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Maximum Sensitivity Based New PID Controller Tuning for Integrating Systems Using Polynomial Method

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Abstract:
This paper presents a new control methodology for a class of integrating systems. A PID controller augmented with a first order lead/lag filter is proposed for improved response. The polynomial approach is employed to derive the controller parameters. The novelty of the proposed method lies in the selection of pole locations. Multiple pole locations are considered where one of the poles is placed for cancelling the zero introduced by controller. The selection of tuning parameter is based on maximum sensitivity (MS). Set point weighting is employed to reduce the overshoot and settling time in the servo response. Various bench marking examples are adopted to evaluate the proposed method in terms of various performance indices. The results are superior to the recently proposed works in terms of both set point tracking and disturbance rejection.

Keywords: integrating systems, time delay, PID controller, maximum sensitivity, total variation

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1 Introduction

PID controller is the most widely used industrial controller strategy because of its well understood structure and working principle. Effective tracking of set point changes and rejection of disturbance of a control loop depends on how well the controller is tuned. A poorly tuned controller may result an oscillatory or unstable response. Following the well-known empirical tuning rules proposed by Ziegler and Nichols [1], many other tuning methods are reported to enhance the performance of a control loop. The difficulty in design increases when the process is associated with time delay. It is possible to compensate the delay effect using smith predictor and improve servo performance. But delay compensation for regulatory response is not possible unless otherwise the disturbances are measured. Disturbance rejection is more important in many chemical processes to ensure the product quality.

More attention is to be paid when the control loop is designed for unstable and integrating processes when compared to inherently stable processes. In the present paper, the authors propose a PID controller with lead/lag filter for various types of integrating processes. Various researchers proposed different control schemes for integrating processes. Some of the noteworthy control schemes proposed based on various methodologies so far are as follows: Internal model control (IMC) [2–7], equating coefficient [8, 9], direct synthesis [10, 11], optimization [12–19], set point overshoot method [20].

Recently, Ajmeri & Ali [21], have proposed a parallel control structure that decouples servo and regulatory responses under nominal conditions. A PD controller is employed as set point tracking controller and a PID controller is employed as disturbance rejection controller. Tuning rules are proposed based on maximum sensitivity (MS).Though this method reported improved performance over the existing methods, a large over shoot in the servo response is observed. This method is not extended for IFOPTD with zero.

In another recent work, Anil & Padma Sree [22], have reported a simple conventional control structure for a class of integrating systems. The controller structure which is derived for various types of integrating systems is as follows: pure PID controller for IPTD; PID with first order lead/lag filter for DIPTD and IFOPTD; PID controller with second order lead/lag filter for IFOPTD with zero. Set point weighting is employed to reduce overshoot in the servo response. This method also used MS based tuning. Though this method is effective when compared to Ajmeri & Ali [21], set point weighting is not effective for IFOPTD with zero and large settling times are observed in disturbance rejection for DIPTD.

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The objective of the present work is to design a control strategy for a class of integrating systems with improved performance. The proposed method describes the design of a PID controller with first order lead/lag filter for various types of integrating systems. The controller is designed using polynomial method treating the transfer function of process as well as controller as a ratio of two polynomials. Unlike the similar controller proposed by Anil & Padma Sree [22], the proposed method is effective for DIPTD and IFOPTD with zero. The proposed method is simple as it employs only one control loop unlike the method proposed by Ajmeri & Ali [21], where two control loops are employed with different controllers. The efficiency of the proposed method is compared in terms of IAE, total variation (TV) and over shoot (OS) with the recently reported methods.

The paper is organized as follows: Section 2 deals with the design aspects of controller for various class of integrating systems. Stability and robust performance of the controller is described in Section 3. In Section 4, simulation results are presented and compared with the other reported methods, followed by conclusion in Section 5.

2 Controller design

2.1 Theoretical background

Consider a generalized transfer function for the class of integrating systems as shown in eq. (1).

$$G_p(s) = \frac{k(1+sz)}{s(\tau s+c)} e^{-\theta s} \tag{1}$$

If $z = \tau = 0$ and $c = 1$, the transfer function becomes IPTD. If $z = c = 0$ and $\tau = 1$, the transfer function becomes DIPTD . If $z = 0, c = 1$ and τ is non zero numeric value, the transfer function becomes IFOPTD, if τ, c, z are non-zero values, the transfer function becomes IFOPTD with zero.

The proposed method employs conventional feedback control structure as shown in Figure 1.

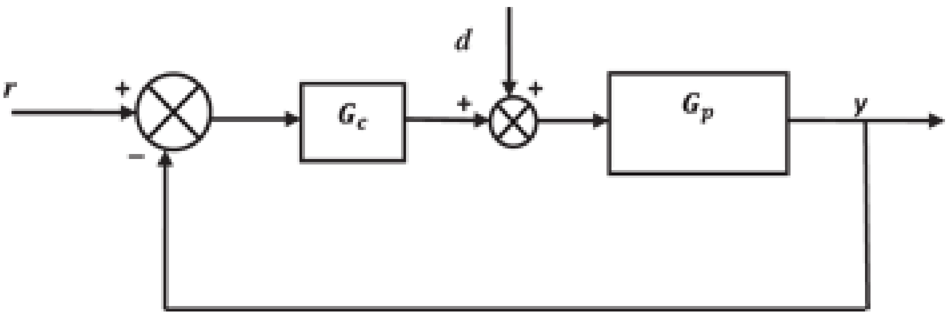

Figure 1: Conventional feedback structure.

From Figure 1, servo and regulatory responses can be derived as shown in eqs. (2) and (3) respectively

$$\frac{y}{r} = \frac{G_c G_p}{1 + G_c G_p} \tag{2}$$

$$\frac{y}{d} = \frac{G_p}{1 + G_c G_p} \tag{3}$$

Where, $G_c$is controller, G_p is process, r is set point, d is disturbance and y is process variable.

The controller is assumed to be a PID controller in series with a lead/lag filter. The controller parameters are derived using polynomial approach in which both the controller and process are represented as a ratio of two polynomials.

The proposed PID controller structure is given in eq. (a)

$$G_c(s) = \frac{q}{p} = \left(k_p + \frac{k_i}{s} + k_d s\right) \frac{s+1}{\beta s + 1} \tag{a}$$

Where,

$$q = \left(k_d s^2 + k_p s + k_i\right)(s+1) \tag{b}$$

$$p = s(\beta s + 1) \tag{c}$$

2.2 *Design* G_c *for IPTD*

With the help of Pade's approximation the process is represented as a ratio of two polynomials *b* and *a* as shown in eq. (d).

$$G_p\,(s) = \frac{b}{a} \tag{d}$$

Where,

$$b = k\left(1 - \frac{s\theta}{2}\right) \tag{e}$$

$$a = s\left(1 + \frac{s\theta}{2}\right) \tag{f}$$

By substituting eq. (4) and eq. (5) in eq. (2) and eq. (3),

$$\frac{y}{r} = \frac{bq}{ap+bq} = \frac{k\left(1-\frac{s\theta}{2}\right)\left(k_d s^2 + k_p s + k_i\right)(s+1)}{s^2\left(1+\frac{s\theta}{2}\right)(\beta s+1) + k\left(1-\frac{s\theta}{2}\right)\left(k_d s^2 + k_p s + k_i\right)(s+1)} \tag{4}$$

$$\frac{y}{d} = \frac{ap}{ap+bq} = \frac{s\left(1+\frac{s\theta}{2}\right)s\,(\beta s+1)}{s^2\left(1+\frac{s\theta}{2}\right)(\beta s+1) + k\left(1-\frac{s\theta}{2}\right)\left(k_d s^2 + k_p s + k_i\right)(s+1)} \tag{5}$$

The characteristic equation (CE), which is the denominator polynomial of both servo and regulatory responses is given in eq. (6).

$$CE = ap + bq = s^2\left(1+\frac{s\theta}{2}\right)(\beta s+1) + k\left(1-\frac{s\theta}{2}\right)(k_d s^2 + k_p s + k_i)(s+1) = 0 \tag{6}$$

In eq. (6), $\left(1-\frac{s\theta}{2}\right)$ is an approximation of $e^{\frac{-s\theta}{2}}$, it is in turn written as

$$(1-\frac{s\theta}{2}) = e^{\frac{-s\theta}{2}} = \frac{e^{\frac{-s\theta}{4}}}{e^{\frac{s\theta}{4}}} = \frac{\left(1-\frac{s\theta}{4}\right)}{\left(1+\frac{s\theta}{4}\right)} \tag{7}$$

By considering $= \theta/4$ and substituting eq. (7) in eq. (6),

$$CE = s^4 + \frac{2\beta+\theta-\frac{kk_d\theta}{2}}{\beta\theta}s^3 + \frac{2kk_d-\frac{kk_p\theta}{2}+2}{\beta\theta}s^2 + \frac{2kk_p-\frac{kk_i\theta}{2}}{\beta\theta}s + \frac{2kk_i}{\beta\theta} = 0 \tag{8}$$

The desired characteristic equation is selected and given in eq. (9)

$$(s+)^3\left(s+\frac{4}{\theta}\right) = 0 \tag{9}$$

From eq. (4), it is clear that the servo response is having a zero at $s = -4/\theta$ which causes overshoot in the servo response. One of the poles of desired CE is placed at $s = -4/\theta$ to mitigate the overshoot in the servo response. The remaining three poles are located at − which is a tuning parameter. However, due to other zeros of the servo response, overshoot still exists. To minimize this overshoot, set point weighting is employed. PID parameters are obtained by comparing eq. (8) and eq. (9). Results are formulated in eq. (g to k).

$$k_p = \frac{12^2\theta\,(\theta+6)}{k\,(^3\theta^3 + 12^2\theta^2 + 48\theta + 16)} \tag{g}$$

$$k_i = \frac{24^3\theta}{k\,(^3\theta^3 + 12^2\theta^2 + 48\theta + 16)} \tag{h}$$

$$k_d = \frac{2\left(^3\theta^3 + 12^2\theta^2 + 12\theta - 8\right)}{k\,(^3\theta^3 + 12^2\theta^2 + 48\theta + 16)} \tag{i}$$

$$\beta = \frac{12\theta}{^3\theta^3 + 12^2\theta^2 + 48\theta + 16} \tag{j}$$

$$\alpha = \frac{\theta}{4} \tag{k}$$

Here, λ is tuned to obtain the desired response. Faster responses can be achieved by selecting the larger values of λ, but robustness can be ensured by smaller values of λ. So the selection of λ is a trade-off between the speed of response and robustness of the system.

2.3 *Design G_c for DIPTD and IFOPTD*

The delay free process is assumed as a ratio of two polynomials as mentioned in eq. (13).

$$G_p(s) = \frac{f}{g} e^{-s\theta} \tag{l}$$

where

$$f = k \tag{m}$$

$$g = s(\tau s + c) \tag{n}$$

Using eq. (2), eq. (3), eq. (4) and eq. (13),

$$\frac{y}{r} = \frac{qf}{pg + qf e^{-s\theta}} e^{-s\theta} = \frac{k\left(k_d s^2 + k_p s + k_i\right)(s+1)}{s^2(\beta s + 1)(\tau s + c) + k\left(k_d s^2 + k_p s + k_i\right)(s+1)e^{-s\theta}} e^{-s\theta} \tag{10}$$

$$\frac{y}{d} = \frac{fp}{pg + qf e^{-s\theta}} e^{-s\theta} = \frac{ks(\beta s + 1)}{s^2(\beta s + 1)(\tau s + c) + k\left(k_d s^2 + k_p s + k_i\right)(s+1)e^{-s\theta}} e^{-s\theta} \tag{11}$$

$$CE = s^2(\beta s + 1)(\tau s + c) + k(k_d s^2 + k_p s + k_i)(s+1)e^{-s\theta} = 0 \tag{12}$$

By considering $= \theta/2$ and using first order Pade's approximation for the delay term, eq. (12) can be modified as

$$CE = s^4 + \frac{\tau + \beta c - \frac{kk_d\theta}{2}}{\beta\theta} s^3 + \frac{kk_d - \frac{kk_p\theta}{2} + c}{\beta\theta} s^2 + \frac{kk_p - \frac{kk_i\theta}{2}}{\beta\theta} s + \frac{kk_i}{\beta\theta} = 0 \tag{13}$$

The desired CE is assumed as mentioned in eq. (14).

$$(s+)^3\left(s + \frac{2}{\theta}\right) = 0 \tag{14}$$

Similar to the design of controller for IPTD system, the poles are accordingly placed to minimize the overshoot in servo response. Comparing eq. (13) and eq. (14), the expressions for PID parameters are derived as

$$k_p = \frac{4^3\tau^2\theta + 2c^3\tau\theta^2 + 12^2\tau^2 + 6c^2\tau\theta}{k(\tau^3\theta^3 + 6\tau^2\theta^2 + 12\tau\theta + 4\tau - 2c\theta)} \tag{o}$$

$$k_i = \frac{4^3\tau^2 + 2c\theta^3\tau}{k(\tau^3\theta^3 + 6\tau^2\theta^2 + 12\tau\theta + 4\tau - 2c\theta)} \tag{p}$$

$$k_d = \frac{2\left(c^2\theta - 3c\tau\theta - 2c\tau + ^3\tau^2\theta^2 + 6^2\tau^2\theta + 6\tau^2\right)}{k(\tau^3\theta^3 + 6\tau^2\theta^2 + 12\tau\theta + 4\tau - 2c\theta)} \tag{q}$$

$$\alpha = \frac{\theta}{2} \tag{r}$$

$$\beta = \frac{\theta(2\tau + c\theta)}{\tau^3\theta^3 + 6\tau^2\theta^2 + 12\tau\theta + 4\tau - 2c\theta} \tag{s}$$

From eq. (o) to eq. (s), it can be understood that every controller parameter except α is a function of λ.

2.4 *Design of G_c for IFOPTD with zero*

Unlike the method followed by Anil & Padma Sree [22], where the zero of the system is omitted in the derivation of the controller parameters, the proposed method does consider the zero in the derivation. Similar to the previous case, the delay free process is assumed as a ratio of two polynomials.

$$G_p(s) = \frac{u}{v} e^{-s\theta} \tag{t}$$

where

$$u = k(1 + sz) \tag{u}$$

$$v = s(\tau s + c) \tag{v}$$

Substituting eq. (4) and eq. (20) in eq. (2) and eq. (3),

$$\frac{y}{r} = \frac{qu}{pv + que^{-s\theta}} e^{-s\theta} = \frac{k(1+sz)\left(k_d s^2 + k_p s + k_i\right)(s+1)}{s^2(\beta s+1)(\tau s+1) + k(1+sz)(k_d s^2 + k_p s + k_i)(s+1)e^{-s\theta}} e^{-s\theta} \tag{15}$$

$$\frac{y}{d} = \frac{pu}{pv + que^{-s\theta}} e^{-s\theta} = \frac{ks(1+sz)(\beta s+1)}{s^2(\beta s+1)(\tau s+1) + k(1+sz)(k_d s^2 + k_p s + k_i)(s+1)e^{-s\theta}} e^{-s\theta} \tag{16}$$

$$CE = s^2(\beta s+1)(\tau s+1) + k(1+sz)(k_d s^2 + k_p s + k_i)(s+1)e^{-s\theta} = 0 \tag{17}$$

Approximating the time delay with Pade's approximation and considering $= \theta/2$,

$$s^4 + \frac{\tau + \beta - k_d\left(\frac{k\theta}{2} - kz\right) - \frac{kk_p\theta z}{2}}{\beta\tau - \frac{kk_d\theta z}{2}} s^3 + \frac{+kk_d - k_p\left(\frac{k\theta}{2} - kz\right) - \frac{kk_i\theta z}{2} + 1}{\beta\tau - \frac{kk_d\theta z}{2}} s^2 + \frac{kk_p - ki\left(\frac{k\theta}{2} - kz\right)}{\beta\tau - \frac{kk_d\theta z}{2}} s + \frac{kk_i}{\beta\tau - \frac{kk_d\theta z}{2}} = 0 \tag{18}$$

The desired CE is assumed to have pole locations as shown in eq. (19).

$$(s+)^3\left(s + \frac{1}{z}\right) = 0 \tag{19}$$

From eq. (15), it can be observed that a zero at $s = -1/z$ is resulting in the servo response. So one of the poles of desired CE is placed at $s = -1/z$ (assuming z is positive) to compensate overshoot and the remaining poles are placed at –λ. The derived PID parameters by comparing eq. (18) and eq. (19) are,

$$k_p = \frac{2(\theta+6)(2\tau+\theta)}{k({}^3\theta^3 + 6^2\theta^2 + 12\theta + 8)} \tag{w}$$

$$k_i = \frac{4^2(2\tau+\theta)}{k({}^3\theta^3 + 6^2\theta^2 + 12\theta + 8)} \tag{x}$$

$$k_d = \frac{2(\tau^3\theta^2 + 6\tau^2\theta + 12\tau - 4)}{k({}^3\theta^3 + 6^2\theta^2 + 12\theta + 8} \tag{y}$$

$$\alpha = \frac{\theta}{2} \tag{z}$$

$$\beta = z \tag{aa}$$

In this case, filter parameters are function of process parameters and completely independent of λ.To achieve good robust and nominal performance, λ should be adjusted to a value which gives good compromise between speed of response and robustness of the system. If z holds a negative value, the desired C.E can be assumed to follow a trajectory as given in eq. (20)

$$(s+)^4 = 0 \tag{20}$$

PID parameters can be derived by comparing eq. (18) and eq. (20).

2.5 Selection of λ

For efficient control of a system, it is always important to derive fine-tuned PID controller. Improperly tuned PID controller results a sluggish output response and induces unnatural variations in the manipulated variable, which in turn increases the fuel cost. The tuning parameter λ is selected as a function of MS, which is a measure of stability margin of the control system. The sensitivity function of the feedback control system is $1/\left(1+G_cG_p\right)$, and its amplitude ratio is given by $\left|1/\left(1+G_cG_p\right)\right|$. A value of MS between 1.2 to 2 implies a satisfactory performance of the control system. However, researchers use MS values greater than 2 in case of unstable and integrating systems. In the proposed method, a value of λ is selected based on MS . The relationship between MS and λ is graphically shown through Figure 2, Figure 3, Figure 4, Figure 5 for various types of integrating systems. Using curve fitting, these graphical relations are formulated and presented in Table 1.

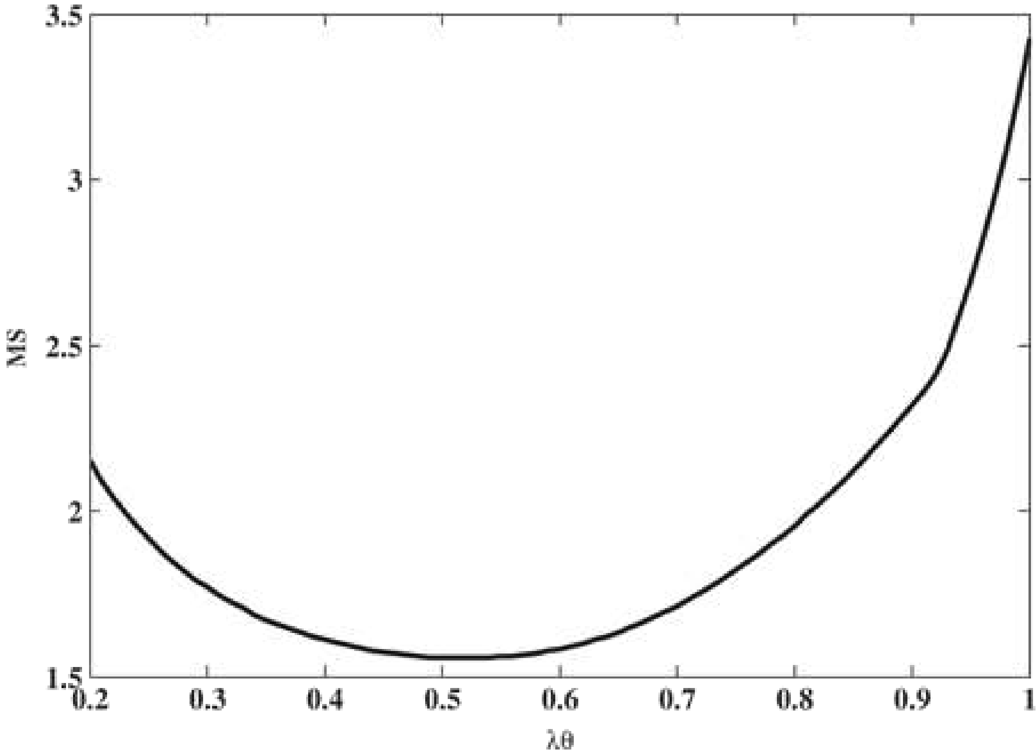

Figure 2: Variation of MS with λ for IPTD.

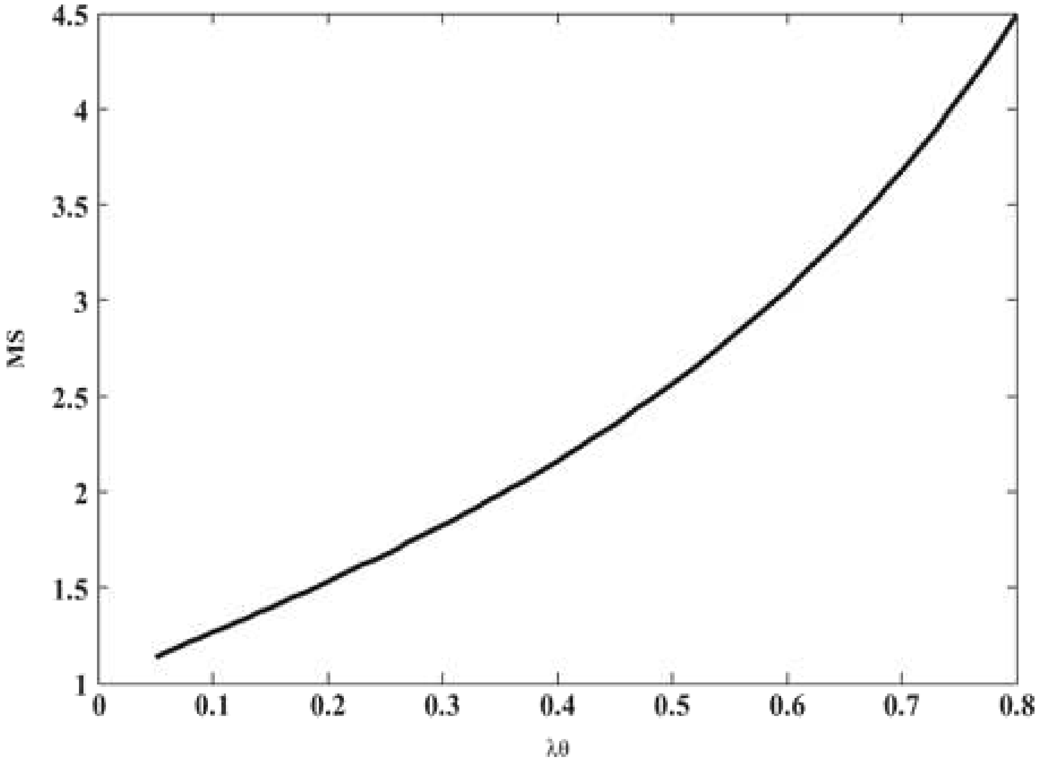

Figure 3: Variation of MS with λ for DIPTD.

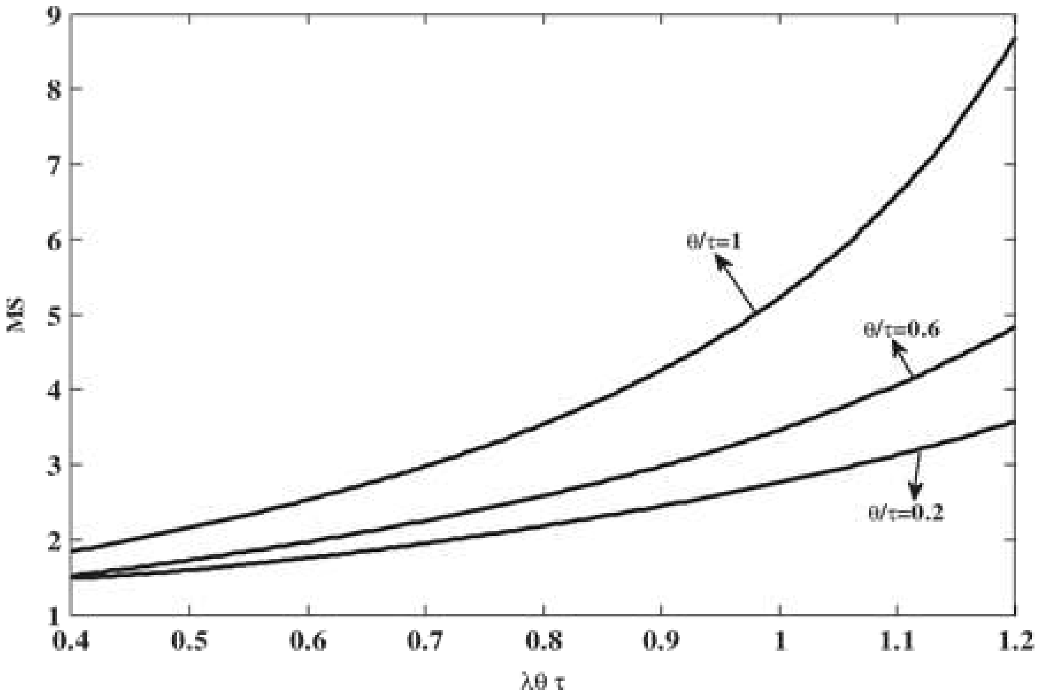

Figure 4: Variation of MS with λ for IFOPTD.

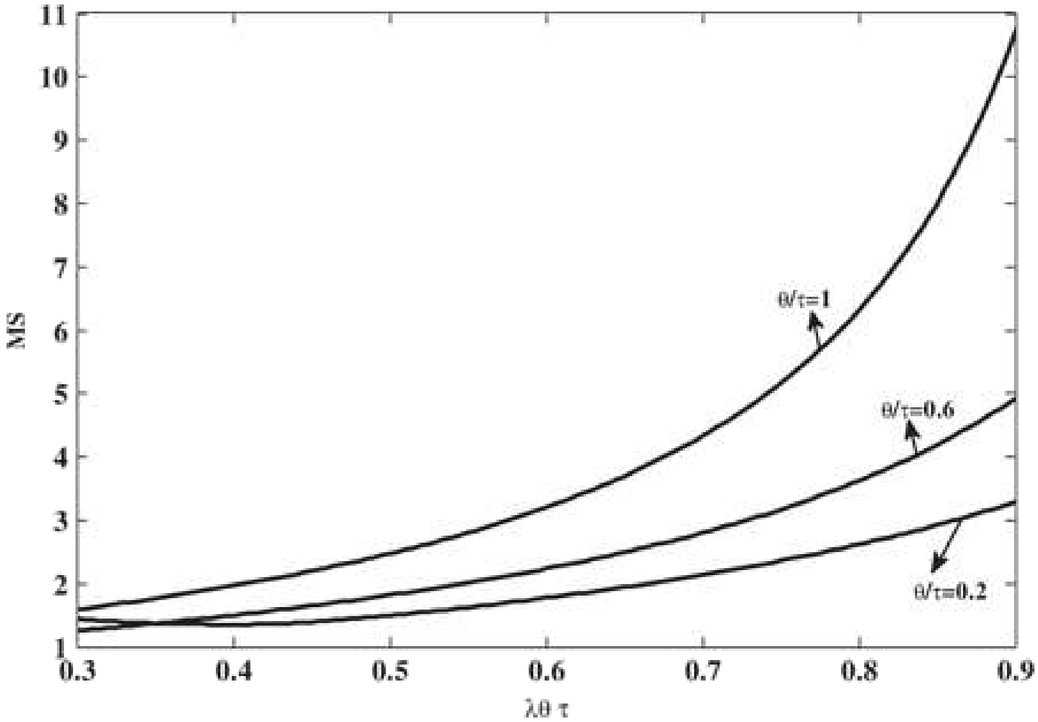

Figure 5: Variation of MS with λ for IFOPTD with zero.

Table 1: Mathematical relation between λ and MS obtained using curve fitting. ($1.5 \leq MS \leq 3.5$).

Process	λ	SSE	R^2
$\frac{k}{s}e^{-s\theta}$	$\frac{1}{\theta}\left(\frac{1.153MS^2-3.183MS+2.164}{MS^2-2.441MS+1.382}\right)$	0.001709	0.9983
$\frac{k}{s^2}e^{-s\theta}$	$\frac{1}{\theta}\left(\frac{2.925MS^2+802.7MS-850.9}{MS^2+533.9MS+1084}\right)$	0.0002858	0.9999
$\frac{k}{s(\tau s+1)}e^{-s\theta}$	$\frac{1}{\theta}\left(P_1+P_2MS+P_3X+P_4MS^2+P_5MSX+P_6X^2\right)$	0.01462	0.9993
$\frac{k(1+sz)}{s(\tau s+1)}e^{-s\theta}$	$\frac{1}{\theta}\left(Q_1+Q_2MS+Q_3X+Q_4MS^2+Q_5MSX+Q_6X^2\right)$	0.0003873	0.9999

$P_1=-0.5421$, $P_2=0.597$, $P_3=0.2543$, $P_4=-0.07363$, $P_5=0.1245$, $P_6=-0.1481$.
$Q_1=-0.3588$, $Q_2=0.465$, $Q_3=0.321$, $Q_4=-0.0605$, $Q_5=0.03874$, $Q_6=-0.06552$.
$X=\theta/\tau$.

For clear understanding of the practitioners, the tuning procedure is elucidated in the flow chat presented in Figure 6

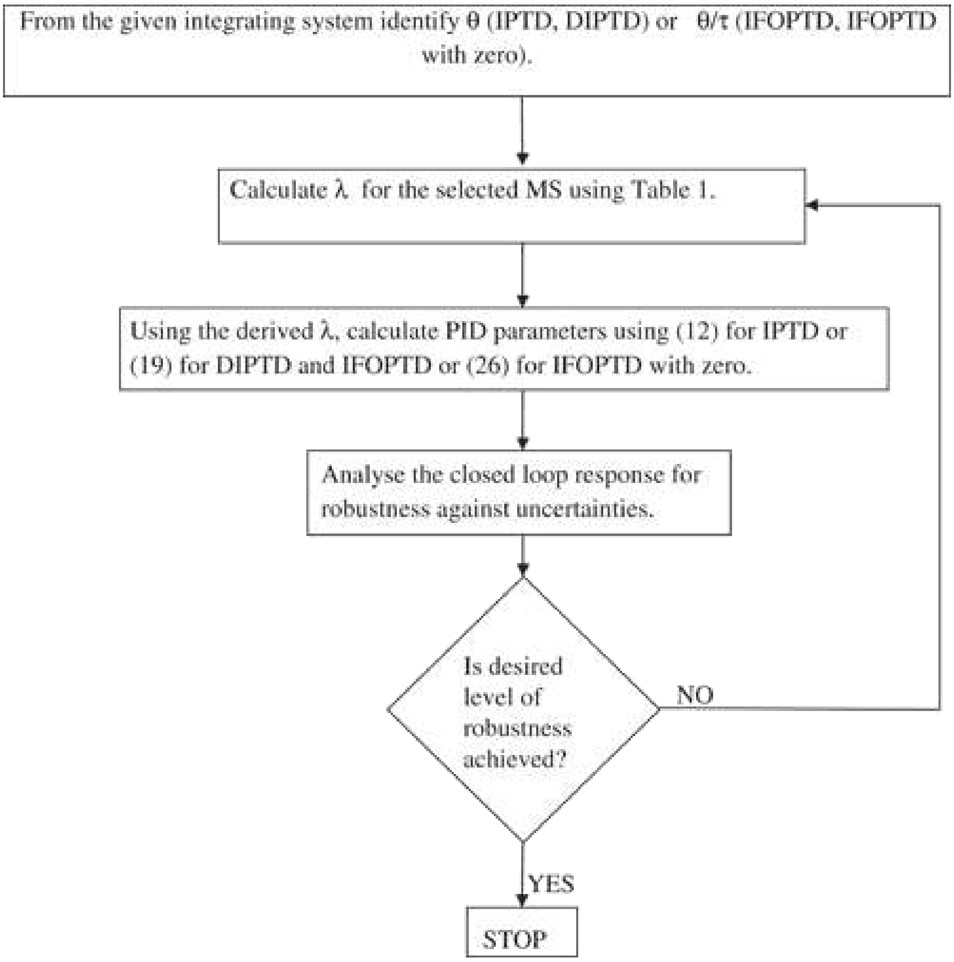

Figure 6: Flow chat of tuning criterion for the proposed method.

2.6 Set point weighting

From eq. (2), it can be understood that the controller introduces zeroes in the servo response which cause overshoot. Set point weighted PID controller is considered in the present work to reduce overshoot and minimize the settling time. The difference between conventional and set point weighted controller lies in how proportional term is considered. In conventional controller proportional gain is multiplied by error (difference between the set point and the controlled variable), whereas for set point weighted controller difference between scaled set point and controlled output will be considered.

$$u(t) = k_c e_p(t) + k_i e(t) + k_d \frac{de(t)}{dt}$$

where

$$e_p(t) = \varepsilon y_{sp} - y$$

and

$$e(t) = y_{sp} - y$$

$$y_{sp} = \text{set point}$$

$u(t)$ = controller output or process input
y = controlled variable

Here, $\mathcal{E}$ is the set point weighting parameter that lies between 0 and 1. The values of $\mathcal{E}$ close to 0 reduce the overshoot significantly at the cost of speed .The values of $\mathcal{E}$ close to 1 offer good speed of response but cause high overshoot. So the selection of $\mathcal{E}$ is a trade-off between the speed of response and over shoot.

3 Stability and robust performance

Many researchers [21, 22] used MS as a measure of stability margin.MS indicates the inverse of the minimum distance between the loop transfer function curve and the critical point in Nyquist plot. The large value of MS corresponds to a less stable system and smaller one to a more stable system. In general, the MS value between 1.2 and 2 is considered as a good choice. Control loops with the MS value close to 1.2 are conservative and the MS values close to 2 correspond to more aggressive.

It is always quite common to have uncertainties in the process parameters due to various reasons like approximation errors, change of operating conditions etc. The robust stability analysis for the proposed method is carried out using complimentary sensitivity function given either in eq. (2) or eq. (3). According to well-known1 small gain theorem, the closed loop system is robustly stable if and only if

$$\|l_m(jw)\,T(jw)\| < 1 \forall \omega\,(-\infty, \infty) \tag{21}$$

where $T(j\omega)$ is the complimentary sensitivity function and $l_m(j\omega)$ is the bound on the process multiplicative uncertainty .The process uncertainty is represented as

$$l_m(j\omega) = \left| \frac{G_p(j\omega)\,e^{-s\theta_p} - G_m(j\omega)\,e^{-s\theta_m}}{G_m(j\omega)\,e^{-s\theta_m}} \right| \tag{22}$$

The complimentary sensitivity function consists of controller parameters, k_p,k_i,k_d, α and β, which are function of λ.

4 Simulation results and comparison

In this section, extensive simulations are carried out to test the proposed method. The servo performance is evaluated by forcing a positive unit step change in the set point at $t = 0s$ and the regulatory performance is evaluated by inducing a positive unit step change in the disturbance at a later time. The performance is compared with the recently reported methods [21, 22] in terms of IAE, TV and OS. Mathematical description of various performance indices are given through eq. (23) to eq. (26).

$$\text{IAE} = \int_0^\infty |e|\,dt \tag{23}$$

$$\text{Integral square error (ISE)} = \int_0^\infty e^2 dt \tag{24}$$

$$\text{Integral time absolute error (ITAE)} = \int_0^\infty t\,|e|\,dt \tag{25}$$

where e is the error. From the mathematical description, it can be understood that IAE criterion treats all the errors equally, ISE criterion penalizes large errors and as time weighted absolute error is considered, ITAE criterion promises faster settling times.

$$\text{TV} = \sum_{i=0}^{\infty} |u_{i+1} - u_i| \tag{26}$$

Where u_i and $u_{i+1}$are the process inputs at i^{th} and $(i+1)^{th}$ instants respectively. TV is a measure of smoothness of the manipulated variable. Smaller value of TV ensures smooth variations in the manipulated variable which cause less wear and tear of the process equipment. A sample period of 0.1 s is considered in the present analysis. Simulation results are compared with the methods proposed by Anil & Padma Sree [22], and/or Ajmeri & Ali [21].

Example1: Many level control problems like a storage tank with pump at the outlet, control of bottom level of a distillation column are the best examples of an IPTD. The following IPTD is considered for the performance analysis of proposed control strategy.

$$G_p(s) = \frac{0.05}{s} e^{-5s}$$

Comparing with eq. (1), $k = 0.05, \theta = 5, \tau = z = 0, c = 1$.For this system, λ is derived as 0.163 using the tuning rules presented in Table 1 to achieve MS=2.The controller parameters are derived using eq. (12). $\mathcal{E}$ is taken as 0.5. Anil & Padma Sree [22], have considered $\mathcal{E}$ as 0.4. The controller parameters are shown in Table 2. With these parameter settings, the simulations are carried out. Disturbance is induced at $80s$. Figure 7 shows the graphical comparison under nominal conditions. Comparison in terms of performance indices is presented in Table 3. To analyze the robust performance, +20 % change is assumed in both the process gain and the time delay of the system. Disturbance is induced at $100s$. Figure 8 shows the response of the system under perturbed conditions. Performance evaluation is presented in Table 4.

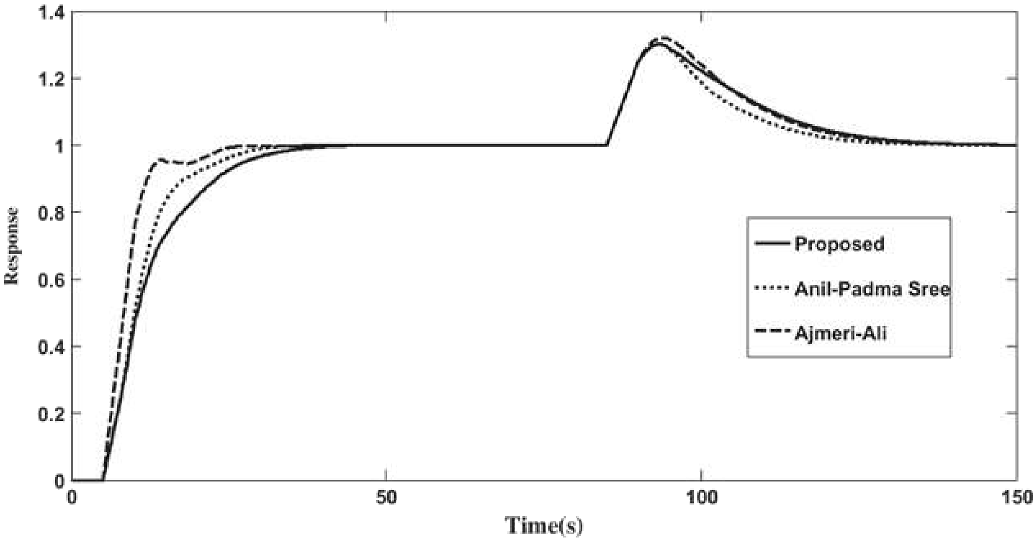

Figure 7: Nominal response of example 1.

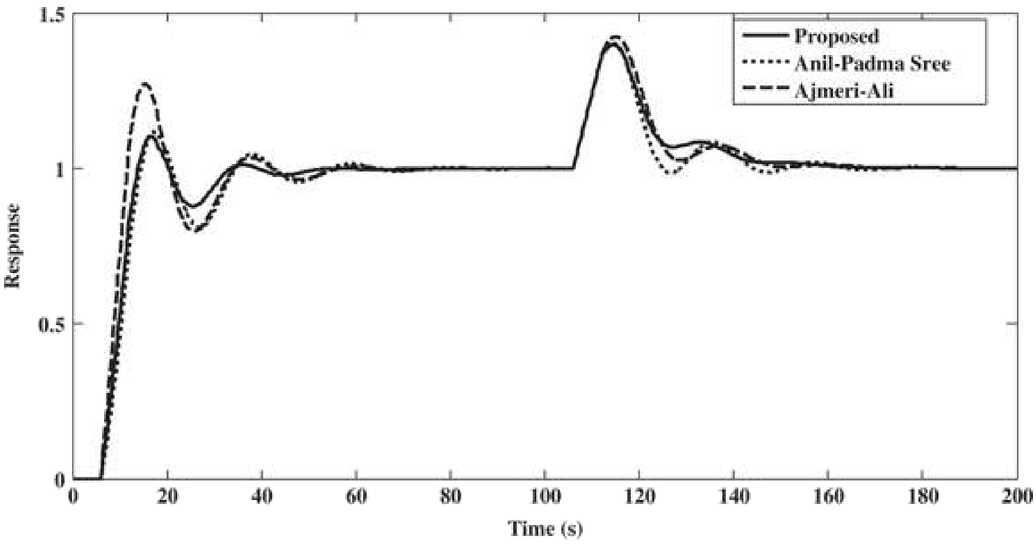

Figure 8: Perturbed response of example 1.

Table 2: Controller parameters for various strategies.

Process	Method	k_p	k_i	k_d	α	β	MS
$0.05\frac{e^{-5s}}{s}$	Proposed	3.4147	0.1633	6.4697	1.25	0.9429	2
	Anil-Padma Sree	3.727	0.1968	7.0440	–	–	2
	Ajmeri-Ali	2.9933	0	7.1189	–	–	2
		3.3209	0.1614	5.8796	–	–	
$\frac{e^{-s}}{s^2}$	Proposed	0.1867	0.0198	0.6469	0.5	0.2207	2
	Anil-Padma Sree	0.1378	0.0142	0.5265	1.0761	1.0392	2
	Ajmeri-Ali	0.0293	0	0.3129	–	–	2
		0.0414	0.0018	0.3246	–	–	
$\frac{0.2e^{-s}}{s(4s+1)}$	Proposed	6.6190	0.9047	13.1965	0.5	0.2111	2

$\frac{(10s+1)e^{-s}}{s(2s+1)}$	Anil-Padma Sree	5.7422	0.9724	11.2082	0.6320	0.4915	2
	Ajmeri-Ali	3.1949	0	9.6792	–	–	2
		3.67	0.3521	9.0902	–	–	
	Proposed	1.3166	0.2353	1.6945	0.5	10	2.35
	Anil-Padma Sree[a]	1.1601	0.2251	1.4452	–	–	2.35
$\frac{0.5(1-0.5s)e^{-0.7s}}{s(0.4s+1)(0.1s+1)(0.5s+1)}$	Proposed	1.0691	0.1543	1.0343	0.64	0.4104	2.81
	Anil-Padma Sree	1.003	0.1572	0.6592	1.1608	0.5490	2.81

[a] PID filter: $\frac{0.5214s+1}{2.674s^2+10.2674s+1}$

Table 3: Performance comparison under nominal conditions.

Process	Method	Servo Response			Regulatory Response		
		IAE	TV	OS	IAE	TV	OS
$0.05\frac{e^{-5s}}{s}$	Proposed	10.95	4.3172	0.012	6.130	2.0860	0.3020
	Anil-Padma Sree	11.41	3.8120	0.001	5.080	2.1104	0.3040
	Ajmeri-Ali	9.069	4.2939	0	6.631	1.7609	0.323
$\frac{e^{-s}}{s^2}$	Proposed	5.690	0.2300	0.001	50.63	2.2486	4.820
	Anil-Padma Sree	5.958	0.1395	0.008	70.58	2.4624	7.033
	Ajmeri-Ali	10.67	3.3287	0.300	543.63	1.736	20.45
$\frac{0.2e^{-s}}{s(4s+1)}$	Proposed	4.759	6.6126	0	1.105	1.9645	0.141
	Anil-Padma Sree	4.543	5.4487	0.095	1.106	2.2456	0.174
	Ajmeri-Ali	4.594	4.1901	0	2.840	1.6735	0.257
$\frac{(10s+1)e^{-s}}{s(2s+1)}$	Proposed	3.368	0.1978	0.001	12.00	2.1600	4.305
	Anil-Padma Sree	4.338	0.3987	0.514	17.18	2.1842	5.052
$\frac{0.5(1-0.5s)e^{-0.7s}}{s(0.4s+1)(0.1s+1)(0.5s+1)}$	Proposed	4.393	1.9860	0.016	6.599	2.6989	1.045
	Anil-Padma Sree	4.094	2.4489	0.021	6.486	2.6940	1.065

Table 4: Performance comparison under perturbed conditions.

Perturbed process	Method	Servo Response			Regulatory Response		
		IAE	TV	OS	IAE	TV	OS
$0.06\frac{e^{-6s}}{s}$	Proposed	11.37	7.6822	0.105	5.880	3.6697	0.399
	Anil-Padma Sree	13.05	8.2170	0.118	5.100	4.3778	0.403
	Ajmeri-Ali	12.57	7.4016	0.272	6.190	3.3297	0.423
$\frac{1.2e^{-1.2s}}{s^2}$	Proposed	5.782	0.3508	0.005	50.768	3.3935	4.974
	Anil-Padma Sree	6.074	0.1921	0.010	71.046	3.3141	7.378
	Ajmeri-Ali	10.75	0.3715	0.366	545.99	2.3423	19.82
$\frac{0.24e^{-1.2s}}{s(4s+1)}$	Proposed	4.758	10.744	0	1.105	3.3244	0.162
	Anil-Padma Sree	4.046	8.0329	0.041	1.055	3.1891	0.200
	Ajmeri-Ali	4.600	9.2122	0	2.84	2.5391	0.289
$\frac{1.2(10s+1)e^{-1.2s}}{s(2s+1)}$	Proposed	3.372	0.3132	0.0034	16.76	5.766	5.785
	Anil-Padma Sree	5.041	0.7464	0.821	29.339	5.7438	6.547
$\frac{0.6(1-0.5s)e^{-0.84s}}{s(0.4s+1)(0.1s+1)(0.5s+1)}$	Proposed	4.570	3.6187	0.058	6.769	5.8847	0.293
	Anil-Padma Sree	4.513	3.5009	0.086	6.737	5.9102	0.316

The proposed method offers better robust performance when compared to the other methods. This is evident from the analysis presented in Table 4 and Figure 8 as the method is superior in terms of IAE, TV and OS compared to the other methods.

Example 2: Fermentation reactors, aero plane dynamics during vertical take-off, DC motors are examples of DIPTD processes. The following system is considered for analysis.

$$G_p(s) = \frac{1}{s^2}e^{-s}$$

Comparing with eq. (1), $k = 1, z = c = 0, \tau = 1, \theta = 1$. λ is calculated as 0.355 to achieve MS = 2 using Table 1. The controller parameters are derived using eq. (19) and are presented in Table 2. $\mathcal{E}$ is selected as 0.4. Anil & Padma Sree [22], have also considered $\mathcal{E}$ as 0.4. Disturbance is considered at $50s$.

The servo and regulatory responses for perfect model match are shown in Figure 9 and Figure 10 respectively. From Table 3, Figure 9, the servo response of the proposed method is superior to the other reported methods in terms of IAE and OS. Anil & Padma Sree [22], have reported low TV which is comparable to that of the proposed method. In regulatory response, the proposed method offers superior response compared to other methods which is evident form Figure 10 and Table 3. So, it can be concluded that the proposed method is very much effective in rejecting disturbance than the other methods. From Figure 11, Figure 12 and Table 4, it is also clear that the proposed method is efficient under perturbed conditions in comparison with the other methods. A +20 % variation in the process gain as well as time delay of the process is considered.

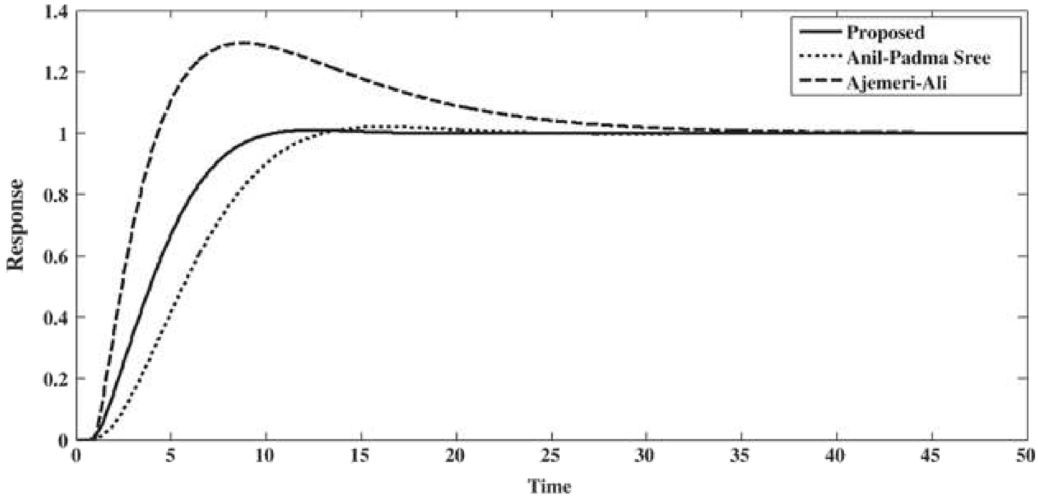

Figure 9: Nominal servo response of example 2.

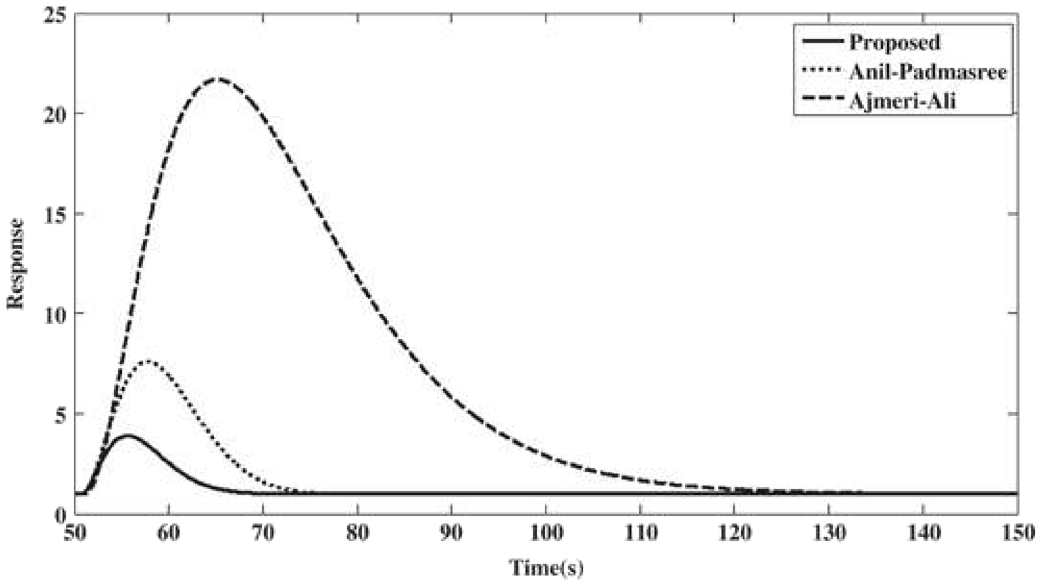

Figure 10: Nominal regulatory response of example 2.

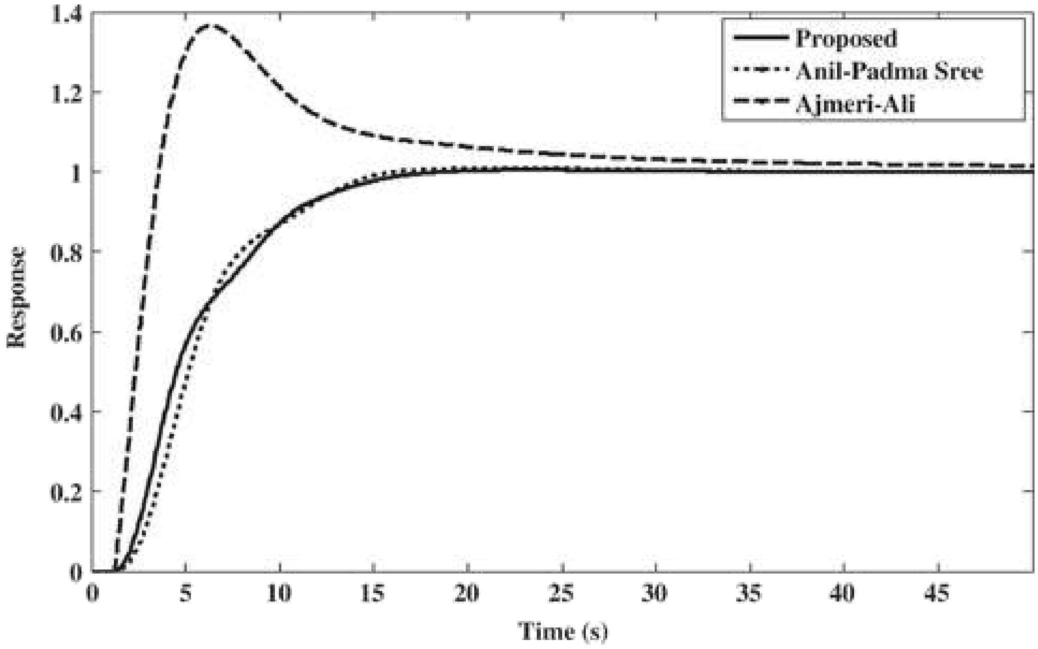

Figure 11: Perturbed servo response of example 2.

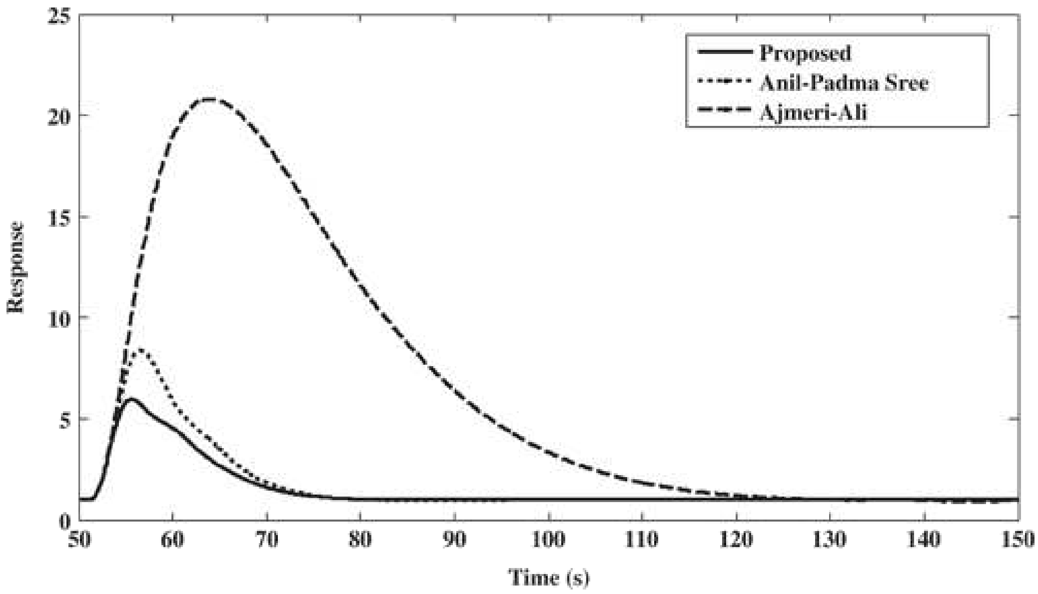

Figure 12: Perturbed regulatory response of example 2.

Example 3: Jacketed CSTR with exothermic reaction, drying process in paper industry are examples of IFOPTD.

$$G_p(s) = \frac{0.2}{s(4s+1)}e^{-s}$$

Comparing the system with eq. (1), $k = 0.2, z = 0, \tau = 4, c = 1$ and $\theta = 1$. λ is taken as 0.475 to achieve MS = 2 with the help of Table 1. $\mathcal{E}$ is taken with a value of 0.35. The controller parameters are derived with the help of equations eq. (19) and are given in Table 2. Anil & Padma Sree [22], have taken $\mathcal{E}$ as 0.4. Disturbance is induced at 40s. The simulation study under nominal conditions is shown in Figure 13 and Table 3. It is observed that all methods are giving comparable performance in set point tracking. In disturbance rejection, the proposed method performs better than the other methods.

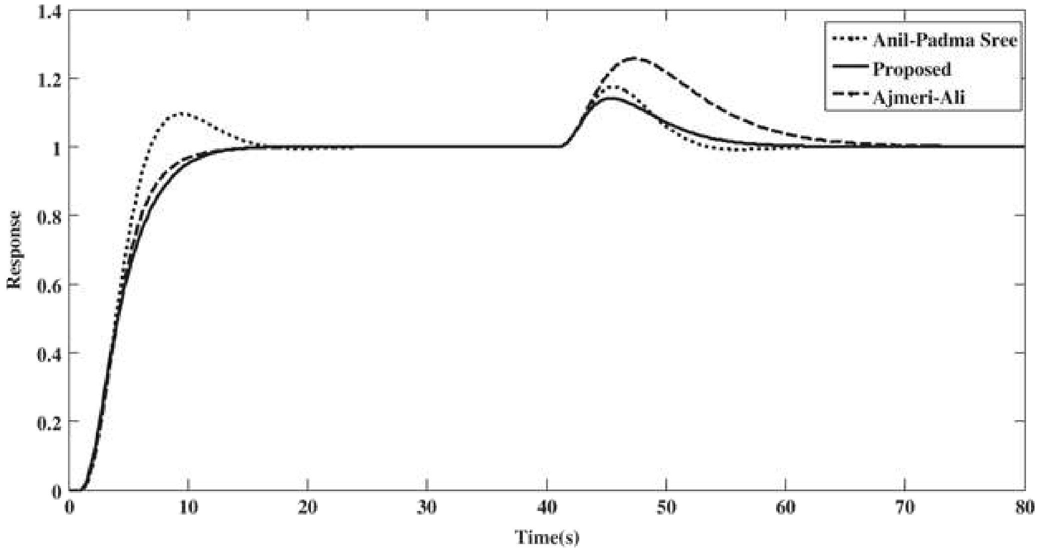

Figure 13: Nominal response of example 3.

The simulation study under perturbations is analyzed for a perturbation of +20 % in both the time delay and process gain of the system. The observed results are presented in Figure 14 and Table 4. The proposed method is less oscillatory and comparable to the strategy of Anil & Padma Sree [22],and performs better than Ajmeri & Ali [21],in disturbance rejection.

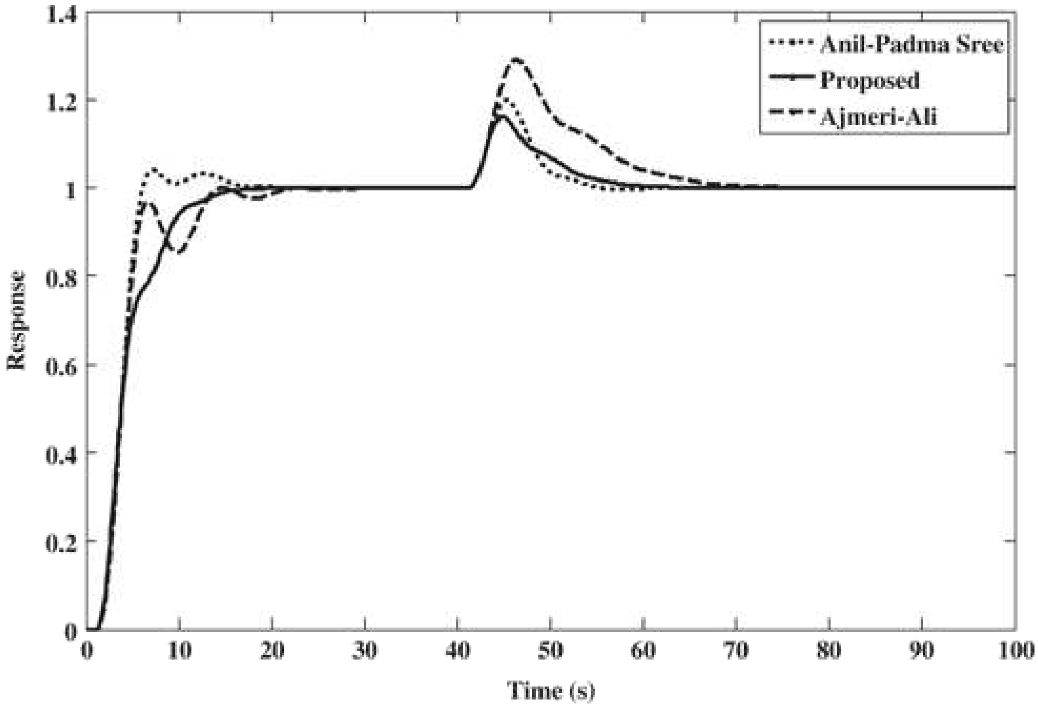

Figure 14: Perturbed response of example 3.

Example 4: An integrating system with both pole and zero is considered in this example

$$G_p(s) = \frac{10s+1}{s(2s+1)}e^{-s}$$

Comparing the system with eq. (1), $k = 1, z = 10, \tau = 2, c = 1$ and $\theta = 1$. A value of 0.5888 of λ is considered to achieve MS= 2.35. $\mathcal{E}$ is chosen as 0.4. The controller parameters are presented in Table 2. Disturbance is considered at 50s.

The simulation analysis of the strategies under nominal conditions is shown in Figure 15. From the graphical analysis shown in Figure 15 and Table 3, it is clear that the proposed method is superior to the method reported by Anil & Padma Sree [22],in both the set point tracking and the disturbance rejection. The other method resulted in higher overshoot and undershoots in disturbance rejection when compared to that of the proposed method. One more important observation is that the other method is not able to reduce the over shoot in servo response even after including set point weighting.

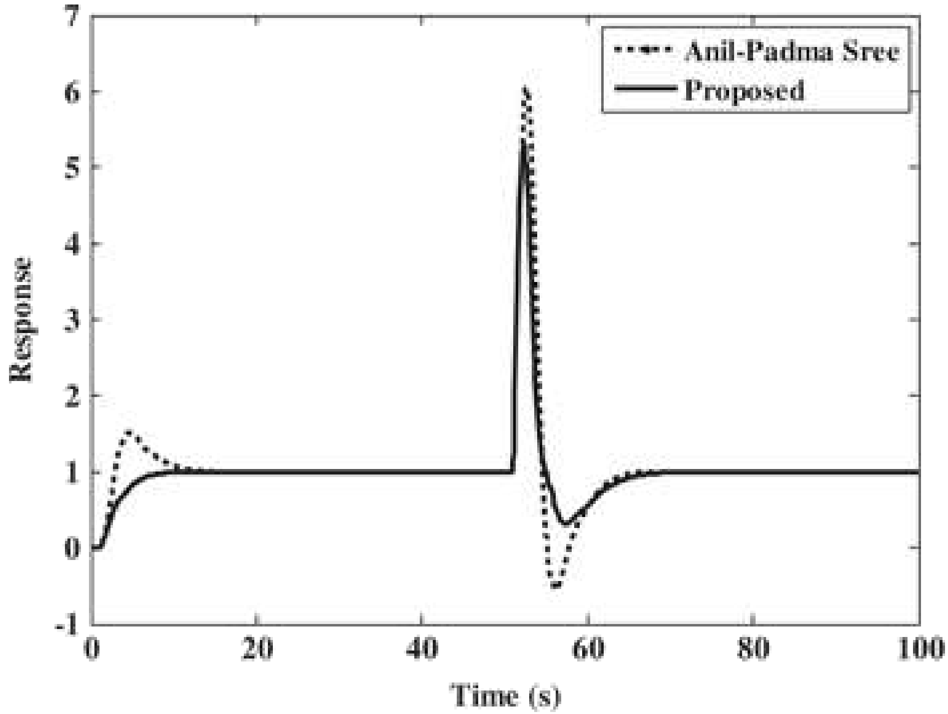

Figure 15: Nominal response of example 4.

From Figure 16 and Table 4, it is also revealed that the proposed method is superior to the method reported by Anil & Padma Sree [22],even in terms of robust performance. A perturbation of +20 % is assumed in the process gain and the time delay of the system.

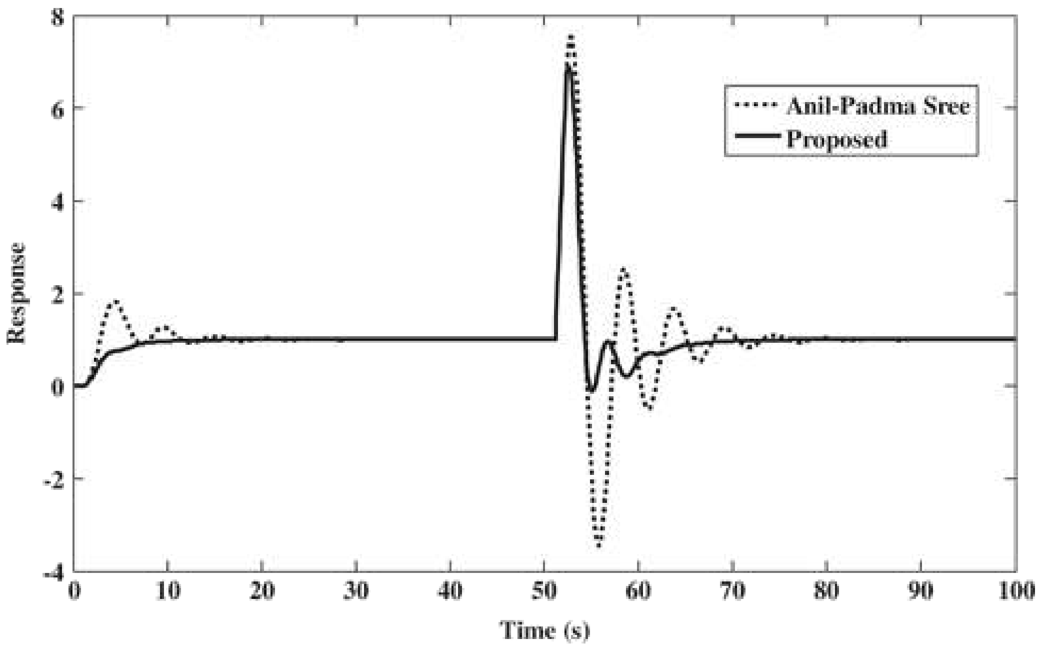

Figure 16: Perturbed response of example 4.

Example 5: In this example, the proposed method is applied to a higher order system by approximating it to an equivalent IFOPTD system.

$$G_p(s) = \frac{0.5\,(1 - 0.5s)}{s\,(0.4s + 1)\,(0.1s + 1)\,(0.5s + 1)} e^{-0.7s}$$

Using model reduction technique [2] the higher order system is reduced to a delay dominant IFOPTD system and is given as

$$G_p(s) = \frac{0.5183}{s(1.1609s + 1)} e^{-1.2799s}$$

For fair comparison with Anil and Padma Sree [22], λ value of 0.59893 is considered which corresponds to MS = 2.81. $\mathcal{E}$ is selected as 0.4. In this example, the reduced order system parameters are used for calculating the controller parameters whereas the actual systems parameters are used for calculating the MS value. Anil & Padma Sree [22], have taken $\mathcal{E}$ as 0.4. Controller settings are presented in Table 2. Disturbance is induced at 50. Nominal responses are compared graphically in Figure 17. From Figure 17 and Table 3, it is observed that both the methods perform almost equally.

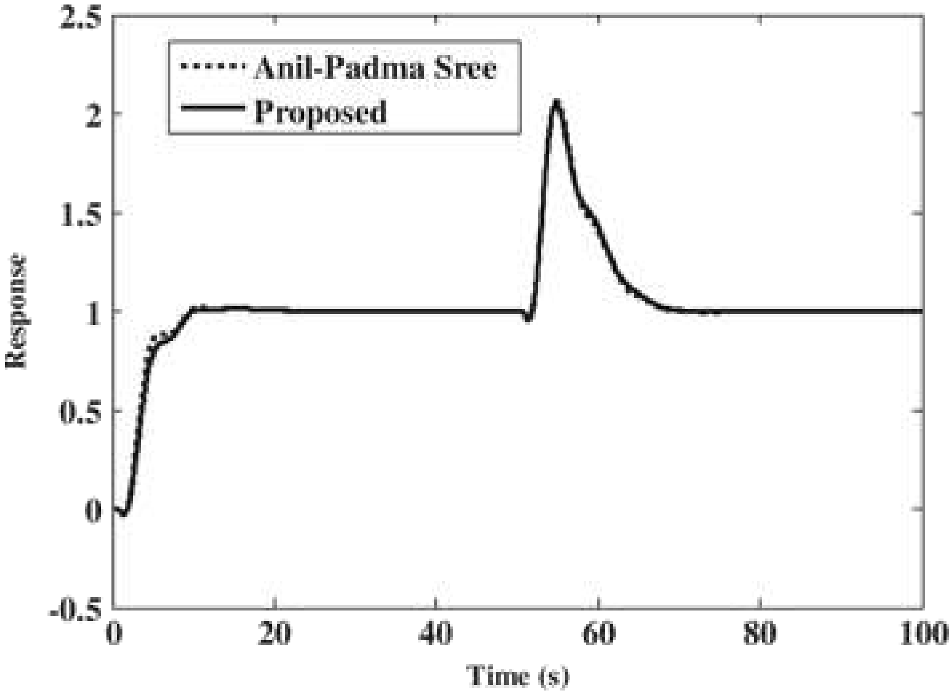

Figure 17: Nominal response of example 5.

From Figure 18 and Table 4, it can be concluded that the performance of the proposed method under perturbed conditions is superior to the method of Anil & Padma Sree [22]. The process gain and the time delay of the system are perturbed by +20 % to analyze the robust performance.

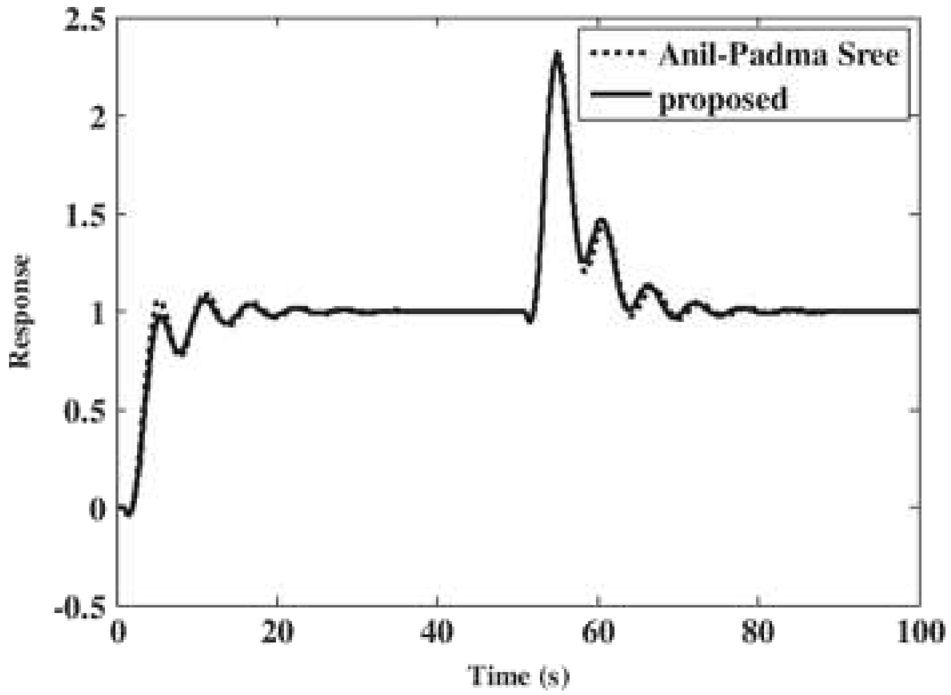

Figure 18: Perturbed response of example 5.

5 Conclusion

A simple control loop is designed for a class of integrating systems. The control loop involves a PID controller with first order lead/lag filter. The controller parameters are derived as a function of MS to achieve desired robust performance. Set point weighting is employed to reduce overshoot in the servo response. The proposed method has shown equal or better performance compared to the other methods. Significantly improved performance is observed especially in the case of DIPTD and IFOPTD with zero. The notable feature of the proposed method is that it offers better performance with single feedback control loop, which eliminates the necessity of multi control structure with separate controllers for servo and regulatory responses and multiple tuning parameters.

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