14} \alpha_1 + T_1 B_{4x} T_{13} \alpha_1 \right] \right\};$$

$$T_{72} = \left\{ -T_{12} - \frac{1}{R} \left[T_1 A_4 \alpha_1 A_{5x} + ET_{12} + \left(\frac{T_1 B_4 T_{14} \alpha_1 - T_1 A_4 T_{13} \alpha_1}{2} \right) - B_5 T_1 B_{4x} - F_x T_{11} \alpha_1 - T_1 A_{4x} T_{13} \alpha_1 - T_1 B_{4x} T_{14} \alpha_1 \right] \right\};$$

$$T_{73} = \left\{ -T_{13} - \frac{1}{R} \left[ET_{13} + \left(\frac{T_1 A_4 T_{12} \alpha_1 + T_1 B_4 T_{11} \alpha_1}{2} \right) - 2F_x T_{14} \alpha_1 - \left(\frac{T_1 A_{4x} T_{12} \alpha_1 + T_1 B_{4x} T_{11} \alpha_1}{2} \right) \right] \right\};$$

$$T_{74} = \left\{ -T_{14} - \frac{1}{R} \left[ET_{14} + \left(\frac{T_1 A_4 T_{11} \alpha_1 + T_1 B_4 T_{12} \alpha_1}{2} \right) - 2F_x T_{13} \alpha_1 - \left(\frac{T_1 A_{4x} T_{11} \alpha_1 + T_1 B_{4x} T_{12} \alpha_1}{2} \right) \right] \right\};$$

$$T_{75} = \left[-\frac{1}{R} (T_1 A_4 \alpha_1 B_{5x} - 2T_1 B_{4x} T_{15} - E_x T_{11} \alpha_1) \right];$$

$$T_{76} = \left[-\frac{1}{R} (T_1 B_4 \alpha_1 B_{5x} - 2T_1 A_{4x} T_{15} - E_x T_{12} \alpha_1) \right];$$

$$T_{77} = \left[-\frac{1}{R} (T_1 A_4 \alpha_1 T_{15}) \right]; \quad T_{78} = \left[-\frac{1}{R} (T_1 B_4 \alpha_1 T_{15}) \right];$$

$$T_{79} = \left\{ -\frac{1}{R} \left[\left(\frac{T_1 A_4 T_{13} \alpha_1 + T_1 B_4 T_{14} \alpha_1}{2} \right) - T_1 A_{4x} T_{13} \alpha_1 - T_1 B_{4x} T_{14} \alpha_1 \right] \right\};$$

$$T_{80} = \left\{ -\frac{1}{R} \left[\left(\frac{T_1 A_4 T_{14} \alpha_1 + T_1 B_4 T_{13} \alpha_1}{2} \right) - T_1 A_{4x} T_{14} \alpha_1 - T_1 B_{4x} T_{13} \alpha_1 \right] \right\}; \quad T_{81} = \frac{2E_x T_{13} \alpha_1}{R};$$

$$T_{82} = \frac{2E_x T_{14} \alpha_1}{R}; \quad T_{83} = \left[-T_{15} - \frac{ET_{15}}{R} + \frac{2E_x T_{15}}{R} \right];$$

$$T_{84} = \left[-B_{5t} - \frac{EB_{5x}}{R} + \left(\frac{E_x B_5 + 2F_x T_{15}}{R} \right) \right];$$

$$T_{85} = \left[-A_{5t} - \frac{1}{R} (EA_{5x}) + \frac{1}{R} \left(\frac{T_1 A_4 \alpha_1 T_{12} - T_1 B_4 \alpha_1 T_{11}}{2} \right) + \frac{1}{R} \left(\frac{T_1 A_{4x} \alpha_1 T_{12} - T_1 B_{4x} \alpha_1 T_{11}}{2} \right) + F_x B_5 \right];$$

$$T_{86} = \frac{R\alpha_1^2}{1 + AM^2}; \quad T_{87} = \frac{RT_{63} - T_{52}}{1 + AM^2};$$

$$T_{88} = \frac{RT_{64} - T_{49}}{1 + AM^2}; \quad T_{89} = \frac{RT_{65} - T_{51}}{1 + AM^2}; \quad T_{90} = \frac{RT_{66} - T_{50}}{1 + AM^2};$$

$$T_{91} = \frac{RT_{59} \alpha_1^2}{1 + AM^2}; \quad T_{92} = \frac{RT_{60} \alpha_1^2}{1 + AM^2}; \quad T_{93} = \frac{T_{86}}{\alpha_1};$$

$$T_{94} = \left(\frac{T_{87}}{\alpha_1} - \frac{2T_{92}}{\alpha_1^2} \right); \quad T_{95} = \left(\frac{-T_{87}}{\alpha_1^2} + \frac{2T_{92}}{\alpha_1^3} + \frac{T_{88}}{\alpha_1} \right);$$

$$T_{96} = \left(\frac{T_{89}}{\alpha_1} - \frac{2T_{91}}{\alpha_1^2} \right); \quad T_{97} = \left(\frac{-T_{89}}{\alpha_1^2} + \frac{2T_{91}}{\alpha_1^3} + \frac{T_{90}}{\alpha_1} \right);$$

$$T_{98} = \frac{T_{91}}{\alpha_1}; \quad T_{99} = \frac{T_{92}}{\alpha_1}; \quad T_{100} = \frac{T_{93}}{\alpha_1};$$

$$T_{101} = \left(\frac{T_{94}}{\alpha_1} - \frac{2T_{99}}{\alpha_1^2} \right); \quad T_{102} = \left(\frac{-T_{94}}{\alpha_1^2} + \frac{T_{95}}{\alpha_1} + \frac{2T_{99}}{\alpha_1^3} \right);$$

$$T_{103} = \left(\frac{T_{96}}{\alpha_1} - \frac{2T_{98}}{\alpha_1^2} \right); \quad T_{104} = \left(\frac{-T_{96}}{\alpha_1^2} + \frac{T_{97}}{\alpha_1} + \frac{2T_{98}}{\alpha_1^3} \right);$$

$$T_{105} = \frac{T_{98}}{\alpha_1}; \quad T_{106} = \frac{T_{99}}{\alpha_1}; \quad T_{107} = \frac{2N_0 l_1 E_c}{R\tau_v};$$

$$T_{108} = Pr T_{107} T_1 B_4 \alpha_1; \quad T_{109} = Pr (T_{107} E \alpha_1 + T_{107} T_{93} B_3);$$

$$T_{110} = Pr T_{107} B_4 \alpha_1;$$

$$T_{111} = Pr [T_{11} + B_3 T_{11} + B_4 \alpha_1 A_{5x} - (A_{3x} T_{12} \alpha_1 + A_4 B_5) + T_{107} ET_{58} + T_{107} T_{97} B_3] - \frac{Pr T_{71} N_0 l_1}{\tau_r l_2};$$

$$T_{112} = Pr T_{107} T_1 A_4 \alpha_1; \quad T_{113} = Pr T_{107} A_4 \alpha_1;$$

$$T_{114} = Pr [T_{12} + B_3 T_{12} + A_4 \alpha_1 A_{5x} - (A_{3x} T_{11} \alpha_1 + B_4 B_5) + T_{107} ET_{57} + T_{107} T_{95} B_3] - \frac{Pr T_{72} N_0 l_1}{\tau_r l_2};$$

$$T_{115} = Pr \left(T_{13} + B_3 T_{13} - 2A_{3x} T_{14} \alpha_1 - \frac{T_{73} N_0 l_1}{\tau_r l_2} \right);$$

$$T_{116} = Pr \left(T_{14} + B_3 T_{14} - 2A_{3x} T_{13} \alpha_1 - \frac{T_{74} N_0 l_1}{\tau_r l_2} \right);$$

$$T_{117} = Pr \left[A_4 \alpha_1 B_{5x} - (B_{3x} T_{11} \alpha_1 + 2B_4 T_{15}) + T_{107} (ET_{62} + T_{96} B_3) - \frac{T_{75} N_0 l_1}{\tau_r l_2} \right];$$

$$T_{118} = Pr [B_4 \alpha_1 B_{5x} - (B_{3x} T_{12} \alpha_1 + 2A_4 T_{15}) + T_{107} (ET_{61} + T_{94} B_3) - \frac{T_{76} N_0 l_1}{\tau_r l_2}];$$

$$T_{119} = Pr T_{107} [T_1 A_4 \alpha_1^2 + T_{93} A_4 \alpha_1];$$

$$T_{120} = Pr T_{107} [T_1 B_4 \alpha_1^2 + T_{93} B_4 \alpha_1];$$

$$T_{121} = Pr [T_{107} (T_1 A_4 \alpha_1 T_{58} + T_1 B_4 \alpha_1 T_{57} + B_4 \alpha_1 T_{95} + A_4 \alpha_1 T_{97})];$$

$$T_{122} = Pr [A_4 \alpha_1 T_{12} - B_4 \alpha_1 T_{11} + T_{107} (T_1 A_4 \alpha_1 T_{57} + A_4 \alpha_1 T_{95})];$$

$$T_{123} = Pr [B_4 \alpha_1 T_{11} - A_4 \alpha_1 T_{12} + T_{107} (T_1 B_4 \alpha_1 T_{58} + B_4 \alpha_1 T_{97})];$$

$$T_{124} = Pr (A_4 \alpha_1 T_{13} - 2B_4 T_{14} \alpha_1);$$

$$T_{125} = Pr (B_4 \alpha_1 T_{13} - 2A_4 T_{14} \alpha_1);$$

$$\begin{aligned}
T_{126} &= P_r(B_4\alpha_1 T_{14} - 2A_4 T_{13}\alpha_1); \\
T_{127} &= P_r\left[A_4\alpha_1 T_{15} + T_{107}(E\alpha_1 T_{60} + T_{99}B_3) - \frac{T_{77}N_0 l_1}{\tau_T l_2}\right]; \\
T_{128} &= P_r\left[B_4\alpha_1 T_{15} + T_{107}(E\alpha_1 T_{59} + T_{98}B_3) - \frac{T_{78}N_0 l_1}{\tau_T l_2}\right]; \\
T_{129} &= P_r\left(\frac{-T_{79}N_0 l_1}{\tau_T l_2}\right); \quad T_{130} = P_r\left(\frac{-T_{80}N_0 l_1}{\tau_T l_2}\right); \\
T_{131} &= P_r\left[-2B_{3x}\alpha_1 T_{13} - \frac{T_{81}N_0 l_1}{\tau_T l_2}\right]; \\
T_{132} &= P_r\left[-2B_{3x}\alpha_1 T_{14} - \frac{T_{82}N_0 l_1}{\tau_T l_2}\right]; \\
T_{133} &= P_r[T_{107}(T_1 A_4 \alpha_1 T_{61} + T_1 B_4 \alpha_1 T_{62} + T_{94} A_4 \alpha_1 + T_{96} B_4 \alpha_1)]; \\
T_{134} &= P_r[T_{107}(T_1 A_4 \alpha_1 T_{62} + T_{96} A_4 \alpha_1)]; \\
T_{135} &= P_r[T_{107}(T_1 A_4 \alpha_1^2 T_{59} + T_1 B_4 \alpha_1^2 T_{60} + T_{98} A_4 \alpha_1 + T_{99} B_4 \alpha_1)]; \\
T_{136} &= P_r(T_{94} B_4 \alpha_1); \quad T_{137} = P_r[T_{107}(T_1 A_4 \alpha_1^2 T_{60} + T_{99} A_4 \alpha_1)]; \\
T_{138} &= P_r[T_{107}(T_1 B_4 \alpha_1^2 T_{59} + T_{98} B_4 \alpha_1)]; \\
T_{139} &= P_r\left(T_{15} + B_3 T_{15} - 2B_{3x} T_{15} - \frac{T_{83}N_0 l_1}{\tau_T l_2}\right); \\
T_{140} &= P_r\left[B_{5r} - (2A_{3x} T_{15} + B_{3x} B_5) + B_3 B_{5x} - \frac{T_{84}N_0 l_1}{\tau_T l_2}\right]; \\
T_{141} &= P_r T_{107} B_3; \quad T_{142} = P_r E T_{107}; \\
T_{143} &= P_r\left[A_{5r} + B_3 A_{5x} - A_{3x} B_{53} - \frac{T_{85}N_0 l_1}{\tau_T l_2}\right]; \\
T_{144} &= \left(T_{111} + \frac{T_{125}}{2}\right); \quad T_{145} = \left(T_{114} - \frac{T_{124}}{2} + \frac{T_{126}}{2}\right); \\
T_{146} &= \left(T_{115} + \frac{T_{122}}{2} + \frac{T_{123}}{2}\right); \quad T_{147} = \left(T_{116} + \frac{T_{121}}{2}\right); \\
T_{148} &= \left(\frac{T_{119}}{2}\right); \quad T_{149} = \left(\frac{T_{120}}{2}\right); \quad T_{150} = \left(\frac{T_{119}}{2}\right); \\
T_{151} &= \left(\frac{T_{120}}{2}\right); \quad T_{152} = \left(\frac{T_{124}}{2} + \frac{T_{126}}{2} + T_{129}\right); \\
T_{153} &= \left(\frac{T_{125}}{2} + T_{130}\right); \quad T_{154} = \left(T_{131} + \frac{T_{133}}{2}\right); \\
T_{155} &= \left(T_{132} + \frac{T_{134}}{2} + \frac{T_{136}}{2}\right); \quad T_{156} = \left(\frac{T_{135}}{2}\right); \\
T_{157} &= \left(\frac{T_{137}}{2} + \frac{T_{138}}{2}\right); \quad T_{158} = \left(-\frac{T_{137}}{2} + \frac{T_{138}}{2} + T_{139}\right); \\
T_{159} &= \left(-\frac{T_{134}}{2} + \frac{T_{136}}{2} + T_{140}\right); \\
T_{160} &= \left(-\frac{T_{122}}{2} + \frac{T_{123}}{2} + T_{143}\right); \\
T_{161} &= \left(\frac{T_{108}}{\alpha_1^2}\right); \quad T_{162} = \left(\frac{T_{109}}{\alpha_1^2}\right); \\
T_{163} &= \left(\frac{T_{110}}{\alpha_1^2}\right); \quad T_{164} = \left(\frac{T_{144}}{\alpha_1^2} - \frac{2T_{117}}{\alpha_1^3} + \frac{6T_{128}}{\alpha_1^4}\right); \\
T_{165} &= \left(\frac{T_{112}}{\alpha_1^2}\right); \quad T_{166} = \left(\frac{T_{113}}{\alpha_1^2}\right); \quad T_{167} = \left(\frac{T_{109}}{\alpha_1^2}\right); \\
T_{168} &= \left(\frac{T_{145}}{\alpha_1^2} - \frac{2T_{118}}{\alpha_1^3} + \frac{6T_{127}}{\alpha_1^4}\right); \\
T_{169} &= \left(\frac{T_{146}}{4\alpha_1^2} - \frac{T_{154}}{4\alpha_1^3} + \frac{3T_{157}}{8\alpha_1^4}\right); \\
T_{170} &= \left(\frac{T_{147}}{4\alpha_1^2} - \frac{T_{155}}{4\alpha_1^3} + \frac{3T_{156}}{8\alpha_1^4}\right); \\
T_{171} &= \left(\frac{T_{117}}{\alpha_1^2} - \frac{4T_{128}}{\alpha_1^3}\right); \quad T_{172} = \left(\frac{T_{118}}{\alpha_1^2} - \frac{4T_{127}}{\alpha_1^3}\right); \\
T_{173} &= \left(\frac{T_{148}}{4\alpha_1^2}\right); \quad T_{174} = \left(\frac{T_{149}}{4\alpha_1^2}\right); \quad T_{175} = \left(\frac{T_{150}}{4\alpha_1^2}\right); \\
T_{176} &= \left(\frac{T_{151}}{4\alpha_1^2}\right); \quad T_{177} = \left(\frac{T_{152}}{9\alpha_1^2}\right); \quad T_{178} = \left(\frac{T_{153}}{9\alpha_1^2}\right); \\
T_{179} &= \left(\frac{T_{154}}{4\alpha_1^2} - \frac{2T_{157}}{4\alpha_1^3}\right); \quad T_{180} = \left(\frac{T_{155}}{4\alpha_1^2} - \frac{T_{156}}{2\alpha_1^3}\right); \\
T_{181} &= \left(\frac{T_{156}}{4\alpha_1^2}\right); \quad T_{182} = \left(\frac{T_{157}}{4\alpha_1^2}\right); \quad T_{183} = \left(\frac{T_{127}}{\alpha_1^2}\right); \\
T_{184} &= \left(\frac{T_{128}}{\alpha_1^2}\right); \quad T_{185} = \left(\frac{T_{158}}{12}\right); \quad T_{186} = \left(\frac{T_{159}}{6}\right); \\
T_{187} &= \left(\frac{T_{141}}{2}\right); \quad T_{188} = \left(\frac{T_{142}}{2}\right); \quad T_{189} = \left(\frac{T_{160}}{2}\right); \\
T_{190} &= \left(\frac{T_{150}}{2}\right); \quad T_{191} = \left(\frac{T_{151}}{2}\right); \quad T_{192} = \left(T_{164} + \frac{T_{71}}{l_2}\right); \\
T_{193} &= \left(T_{168} + \frac{T_{72}}{l_2}\right); \quad T_{194} = \left(T_{169} + \frac{T_{73}}{l_2}\right); \\
T_{195} &= \left(T_{170} + \frac{T_{74}}{l_2}\right); \quad T_{196} = \left(T_{171} + \frac{T_{75}}{l_2}\right); \\
T_{197} &= \left(T_{172} + \frac{T_{76}}{l_2}\right); \quad T_{198} = \left(T_{177} + \frac{T_{79}}{l_2}\right); \\
T_{199} &= \left(T_{178} + \frac{T_{80}}{l_2}\right); \quad T_{200} = \left(T_{179} + \frac{T_{81}}{l_2}\right); \\
T_{201} &= \left(T_{180} + \frac{T_{82}}{l_2}\right); \quad T_{202} = \left(T_{183} + \frac{T_{77}}{l_2}\right); \\
T_{203} &= \left(T_{184} + \frac{T_{78}}{l_2}\right); \quad T_{204} = \left(T_{189} + \frac{T_{83}}{l_2}\right); \quad T_{205} = \left(\frac{T_{84}}{l_2}\right); \\
T_{206} &= \left(\frac{T_{85}}{l_2}\right); \quad T_{207} = \left(\alpha_1^3 + \frac{P}{R} T_{93} - P\alpha_1 - M^2 \alpha_1\right); \\
T_{208} &= \left(T_{67} - B_{3x} A_4 \alpha_1^2 + \frac{P}{R} T_{94} - P T_{61} - M^2 T_{61}\right); \\
T_{209} &= \left(T_{68} + B_3 A_{4x} \alpha_1 + A_4 \alpha_1 B_{3x} + \frac{P}{R} T_{95} - P T_{57} - A_{3x} B_4 \alpha_1^2 - A_4 \alpha_1 - M^2 T_{57}\right); \\
T_{210} &= \left(T_{69} - B_{3x} B_4 \alpha_1^2 - P T_{62} - M^2 T_{62}\right);
\end{aligned}$$

$$T_{211} = \left(T_{70} + B_3 B_{4x} \alpha_1 + B_4 \alpha_1 B_{3x} + \frac{P}{R} T_{97} - P T_{58} - A_{3x} A_4 \alpha_1^2 - B_{4t} \alpha_1 - M^2 T_{58} \right);$$

$$T_{212} = \left(T_{59} \alpha_1^3 + \frac{P}{R} T_{98} - P \alpha_1 T_{59} - M^2 \alpha_1 T_{59} \right);$$

$$T_{213} = \left(T_{60} \alpha_1^3 + \frac{P}{R} T_{99} - P \alpha_1 T_{60} - M^2 \alpha_1 T_{60} \right);$$

$$T_{214} = (-A_4 \alpha_1^2 A_{4x} - B_{4x} B_4 \alpha_1^2);$$

$$T_{215} = (-B_4 \alpha_1^2 B_{4x} - A_{4x} A_4 \alpha_1^2); \quad T_{216} = (B_{3t} + B_3 B_{3x});$$

$$T_{217} = \left(\frac{T_{214}}{2} + \frac{T_{215}}{2} \right); \quad T_{218} = \left(\frac{-T_{214}}{2} + \frac{T_{215}}{2} - T_{216} \right);$$

$$T_{219} = \left[\frac{P}{R} T_{93} - T_{207} - \alpha_1 (P + M^2) \cosh \alpha_1 \eta \right];$$

$$T_{220} = \left[-\frac{P}{R} T_{96} \eta + T_{210} \eta + (P + M^2) T_{62} \eta \right];$$

$$T_{221} = \left[-\frac{P}{R} (T_{97} + T_{89} \eta^2) + T_{211} + T_{212} \eta^2 + (P + M^2) (T_{58} + \alpha_1 T_{59} \eta^2) \right];$$

$$a_{10} = [\cosh \alpha_1 \eta (B_8 T_{161} + B_9 T_{162} + G T_{163} + T_{184} \eta^2)];$$

$$a_{11} = [\cosh 2\alpha_1 \eta (A_9 T_{175} + B_9 T_{176} + T_{169} + T_{182} \eta^2)];$$

$$a_{12} = [\eta^2 (G T_{187} + B_8 T_{188} + A_9 T_{190} + B_9 T_{191} + T_{189})];$$

$$b_{10} = [\sinh \alpha_1 \eta (B_8 T_{165} + G T_{166} + A_9 T_{167} + T_{202} \eta^2 + T_{193})];$$

$$b_{11} = [\sinh 2\alpha_1 \eta (B_9 T_{173} + A_9 T_{174} + T_{195})];$$

$$b_{12} = [T_{201} \eta \cosh 2\alpha_1 \eta + T_{181} \eta^2 \sinh 2\alpha_1 \eta + T_{186} \eta^3 + T_{205} \eta];$$

$$A_1 = \frac{\alpha^2 - 2K_1 S_{c1}}{\alpha^2 (2 \cosh \alpha \eta)};$$

$$B_1 = \frac{1}{\sinh \alpha \eta} \left[1 - \left(\frac{K_1 S_{c1}}{\alpha^2} + A_1 \cosh \alpha \eta \right) \right];$$

$$A_2 = \frac{\beta^2 - 2K_2 S_{c2}}{\beta^2 (2 \cosh \beta \eta)};$$

$$B_2 = \frac{1}{\sinh \beta \eta} \left[1 - \left(\frac{K_2 S_{c2}}{\alpha^2} + A_2 \cosh \beta \eta \right) \right];$$

$$A_3 = -A_4; \quad B_3 = -B_4 \alpha_1 \cosh \alpha_1 \eta;$$

$$E = -T_1 B_4 \alpha_1 \cosh \alpha_1 \eta; \quad F = -T_1 A_4;$$

$$A_6 = \frac{-1}{\cosh \alpha \eta} [\sinh \alpha \eta (T_{25} \eta + T_{34} \eta) + \cosh(\alpha_1 + \alpha) \eta T_{29} + \cosh(\alpha_1 - \alpha) \eta T_{30} + \cosh \alpha \eta (-T_{33} \eta^2)];$$

$$B_6 = \frac{-1}{\sinh \alpha \eta} [\sinh \alpha \eta (T_{25} \eta + T_{34} \eta + T_{31} \eta^2) + \cosh(\alpha_1 + \alpha) \eta T_{29} + \cosh(\alpha_1 - \alpha) \eta T_{30} + \cosh \alpha \eta (A_6 + T_{26} \eta - T_{32} \eta - T_{33} \eta^2) + \sinh(\alpha_1 + \alpha) \eta T_{27} + \sinh(\alpha_1 - \alpha) \eta T_{28}];$$

$$A_7 = \frac{-1}{\cosh \beta \eta} [\cosh(\alpha_1 + \beta) \eta T_{44} + \cosh(\beta - \alpha_1) \eta T_{45} + T_{47} \eta \sinh \beta \eta + S_3 \eta^2 \cosh \beta \eta];$$

$$B_7 = \frac{-1}{\sinh \beta \eta} [\cosh \beta \eta (A_7 + T_{46} \eta) + \sinh(\alpha_1 + \beta) \eta T_{42} + \sinh(\beta - \alpha_1) \eta T_{43} + \cosh(\alpha_1 + \beta) \eta T_{44} + \cosh(\beta - \alpha_1) \eta T_{45} + T_{47} \eta \sinh \beta \eta + S_3 \eta^2 \cosh \beta \eta + S_4 \eta^2 \sinh \beta \eta];$$

$$A_8 = -A_9;$$

$$B_8 = \frac{1}{P + M^2} [B_9 T_{207} \cosh \alpha_1 \eta + \frac{P}{R} G + T_{210} \eta \sinh \alpha_1 \eta + T_{211} \cosh \alpha_1 \eta + T_{212} \eta^2 \cosh \alpha_1 \eta + T_{217} \cosh 2\alpha_1 \eta + T_{218}];$$

$$A_9 = \frac{-1}{\alpha_1 \sinh \alpha_1 \eta} [T_{61} \eta \cosh \alpha_1 \eta + T_{57} \sinh \alpha_1 \eta + \alpha_1 T_{60} \eta^2 \sinh \alpha_1 \eta];$$

$$B_9 = \frac{1}{T_{219}} [T_{220} \sinh \alpha_1 \eta + T_{221} \cosh \alpha_1 \eta + T_{217} \cosh 2\alpha_1 \eta + T_{218}];$$

$$A_{10} = -[a_{10} + T_{171} \eta \sinh \alpha_1 \eta + T_{179} \eta \sinh 2\alpha_1 \eta + a_{11} + T_{178} \cosh 3\alpha_1 \eta + a_{12}];$$

$$B_{10} = -\frac{1}{\eta} [b_{10} + T_{197} \eta \cosh \alpha_1 \eta + b_{11} + T_{198} \sinh 3\alpha_1 \eta + b_{12}];$$

$$G = -[B_9 T_{93} \cosh \alpha_1 \eta + T_{96} \eta \sinh \alpha_1 \eta + T_{97} \cosh \alpha_1 \eta + T_{98} \eta^2 \cosh \alpha_1 \eta];$$

$$H = -(A_9 T_{100} + T_{102});$$

$$A_4 = \frac{(E + T_1 B_4 \alpha_1 \cosh \alpha_1 \eta)}{T_1 \alpha_1 \sinh \alpha_1 \eta}; \quad B_4 = \frac{[(E_1 + E_2) 8\pi^3 \varepsilon \cos 2\pi(x-t) - 4\pi^2 E_3 \varepsilon \sin 2\pi(x-t) - 32\pi^5 E_4 \varepsilon \cos 2\pi(x-t) - 2\pi E_5 \varepsilon \cos 2\pi(x-t)]}{\cosh \alpha_1 \eta \left(\frac{P}{R} T_1 \alpha_1 - T_{16} - (P + M^2) \alpha_1 \right)};$$

$$A_5 = \frac{1}{2} [1 - 2(T_{11} \cosh \alpha_1 \eta + T_{13} \cosh 2\alpha_1 \eta + T_{15} \eta^2)];$$

$$B_5 = \frac{1 - (A_5 + T_{11} \cosh \alpha_1 \eta + T_{12} \sinh \alpha_1 \eta + T_{13} \cosh 2\alpha_1 \eta + T_{14} \sinh 2\alpha_1 \eta + T_{15} \eta^2)}{\eta};$$

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