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On Neutrosophic $\alpha\psi$-Closed Sets

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Abstract: The aim of this paper is to introduce the concept of $\alpha\psi$-closed sets in terms of neutrosophic topological spaces. We also study some of the properties of neutrosophic $\alpha\psi$-closed sets. Further, we introduce continuity and contra continuity for the introduced set. The two functions and their relations are studied via a neutrosophic point set.

Keywords: neutrosophic topology; neutrosophic $\alpha\psi$-closed set; neutrosophic $\alpha\psi$-continuous function; neutrosophic contra $\alpha\psi$-continuous mappings

MSC: 54 A 40; 03 F 55

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1. Introduction

Zadeh [1] introduced and studied truth (t), the degree of membership, and defined the fuzzy set theory. The falsehood (f), the degree of nonmembership, was introduced by Atanassov [2–4] in an intuitionistic fuzzy set. Coker [5] developed intuitionistic fuzzy topology. Neutrality (i), the degree of indeterminacy, as an independent concept, was introduced by Smarandache [6,7] in 1998. He also defined the neutrosophic set on three components $(t, f, i) = (truth, falsehood, indeterminacy)$. The Neutrosophic crisp set concept was converted to neutrosophic topological spaces by Salama et al. in [8]. This opened up a wide range of investigation in terms of neutosophic topology and its application in decision-making algorithms. Arokiarani et al. [9] introduced and studied α-open sets in neutrosophic topoloical spaces. Devi et al. [10–12] introduced $\alpha\psi$-closed sets in general topology, fuzzy topology, and intutionistic fuzzy topology. In this article, the neutrosophic $\alpha\psi$-closed sets are introduced in neutrosophic topological space. Moreover, we introduce and investigate neutrosophic $\alpha\psi$-continuous and neutrosophic contra $\alpha\psi$-continuous mappings.

2. Preliminaries

Let neutrosophic topological space (NTS) be(X, τ). Each neutrosophic set(NS) in (X, τ) is called a neutrosophic open set (NOS), and its complement is called a neutrosophic open set (NOS).

We provide some of the basic definitions in neutrosophic sets. These are very useful in the sequel.

Definition 1. *[6] A neutrosophic set (NS) A is an object of the following form*

$$U = \{\langle x, \mu_U(x), \nu_U(a), \omega_U(x)\rangle : x \in X\}$$

where the mappings $\mu_U : X \to I$, $\nu_U : X \to I$, *and* $\omega_U : X \to I$ *denote the degree of membership (namely* $\mu_U(x)$*), the degree of indeterminacy (namely* $\nu_U(x)$*), and the degree of nonmembership (namely* $\omega_U(x)$*) for each element* $x \in X$ *to the set U, respectively, and* $0 \le \mu_U(x) + \nu_U(x) + \omega_U(x) \le 3$ *for each* $a \in X$.

Definition 2. *[6] Let U and V be NSs of the form* $U = \{\langle a, \mu_U(x), \nu_U(x), \omega_U(x)\rangle : a \in X\}$ *and* $V = \{\langle x, \mu_V(x), \nu_V(x), \omega_V(x)\rangle : x \in X\}$. *Then*

(i) $U \subseteq V$ *if and only if* $\mu_U(x) \le \mu_V(x)$, $\nu_U(x) \ge \nu_V(x)$ *and* $\omega_U(x) \ge \omega_V(x)$;
(ii) $\overline{U} = \{\langle x, \nu_U(x), \mu_U(x), \omega_U(x)\rangle : x \in X\}$;
(iii) $U \cap V = \{\langle x, \mu_U(x) \wedge \mu_V(x), \nu_U(x) \vee \nu_V(x), \omega_U(x) \vee \omega_V(x)\rangle : x \in X\}$;
(iv) $U \cup V = \{\langle x, \mu_U(x) \vee \mu_V(x), \nu_U(x) \wedge \nu_V(x), \omega_U(x) \wedge \omega_V(x)\rangle : x \in X\}$.

We will use the notation $U = \langle x, \mu_U, \nu_U, \omega_U\rangle$ *instead of* $U = \{\langle x, \mu_U(x), \nu_U(x), \omega_U(x)\rangle : x \in X\}$. *The NSs* $0_\sim$ *and* $1_\sim$ *are defined by* $0_\sim = \{\langle x, \underline{0}, \underline{1}, \underline{1}\rangle : x \in X\}$ *and* $1_\sim = \{\langle x, \underline{1}, \underline{0}, \underline{0}\rangle : x \in X\}$.

Let $r, s, t \in [0,1]$ *such that* $r + s + t \le 3$. *A neutrosophic point (NP)* $p_{(r,s,t)}$ *is neutrosophic set defined by*

$$p_{(r,s,t)}(x) = \begin{cases} (r,s,t)(x) & if\ x = p \\ (0,1,1) & otherwise. \end{cases}$$

Let f be a mapping from an ordinary set X into an ordinary set Y. If $V = \{\langle y, \mu_V(y), \nu_V(y), \omega_V(y)\rangle : y \in Y\}$ *is an NS in Y, then the inverse image of V under f is an NS defined by*

$$f^{-1}(V) = \{\langle x, f^{-1}(\mu_V)(x), f^{-1}(\nu_V)(x), f^{-1}(\omega_V)(x)\rangle : x \in X\}.$$

The image of NS $U = \{\langle y, \mu_U(y), \nu_U(y), \omega_U(y)\rangle : y \in Y\}$ *under f is an NS defined by* $f(U) = \{\langle y, f(\mu_U)(y), f(\nu_U)(y), f(\omega_U)(y)\rangle : y \in Y\}$ *where*

$$f(\mu_U)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_U(x), & if\ f^{-1}(y) \neq 0 \\ 0 & otherwise \end{cases}$$

$$f(\nu_U)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_U(x), & if\ f^{-1}(y) \neq 0 \\ 1 & otherwise \end{cases}$$

$$f(\omega_U)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \omega_U(x), & if\ f^{-1}(y) \neq 0 \\ 1 & otherwise \end{cases}$$

for each $y \in Y$.

Definition 3. *[8] A neutrosophic topology (NT) in a nonempty set X is a family* τ *of NSs in X satisfying the following axioms:*

(NT1) $0_\sim, 1_\sim \in \tau$;
(NT2) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$;
(NT3) $\cup G_i \in \tau$ *for any arbitrary family* $\{G_i : i \in J\} \subseteq \tau$.

Definition 4. *[8] Let U be an NS in NTS X. Then*
$Nint(U) = \cup\{O : O$ *is an NOS in X and* $O \subseteq U\}$ *is called a neutrosophic interior of U;*
$Ncl(U) = \cap\{O : O$ *is an NCS in X and* $O \supseteq U\}$ *is called a neutrosophic closure of U.*

Definition 5. *[8] Let* $p_{(r,s,t)}$ *be an NP in NTS X. An NS U in X is called a neutrosophic neighborhood (NN) of* $p_{(r,s,t)}$ *if there exists an NOS V in X such that* $p_{(r,s,t)} \in V \subseteq U$.

Definition 6. *[9] A subset U of a neutrosophic space* (X, τ) *is called*

1. *a neutrosophic pre-open set if* $U \subseteq Nint(Ncl(U))$*, and a neutrosophic pre-closed set if* $Ncl(Nint(U)) \subseteq U$*,*
2. *a neutrosophic semi-open set if* $U \subseteq Ncl(Nint(U))$*, and a neutrosophic semi-closed set if* $Nint(Ncl(U)) \subseteq U$*,*
3. *a neutrosophic* α*-open set if* $U \subseteq Nint(Ncl(Nint(U)))$*, and a neutrosophic* α*-closed set if* $Ncl(Nint(Ncl(U))) \subseteq U$*.*

The pre-closure (respectively, semi-closure and α*-closure) of a subset* U *of a neutrosophic space* (X, τ) *is the intersection of all pre-closed (respectively, semi-closed,* α*-closed) sets that contain* U *and is denoted by* $Npcl(U)$ *(respectively,* $Nscl(U)$ *and* $N\alpha cl(U)$*).*

Definition 7. *A subset* A *of a neutrosophic topological space* (X, τ) *is called*

1. *a neutrosophic semi-generalized closed (briefly, Nsg-closed) set if* $Nscl(U) \subseteq G$ *whenever* $U \subseteq G$ *and* G *is neutrosophic semi-open in* (X, τ)*;*
2. *a neutrosophic* $N\psi$*-closed set if* $Nscl(U) \subseteq G$ *whenever* $U \subseteq G$ *and* G *is Nsg-open in* (X, τ)*.*

3. On Neutrosophic $\alpha\psi$-Closed Sets

Definition 8. *A neutrosophic* $\alpha\psi$*-closed (*$N\alpha\psi$*-closed) set is defined as if* $N\psi cl(U) \subseteq G$ *whenever* $U \subseteq G$ *and* G *is an* $N\alpha$*-open set in* (X, τ)*. Its complement is called a neutrosophic* $\alpha\psi$*-open (*$N\alpha\psi$*-open) set.*

Definition 9. *Let* U *be an NS in NTS* X*. Then*
$N\alpha\psi int(U) = \cup\{O : O \text{ is an } N\alpha\psi OS \text{ in } X \text{ and } O \subseteq U\}$ *is said to be a neutrosophic* $\alpha\psi$*-interior of* U*;*
$N\alpha\psi cl(U) = \cap\{O : O \text{ is an } N\alpha\psi CS \text{ in } X \text{ and } O \supseteq U\}$ *is said to be a neutrosophic* $\alpha\psi$*-closure of* U*.*

Theorem 1. *All* $N\alpha$*-closed sets and N-closed sets are* $N\alpha\psi$*-closed sets.*

Proof. Let U be an $N\alpha$-closed set, then $U = N\alpha cl(U)$. Let $U \subseteq G$, where G is $N\alpha$-open. Since U is $N\alpha$-closed, $N\psi cl(U) \subseteq N\alpha cl(U) \subseteq G$. Thus, U is $N\alpha\psi$-closed. □

Theorem 2. *Every Nsemi-closed set in a neutrosophic set is an* $N\alpha\psi$*-closed set.*

Proof. Let U be an Nsemi-closed set in (X, τ), then $U = Nscl(U)$. Let $U \subseteq G$, where G is $N\alpha$-open in (X, τ). Since U is Nsemi-closed, $N\psi cl(U) \subseteq Nscl(U) \subseteq G$. This shows that U is $N\alpha\psi$-closed set.

The converses of the above theorems are not true, as can be seen by the following counter example. □

Example 1. *Let* $X = \{u, v, w\}$ *and neutrosophic sets* G_1, G_2, G_3, G_4 *be defined by*

$$
\begin{aligned}
G_1 &= \left\langle x, \left(\tfrac{u}{0.3}, \tfrac{v}{0.4}, \tfrac{w}{0.2}\right), \left(\tfrac{u}{0.5}, \tfrac{v}{0.1}, \tfrac{w}{0.2}\right), \left(\tfrac{u}{0.2}, \tfrac{v}{0.5}, \tfrac{w}{0.6}\right) \right\rangle \\
G_2 &= \left\langle x, \left(\tfrac{u}{0.6}, \tfrac{v}{0.3}, \tfrac{w}{0.4}\right), \left(\tfrac{u}{0.1}, \tfrac{v}{0.5}, \tfrac{w}{0.1}\right), \left(\tfrac{u}{0.3}, \tfrac{v}{0.2}, \tfrac{w}{0.5}\right) \right\rangle \\
G_3 &= \left\langle x, \left(\tfrac{u}{0.6}, \tfrac{v}{0.4}, \tfrac{w}{0.4}\right), \left(\tfrac{u}{0.1}, \tfrac{v}{0.1}, \tfrac{w}{0.1}\right), \left(\tfrac{u}{0.2}, \tfrac{v}{0.2}, \tfrac{w}{0.5}\right) \right\rangle \\
G_4 &= \left\langle x, \left(\tfrac{u}{0.3}, \tfrac{v}{0.3}, \tfrac{w}{0.2}\right), \left(\tfrac{u}{0.5}, \tfrac{v}{0.5}, \tfrac{w}{0.2}\right), \left(\tfrac{u}{0.3}, \tfrac{v}{0.5}, \tfrac{w}{0.6}\right) \right\rangle \\
G_5 &= \left\langle x, \left(\tfrac{u}{0.3}, \tfrac{v}{0.3}, \tfrac{w}{0.3}\right), \left(\tfrac{u}{0.5}, \tfrac{v}{0.5}, \tfrac{w}{0.4}\right), \left(\tfrac{u}{0.3}, \tfrac{v}{0.5}, \tfrac{w}{0.3}\right) \right\rangle \\
G_6 &= \left\langle x, \left(\tfrac{u}{0.6}, \tfrac{v}{0.4}, \tfrac{w}{0.5}\right), \left(\tfrac{u}{0.1}, \tfrac{v}{0.3}, \tfrac{w}{0.1}\right), \left(\tfrac{u}{0.3}, \tfrac{v}{0.3}, \tfrac{w}{0.4}\right) \right\rangle \\
G_7 &= \left\langle x, \left(\tfrac{u}{0.2}, \tfrac{v}{0.3}, \tfrac{w}{0.3}\right), \left(\tfrac{u}{0.5}, \tfrac{v}{0.5}, \tfrac{w}{0.2}\right), \left(\tfrac{u}{0.3}, \tfrac{v}{0.3}, \tfrac{w}{0.5}\right) \right\rangle.
\end{aligned}
$$

Let $\tau = \{0_\sim, G_1, G_2, G_3, G_4, 1_\sim\}$*. Here,* G_6 *is an* $N\alpha$ *open set, and* $N\psi cl(G_5) \subseteq G_6$*. Then* G_5 *is* $N\alpha\psi$*-closed in* (X, τ) *but is not* $N\alpha$*-closed; thus, it is not N-closed and* G_7 *is* $N\alpha\psi$*-closed in* (X, τ)*, but not Nsemi-closed.*

Theorem 3. *Let (X, τ) be an NTS and let $U \in NS(X)$. If U is an $N\alpha\psi$-closed set and $U \subseteq V \subseteq N\psi cl(U)$, then V is an $N\alpha\psi$-closed set.*

Proof. Let G be an $N\alpha$-open set such that $V \subseteq G$. Since $U \subseteq V$, then $U \subseteq G$. But U is $N\alpha\psi$-closed, so $N\psi cl(U) \subseteq G$, since $V \subseteq N\psi cl(U)$ and $N\psi cl(V) \subseteq N\psi cl(U)$ and hence $N\psi cl(V) \subseteq G$. Therefore V is an $N\alpha\psi$-closed set. □

Theorem 4. *Let U be an $N\alpha\psi$-open set in X and $N\psi int(U) \subseteq V \subseteq U$, then V is $N\alpha\psi$-open.*

Proof. Suppose U is $N\alpha\psi$-open in X and $N\psi int(U) \subseteq V \subseteq U$. Then $\overline{U}$ is $N\alpha\psi$-closed and $\overline{U} \subseteq \overline{V} \subseteq N\psi cl(\overline{U})$. Then $\overline{U}$ is an $N\alpha\psi$-closed set by Theorem 3.5. Hence, V is an $N\alpha\psi$-open set in X. □

Theorem 5. *An NS U in an NTS (X, τ) is an $N\alpha\psi$-open set if and only if$V \subseteq N\psi int(U)$ whenever V is an $N\alpha$-closed set and $V \subseteq U$.*

Proof. Let U be an $N\alpha\psi$-open set and let V be an $N\alpha$-closed set such that $V \subseteq U$. Then $\overline{U} \subseteq \overline{V}$ and hence $N\psi cl(\overline{U}) \subseteq \overline{V}$, since $\overline{U}$ is $N\alpha\psi$-closed. But $N\psi cl(\overline{U}) = \overline{N\psi int(U)}$, so $V \subseteq N\psi int(U)$. Conversely, suppose that the condition is satisfied. Then $\overline{N\psi int(U)} \subseteq \overline{V}$ whenever $\overline{V}$ is an $N\alpha$-open set and $\overline{U} \subseteq \overline{V}$. This implies that $N\psi cl(\overline{U}) \subseteq \overline{V} = G$, where G is $N\alpha$-open and $\overline{U} \subseteq G$. Therefore, $\overline{U}$ is $N\alpha\psi$-closed and hence U is $N\alpha\psi$-open. □

Theorem 6. *Let U be an $N\alpha\psi$-closed subset of (X, τ). Then $N\psi cl(U) - U$ does not contain any non-empty $N\alpha\psi$-closed set.*

Proof. Assume that U is an $N\alpha\psi$-closed set. Let F be a non-empty $N\alpha\psi$-closed set, such that $F \subseteq N\psi cl(U) - U = N\psi cl(U) \cap \overline{U}$. i.e., $F \subseteq N\psi cl(U)$ and $F \subseteq \overline{U}$. Therefore, $U \subseteq \overline{F}$. Since $\overline{F}$ is an $N\alpha\psi$-open set, $N\psi cl(U) \subseteq \overline{F} \Rightarrow F \subseteq (N\psi cl(U) - U) \cap (\overline{N\psi cl(U)}) \subseteq N\psi cl(U) \cap \overline{N\psi cl(U)}$. i.e., $F \subseteq \phi$. Therefore, F is empty. □

Corollary 1. *Let U be an $N\alpha\psi$-closed set of (X, τ). Then $N\psi cl(U)$-U does not contain anynon-empty N-closed set.*

Proof. The proof follows from the Theorem 3.9. □

Theorem 7. *If U is both $N\psi$-open and $N\alpha\psi$-closed, then U is $N\psi$-closed.*

Proof. Since U is both an $N\psi$-open and $N\alpha\psi$-closed set in X, then $N\psi cl(U) \subseteq U$. We also have $U \subseteq N\psi cl(U)$. Thus, $N\psi cl(U) = U$. Therefore, U is an $N\psi$-closed set in X. □

4. On Neutrosophic $\alpha\psi$-Continuity and Neutrosophic Contra $\alpha\psi$-Continuity

Definition 10. *A function $f : X \to Y$ is said to be a neutrosophic $\alpha\psi$-continuous (briefly, $N\alpha\psi$-continuous) function if the inverse image of every open set in Y is an $N\alpha\psi$-open set in X.*

Theorem 8. *Let $g : (X, \tau) \to (Y, \sigma)$ be a function. Then the following conditions are equivalent.*

(i) *g is $N\alpha\psi$-continuous;*
(ii) *The inverse $f^{-1}(U)$ of each N-open set U in Y is $N\alpha\psi$-open set in X.*

Proof. The proof is obvious, since $g^{-1}(\overline{U}) = \overline{g^{-1}(U)}$ for each N-open set U of Y. □

Theorem 9. *If $g : (X, \tau) \to (Y, \sigma)$ is an $N\alpha\psi$-continuous mapping, then the following statements hold:*

(i) *$g(N\alpha\psi Ncl(U)) \subseteq Ncl(g(U))$, for all neutrosophic sets U in X;*

(ii) $N\alpha\psi Ncl(g^{-1}(V)) \subseteq g^{-1}(Ncl(V))$, *for all neutrosophic sets V in Y.*

Proof.

(i) Since $Ncl(g(U))$ is a neutrosophic closed set in Y and g is $N\alpha\psi$-continuous, then $g^{-1}(Ncl(g(U)))$ is $N\alpha\psi$-closed in X. Now, since $U \subseteq g^{-1}(Ncl(g(U)))$, $N\alpha\psi cl(U) \subseteq g^{-1}(Ncl(g(U)))$. Therefore, $g(N\alpha\psi Ncl(U)) \subseteq Ncl(g(U))$.

(ii) By replacing U with V in (i), we obtain $g(N\alpha\psi cl(g^{-1}(V))) \subseteq Ncl(g(g^{-1}(V))) \subseteq Ncl(V)$. Hence, $N\alpha\psi cl(g^{-1}(V)) \subseteq g^{-1}(Ncl(V))$.

□

Theorem 10. *Let g be a function from an NTS (X, τ) to an NTS (Y, σ). Then the following statements are equivalent.*

(i) *g is a neutrosophic $\alpha\psi$-continuous function;*
(ii) *For every NP $p_{(r,s,t)} \in X$ and each NN U of $g(p_{(r,s,t)})$, there exists an $N\alpha\psi$-open set V such that $p_{(r,s,t)} \in V \subseteq g^{-1}(U)$.*
(iii) *For every NP $p_{(r,s,t)} \in X$ and each NN U of $g(p_{(r,s,t)})$, there exists an $N\alpha\psi$-open set V such that $p_{(r,s,t)} \in V$ and $g(V) \subseteq U$.*

Proof. $(i) \Rightarrow (ii)$. If $p_{(r,s,t)}$ is an NP in X and if U is an NN of $g(p_{(r,s,t)})$, then there exists an NOS W in Y such that $g(p_{(r,s,t)}) \in W \subset U$. Thus, g is neutrosophic $\alpha\psi$-continuous, $V = g^{-1}(W)$ is an $N\alpha\psi Oset$, and

$$p_{(r,s,t)} \in g^{-1}(g(p_{(r,s,t)})) \subseteq g^{-1}(W) = V \subseteq g^{-1}(U).$$

Thus, (ii) is a valid statement.

$(ii) \Rightarrow (iii)$. Let $p_{(r,s,t)}$ be an NP in X and let U be an NN of $g(p_{(r,s,t)})$. Then there exists an $N\alpha\psi Oset$ U such that $p_{(r,s,t)} \in V \subseteq g^{-1}(U)$ by (ii). Thus, we have $p_{(r,s,t)} \in V$ and $g(V) \subseteq g(g^{-1}(U)) \subseteq U$. Hence, (iii) is valid.

$(iii) \Rightarrow (i)$. Let V be an NO set in Y and let $p_{(r,s,t)} \in g^{-1}(V)$. Then $g(p_{(r,s,t)}) \in g(g^{-1}(V)) \subset V$. Since V is an NOS, it follows that V is an NN of $g(p_{(r,s,t)})$. Therefore, from (iii), there exists an $N\alpha\psi Oset$ U such that $p_{(r,s,t)} \in U$ and $g(U) \subseteq V$. This implies that

$$p_{(r,s,t)} \in U \subseteq g^{-1}(g(U)) \subseteq g^{-1}(V).$$

Therefore, we know that $g^{-1}(V)$ is an $N\alpha\psi Oset$ in X. Thus, g is neutrosophic $\alpha\psi$-continuous. □

Definition 11. *A function is said to be a neutrosophic contra $\alpha\psi$-continuous function if the inverse image of each NOS V in Y is an $N\alpha\psi C$ set in X.*

Theorem 11. *Let $g : (X, \tau) \to (Y, \sigma)$ be a function. Then the following assertions are equivalent:*

(i) *g is a neutrosophic contra $\alpha\psi$-continuous function;*
(ii) *$g^{-1}(V)$ is an $N\alpha\psi$ C set in X, for each NOS V in Y.*

Proof. $(i) \Rightarrow (ii)$ Let g be any neutrosophic contra $\alpha\psi$-continuous function and let V be any NOS in Y. Then $\overline{V}$ is an NCS in Y. Based on these assumptions, $g^{-1}(\overline{V})$ is an $N\alpha\psi Oset$ in X. Hence, $g^{-1}(V)$ is an $N\alpha\psi Cset$ in X.

The converse of the theorem can be proved in the same way. □

Theorem 12. *Let $g : (X, \tau) \to (Y, \sigma)$ be a bijective mapping from an NTS(X, T)into an NTS(Y, T). The mapping g is neutrosophic contra $\alpha\psi$-continuous, if $Ncl(g(U)) \subseteq g(N\alpha\psi int(U))$, for each NS U in X.*

Proof. Let V be any NCS in X. Then $Ncl(V) = V$, and g is onto, by assumption, which shows that $g(N\alpha\psi int(g^{-1}(V))) \supseteq Ncl(g(g^{-1}(V))) = Ncl(V) = V$. Hence, $g^{-1}(g(N\alpha\psi int(g^{-1}(V)))) \supseteq g^{-1}(V)$. Since g is an into mapping, we have $N\alpha\psi int(g^{-1}(V)) = g^{-1}(g(N\alpha\psi int(g^{-1}(V)))) \supseteq g^{-1}(V)$. Therefore, $N\alpha\psi int(g^{-1}(V)) = g^{-1}(V)$, so $g^{-1}(V)$ is an $N\alpha\psi$O set in X. Hence, g is a neutrosophic contra $\alpha\psi$-continuous mapping. □

Theorem 13. *Let $g : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following statements are equivalent:*

(i) *g is a neutrosophic contra $\alpha\psi$-continuous mapping;*
(ii) *for each NP $p_{(r,s,t)}$ in X and NCS V containing $g(p_{(r,s,t)})$ there existsanN$\alpha\psi$Oset U in X containing $p_{(r,s,t)}$ such that $A \subseteq f^{-1}(B)$;*
(iii) *for each NP $p_{(r,s,t)}$ in X and NCS V containing $p_{(r,s,t)}$ there existsanN$\alpha\psi$Oset U in X containing $p_{(r,s,t)}$ such that $g(U) \subseteq V$.*

Proof. $(i) \Rightarrow (ii)$ Let g be a neutrosophic contra $\alpha\psi$-continuous mapping, let V be any NCS in Y and let $p_{(r,s,t)}$ be an NP in X and such that $g(p_{(r,s,t)}) \in V$. Then $p_{(r,s,t)} \in g^{-1}(V) = N\alpha\psi int(g^{-1}(V))$. Let $U = N\alpha\psi int(g^{-1}(V))$. Then U is an $N\alpha\psi Oset$ and $U = N\alpha\psi int(g^{-1}(V)) \subseteq g^{-1}(V)$.

$(ii) \Rightarrow (iii)$ The results follow from evident relations $g(U) \subseteq g(g^{-1}(V)) \subseteq V$.

$(iii) \Rightarrow (i)$ Let V be any NCS in Y and let $p_{(r,s,t)}$ be an NP in X such that $p_{(r,s,t)} \in g^{-1}(V)$. Then $g(p_{(r,s,t)}) \in V$. According to the assumption, there exists an $N\alpha\psi OS$ U in X such that $p_{(r,s,t)} \in U$ and $g(U) \subseteq V$. Hence, $p_{(r,s,t)} \in U \subseteq g^{-1}(g(U)) \subseteq g^{-1}(V)$. Therefore, $p_{(r,s,t)} \in U = \alpha\psi int(U) \subseteq N\alpha\psi int(g^{-1}(V))$. Since $p_{(r,s,t)}$ is an arbitrary NP and $g^{-1}(V)$ is the union of all NPs in $g^{-1}(V)$, we obtain that $g^{-1}(V) \subseteq N\alpha\psi int(g^{-1}(V))$. Thus, g is a neutrosophic contra $N\alpha\psi$-continuous mapping. □

Corollary 2. *Let X, X_1 and X_2 be NTS sets, $p_1 : X \to X_1 \times X_2$ and $p_2 : X \to X_1 \times X_2$ are the projections of $X_1 \times X_2$ onto X_i, $(i = 1, 2)$. If $g : X \to X_1 \times X_2$ is a neutrosophic contra $\alpha\psi$-continuous, then $p_i g$ are also neutrosophic contra $\alpha\psi$-continuous mapping.*

Proof. This proof follows from the fact that the projections are all neutrosophic continuous functions. □

Theorem 14. *Let $g : (X_1, \tau) \to (Y_1, \sigma)$ be a function. If the graph h: $X_1 \to X_1 \times Y_1$ of g is neutrosophic contra $\alpha\psi$-continuous, then g is neutrosophic contra $\alpha\psi$-continuous.*

Proof. For every NOS, V in Y_1 holds $g^{-1}(V) = 1 \wedge g^{-1}(V) = h^{-1}(1 \times V)$. Since h is a neutrosophic contra $\alpha\psi$-continuous mapping and $1 \times V$ is an NOS in $X_1 \times Y_1$, $g^{-1}(V)$ is an $N\alpha\psi Cset$ in X_1, so g is a neutrosophic contra $\alpha\psi$-continuous mapping. □

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