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Kinematic and Dynamic Analysis of 3PUU Parallel Manipulator for Medical Applications

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Abstract

This paper presents an explicit work on developing a parallel manipulator for medical application. There are many applications in medicine that requires the technology of robotics like injecting electrodes, drilling of bone, medical transportation, to carry out different tests and so on. Surgeons divert their focus on improving the quality of the surgical procedure that includes accuracy, security, low morbidity and mortality. This work mainly concentrates on designing and developing a kinematic and dynamic model of parallel manipulator for bone drilling application which is required for prosthetics operations. According to the requirement of the application a three prismatic-universal-universal (3PUU) parallel manipulator is designed which rotates about x and y axis and translates along z axis. This research work involves in developing a conceptual design, deriving kinematics both inverse and forward, workspace and dynamics of the parallel manipulator. Inverse kinematics is obtained by the geometrical approach whereas inverse dynamics is obtained using Langrangian method. Further, the tool inverse kinematic solutions, overall workspace, dynamics of the three legs are determined. This study will help the researchers for the further development of parallel manipulators in medical assistance.

Keywords: parallel manipulator; kinematics; workspace; dynamics; langrangian;

1. Introduction

Bone drilling is a common step in operative fracture treatment and reconstructive surgery. Bone fracture treatment usually involves restoring the fractured parts to their initial positions and constraining their movements till healing takes place. Conventionally fractured bones are healed by setting and immobilizing the part from outside like drape band, later on the focus went on to heal the parts internally using screws, wires and plates. Figure 1 shows the conventional method of drilling by a surgeon. It is highly time consuming for the surgeons to determine the location

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to be drilled. The surgeons have to do various calculations in order to locate the special coordinates from the MRI or CT scans. Surgeons spend 80% of their time in preparing and the rest 20% goes into actual operation. Presently, surgeons divert their focus on improving the quality of the surgical procedure that includes accuracy, security, low morbidity and mortality. Surgeons use stereotactic frames on patient in order to perform surgical action on the desired location precisely.

Fig. 1. Bone drilling by a surgeon

There are many studies that are being implemented on parallel manipulators (PMs) for medical purpose. Parallel manipulator is a closed loop kinematic chain mechanism that is connected to the base via multiple independent kinematic chains. PMs have always proved their excellence over serial manipulators. Compact surgical robot system for image-guided orthopedic surgery was developed [1], robot-assisted spine and trauma surgery was proposed in [2] utilizing a designed six-degree-of-freedom (6-DOF) PM, the mouth opening and closing training for the rehabilitation of patients was suggested in [3] with a 6-DOF parallel robot, a 4-DOF parallel wire driven mechanism was presented in [4] with applications to leg rehabilitation, an idea of applying parallel robots to surgical treatments with monitoring real time images was proposed in [5] and cardiopulmonary resuscitation was performed using a 3PUU PM [6]. The remainder of the paper is organized in the following way: the conceptual design and the architecture of the PM for bone drilling application are discussed in Section 2. The inverse kinematic models of the platform as well as the tool are derived; the forward kinematics is discussed in the Section 3. The velocity equations and Jacobian matrix generation are included in Section 4. The singularities and workspace of the PM is discussed in Section 5 and then the dynamic analysis is explained in Section 6 and finally concluding remarks are given in Section 7.

2. Conceptual design of the 3-PUU PM

Figure 2 shows the proposed PM (PM) CAD model for bone drilling application. It consists of a fixed platform, a moving platform which is connected by three limbs which are symmetric about 120 degrees and are identical kinematic structure.

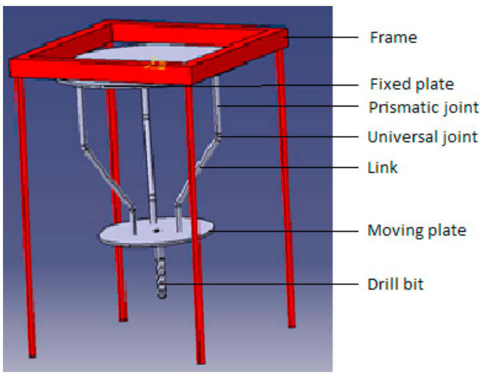

Fig. 2. 3PUU PM CAD model.

Each limb has three joints i.e. Prismatic joint (P) and two Universal joints (U), where the P joint is driven by the actuators. The proposed work represents 3PUU mechanism which can achieve rotational angles α and β about x and y axes respectively and translation along z axis. The primary motion of the manipulator is the translation along z axis, the rotations help in orientation of the manipulator to the desired position for drilling. The fixed actuators make it possible that the moving components of the manipulator do not bear the load of the actuators. This enables large powerful actuators to drive relatively small structures, which facilitates the design of the manipulator with faster, stiffer and stronger characteristics. Table 1 shows the architectural parameters of bone drilling PM.

Table 1: Architectural parameters

Parameters	Values	Units
Initial angle	70	Degrees
Radius of moving platform	75	mm
Length of the arms	70	mm

3. Kinematics

Kinematic modelling includes the study of manipulator positions without consideration of the external forces and moments. Inverse kinematics is the inverse of forward kinematics i.e. obtaining the joint angles for the necessary and desired position of the end effector. Figure 3a and 3b represents the schematic diagram of 3PUU PM. For the kinematic analysis, Cartesian coordinate system has been assigned to the kinematic model as shown in Figure 3a. The inverse kinematics with the tool is shown in Figure 3b. The fixed base is denoted by a global reference system O{x, y, z} which is located at the center of the base platform. Similarly, the moving platform is assigned with a moving Cartesian frame O' {x', y', z'} which is located at the center of moving platform. Where, b_1, b_2 and b_3 are the vertices of the base joints and a_1, a_2 and a_3 are the vertices of moving joints. The global reference system and the moving frame are parallel to each other [13][14].

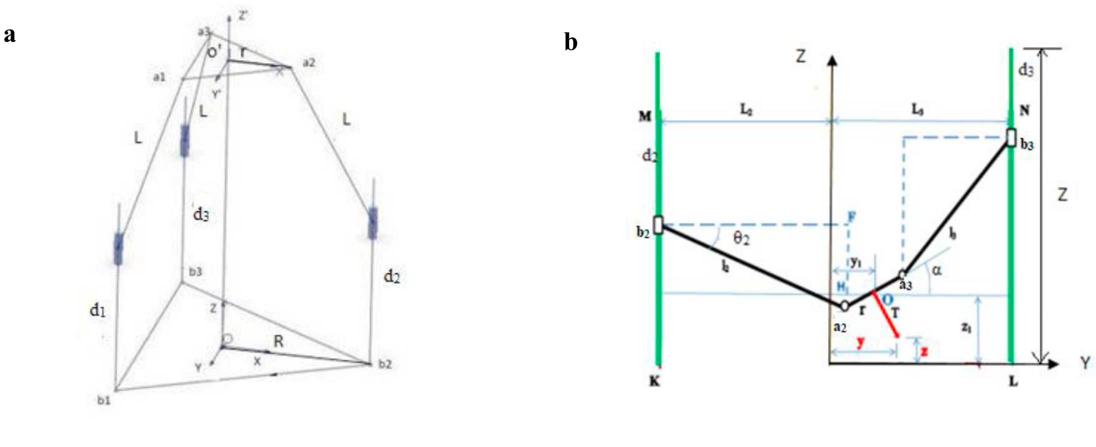

Fig. 3. (a) Kinematic model of 3-PUU PM; (b) 3-PUU PM with tool when rotated about x axis.

The desired position and orientation of the end effector position is to rotate about x and y axes and translates along z axis. Using the constraint $||a_i-b_i||$ the actuator position is calculated by solving the constraint using advanced linear algebraic method. The actuator positions are given in equations 1 to 3 and the kinematic equations shows that there will be two solutions are generated.

$$d_1 = rcos\alpha sin\beta + z - \sqrt{L^2 - (rcos\alpha - R)^2 + (rsin\alpha sin\beta)^2} \quad (1)$$

$$d_2 = 0.5rcos\alpha sin\beta - 0.866rsin\alpha + z - \sqrt{L^2 - (0.5R - 0.5rcos\alpha)^2 + (0.866R - 0.5rsin\alpha sin\beta - 0.866rcos\alpha)^2} \quad (2)$$

$$d_3 = 0.5rcos\alpha sin\beta + 0.866rsin\alpha + z - \sqrt{L^2 - (0.5R - 0.5rcos\alpha)^2 + (0.866rcos\alpha - 0.866R - 0.5rsin\alpha sin\beta)^2} \quad (3)$$

The radius of the moving platform is considered to be 50mm and the length of the arm is considered to be 70mm. The initial angles that are considered is 70° and z=65.77mm. From the inverse kinematic study it is observed that to obtain manipulator tilt β with respect to y axis the leg 2 and leg 3 must be actuated to same position and the leg 1 to a different position. In order to obtain manipulator tilt α with respect to x axis the leg 1 always remains in the positive direction whereas leg 2 and leg 3 varies from positive to negative directions. Similarly, the kinematic equations to determine the actuator positions of the prismatic joints are obtained for the given tool positions and are derived in equations 4 to 6.

$$d_1 = Z - (\, z_1 + \sqrt{l^2 + (L_1 + y_1 - rcos\alpha)^2} + rsin\beta) \quad (4)$$

$$d_2 = \; Z - (z_1 + \sqrt{l^2 + (L_2 + y_1 - rcos\alpha)^2} - rsin\alpha) \quad (5)$$

$$d_3 = \; Z - (z_1 + \sqrt{l^2 + (L_3 + y_1 - rcos\alpha)^2} + rsin\alpha) \quad (6)$$

Forward kinematics is where the position and orientation of the moving platform is obtained with the help of the known actuator positions. With the help of equations 4 to 6 the forward kinematics is derived. To obtain α, it is assumed that β and translation along z axis is zero. To obtain β, it is assumed that α and translation along z is zero. Similarly for z, α and β are zero. By inversing the inverse kinematics, α β and z are obtained and are given in equations 7 to 9.

$$\alpha = \cos^{-1}\left[\frac{R+\sqrt{L^2-(z-D)^2}}{r}\right] \quad (7)$$

$$\beta = \sin^{-1}\left[\frac{D-z+\sqrt{L^2-(r-R)^2}}{r}\right] \quad (8)$$

$$z = D + \sqrt{L^2 - (r-R)^2} \quad (9)$$

4. Velocity Analysis

Velocity equations are derived in order to obtain the Jacobian matrix. By differentiating equations 4 to 6 and rearranging the terms the velocity equation is derived and it is given in equation 10.

$$A\dot{U} = B\dot{V} \quad (10)$$

Where,

A and B are the Jacobian matrices which relates the output velocity to the actuated joint rates.

$$A = \begin{bmatrix} rf + z - d_1 & 0 & 0 \\ 0 & 0.5rf - d_2 + z + 0.866rsin\alpha & 0 \\ 0 & 0 & 0.5rf - d_3 + z - 0.866rsin\alpha \end{bmatrix}$$

$$B = \begin{bmatrix} B_1 & B_2 & B_3 \\ B_4 & B_5 & B_6 \\ B_7 & B_8 & B_9 \end{bmatrix},\ \dot{U} = \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix},\ \dot{V} = \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{z} \end{bmatrix}$$

$$B_1 = d_1rb + rbz - r^2e - r^2cf - r^2fb$$

$$B_2 = r^2fa + rza - rad_1 - r^2bc$$

$$B_3 = rf + z - d_1$$

$$B_4 = 0.5d_2rb + 0.866rd_2cos\alpha - 0.25r^2fb - 0.48rfcos\alpha - 0.5rzb - 0.866zrcos\alpha + 0.48r^2bsin\alpha + 0.64rc$$
$$-0.25rRsin\alpha + 0.25r^2c + 0.25rfb - 0.48r^2bsin\alpha + 0.48r^2fcos\alpha - 0.644r^2c - 0.48rRf + 0.64Rrsin\alpha$$

$$B_5 = -0.5d_2ra + 0.25r^2fa + 0.5rza - 0.48ar^2sin\alpha + 0.25r^2be - 0.48rRe + 0.48r^2ecos\alpha$$

$$B_6 = 0.5rf - d_2 - 0.866rsin\alpha$$

$$B_7 = 0.5d_2rb + 0.866rd_2cos\alpha - 0.25r^2fb + 0.48rfcos\alpha - 0.5rzb - 0.866zrcos\alpha - 0.48r^2bsin\alpha - 0.64rc$$
$$-0.25rRsin\alpha + 0.25r^2c + 0.25rfb + 0.48r^2bsin\alpha + 0.48r^2fcos\alpha - 0.644r^2c - 0.48rRf - 0.64Rrsin\alpha$$

$$B_8 = -0.5d_2ra + 0.25r^2fa + 0.5rza + 0.48ar^2sin\alpha + 0.25r^2be - 0.48rRe - 0.48r^2ecos\alpha$$

$$B_9 = 0.5rf - d_2 + 0.866rsin\alpha$$

$$a = cos\alpha cos\beta, b = sin\alpha sin\beta, c = cos\alpha sin\alpha, d = cos\beta sin\beta, e = sin\alpha cos\beta, f = cos\alpha sin\beta$$

5. Singularity and Workspace

Singularity configurations are particular poses of the end-effector, for which manipulators lose their inherent infinite rigidity, and in which the end-effector will have uncontrollable degrees of freedom. Most manipulators have singularities at the boundary of their workspace, and some have singularities inside their workspace [7]. When a manipulator is in a singular configuration, it becomes failed at the moment. It is very important to avoid such situations when designing a manipulator. Singularities occur when the Jacobian matrices A and B (discussed in Section 4) becomes singular. Singularities are of three types depending on the Jacobian matrices. Type 1 occurs when det (A) =0 and det (B) $\neq$ 0, at this condition the manipulator loses one or two degrees of freedom. It's also called inverse kinematic singularities. Type 2 occurs when det (A) $\neq$ 0 and det (B) =0, at this condition the manipulator gains one or more degrees of freedom. It's also called forward kinematic singularities. Type 3 is the mixture of type 1 and type 2 where the Jacobian matrices A and B both become singular.

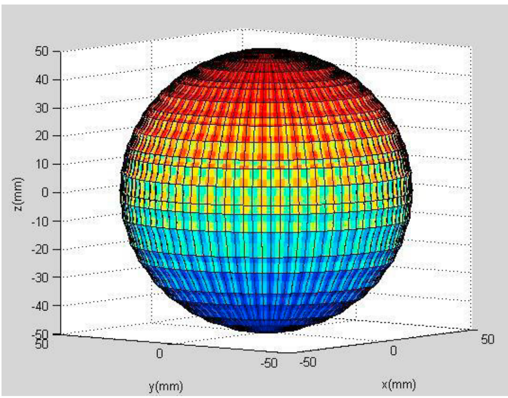

Fig. 4. Workspace of the bone drilling PM.

The reachable workspace can be determined by the centre point of the moving platform [8]. Before determining the workspace, some physical constraints must be considered. Based on the forward kinematics results for different link lengths and the maximum tilt of the tool, the workspace is plotted using MATLAB and it is shown in Figure 4.

6. Dynamics

Dynamics is determining the desired position or desired trajectory of the end effector with the consideration of external forces and moments. Dynamics is more difficult than the kinematics in case of PMs. Dynamics is also of two types forward and inverse dynamics. Forward dynamics is directly obtaining the trajectory or desired position with the known torques of the arms. Inverse dynamics is obtaining the torques and forces of the arms with the known position values. Forward dynamics approach is found to be difficult similar to forward kinematics thereby inverse dynamic modelling is carried out in this study. Generally, the inverse dynamics is solved by three main methods; Newton-Euler formulation [9], Langargian method [10] and virtual work method [11], there are other methods to obtain the inverse dynamic model as discussed by khan [12]. The dynamics of the bone drilling PM is obtained by the Langrangian method.

The langrangian formula for determining the forces is given by $\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta_i}}\right) - \left(\frac{\partial L}{\partial \theta_i}\right)$, where L=K-P and where K is the total kinetic energy and P is the total potential energy. The kinetic energies for individual legs is $K_i = \frac{mL}{4}\dot{\theta}_i^2 + \frac{I}{2}\dot{\theta}_i^2 + \frac{m}{2}\dot{d}_i^2$ and the kinetic energy of the moving platform is$K_4 = \frac{mL}{2}\theta_3^2 + \frac{I}{2}\alpha^2 + \frac{I}{2}\beta^2 + \frac{m}{2}\dot{d}_3^2$. The kinetic energies of the individual legs and the platform is computed. Once kinetic energy is determined the potential energy of the legs are calculated. And now L becomes
$L = \left[\frac{mL}{4} + \frac{I}{2}\right]\dot{\theta}_1^2 + \left[\frac{mL}{4} + \frac{I}{2}\right]\dot{\theta}_2^2 + \left[\frac{mL}{4} + \frac{I}{2}\right]\dot{\theta}_3^2 + \frac{m}{2}(\dot{d}_1^2 + \dot{d}_2^2 + \dot{d}_3^2 + \frac{I_p}{m}\dot{\alpha} + \frac{I_p}{m}\dot{\beta}) + mg\frac{L}{2}(sin\theta_1 + sin\theta_2 + 3sin\theta_3)$.
Using all these equations the required actuator forces/torques are calculated and the derived equations are given in 11 to 13.

$$\tau_1 = \left[\frac{mL}{2} + I\right]\ddot{\theta}_1 + \frac{d}{dt}\left(\frac{\partial P}{\partial \dot{\theta_i}}\right) - \frac{mgL}{2}cos\theta_1\dot{\theta}_1 \quad (11)$$

$$\tau_2 = \left[\frac{mL}{2} + I\right]\ddot{\theta}_2 + \frac{d}{dt}\left(\frac{\partial P}{\partial \dot{\theta}_2}\right) - \frac{mgL}{2}cos\theta_2\dot{\theta}_2 \quad (12)$$

$$\tau_3 = \left[\frac{mL}{2} + I\right]\ddot{\theta}_3 + \frac{d}{dt}\left(\frac{\partial P}{\partial \dot{\theta}_3}\right) - \frac{mgL}{2}cos\theta_3\dot{\theta}_3 \quad (13)$$

Where, $P = \frac{m}{2}(\dot{d}_1^2 + \dot{d}_2^2 + \dot{d}_3^2 + \frac{I_p}{m}\dot{\alpha} + \frac{I_p}{m}\dot{\beta})$ and $\dot{d}_1^2, \dot{d}_2^2$ and $\dot{d}_3^2$ are $obtained$ $from$ $section$ 4

Equations 11 to 13 represent the joint space dynamic model of 3PUU PM. The simulation of the dynamic model is obtained by incorporating the dynamic equations in MATLAB and the necessary displacement, velocity, acceleration and joint forces of the legs are obtained.

7. Results and Discussion

Kinematic analysis was carried out for the proposed 3PUU PM. The PM actuator positions are varied with α and β for different initial angles and the results are shown in Figure 5. From the results, it is observed that the actuator inputs keep decreasing for the same input range of α. As the prismatic sliding is constrained from -50mm to +50mm the range of α is found to be -40° to 60°. Further, the kinematic analysis also shows that as the initial angles increases, the actuator positions decreases to obtain the same angle of tilt with respect to x axis. Similarly, for the prismatic sliding constraints -50mm to +50mm, the range of β is found to be -75° to 120°. It is found that as the initial angles increases the actuator positions decreased slightly for leg 2 and leg 3 whereas leg 1 remains the same to obtain the same angle of tilt with respect to y axis.

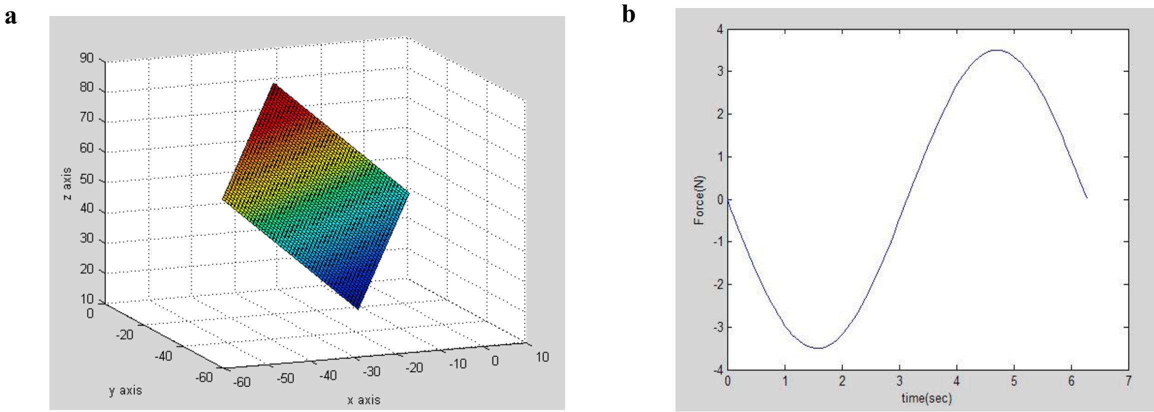

Fig. 5. (a) For β=0° and θ= 70°; (b) For α=0° and θ= 70°.

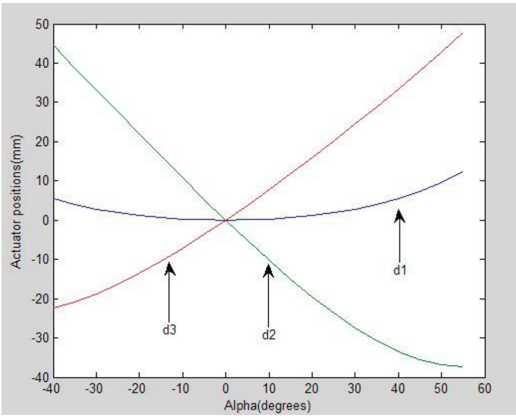

Fig. 6. Workspace of the PM.

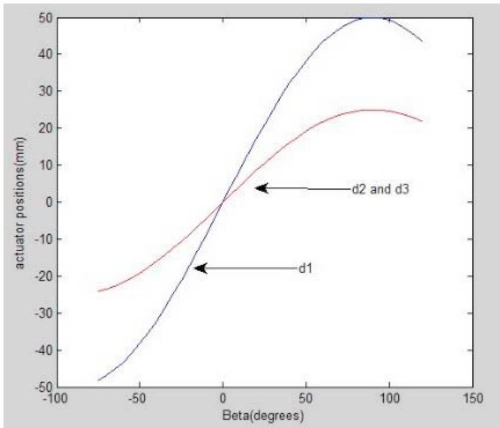

Fig. 7. Joint Force for leg 1.

Figure 6 shows the workspace generated by the proposed bone drilling PM. The moving platform is oriented to the maximum angles of tilt in x and y direction. The maximum angle of the moving platform is found to rotate 45° about x axis and 52° about y axis. Figure 7 shows the joint force for the leg 1 based on the dynamic simulation. Similarly the joint force, acceleration, velocity and displacement are found using Sim-Mechanics tool.

8. Conclusions

A 3PUU parallel manipulator for bone drilling application is designed and analyzed. Based on the study, the following points are concluded,

- The inverse and forward kinematics of the proposed PM have been derived in a closed loop form and discussed. From the forward kinematic study, the maximum and minimum moving platform tilt is obtained for different initial angles and different actuator displacements. It is observed that as the link length increases the minimum and maximum angle of tilt about x axis decreases whereas the minimum and maximum angle of tilt about y axis increases. As the initial angle increases, the maximum angle of tilt about x axis decreases and increases about y axis. For the increase in link length from 50mm to 100mm, it is found that there is a decrease in the maximum moving platform tilt in x axis by 9.7%. Similarly, for the increase in link length from 50mm to 100mm, there is a decrease in the maximum moving platform tilt in y axis by 99.7%.

- Based on the inverse kinematics and the velocity analysis the occurrence of singularities are identified. The tool kinematics as well as workspace is obtained using MATLAB.
- The dynamics of the PM is derived using Lagrangian method and the joint force, acceleration, velocity and displacement of the PM are studied.
- The studies presented here provide a PM design for bone drilling medical application. Further work will be focused on developing a control algorithm based on the dynamic model.

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