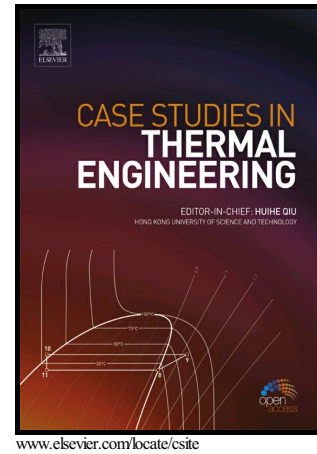


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# Magneto-Marangoni nano-boundary layer flow of water and ethylene glycol based $\gamma$ $\text{Al}_2\text{O}_3$ nanofluids with non-linear thermal radiation effects

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## Abstract

For the first time, a numerical investigation is performed to study the influences of magnetic field on Marangoni boundary layer flow of water and ethylene glycol based  $\gamma$   $\text{Al}_2\text{O}_3$  nanofluids over a flat surface in the presence of non-linear thermal radiation. Experimental based thermo-physical properties and an effective Prandtl number model for  $\gamma$   $\text{Al}_2\text{O}_3$  nanofluids are considered to analyse the Marangoni convection. To study the magnetic field effects, the electric conductivities of both nanoparticles and base fluids are taken into account. Numerical solutions of resulted equations are obtained using fourth order Runge-Kutta method with shooting technique. The combined effect of magnetic parameter with other involved parameters is discussed on velocity and temperature distributions and the local Nusselt number via graphical illustrations.

**Keywords:** Effective Prandtl number; Magnetic field; Marangoni boundary layer;  $\gamma$ - $\text{Al}_2\text{O}_3$  nanoparticles; non-linear radiation;

**Nomenclature**

$B_0$	magnetic field strength (Tesla)	$\sigma_0$	the surface tension at the interface (N/m)
$k^*$	mean absorption coefficient ( $m^{-1}$ )	$\overline{\sigma}$	rate of change of surface tension with temperature
$M_n$	magnetic parameter (-)	$\sigma_{nf}$	the electric conductivity of the nanofluid ( $S m^{-1}$ )
$k_{nf}$	thermal conductivity of the nanofluid (W/m.K)	$\sigma_s$	the electric conductivity of the nanoparticles ( $S m^{-1}$ )
$k_f$	thermal conductivity of the base fluid (W/m.K)	$\sigma_f$	the electric conductivity of the base fluid ( $S m^{-1}$ )
$Pr_{nf}$	Prandtl number of nanofluid (-)	$\sigma^*$	Stefan- Boltzman constant (-)
$Pr_f$	Prandtl number of base fluid (-)	$\phi$	solid volume fraction of nanofluid (-)
$q_r$	Radiative heat flux ( $W/m^2$ )	$\mu_{nf}$	effective dynamics viscosity of the nanofluid (N. s/ $m^2$ )
		$\mu_f$	Dynamics viscosity of the base fluid(N. s/ $m^2$ )
$R_d$	radiation parameter (-)	$\rho_{nf}$	the effective density of the nanofluid ( $kg m^{-3}$ )
$Nu_x$	local Nusselt number (-)	$\rho_s$	density of the base fluid ( $kg m^{-3}$ )
$T$	local temperature of the fluid (K)	$\rho_f$	density of the nanoparticles( $kg m^{-3}$ )
$T_w$	temperature at the wall (K)	$\theta$	dimensionless temperature (-)
$T_\infty$	ambient temperature (K)	$\theta_w$	temperature ratio parameter (-)
$u, v$	velocity components in $x$ and $y$ directions ( $m s^{-1}$ )	$\eta$	space variable (-)
$x, y$	coordinates along and perpendicular to the surface (m)		

**1. Introduction**

In recent years, magneto-Marangoni convection has become a popular research topic because of its various applications in many engineering fields such as mechanical, aerospace, chemical engineering, crystal growth processing, thin liquid films, materials science engineering and so on. The magneto-Marangoni convection of nanofluids can be defined as interfacial electrically conducting nanoliquid flow driven by the surface tension gradient under the influence of magnetic field.

Many attentions have been taken to study the heat transfer characteristics of different types of nanofluids and one can get clear and brief introduction and applications about nanofluids in the following publications [1-22]. Recently, both theoretical and experimental

researchers have turned their interest to study the  $\gamma$   $Al_2O_3$  nanofluids because of its applications in cooling processes [23-32]. Very recently, Moghaieb et al. [33] used  $\gamma$   $Al_2O_3$ - $H_2O$  nanofluid as an engine coolant in their study. A comparative study of  $Al_2O_3$  and  $\gamma$   $Al_2O_3$  nanofluids over a stretching sheet has been conducted by Vishnu Ganesh et al. [34]. Rashidi et al. [35] studied the effects of an effective Prandtl number model on the boundary layer flow of  $\gamma$   $Al_2O_3$ - $H_2O$  and  $\gamma$   $Al_2O_3$ - $C_2H_6O_2$  over a vertical stretching sheet. Recently, the Marangoni boundary layer flow of nanofluid has been discussed with various physical effects using well known thermo-physical properties model for base fluid and nanoparticles without any effective Prandtl number [36-40].

No attempt has been made to study the magneto-Marangoni boundary layer flow of water and ethylene glycol based  $\gamma$   $Al_2O_3$  nanofluids with nonlinear thermal radiation effects. Experimental based thermo-physical properties and an effective Prandtl number model for  $\gamma$   $Al_2O_3$  nanofluids are considered to analyse the Marangoni convection. The electric conductivities of both nanoparticles and base fluids are taking into account to study the magnetic field effects.

## 2. Formulation of the Problem

Consider the steady two-dimensional incompressible Marangoni boundary layer flow over a flat surface in a water/ethylene glycol based nanofluid containing  $\gamma$   $Al_2O_3$  nanoparticles in the presence of non-linear thermal radiation. It is assumed that the flow is laminar and the base fluid and the nanoparticles are in thermal equilibrium. A constant magnetic field strength of  $B_0$  is applied in the transverse direction and also the external electric field is assumed to be zero and the magnetic Reynolds number is assumed to be small (The induced magnetic field is negligible compared to applied magnetic field). The thermo-physical properties are considered as given in Table.1. We consider the a Cartesian coordinate system  $(x,y)$ , where  $x$  and  $y$  are the coordinates measured along the stretching

sheet and normal to it. The flow takes place at  $y \geq 0$ . In addition,  $T_w$  and  $T_\infty$  are the temperature of the surface and ambient fluid, respectively.

Furthermore, it is assumed that the surface tension  $\sigma$  is vary linearly with temperature as [37]:

$$\sigma = \sigma_0 \left[ 1 - \bar{\sigma} (T - T_\infty) \right] \quad (1)$$

where  $\sigma_0$  is the surface tension at the interface and  $\bar{\sigma}$  is the rate of change of surface tension with temperature.

Taking the above assumptions into consideration, the steady boundary layer equations governing the convective flow and heat transfer for a nanofluid in the presence of magnetic field and thermal radiation can be written as [6, 34, 35]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{nf} B_0^2 u}{\rho_{nf}}, \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{nf}} \left( \frac{\partial q_r}{\partial y} \right). \quad (4)$$

The corresponding boundary conditions are

$$\begin{aligned} v = 0, \quad T = T_\infty + bx^2, \quad \mu_{nf} \left( \frac{\partial u}{\partial y} \right) &= \frac{\partial \sigma}{\partial T} \frac{\partial T}{\partial x} & \text{at} \quad y = 0, \\ u \rightarrow 0, \quad T \rightarrow T_\infty & & \text{as} \quad y \rightarrow \infty, \end{aligned} \quad (5)$$

where  $u$  and  $v$  are the velocity components along the axis  $x$  and  $y$ , respectively,  $T$  is the temperature of the nanofluid,  $T_\infty$  is the temperature of the nanofluid far away from the wall,  $\sigma_{nf}$  is the electric conductivity of the nanofluid and  $q_r$  is the radiative heat flux.

Using the Rosseland approximation for non-linear thermal radiation, the radiative heat flux is given by [22]

$$q_r = -\frac{4\sigma^* \partial T^4}{3k^* \partial y} = -\frac{16\sigma^*}{3k^*} T^3 \frac{\partial T}{\partial y} \quad (6)$$

where  $\sigma^*$  and  $k^*$  are the Stefan- Boltzman constant and the mean absorption coefficient, respectively. Now Eq. (4) can be expressed as

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[ \left( \frac{k_{nf}}{(\rho C_p)_{nf}} + \frac{16\sigma^* T^3}{3(\rho C_p)_{nf} k^*} \right) \frac{\partial T}{\partial y} \right] \quad (7)$$

### 3. Thermo-physical Properties of $\gamma$ $Al_2O_3$ - $H_2O$ and $\gamma$ $Al_2O_3$ - $C_2H_6O_2$ Nanofluids

The effective dynamic density ( $\rho_{nf}$ ), the heat capacitance ( $(\rho C_p)_{nf}$ ) and the electric conductivity ( $\sigma_{nf}$ ) of the nanofluid are defined as

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s,$$

$$\frac{\sigma_{nf}}{\sigma_f} = \left[ 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} \right] \quad (8)$$

where  $\phi$  is the solid volume fraction of nanofluid.

The dynamic viscosity of  $\gamma$   $Al_2O_3$  nanofluids is given by [23-25]

$$\frac{\mu_{nf}}{\mu_f} = 123\phi^2 + 7.3\phi + 1, \quad (\text{for } \gamma \text{ Al}_2\text{O}_3\text{-H}_2\text{O}), \quad (9)$$

$$\frac{\mu_{nf}}{\mu_f} = 306\phi^2 - 0.19\phi + 1, \quad (\text{for } \gamma \text{ Al}_2\text{O}_3\text{-C}_2\text{H}_6\text{O}_2). \quad (10)$$

The effective thermal conductivity of  $\gamma \text{ Al}_2\text{O}_3$  nanofluids is given by [23-25]

$$\frac{k_{nf}}{k_f} = 4.97\phi^2 + 2.72\phi + 1, \quad (\text{for } \gamma \text{ Al}_2\text{O}_3\text{-H}_2\text{O}), \quad (11)$$

$$\frac{k_{nf}}{k_f} = 28.905\phi^2 + 2.8273\phi + 1, \quad (\text{for } \gamma \text{ Al}_2\text{O}_3\text{-C}_2\text{H}_6\text{O}_2). \quad (12)$$

The effective Prandtl number of  $\gamma \text{ Al}_2\text{O}_3$  nanofluids is given by [26]

$$\frac{\text{Pr}_{nf}}{\text{Pr}_f} = 82.1\phi^2 + 3.9\phi + 1, \quad (\text{for } \gamma \text{ Al}_2\text{O}_3\text{-H}_2\text{O}), \quad (13)$$

$$\frac{\text{Pr}_{nf}}{\text{Pr}_f} = 254.3\phi^2 - 3\phi + 1, \quad (\text{for } \gamma \text{ Al}_2\text{O}_3\text{-C}_2\text{H}_6\text{O}_2). \quad (14)$$

Equation (8) is the general relationship used to calculate the density, specific heat and electric conductivity for nanofluids. Eqs. (9) and (10) are the dynamic viscosity of nanofluids that have been obtained by performing a least-square curve fitting of some scarce experimental data available for the mixtures considered [41-43]. It has been found that these formulas considerably underestimate the viscosity of the  $\gamma \text{ Al}_2\text{O}_3$  nanofluids under consideration with respect to the measured experimental data as shown by Maiga et al. [23]. Eqs. (11) and (12) are obtained from the well-known model proposed by Hamilton and Crosser [44], regarding the thermal conductivity of the nanofluids, the same situation does

exist for  $\gamma$   $Al_2O_3$  nanofluids [23]. Eqs. (13) and (14) are the effective Prandtl number of  $\gamma$   $Al_2O_3$  nanofluids which are obtained by a curve fitting using regression laws [26].

#### 4. Similarity Transformations and Non-dimensionalization

By using the following non- dimensional variables [37]

$$\psi(\eta) = \xi_2 x f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \xi_1 y \quad (15)$$

$$\text{where } \xi_1 = \left( \frac{\sigma_0 \bar{\sigma} a \rho_f}{\mu_f^2} \right)^{1/3}, \quad \xi_2 = \left( \frac{\sigma_0 \bar{\sigma} a \mu_f}{\rho_f^2} \right)^{1/3},$$

the governing boundary layer equations (3) and (7) are transformed to non-dimensional ordinary differential equations as follow:

##### 4.1. Momentum equation

$$f'''' + \frac{1}{(123\phi^2 + 7.3\phi + 1)} \left[ \left( 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right) (f f''' - f'^2) - \left[ 1 + \frac{3 \left( \frac{\sigma_s}{\sigma_f} - 1 \right) \phi}{\left( \frac{\sigma_s}{\sigma_f} + 2 \right) - \left( \frac{\sigma_s}{\sigma_f} - 1 \right) \phi} \right] Mn f' \right] = 0$$

(for  $\gamma$   $Al_2O_3$ - $H_2O$ ),

$$f'''' + \frac{1}{(306\phi^2 - 0.19\phi + 1)} \left[ \left( 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right) (f f''' - f'^2) - \left[ 1 + \frac{3 \left( \frac{\sigma_s}{\sigma_f} - 1 \right) \phi}{\left( \frac{\sigma_s}{\sigma_f} + 2 \right) - \left( \frac{\sigma_s}{\sigma_f} - 1 \right) \phi} \right] Mn f' \right] = 0$$

(for  $\gamma$   $Al_2O_3$ -  $C_2H_6O_2$ ).

(16)



## 4.2. Energy Equation

$$\theta'' \left[ 1 + R_d A (\theta (\theta_w - 1) + 1)^3 \right] + R_d A \left[ 3 \theta'^2 (\theta_w - 1) (\theta (\theta_w - 1) + 1)^2 \right] + B (f \theta' - 2 \theta f') = 0, \quad (17)$$

where

$$A = (4.97 \phi^2 + 2.72 \phi + 1)^{-1} \quad \text{and} \quad B = \frac{\text{Pr}_f \left( 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right) (82.1 \phi^2 + 3.9 \phi + 1)}{123 \phi^2 + 7.3 \phi + 1}$$

(for  $\gamma \text{Al}_2\text{O}_3\text{-H}_2\text{O}$ ),

$$A = (28.905 \phi^2 + 2.8273 \phi + 1)^{-1} \quad \text{and} \quad B = \frac{\text{Pr}_f \left( 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right) (254.3 \phi^2 - 3 \phi + 1)}{306 \phi^2 - 0.19 \phi + 1}$$

(for  $\gamma \text{Al}_2\text{O}_3\text{-C}_2\text{H}_6\text{O}_2$ ),

The corresponding boundary conditions are

$$\begin{aligned} f(0) = 0, \quad f'''(0) = \frac{-2}{C}, \quad f'(\infty) = 0 \\ \theta(0) = 1 \quad \text{and} \quad \theta(\infty) = 0. \end{aligned} \quad (18)$$

where

$$C = 123 \phi^2 + 7.3 \phi + 1 \quad (\text{for } \gamma \text{Al}_2\text{O}_3\text{-H}_2\text{O}),$$

$$C = 306 \phi^2 - 0.19 \phi + 1 \quad (\text{for } \gamma \text{Al}_2\text{O}_3\text{-C}_2\text{H}_6\text{O}_2), \quad Mn = \frac{\sigma_f B_0^2}{\rho_f \xi_1 \xi_2} \text{ is the magnetic}$$

parameter,  $R_d = \frac{16 \sigma^* T_\infty^3}{3k^* k_f}$  is the radiation parameter and  $\theta_w = \frac{T_w}{T_\infty}$  is the temperature ratio

parameter.

The surface velocity is defined as

$$u_w = \left[ \frac{(\sigma_0 \bar{\sigma} a)^2}{\rho_f \mu_f} \right]^{1/3} x f'(0).$$

The local Nusselt number  $Nu_x$  can be defined as

$$Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)}, \quad (19)$$

where  $q_w = -k_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0} + (q_r)_w$  is the local surface heat flux.

Using (15), we obtain the following Nusselt number

$$Nu_x = D \theta'(0) \xi_1 x \quad (20)$$

Where

$$D = -(4.97 \phi^2 + 2.72 \phi + 1) \left[ 1 + \frac{R_d (\theta_w)^3}{(4.97 \phi^2 + 2.72 \phi + 1)} \right] \text{ (for } \gamma Al_2O_3-H_2O \text{) and}$$

$$D = -(28.905 \phi^2 + 2.8273 \phi + 1) \left[ 1 + \frac{R_d (\theta_w)^3}{(28.905 \phi^2 + 2.8273 \phi + 1)} \right] \text{ (for } \gamma Al_2O_3-C_2H_6O_2 \text{),}$$

## 5. Numerical Technique

The governing non- dimensionless ODEs ((16) and (17) subject to the BC's (18) have been solved numerically by fourth order RK method with shooting technique. The governing equations have been reduced into a set of first order ODEs.

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \\ \dot{y}_5 \end{bmatrix} = \begin{bmatrix} y_2 \\ y_3 \\ -C^{-1} \left[ \left( 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right) (y_1 y_3 - y_2^2) - 1 + \frac{3 \left( \frac{\sigma_s}{\sigma_f} - 1 \right) \phi}{\left( \frac{\sigma_s}{\sigma_f} + 2 \right) - \left( \frac{\sigma_s}{\sigma_f} - 1 \right) \phi} \right] Mn y_2 \\ y_4 \\ \frac{-R_d A \left[ 3 y_5^2 (\theta_w - 1) (y_4 (\theta_w - 1) + 1)^2 \right] - B [y_1 y_5 - 2 y_4 y_2]}{1 + R_d A [y_4 (\theta_w - 1) + 1]^3} \end{bmatrix} \quad (21)$$

and the corresponding initial conditions are:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 0 \\ g_1 \\ -2/C \\ 1 \\ g_2 \end{bmatrix} \quad (22)$$

First order Equations in (21) with initial conditions (22) are solved using fourth order Runge–Kutta integration technique. Suitable guessing values of the unknown initial conditions  $g_1$  and  $g_2$  are approximated by shooting method until the boundary conditions at  $f'(\infty)=0$  and  $\theta(\infty)=0$  are satisfied with the accuracy of  $10^{-6}$ .

## 6. Results and Discussion

The numerical results of non-dimensional velocity profile, non-dimensional temperature profile and reduced Nusselt number are obtained by above explained numerical method. The results are plotted to show the combined effect of magnetic parameter and other pertinent physical parameters on the flow of water and ethylene glycol based  $\gamma$   $Al_2O_3$  nanofluids. The velocity and temperature profiles of nanofluid are discussed for two cases, namely in the presence of magnetic field ( $Mn = 0$ ) and in the absence of magnetic field ( $Mn=$

Positive number). The Prandtl number of water and ethylene glycol is fixed as 6.96 and 204 respectively.

The magnetic parameter  $Mn$  is a key parameter to understand the flow behaviour of the nanofluid under the influence of magnetic field. The combined effects of magnetic parameter and the nanoparticle volume fraction of  $\gamma\text{Al}_2\text{O}_3$  nanofluids on the Marangoni velocity profile are shown in Figs 1(a) and 1(b). It is noted that, an increment in the nanoparticle volume fraction accelerates the velocity profile far away from the wall of the surface and decelerates the velocity profile near the wall of the surface. It is also noted that the velocity profile of both water and ethylene glycol based  $\gamma\text{Al}_2\text{O}_3$  nanofluids decreases in the presence of magnetic field and increases in the absence of magnetic field. This is due to the fact that the presence of magnetic field leads to produce a resistive type force (Lorentz force) in the flow region, which arrests the motion of the fluid. The combined effect of magnetic and nanoparticle volume fraction parameters decrease the thickness of the nano-momentum boundary layer. On comparing these figures, the nano-momentum boundary layer thickness of ethylene glycol based nanofluid is higher than the thickness of the nano-momentum boundary layer of water based nanofluid.

The combined effects of magnetic parameter and the nanoparticle volume fraction of  $\gamma\text{Al}_2\text{O}_3$  nanofluids on the Marangoni temperature profile are elucidated in Figs. 2(a) and 2(b). It is noted that an increment in the nanoparticle volume fraction diminishes the temperature profile. It is also observed that the temperature profile of both water and ethylene glycol based  $\gamma\text{Al}_2\text{O}_3$  nanofluids enhances in the presence of magnetic field and diminishes in the absence of magnetic field. The combined effects of magnetic and nanoparticle volume fraction parameters increase the thickness of the nano-thermal boundary layer. On comparing these figures, the nano-thermal boundary layer thickness of water based nanofluid is higher than the thickness of the nano-thermal boundary layer of the ethylene glycol based nanofluid.

Fig. 3(a) and 3(b) demonstrate the combined effect of magnetic and the radiation parameters in the Marangoni temperature profile of water and ethylene glycol based  $\gamma$   $\text{Al}_2\text{O}_3$  nanofluids, respectively. It is clear that the temperature profile of the nanofluid increases as radiation parameter increases. The absorption coefficient decreases whenever radiation parameter increases. This fact leads to increase the temperature profile of both the nanofluids. The combined effects of magnetic and the radiation parameters lead to increase the thickness of the nano-thermal boundary layer.

The variation of local Nusselt number with physical parameters displayed in the Figs. 4 and 5. The magnetic parameter and the reduced Nusselt number are chosen as  $x$  and  $y$  axes respectively. From these figures, It is seen that the magnitude of local Nusselt number decreases with magnetic parameter and increase with nanoparticle volume fraction, radiation and temperature ratio parameters. On comparing the figures 4(a), 4(b), 5(a) and 5(b), it is clear that the magnitude reduced Nusselt number of ethylene glycol based  $\gamma$   $\text{Al}_2\text{O}_3$  nanofluid is higher than that of water based  $\gamma$   $\text{Al}_2\text{O}_3$  nanofluid.

## 7. Conclusion

Marangoni boundary layer flow of water and ethylene glycol based  $\gamma$   $\text{Al}_2\text{O}_3$  nanofluids under the influence of constant magnetic field and non-linear thermal radiation is investigated. The governing non- dimensional equations are solved numerically. The following significant results are noticed.

- The nanoparticle volume fraction parameter decelerates velocity profile near the wall of the surface and accelerates the velocity profile far away from the wall of the surface. The temperature profile diminishes with nanoparticle volume fraction.

- The combined effect of magnetic and nanoparticle volume fraction parameters decrease the thickness of the nano-momentum boundary layer and increase the thickness of the nano-thermal boundary layer. The combined effects of magnetic and the radiation parameters lead to increase the thickness of the nano-thermal boundary layer.
- The nano-momentum boundary layer thickness of ethylene glycol-based  $\gamma$   $\text{Al}_2\text{O}_3$  nanofluid is higher than the thickness of the nano-momentum boundary layer of the water based  $\gamma$   $\text{Al}_2\text{O}_3$  nanofluid and an opposite phenomenon has been observed in nano-thermal boundary layer thickness.
- The local Nusselt number decreases with magnetic parameter and increases with nanoparticle volume fraction, radiation and temperature ratio parameters.

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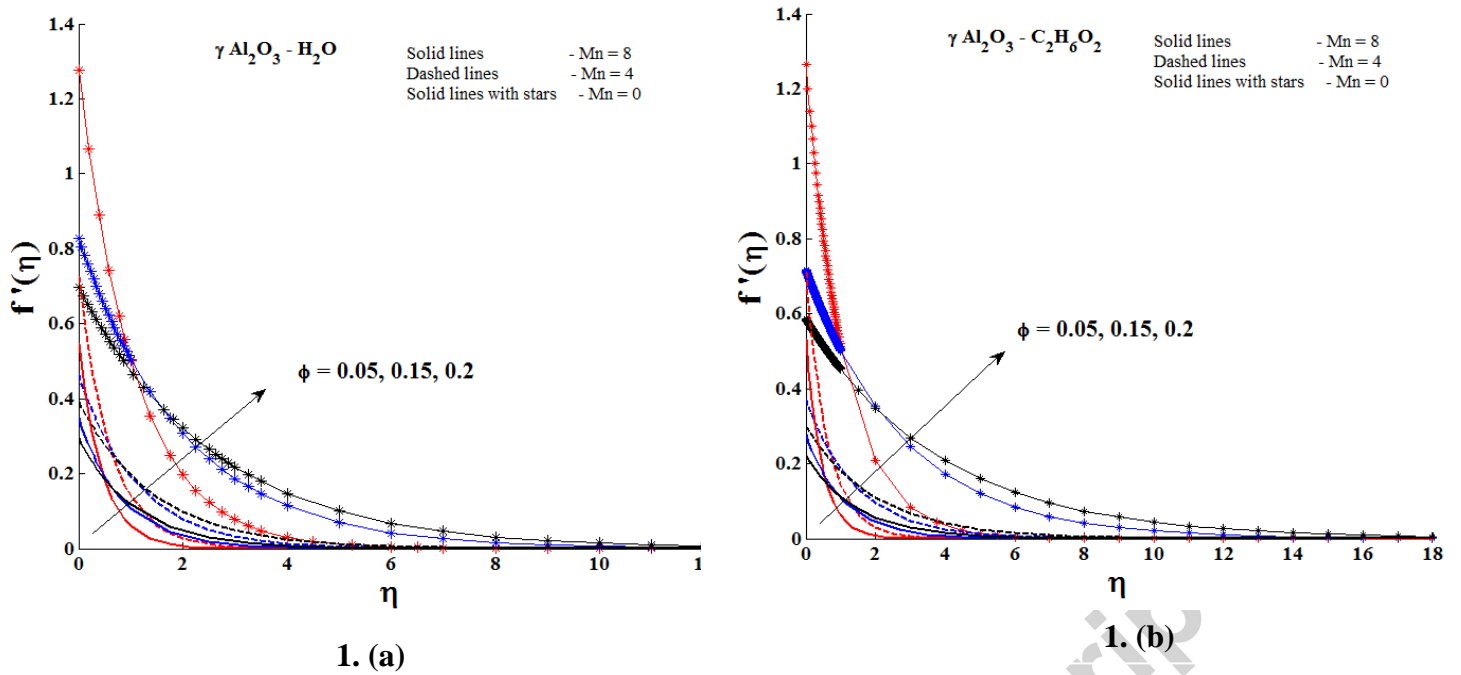
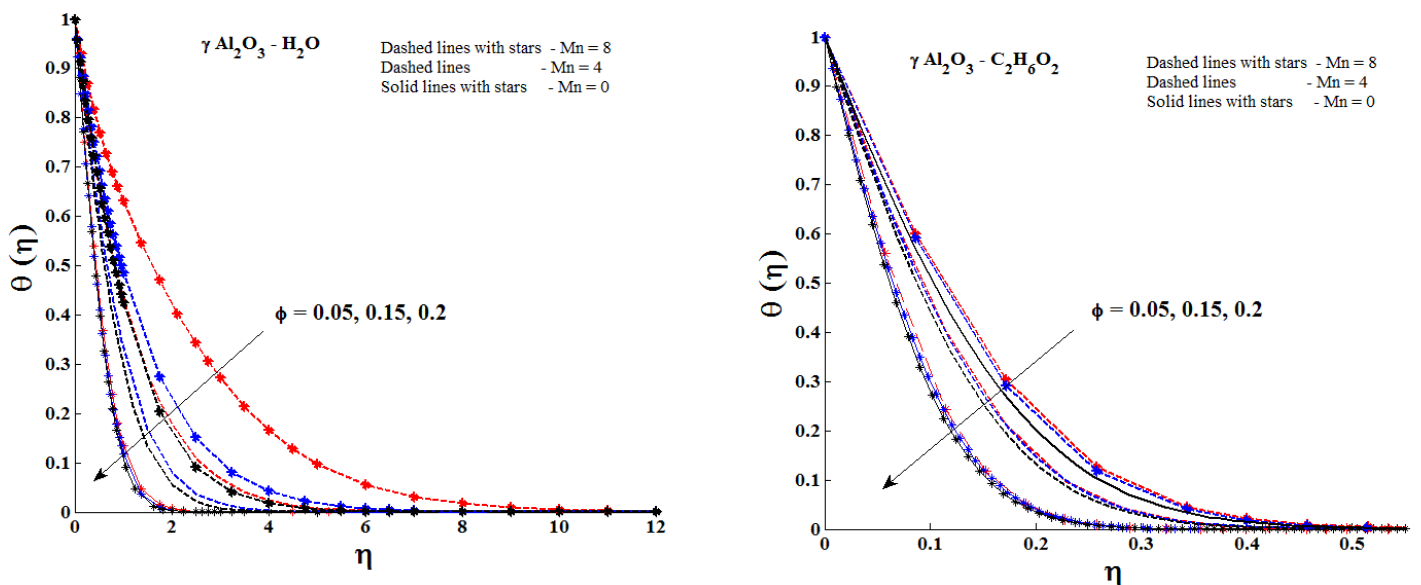


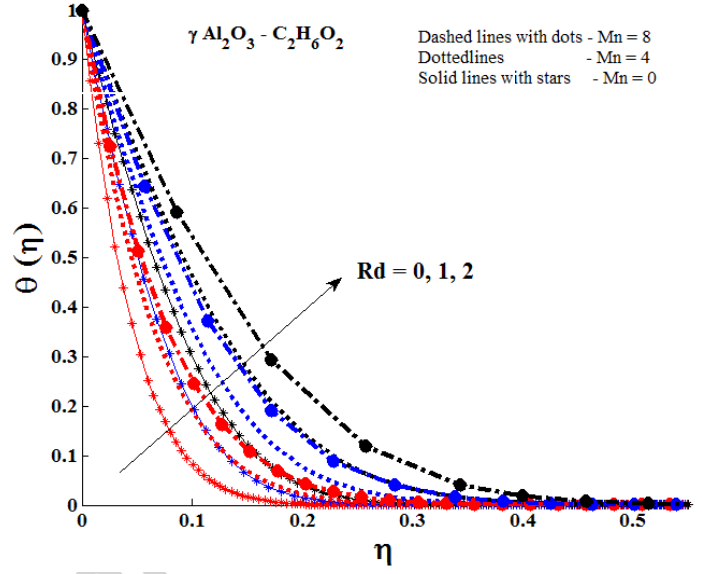
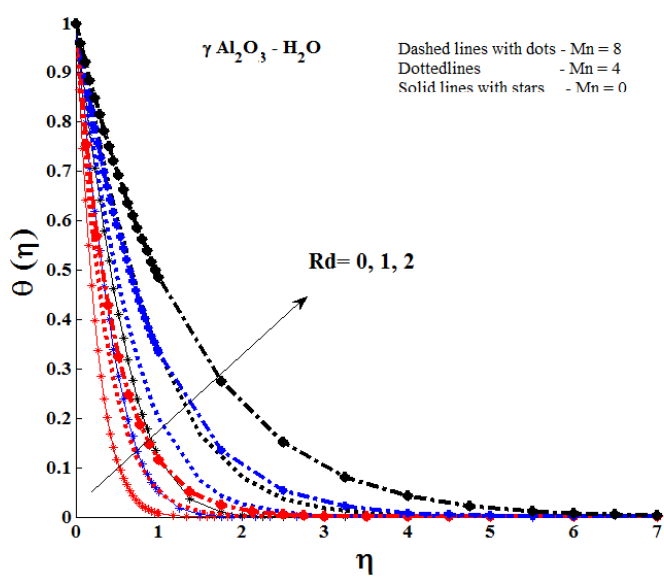
Fig. 1. Combined effect of nanoparticle volume fraction ( $\phi$ ) and magnetic parameter (Mn) on velocity profile with  $\text{Rd} = 2$  and  $\theta_w = 1.5$  (a)  $\gamma\text{Al}_2\text{O}_3\text{-H}_2\text{O}$  with  $\text{Pr} = 6.96$  (b)  $\gamma\text{Al}_2\text{O}_3\text{-C}_2\text{H}_6\text{O}_2$  with  $\text{Pr} = 204$ .



2. (a)

2. (b)

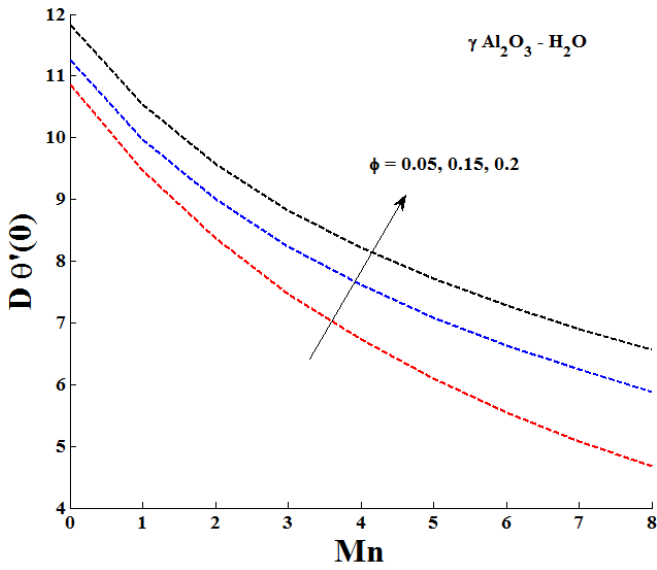
Fig. 2. Combined effect of nanoparticle volume fraction ( $\phi$ ) and magnetic parameter (Mn) on temperature profile with  $Rd=2$  and  $\theta_w=1.5$  (a)  $\gamma \text{ Al}_2\text{O}_3\text{-H}_2\text{O}$  with  $\text{Pr}=6.96$  (b)  $\gamma \text{ Al}_2\text{O}_3\text{-C}_2\text{H}_6\text{O}_2$  with  $\text{Pr}=204$ .



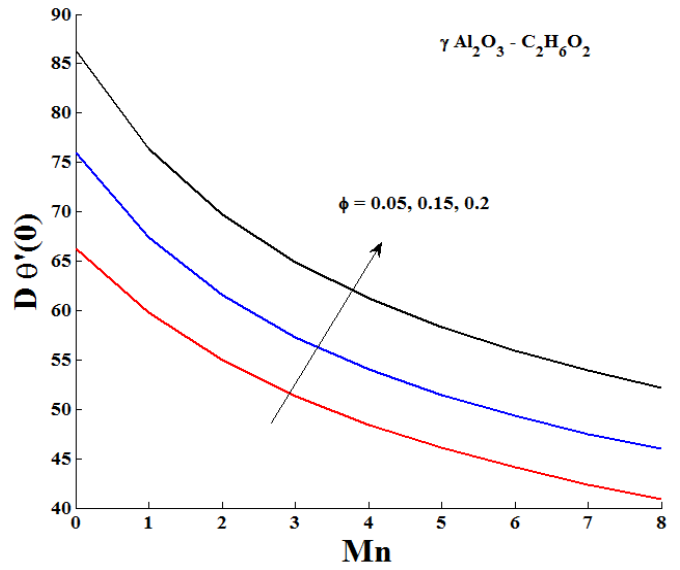
3. (a)

3. (b)

Fig. 3. Combined effect of radiation parameter (Rd) and magnetic parameter (Mn) on temperature profile with  $\phi=0.15$  and  $\theta_w=1.5$  (a)  $\gamma \text{ Al}_2\text{O}_3\text{-H}_2\text{O}$  with  $\text{Pr}=6.96$  (b)  $\gamma \text{ Al}_2\text{O}_3\text{-C}_2\text{H}_6\text{O}_2$  with  $\text{Pr}=204$ .

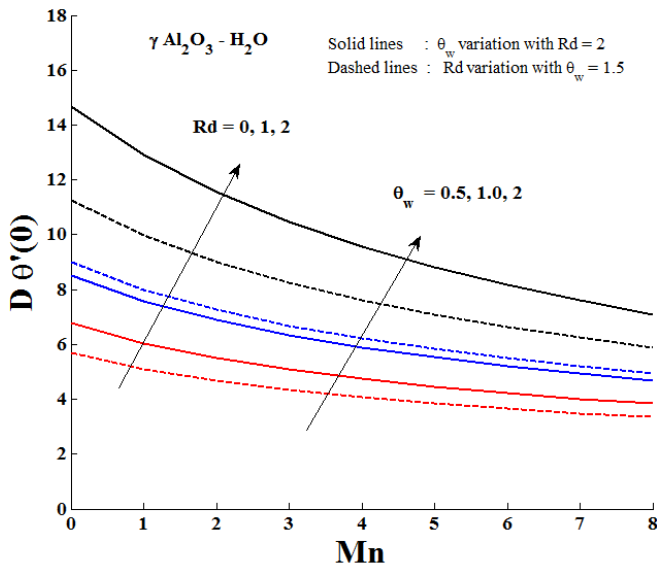


4. (a)

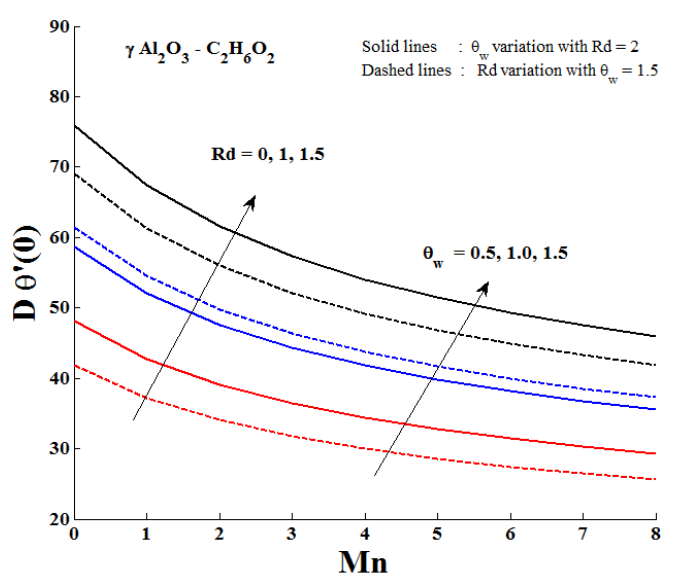


4. (b)

Fig. 4. Combined effect of nanoparticle volume fraction ( $\phi$ ) and magnetic parameter ( $Mn$ ) on reduced Nusselt number with  $Rd=2$  and  $\theta_w=1.5$  (a)  $\gamma Al_2O_3 - H_2O$  with  $Pr=6.96$  (b)  $\gamma Al_2O_3 - C_2H_6O_2$  with  $Pr=204$ .



5. (a)



5. (b)

Fig. 5. Combined effect of radiation (Rd), temperature ratio ( $\theta_w$ ) and magnetic parameters (Mn) on reduced Nusselt number with  $\phi=0.15$  (a)  $\gamma \text{ Al}_2\text{O}_3 - \text{H}_2\text{O}$  with Pr = 6.96 (b)  $\gamma \text{ Al}_2\text{O}_3 - \text{C}_2\text{H}_6\text{O}_2$  with Pr=204 .

Table 1. Thermo-physical properties of water, ethylene glycol and alumina [21]

	$\rho$ (kg/m <sup>3</sup> )	$C_p$ (J/kg K)	$k$ (W/m K)	$\sigma$ ( $\Omega \cdot \text{m}$ ) <sup>-1</sup>	Pr
Pure water (H <sub>2</sub> O)	998.3	4182	0.60	0.05	6.96
Ethylene glycol (C <sub>2</sub> H <sub>6</sub> O <sub>2</sub> )	1116.6	2382	0.249	$1.07 \times 10^{-7}$	204
Alumina (Al <sub>2</sub> O <sub>3</sub> )	3970	765	40	$10^{-12}$	-

### Highlights

- Magnetic field effect on  $\gamma \text{ Al}_2\text{O}_3$  nanoparticles investigated first time.
- Maragoni boundary layer flow along with non –linear thermal radiation considered.
- Water and Ethylene glycol are taken as base fluids.
- The experimental based thermo physical properties are used.
- Results discussed for  $\gamma \text{ Al}_2\text{O}_3$  nanoparticles with an effective Prandtl number model.