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Original article

Modelling and outcome analysis of a competitive environment – A probabilistic approach

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ABSTRACT

Competition is one of the underlining phenomena in the world that has been continuously deforming the life and history of human beings, animals, plants and even microorganisms. The results of different competitions are fascinating from explanation of evolution to the dramatic industrial revolution. The article proposes a mathematical model to estimate the outcome probability of different competitors in a competition system on account of the number of failure gates associated. The article includes special cases like the tournament model considering the number of failure gates which are likely at various levels of competition. The study additionally gives the idea of identical probability curves on account of eight member competitions which is highly relevant in strategy and decision making process and there by sustainability of an individual competitor gets ensured. The concept of identical probability utilised here will help to understand the different scenarios in a competitive environment, with similar chance for occurrence. The model developed will definitely be a more practically relevant substitute for conventional probabilistic tools, including Bayesian approaches and whilst the character of considering bulk of competitors all together, enabling the model itself to find further extensions in the dimension of modern game theory as well. The time-dependent behaviour of number of failure gate envisaged provides more practical scope for the mathematical model presented. The model is applicable to analyse the trust related issues in supply chain with huge number of suppliers participating and also in the areas of decision making in the field of industrial management and completion science as a whole.

1. Introduction

The profound knowledge about the art of competition with a suitable scientific tool is a necessity of today. In nature, competitions can be viewed in various forms, with different intensities diverges from microscopic level to a cosmic domain. The craving for success in each competitor can be considered as a major drive for any competition in general. However, the external factors, networks in between different competitors with in the competition etc. are some other reasons of real-world outcomes. Hence the success and failure of every competitor can be treated as a succession of different elements. Competition can also be defined as an interaction between two or more individuals under a set of common rules and different criteria which results in a series of ranks. Every competition can be basically classified into two types, single step and compound step. Single step is a type of competition between two competitors and results in a winner and a looser. Compound step is another sort of competition between multiple individuals resulting in a progression of ranks or positions, if there are N contenders there will be N positions. In many cases compound steps are unrealistic to locate an ultimate winner, and hence we can lead a great deal of single steps to reach at the ultimate winner. It is beyond the realm of imagination to expect to locate an ultimate list of ranks for all competitors in a single competition, yet it is conceivable through chain of such criteria.

In "Probability and Uncertainty in Economic Modelling" by Itzhak Gilboa et al. (2008) point out the limitations and restrictions of standard expected utility model, Bayesian approach and also discussing the possibility of other models in the economics. Enrico Scala's (2008) quoted the basic probabilistic ideas in the economic studies of probabilistic theory and also explaining different termi-

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nologies in probabilistic economics. Many authors (2009–2010) studied about application of advanced probabilistic ideas in the field of economical oriented topics like random variable transformations, probabilistic distributions. Xintao Xia et al. (2016) introduced a new methodology to calculate reliability of a running manufacturing system which may or may not have sample size with probability distribution. Author Senol (2007) proposed the Poisson process approach to identify the degree for failure mode and effect in reliability analysis. Also Lin (2005) introduced a process reliability assessment with a Bayesian approach which can predict whether the process holds the quality reliability requirements. Reliability Modelling and Optimization Strategy for Manufacturing System Based on RQR Chain by Yihai He (2015), proposing a mathematical model which can be applied in calculating the reliability and optimizing the manufacturing with respect to product qualified probability and RQR chain (namely, manufacturing system reliability (R), manufacturing process quality (Q), and the produced product reliability (R)). Sunil S Bhamare et al. (2008) discussed about the evolution of reliability engineering in the last six decades and the various statistical and mathematical models that has been developed and using the same as an overview.

Evolution and Game Theory by Samuelson (2002) is found to be one of the underlining research articles which mentions possibilities and modelling of evolutionary game theory in economic science. Researchers Jianya et al. (2015) developed an asymmetric competition game model in E-Marketplace, the model also study the competition between sellers and buyers together. Friedman's (1998) is another introductory literature regarding the implementation of evolution game theory in modern economics and also discussing the purpose and benefits of evolutionary game theory in economics. Ozkan-Canbolat et al., (2016) focus on circumstances in which the bandwagon diffusion of an innovation happens in a jointly even though, on an average, organizations jointly assess negative outcomes through adapting a particular innovation. Also Bellomo and Delitala (2008) introduced a medical application of game theory and the analysis on the mathematical kinetic theory of active particles applied to the modelling of the very early stage of cancer phenomena, specifically mutations, onset, progression of cancer cells. Some authors (2005, 2012) studied about game theory and focused on modelling of warfare. Jorswieck et al. (2009) introduced an application of game theory in the field of signal processing and communication engineering. Noguchi et al. (2014), designed a rational solution obtained by game theory in a multiobjective electromagnetic apparatus. Anastasopoulos et al. (2012) developed a model which can be implemented in feedback suppression system on a multicast-satellite. Srinivas et al. (2019) studied the effect of independent dummy variable in a competitive environment and highlighted the application of the number of failure gates in e-martketing.

The literature survey point out some scientific involvements happened in recent decades. The engineering and pure managerial methodologies cope up with the game theory and conventional probabilistic approaches have difficulties to find their position in all engineering and economics issues in the current social environs. Conventional game theory fails to hold more number of players (competitors) simultaneously. And the limitation of Bayesian approach which has inadequate flexibility to address some complicated problems in engineering and economics as well noticed. Even though modern game theory has the ability to consider two-three players at the same time, its complexity is a reasonable question. This promotes the necessity of a multiplayer probabilistic model which can be assimilated with the existing game theory to evolve as a wide-ranging strategy tool for future days.

In order to find the ultimate n^{th} position in a rank list, it is required to use the methods like multiple chains, success-failure gateway. The objective of multiple chain method is to reduce the number of chains to find the ultimate n^{th} position. i.e., after the 1st tournament eliminate the first and repeat the same criteria, which will result in the ultimate 2nd. Optimising the number of chains required is the task here and the objective of the Success-failure gateway method is to find the probability of pass through for n^{th} position with in a combination of success failure gateways (contains both success and failure gate). Success gates are certain control gate which will promote the winner to the next level, failure gates are those gates which deny the winner from entering to the next level or it promotes the looser to the next level. This work makes use of knockout criteria. The number of competitors should be even and in order to complete the tournament the number of competitors should be in the order of two.

The article comprises of mathematical modelling including single failure gate model and λ number of failure gate model. In addition to this basic model a particular case of tournament model is studied as well. The Sections 2.4 and 2.5 focused on the possibility for multiple number of failure gates using binomial theorem and the general solution for R_M. Section 3 is dedicated to the case study with the concept of failure gate as a variable and the idea of identical probability, in an eight member competition environment. Finally the practical relevance of the study, especially those relating to the decision making process and so forth is highlighted in the Section 4 as the concluding remarks.

2. Mathematical model

The mathematical modelling associated with this article incorporates the multiple competitor system using number of failure gate concepts. The Fig. 1 may be viewed as a detailed pictorial representation of a competitive environment in general. The alphabets (A-J) inside the blue coloured circle represent the competitors and inter networks between them as a system. The yellow coloured cloud represents the failure gates.

Fig. 1 can be considered as a detailed pictorial representation of a competitive environment in general. The alphabets (A-J) inside the blue coloured circle represent the competitors and inter networks between them as a system. The yellow coloured cloud represents the failure gates. The purpose of failure gate is that the probabilistic gain and loose of each competitor is effected only through the failure gate and the failure gates is linked between all other factors with in the competition and also external. The strategic changes may also have the possibility to change the frequency distribution of different ranks in the system and both are

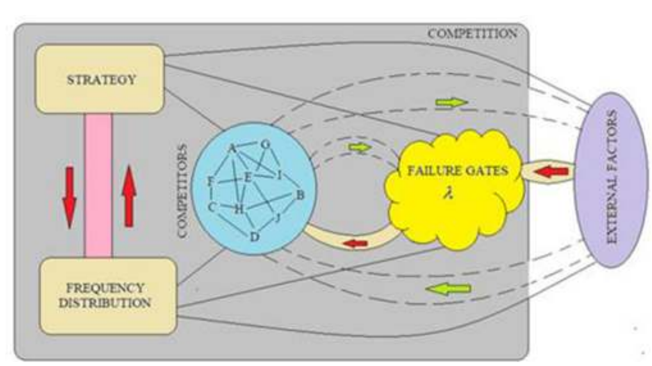

Fig. 1. Represents overall competition including failure gates and external factors.

interlinked. However everything is considered with the failure gates as well. The red arrow represents possible directions where intense variation may occur and green line with limited variation based on social situation. The following diagram represents all possible success-failure gateways for a four ranked four member system that has an initial random order of 1, 3, 2 and 4.

The probabilistic value corresponding to each rank at different number of failure gates can be observed in the Fig. 5 given above. The number of failure gates and number of success and failure gates distribution will be exactly mirror image due to symmetry. The Fig. 3 is a simplest case of non-frequency distribution rank models with only four competitors competing in the competition (refer Fig. 2). The complexity of the system will increase with increase in the number of competitors participating and their frequency. A general competition probabilistic model is developed in this article.

2.1. Single failure gate model

In this section we consider a competition model with a single failure gateway in ground level. This gateway is sufficient for altering the probabilistic outcome of getting least rank from zero to non-zero, the guarantied positive outcome of best rank and also the probabilistic intermediate rank in a probabilistic region. The pictorial representation of this system is given below (Fig. 4(a–c)) and also the mathematical model equations of this competition system with single failure gate are given in (2.1)–(2.3).

$$P_b(n) = 1 - \frac{(M-1).R_{M-2}}{R_M} \quad (2.1)$$

$$P_l(n) = \frac{(M-1).R_{M-2}}{R_M} \quad (2.2)$$

$$P(n) = \frac{n_1.R_{M-2} + \left(\frac{M}{2}-1\right)n_2 R_{M-2}}{R_M} \quad (2.3)$$

where M represents total number of competitors, R_M represents the number of possible competition arrangement for M competitors, R_{M-2} represents the number of possible competition arrangement for M competitors by keeping one competitor fixed in apposition, n_1 and n_2 represents number of competitors in front of n^{th} position and behind n^{th} position respectively. Where M_{C_2} is number of combinations of competitors. In (2.1), $P_b(n)$ represents probability of the best rank to pass through to the next level, $P_l(n)$ in (2.2) represents the outcome probability of least rank and $P(n)$ in (2.3) represents an intermediate rank to pass through to the next level. Here $M = n_1 + n_2 + 1$, $R_M = M_{C_2}.R_{M-2}$

2.2. Generalised model with λ failure gates

We know that the total possible passes through gateways are half the number of competitors. Let λ is the number of failure gates. Therefore (M/2 – λ) is the number of success gates.

$$P_b(n) = \frac{\left(\frac{M}{2}-\lambda\right)(M-1)R_{M-2}}{R_M} \quad (2.4)$$

We know that the total possible pass-through gateways are half the number of competitors. Let λ is the number of failure gates.

$$P_l(n) = \frac{\lambda(M-1).R_{M-2}}{R_M} \quad (2.5)$$

By combining (2.4) and (2.5) and if n is in an intermediate position, then we can split, the position into two parts as follows.

Let λ is the number of failure gates then

$$P(n) = \frac{\lambda n_1 R_{M-2} + \left(\frac{M}{2}-\lambda\right)n_2.R_{M-2}}{R_M} \quad (2.6)$$

where

$$R_M = M_{C_2}.R_{M-2} \quad (2.7)$$

2.3. Particular case: (tournament model)

In order to make a complete tournament, number of competitors should be in the order of 2 i.e. $M = 2^\mu$. Then, the general outcome probability Eqs. (2.5)–(2.7) becomes,

$$P_b(n) = \frac{(2^{\mu-1}-\lambda)(2^\mu-1).R_{M-2}}{R_M} \quad (2.8)$$

$$P_l(n) = \frac{\lambda(2^\mu-1).R_{M-2}}{R_M} \quad (2.9)$$

$$P(n) = \frac{\lambda n_1.R_{M-2} + \left(2^{\mu-1}-\lambda\right)n_2.R_{M-2}}{R_M}; \quad n_1 + n_2 + 1 = 2^\mu \quad (2.10)$$

If the number of failure gates is different in different levels then the total outcome probability of least rank will be the product of individual level probabilities. The following figure is pictorial representation of tournament model with different λ values. The number failure gates will be different in different levels implies a real-time situation in a social dilemma.

The inverted pyramid shown in Fig. 6 is the representation of reduction in maximum number of failure gates in different levels, for that the maximum number of failure gate is directly related to the total number of competitors participating in the competition. In the initial level, number of failure gates will be at the maximum

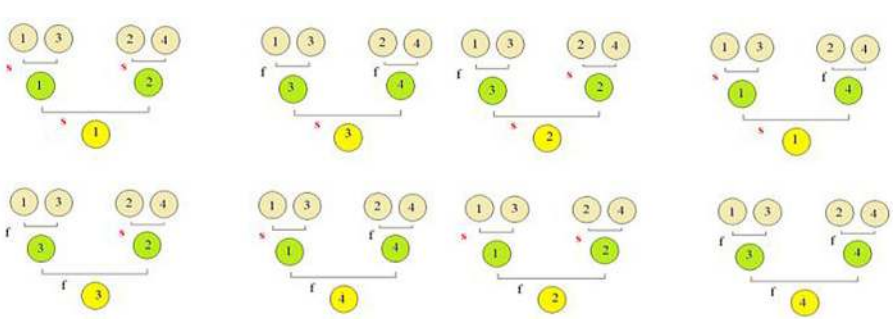

Fig. 2. Represents different possible arrangements of success-failure gate ways. ***Observation***: rank 1-only comes through success gateways, rank 2-have possibility of both success and failure gateways, rank 3-have possibility of both success and failure gateways; rank 4- only comes through failure gates.

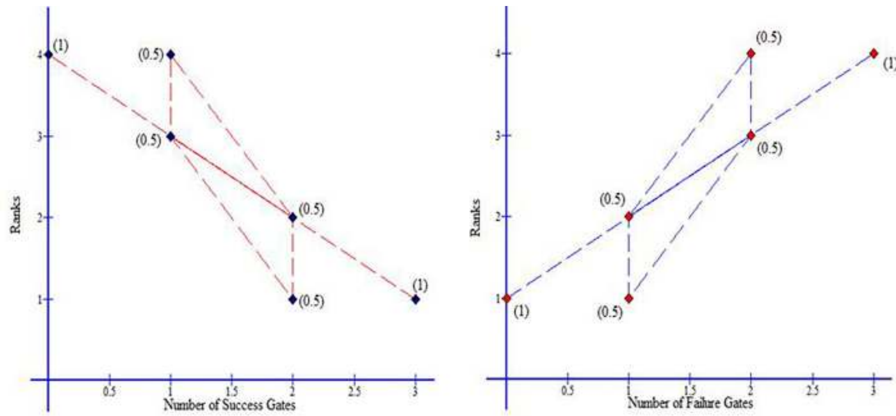

Fig. 3. Represents the rank distributions with probability corresponding to number of success and failure gates.

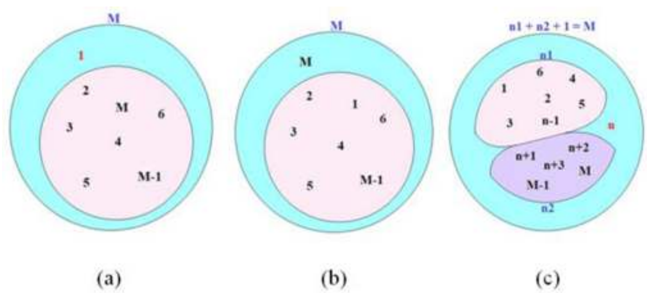

Fig. 4. **(a)** represents competitive environment for a best rank in a *M* member competition. **(b)** Represents competitive environment for a least rank in a *M* member competition. **(c)** Represents competitive environment for intermediate rank in a *M* member competition.

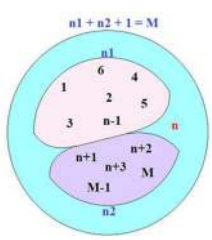

Fig. 5. Represents general representation for intermediate ranks in *M* member competition.

and the for a single winner tournament model number of failure gates will end with solitary for final level. Hence the total outcome probability of a n^{th} rank to become a winner is the product of outcome probabilities in each level. The total outcome probability for least rank is given in Eq. (2.11), here $n \to 2^{\mu}$

$$P(\lambda, 2^{\mu}) = \prod_{i=1}^{\mu} \frac{\lambda_i}{2^{\mu-i}} \tag{2.11}$$

2.4. Binomial expansion for failure gate

If the probability of happening a single independent failure gate is known, then the same probability for y failure gates can be obtained through binomial theory, let $P(f)$ be the probability for getting single independent failure gate, then probability for obtaining y failure gate is

$$p(\lambda = y) = {}^{\left(\frac{M}{2}\right)} C_y p(f)^y (1 - p(f))^{\left(\frac{M}{2} - y\right)} \tag{3.1}$$

2.5. General solution of R_M

In this section, we are finding the value for total possible competitions for M competitors (R_M) as follows. From (2.7), we have

$$R_M = M_{C_2}.(M-2)_{C_2}.(M-4)_{C_2}........R_2$$

$$R_M = \prod_{K=0}^{\frac{M}{2}-1} (M - 2K)C_2 = \frac{M!}{2^{M/2}} \tag{4.1}$$

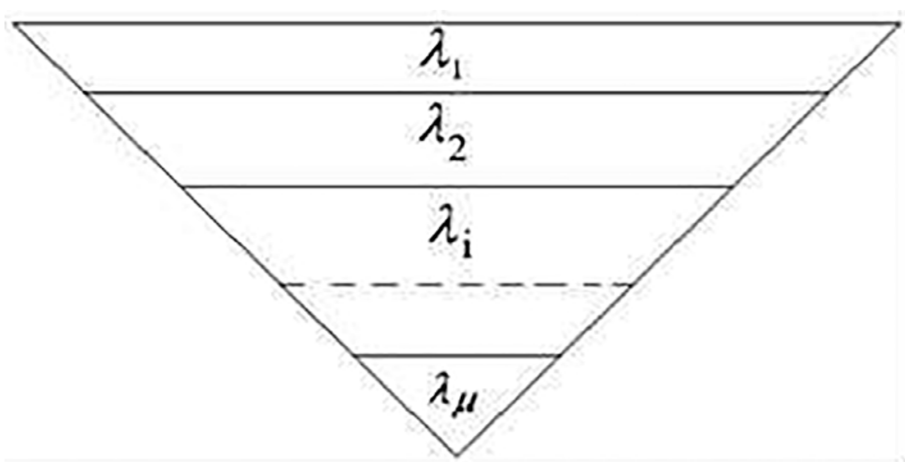

Fig. 6. Represents the pictorial structure for different number of failure gates in different levels.

3. Case study

In associated with the physical aspects of the concept of number of failure gates, these failure gates may be broadly affected by various physical quantities. However, these failure gates may physically appear through the six pillars as explained below. These influencing factors and there relative impact on an associated problem is often somewhat subjective and purely practical oriented or situation based. Through the better understanding of this in-bond relation between this real time physical factors and the number of failure gates one will definitely arrive at proper decision making. The number of failure gates may be relating to various contributing factors is included in the Fig. 7 given below.

In the above figure the various factors that can bring changes the number of failure gates is discussed. This can be classified into six major factors including constant number of failure gates, its dependence on the individual competitor, the uncertain situations influencing the probability, total number of competitors, the frequency distribution of ranks and finally other external factors. The consideration of all these factors will help to analyse a real time problem using the model developed.

The number of failure gates may not be a constant for all situations. It might be a function of time or a function of number of competitors or both.

3.1. 8-Member competition systems

The Fig. 8 as shown below represents the probability distribution in the case of constant λ values in an 8 member competition for different values of λ. Here we can see various prob-

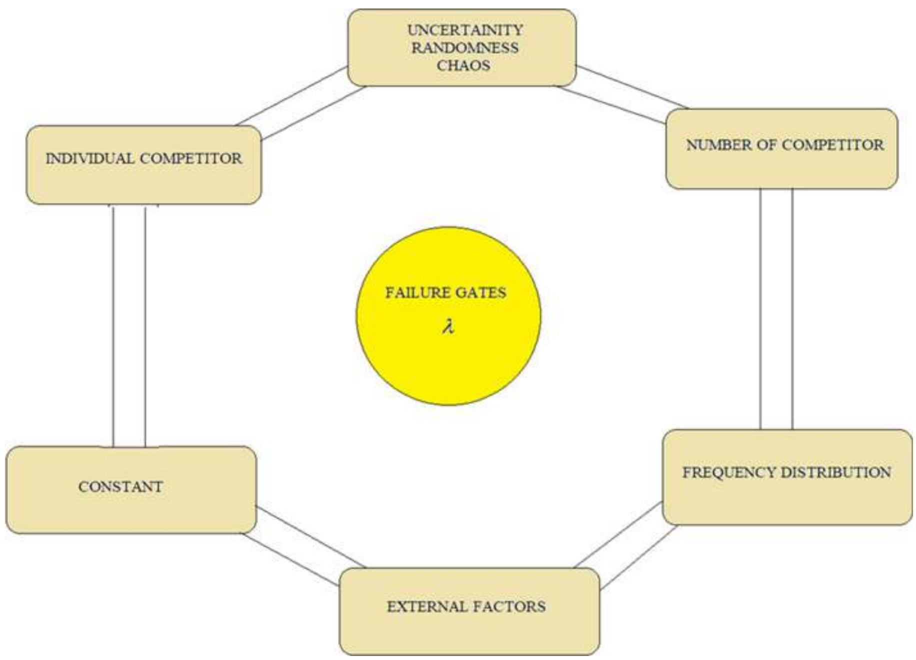

Fig. 7. Represents the various possible factors affecting the number of failure gates.

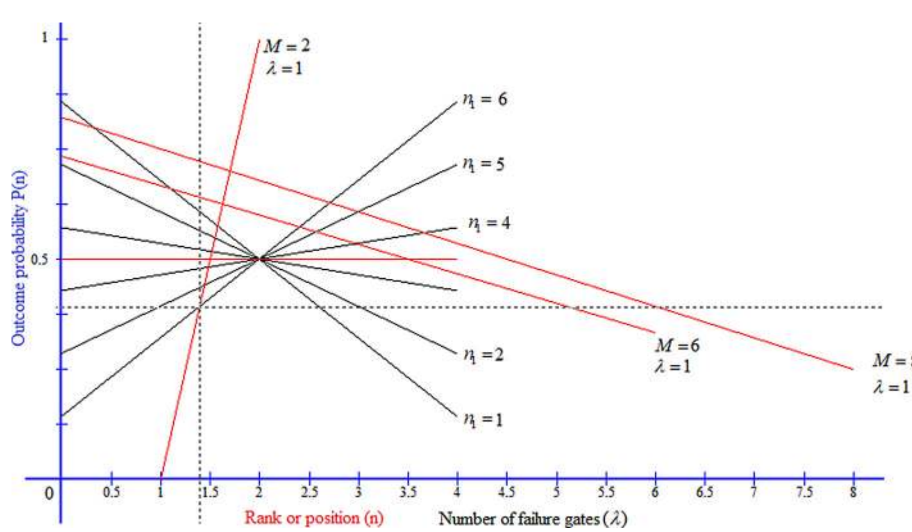

Fig. 8. Represents probability distribution on account of constant number of failure gates (λ) in an 8-member competition for various estimated number of failure gates.

ability intersection points in the graphical representation given below.

Three different types of intersection points are considered in this diagram; first one is the intersection between rank curves and the curves representing the number of failure gates. Such intersections are identified as Black-Red coloured (B-R) intersections. Correspondingly Red- Red (R-R) intersections is also recognized. And finally a Pivoted intersection is noticed in between all curves representing the number of failure gates. Since both the position and number of failure gates cannot be interpreted simultaneously for both red and black curves on B-R intersections, the net values of the position as well as number of failure gates is calculated by taking averages.

The Fig. 8 gives a profound idea about the identical outcome probabilities associated with different situations based up on the total number of competitors and the number of failure gates. The intersection points indicate the identical outcome probabilities for an 8 member competition. The black line refers the plot between the number of failure gate against outcome probability of n^{th} rank for different positions (n varies from 2 to 7 or n_1 varies from 1 to 6). The red line is the graphical representation for the n^{th} rank in a competition against outcome probability of the n^{th} rank based on Eq. (2.6). The two plots are combined to understand the identical probability intersection points involved in the competition. The study focuses on the two different intersections and one pivot point for the number of failure gate against outcome probability of n^{th} rank plotted for different positions. The detailed description of these intersection points are given in Table 1. This analysis highlights the possibility of identical probabilistic lines which will definitely find its position in decision making and hence provides better understanding about a competitive environments as a whole.

The 3D surface plot and 2D contour plot is studied for an eight member competition and the probabilistic and non-probabilistic zones based on the maximum possibility of number of failure gates (which is equal to 4 for an 8 member competition) classified. The red lines in the Fig. 9(a–c) is the representation of identical probabilities in the system.

3.2. Time-dependent number of failure gates

We can also assume that λ is a function of M or t i.e $\lambda = f(M)$ or, then the rate of change in failure gates (λ) can be represented as

$$\lambda' = \frac{\partial \lambda}{\partial t} = g'(t) \tag{3.1}$$

Also, we are assuming that M is also a function of t, then $M = h(t)$. The rate of change of number of competitors can be represented as

$$M' = \frac{\partial M}{\partial t} = h'(t) \tag{3.2}$$

Table 1
Results obtained in the intersection points from Fig. 10.

M Black	M Red	M Net	n Red	n Black	n Net	λRed	λ Black	λ Net	P(n)	Int.sec
8	8	8	0.333	2	1.166	1	0.333	0.666	0.797	B-R
8	6	7	1.106	2	1.553	1	1.106	1.053	0.659	B-R
8	2	5	1.575	2	1.787	1	1.575	1.287	0.575	B-R
8	2	5	1.548	3	2.274	1	1.548	1.274	0.548	B-R
8	2	5	1.481	5	3.240	1	1.481	1.240	0.481	B-R
8	6	7	2.976	5	3.988	1	2.976	1.988	0.534	B-R
8	8	8	3.666	5	4.333	1	3.666	2.333	0.559	B-R
8	2	5	1.440	6	3.720	1	1.440	1.220	0.440	B-R
8	6	7	2.575	6	4.287	1	2.575	1.787	0.561	B-R
8	8	8	3.000	6	4.500	1	3.000	2.000	0.607	B-R
8	2	5	1.391	7	4.195	1	1.391	1.195	0.391	B-R
8	6	7	2.407	7	4.703	1	2.407	1.703	0.572	B-R
8	8	8	2.714	7	4.857	1	2.714	1.857	0.627	B-R
6	2	4	1.625			1			0.625	R-R
8	2	5	1.700			1			0.700	R-R
8	4		4	–		1	–		0.500	Pivot

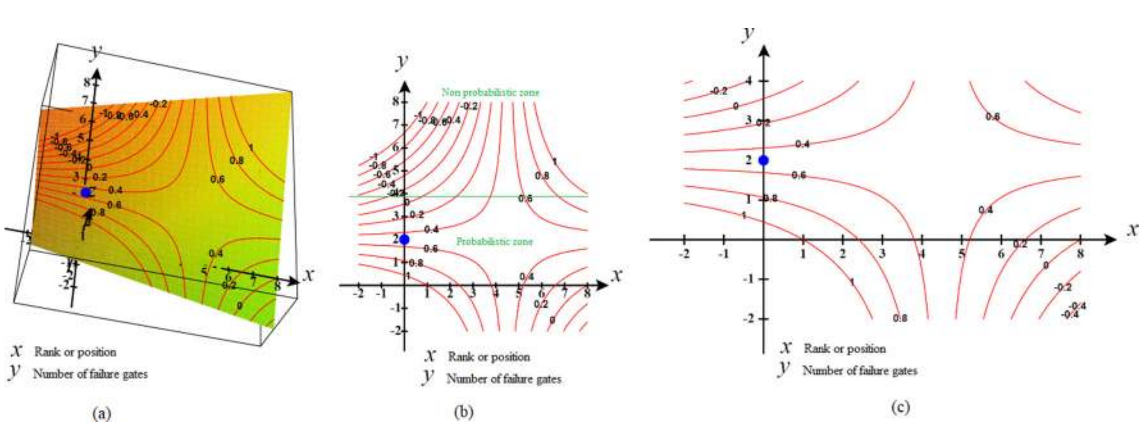

Fig. 9. Represents the graphical representation of ranks against number of failure gates with identical probability curves for an 8 member competition. (a) Represents the 3D surfaces in general. (b) Represents the probabilistic and non-probabilistic zones based on the maximum number of failure gates. (c) Represents the probabilistic region alone with identical probability curves.

By combining (2.8) and (2.9), the rate of change of failure gates with respect to number of competitors can be represented as

$$\frac{\partial\lambda}{\partial M} = \frac{g'(t)}{h'(t)} = \gamma(t) \quad (3.3)$$

Maximum possible limit for λ is

$$\lim_{t\to a}\lambda(t) = \lambda(a) \leqslant \frac{M}{2}, \forall a \in t \quad (3.4)$$

From (2.8)–(2.10) and (2.4)–(2.6), it is observed that the increasing and decreasing failure gate will affect the outcome probability of each position (ranks) with respect to time.

$$P(n,t) = \frac{g(t)n_1R_{h(t)-2} + \left(\frac{h(t)}{2} - g(t)\right)n_2.R_{h(t)-2}}{R_{h(t)}} \quad (3.5)$$

with

$$n_1 + n_2 + 1 = h(t)n_1 = n - 1 \quad (3.6)$$

If λ is only a function of M, then

$$P(n,M) = \frac{f(M)n_1R_{M-2} + \left(\frac{M}{2} - f(M)\right)n_2.R_{M-2}}{R_M} \quad (3.7)$$

with

$$n_1 + n_2 + 1 = Mn_1 = n - 1 \quad (3.8)$$

The following Fig. 10 is the graphical representation of increasing and decreasing number of failure gates with respect to number of competitors. The Fig. 11 represents the effect of number of competitors in a 8 member competition with a single failure gate.

The Fig. 10 explains the possibility of a non-constant number of failure gates, which is more relevant in practical environments. The number of failure gates can be either increasing or decreasing or a combination of both associated with the factors assigned (the tendency of this is morally based on the situation of the competitive environment)

The Fig. 11 provides some non-linear character that arises in the case of first eight positions. The results point out the equilibrium point at the fourth position which is as similar as a pivot intersection discussed earlier in Fig. 8 and in Table 1as well. The concept of these pivot points can be applied in addressing the different real time social dilemma and the same can be solved using the model described in the present article.

4. Concluding remarks

The outcome probability of an individual positions or ranks in a 2 N competition is studied under single failure gate model (where $\lambda = 1$) pursued by general failure gate model with the estimation of λ other than 1. The study observed the relation between outcome probability and the corresponding number of failure gates in the simplest level of solitary failure gate. This model is suitable in high precision competitive environments, where the sustainability of each position can be analysed using this model. The outcome probability for happening y number of failure gates is examined, with the assistance of probability of single failure gate utilizing binomial distribution. This improved the down to earth plausibility of the

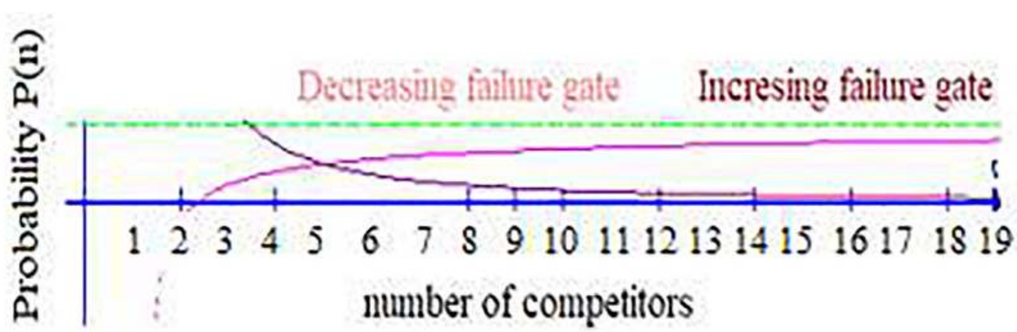

Fig. 10. Represents graphical view of expanding and diminishing number of failure gates for number of competitors.

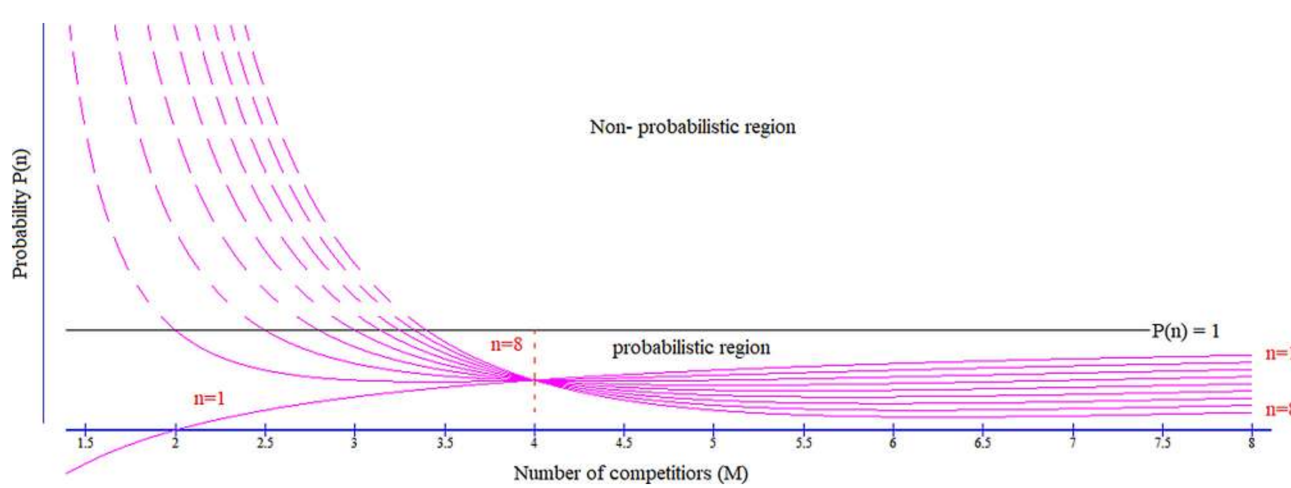

Fig. 11. Represents the impact of number of competitors in an 8-member competition having a solitary number of failure gates ($\lambda = 1$).

model in a wide range, on the grounds that the number of failure gates may not be solitary constantly. Through binomial expansion we can analyse the probability of occurrence of multiple number of failure gates acting simultaneously.

A contextual investigation of a tournament model with 2^{μ} competitors done and equations for outcome probability of best, least and intermediate ranks reached at. The number of failure gates treated as a component of different variables like time and number of competitors and probability equations are derived for each positions. The work can likewise be applied in the fields of ecology, psychology, economics, quality and inspection engineering. The study can be extended in terms of different strategies other than knockout and also in terms of 3 or more competitors in a single step of advancing. The intersecting points and corresponding identical probabilities for an eight member competition is considered and plotted the results obtained. This procedure is extremely valuable to comprehend the sustainability of each position in a competition with the reference of outcome probabilities.

The model is capable to capture n number of competitors concurrently with in a probabilistic boundary using the concepts of outcome probability, number of failure gates, identical probability curves so on. The limitations of existing mathematical tools being used in engineering and economics can be addressed by employing the model given. This will facilitate creation of a proper decision making tool in many engineering problems like trust issues between different suppliers in supply chain, with huge number of suppliers involved.

Disclosure of interest

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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